Vibration research of cable-stayed bridge with tower-girder consolidation

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Received 15 June 2014, www.cmnt.lv

Abstract

Cable-stayed bridge is a combinative system of bridge towers, beams, piers and cables. Researches on the vibration of the structure generally use finite element method. And the vibration analysis of structure with the specific geometry is easy to get, while the research on the self-vibration considering the change of the bridge geometric parameters is difficult. Based on the Hamilton principle, this paper studied self-vibration frequency of a cable-stayed bridge whose tower and girder were consolidated. The motion equation of a cable-stayed bridge under the symmetric vibration was gained. Considering the changes of the geometric parameters, comparative analysis of the influences on natural vibration frequency were carried out. Results showed that the natural vibration frequency turned approximately linearly when the stiffness and the cable force changed. And the influence with beam changes on the natural vibration frequency was likely a parabola.

Keywords: Hamilton principle, cable-stayed bridge, natural vibration frequency

1 Introduction

Cable-stayed bridge is a composite structure consist of girders, cables and towers. It is a kind of bridge whose girders are under pressure, and its support system is under tension [1]. The deck system is composed of stiffening girder while steel cables comprise the support systems. The main character of the cable-stayed bridge is that inclined cable pylon is used as an elastic intermediate support to reduce the bending moment, the beam weight, and to improve the beam span ability [2].

As a kind of flexible structure, the vibration of towers and deck, cable tension will periodically change due to the vibration of deck and pylons under the wind, earthquake and traffic loads, which will lead the problem of large amplitude vibration and destroy the safety and durability of cable-stayed bridge greatly. Therefore, study on the vibration characteristics of cable-stayed bridge is necessary [3-8]. It will lay a foundation for the design and control of cable-stayed bridge

At present, researches on vibration of cable-stayed bridges mainly used finite element software [9]. Multi analog cable elements are used to achieve the accurate calculation of the structural model. Through dynamic analysis and calculation of kinetic energy ratio of modal participation, the identification modal function is realized, also the natural vibration frequencies and the modes of the cable-stayed bridge are generated [10, 11].With this method, the cable-stayed bridge dynamic vibration can be analysed clearly. It is difficult to make a thorough illustration of the energy conversion and the vibration mechanism during the vibration process. As for this problem, a new method is proposed.

FIGURE 1 Sutong Bridge

2 Theoretical basis

In this paper. The Hamilton principle is used [12] to analyse the vibration of the cable-stayed bridge. The principle expresses that the variation of kinetic energy, potential energy differences and non-conservative force work during the time interval of t1 to t2 is equal to zero. And the formula is expressed as:

$$\int_{t_1}^{t_2} \delta[T(t)-V(t)]dt + \int_{t_1}^{t_2} \delta W(t)dt = 0, \quad (1)$$

where T(t) is kinetic energy at time t, V(t) is potential energy at time t and W(t) is no conservative force work at time t

Based on this principle, the motion equations can be directly derived for any given system. Also most kinetic energy of the structure system can be expressed with the generalized coordinates and their first derivative. And the generalized coordinates are used individually to express the potential energy. In addition, variable branch of non-reactive conservative work caused by virtual displacement

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under generalized coordinates can be expressed as a linear function with the variables:

$$\left.\begin{aligned} &T=T(q_1,q_2,...,q_N,\dot{q}_1,\dot{q}_2,...,\dot{q}_N)\\ &V=V(q_1,q_2,...,q_N)\\ &\delta W_{nc}=Q_1\delta q_1+Q_2\delta q_2+...+Q_N\delta q_N \end{aligned}\right\} \quad , \tag{2}$$

where $Q_1,Q_2,...Q_N$ refers to the generalized force function that correspond to the coordinate $q_1,q_2,...q_N$, respectively. Put equation 2 into equation 1, one can be got:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right)-\frac{\partial T}{\partial q_i}+\frac{\partial V}{\partial q_i}=Q_i\,. \tag{3}$$

For the free vibration of the system, the generalized force function is zero. Therefore, the key of the free vibration analysis is the potential energy and kinetic energy solution system.

3 Vibration analysis

According to the cable-stayed bridge whose pylon and beam are consolidated, whose tower and pier are separated, The Hamilton principle is used to carry on the following research.

3.1 CALCULATION MODEL

Figure 2 shows the layout of a typical cable-stayed bridge with twin towers

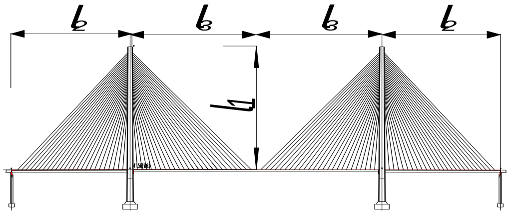

FIGURE 2 Cable-stayed bridge vertical layout

For the analysis of the system, it is assumed that the deformation is small, the damping and shear deformation can be ignored. The process of structural vibration is actually a conversion between kinetic energy and potential energy. Kinetic and potential energy need to be solved in the specific vibration type. The following issue takes the symmetric vibration of a cable-stayed bridge as an example to elaborate the article thought.

For symmetric vibration of the system, in accordance with the concept of structure conversion presented by the structure mechanics [13], a simpler structure is converted, as shown in Figure2.

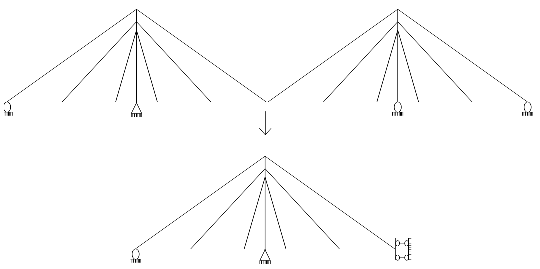

FIGURE 3 A simplified structure diagram

The system is essentially a continuous beam system. It is difficult to use a function to express the mode because of the tower, girder and cable of the cable-stayed bridge having no unified deformation. The vibration system model established in this paper is based on the concept of generalized degree of freedom.

3.2 VIBRATION MODE FUNCTION

The tower vibration can be considered as the driving force of the vibration system when the system vibrates [14]. And then the vibration will cause the deformation of beams and cables. The mode of vibration can be shown in Figure4.

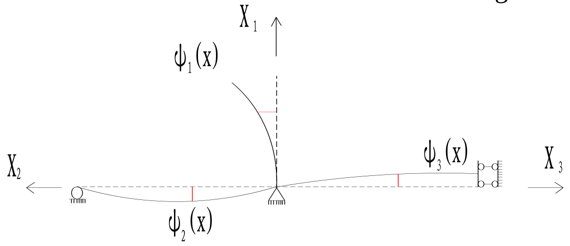

FIGURE 4 Coordinate establishment

When the vibration occurs in the system, the tower, the beam, and the cable deform with the same pace .Therefore it only needs to select one of them as a reference, the other two represented by relative deformation. In this paper the tower vibration is chosen as the reference vibration shape.

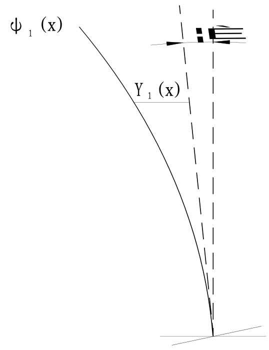

FIGURE 5 Mode of tower vibration

The mode of vibration of Figure5 by the following:

$$\varphi_1(x)=Y_1(x)+\theta x=1-\cos\frac{\pi x}{2l_1}+\theta x\,, \tag{4}$$

where θ can be solved according to the displacement method.

$$\left.\begin{aligned} &\theta=\frac{M}{3i_2+i_3}\\ &M=\frac{i_1\pi^2}{4l_1} \end{aligned}\right\}, \tag{5}$$

where 1 is the tower, 2 refers to the left span, and 3 stands for the middle span.

When the tower deforms as shown in Figure 5, as a result of which will drive the beam to rotate, and then generate the deformation as shown in Figure 4.Need to find two functions to describe the vibration modes of the left and the right span. It is assumed that the searched function conform to the following boundary conditions:

φ_2 satisfy the boundary condition as follow:

1) $x = 0, \varphi_2(0) = 0, \varphi_2{}'(0) = \theta$

2) $x = l_2/2, \varphi_2(l_2/2) = \Delta_2$

3) $x = l_2, \varphi_2(l_2) = 0$

Δ_2 stands for displacement at middle point of left span when rotate at θ angle,

φ_3 satisfy the boundary condition as follow:

1) $x = 0, \varphi_3(0) = 0, \varphi_3{}'(0) = \theta$

2) $x = l_3, \varphi_3(l_3) = \Delta_3$

Δ_3 stands for displacement at end of right span when rotate at θ angle.

According to the above boundary conditions, corresponding functions can be found out as follows:

$$\left.\begin{aligned} &\varphi_2(x) = (a + bx)\sin\frac{\pi x}{l_2} \\ &a = \frac{\theta l_2}{\pi} \\ &b = \frac{2(\Delta_2 - a)}{l_2} \end{aligned}\right\}, \tag{6}$$

$$\left.\begin{aligned} &\varphi_3(x) = (c + dx)\sin\frac{\pi x}{2l_3} \\ &c = \frac{2\theta l_3}{\pi} \\ &b = \frac{\Delta_3 - c}{l_3} \end{aligned}\right\}. \tag{7}$$

3.3 MOTION EQUATION

On the basis of the above modal function, the vibration was gained as shown in Figure 6.

The arrows give the positive movement direction in the figure. All the displacement functions are related in the following formula.

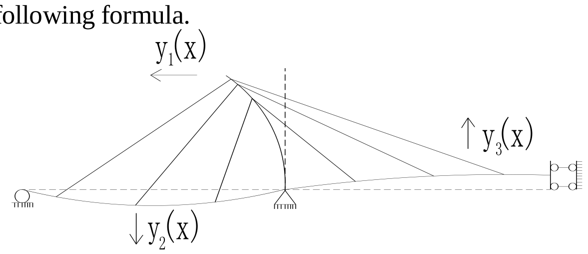

FIGURE 6 Mode of cable-stayed bridge vibration

$$\begin{aligned} &y_i(x,t) = \varphi_i(x)\,\mathrm{Z(t)} \\ &i = 1,2,3. \end{aligned} \tag{8}$$

Calculate the kinetic and potential energy of the system, Due to the small deformation assumption, the kinetic energy of the cables is not considered.

$$T = \frac{1}{2}\sum_{i=1}^{3}\int_0^{l_i} m_i v_i^2 dx = \frac{1}{2}\sum_{i=1}^{3}\int_0^{l_i} m_i(\dot{y}_i)^2 dx, \tag{9}$$

$$V = \frac{1}{2}\sum_{i=1}^{3}\int_0^{l_i} EI_i(y_i'')^2 dx + V_s, \tag{10}$$

where V_s represents cable potential energy.

It is assumed that the cable force is almost unchanged in the process of vibration, l_{s0} stands for the non-stress length of cable and p is the cable force.

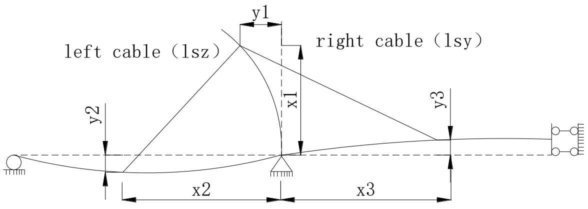

FIGURE 7 The calculation diagram of cable

According to the left and right cables' elongation Equations give:

$$\left.\begin{aligned} \Delta l_{sz} &= \sqrt{(x_2 - y_1)^2 + (x_1 + y_2)^2} - l_{sz0} \\ \Delta l_{sy} &= \sqrt{(x_3 + y_1)^2 + (x_1 - y_3)^2} - l_{sy0} \end{aligned}\right\}. \tag{11}$$

The cable potential energy can be calculated as:

$$V_s = \sum_{i=1}^{n} \mathrm{p}_i \cdot \Delta l_i, \tag{12}$$

where n refers to the cable number.

Put the equation 9, 10, 12 into the equation 3 leads to:

$$\left.\begin{aligned} &\ddot{Z}(t)\sum_{i=1}^{3}\int_0^{l_i} m_i\varphi_i^2 dx + Z(t)\sum_{i=1}^{3}\int_0^{l_i} EI_i(\varphi_i'')^2 dx \\ &+\frac{p_{zi}}{l_{zi}}(-x_{i2}\varphi_1(x_{i1}) + x_{i1}\varphi_2(x_{i2}) + Z(t)(\varphi_2(x_{i2}))^2 + Z(t)(\varphi_1(x_{i1}))^2) \\ &+\frac{p_{yi}}{l_{yi}}(x_{i3}\varphi_1(x_{i1}) - x_{i1}\varphi_3(x_{i3}) + Z(t)(\varphi_3(x_{i3}))^2 + Z(t)(\varphi_1(x_{i1}))^2) = 0 \\ &l_{zi} = \sqrt{x_{i1}{}^2 + x_{i2}{}^2} \\ &l_{yi} = \sqrt{x_{i1}{}^2 + x_{i3}{}^2} \end{aligned}\right\}. \tag{13}$$

In Equation 13, $p_{zi}, p_{yi}, x_{i1}, x_{i2}, x_{i3}$ are shown as follows:

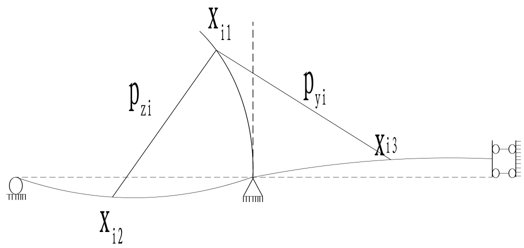

FIGURE 8 The coordinate parameters of the cable

The general motion equation is expressed as:

$$\bar{m}\ddot{Z}(t) + \bar{k}Z(t) + \bar{p} = 0, \tag{14}$$

$$\left.\begin{aligned}
&\bar{m}=\sum_{i=1}^{3}\int_{0}^{l_i} m_i\varphi_i^{2}dx\\
&\bar{k}=\sum_{i=1}^{3}\int_{0}^{l_i} EI_i(\varphi_i'')^{2}dx+\frac{p_{zi}}{l_{zi}}((\varphi_2(x_{i2}))^{2}+(\varphi_1(x_{i1}))^{2})\\
&+\frac{p_{yi}}{l_{yi}}((\varphi_3(x_{i3}))^{2}+(\varphi_1(x_{i1}))^{2}\\
&\bar{p}=\frac{p_{zi}}{l_{zi}}(-x_{i2}\varphi_1(x_{i1})+x_{i1}\varphi_2(x_{i2}))\\
&+\frac{p_{yi}}{l_{yi}}(x_{i3}\varphi_1(x_{i1})-x_{i1}\varphi_3(x_{i3}))
\end{aligned}\right\}. \tag{15}$$

Under the natural vibration, the frequency of the system is:

$$\omega=\sqrt{\frac{\bar{k}}{\bar{m}}}. \tag{16}$$

4 Example analysis

The calculation proposed in this paper is complex, so the matlab [15] programming is used to execute the calculation. The initial values of the parameters are listed in the following table. Concrete material is employed in the towers. The beam girder is made up of steel and the cable adopts the symmetrical layout, with 6 cables on each side.

TABLE 1 Parameters of Tower and beam

	Tower	**Left girder**	**Right girder**
Area (m^2)	9.2	0.4	0.4
Density (Kg/m^3)	2500	7800	7800
Moment of inertia (m^4)	8.12	4.76	4.76
Length (m)	150	180	190
Elastic modulus (10^{10}) (Pa)	2.50	21	21

TABLE 2 Parameters of cable

Cable number	**1**	**2**	**3**	**4**	**5**	**6**
X1	100	110	120	130	140	150
X2	20	60	90	120	150	180
X3	20	60	90	120	150	180
Cable force (10^6) (N)	2.0	2.1	2.2	2.3	2.4	2.5

Based on the theory in this paper and with the parameters listed in Tab.1, the natural vibration frequency of the cable-stayed bridge is 0.6119. In order to verify the rationality of the calculation, a calculation of the cable-stayed bridge was performed through ANSYS. The first order mode is shown in Figure 9.

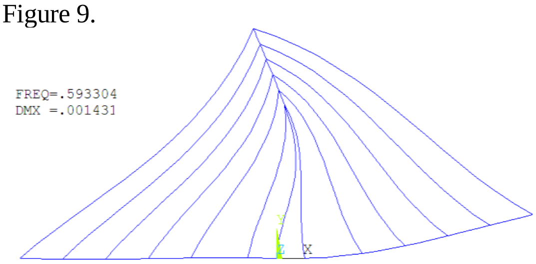

FIGURE 9 The finite element calculation result

As can be seen from Figure 9, the first-order natural frequency is 0.593304, which is close to the calculation result of 0.6819. There is some deviation, it is because that the vibration model established in this paper is different from the finite element model.

The influences of parameters changes to the self-variation frequency of the cable-stayed bridge are analysed as follows.

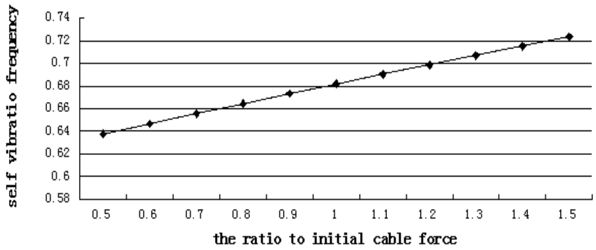

FIGURE 10 Impact of cable force change on the self-vibration frequency

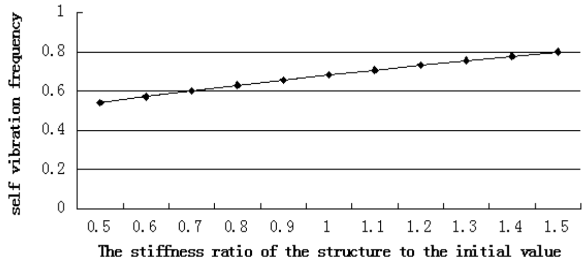

FIGURE 11 Impact of Structure stiffness change on the self-vibration frequency

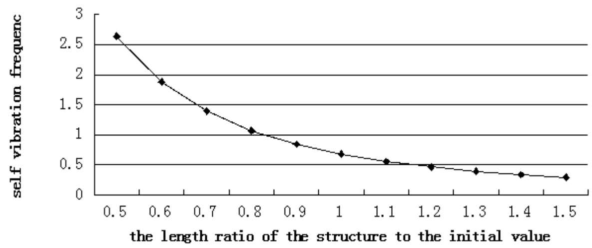

FIGURE 12 Impact of Length change on the self-vibration frequency

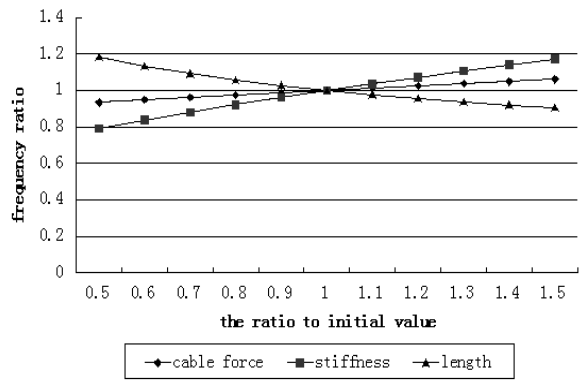

FIGURE 13 The comparison of impacts of various parameters on the self-vibration frequency

From the above figures, the results can be concluded:

(1)The influence of the stiffness and cable force change on the natural vibration frequency is approximately linear. The natural vibration frequency increases with the increasing of the ratio, however, the magnitude of change is relatively small. The change of the structure length has a great impact on the natural vibration frequency, increases with the decrease of its ratio, which presents a parabolic variation law.

(2)The influence of the parameters change on the self-frequency of the structure is different. The minimum influence is cable force change. As the ratio increases, the self-vibration frequency will get larger. An approximate linear variation is presented.

Due to the lack of space, this paper only focuses on the changes of the above parameters on natural vibration frequency.

5 Conclusions

This paper studied the self-vibration frequency calculation of the cable-stayed bridge based on the Hamilton principle. The motion equation of the cable-stayed bridge in the symmetric vibration is generated, and the basic frequencies are calculated. Also comparative analysis of the influences on the geometric parameters changes to the vibration frequency are carried out. The calculation results show that the influence of the stiffness and the cable force of the cable-stayed bridge on the natural vibration frequency variation is approximately a linear change. While the influence of tower and beam length is likely a parabola. Among the influence parameters, the cable force change has less impact, and the greatest impact lays in the tower and beam length change.

In this paper, the calculation model was simplified, just considered the main factors. And the influence of cable sag or elastic modulus on natural vibration frequency needs to be further studied.

References

[1] YAN G M 1979 *Modern Cable-stayed Bridge* Southwest Jiao Tong University Press: Chengdu

[2] Bridge Research Institute of MBEC 1992 *Cable-stayed Bridge* Science and Technology Literature Press Beijing

[3] WANG F J 2007 *Cable parameter vibration and dynamic analysis of Cable-stayed bridge* Southwest Jiao Tong University Chengdu

[4] Michel Virlogeux 1998 Cable vibration in cable-stayed bridges *Bridge Aerodynamics* **154**(1), 121-139

[5] WU Q,TAKAHASHI K, OKABAYASHI T, et al. 2003 Response characteristics of load vibration in stayed cables on existing cable-stayed bridge *Journal of Sound and Vibration* 261(3),403-420

[6] A. Bosdogianni et al. 1996 Wind-and rain-induced oscillations of cables of stayed bridges *Wind Eng. and Industrial Aerodynamics*, **64**(2), 171-185

[7] WANG X Z, ZHOU S X, LIU Q 2007 Dynamic characteristic analysis of single column pylon of single cable plane cable stayed bridge *Journal of Chongqing Jiaotong University* **26**(3), 10-13.

[8] KANG Z Z Wan X 1998 Numerical study on parametric resonance of cable in cable stayed bridge *China Civil Engineering Journal* **31**(4), 14-22

[9] ZHONG Y F HU X L 2008 Mode shape analysis and identification of long span cable stayed bridge based on the method of computing the proportion of kinetic energy in each mode shape *Highway Traffic Science and Technology* **25**(9), 93-95

[10] ZHANG H, CHEN Y B 2007 Modal test and analysis of long span cable stayed bridge *Journal of Wuhan University of Technology* 29(4),79-81

[11] WANG Z G, SUN B N 2002 Parametic Vibration of the Cable in Cable stayed Bridge *China Civil Engineering Journal*, **35(**5),28-33

[12] Clough R, J Peng Jin 2007 *Structural Dynamics (Second Edition)* Higher Education Press: Beijing

[13] ZHU C M, ZHANG W P 2009 *Structural Mechanics (I) (Second Edition)* Higher Education Publishing House: Beijing

[14] WANG B 2008 *Study on long span cable-stayed bridge cable local vibration characteristics and its effect* Huazhong University of Science and Technology

[15] ZHANG Z Y, YANG Z Y 2010 *MATLAB tutorial R2012a* Beijing University of Aeronautics and Astronautics Press: Beijing

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