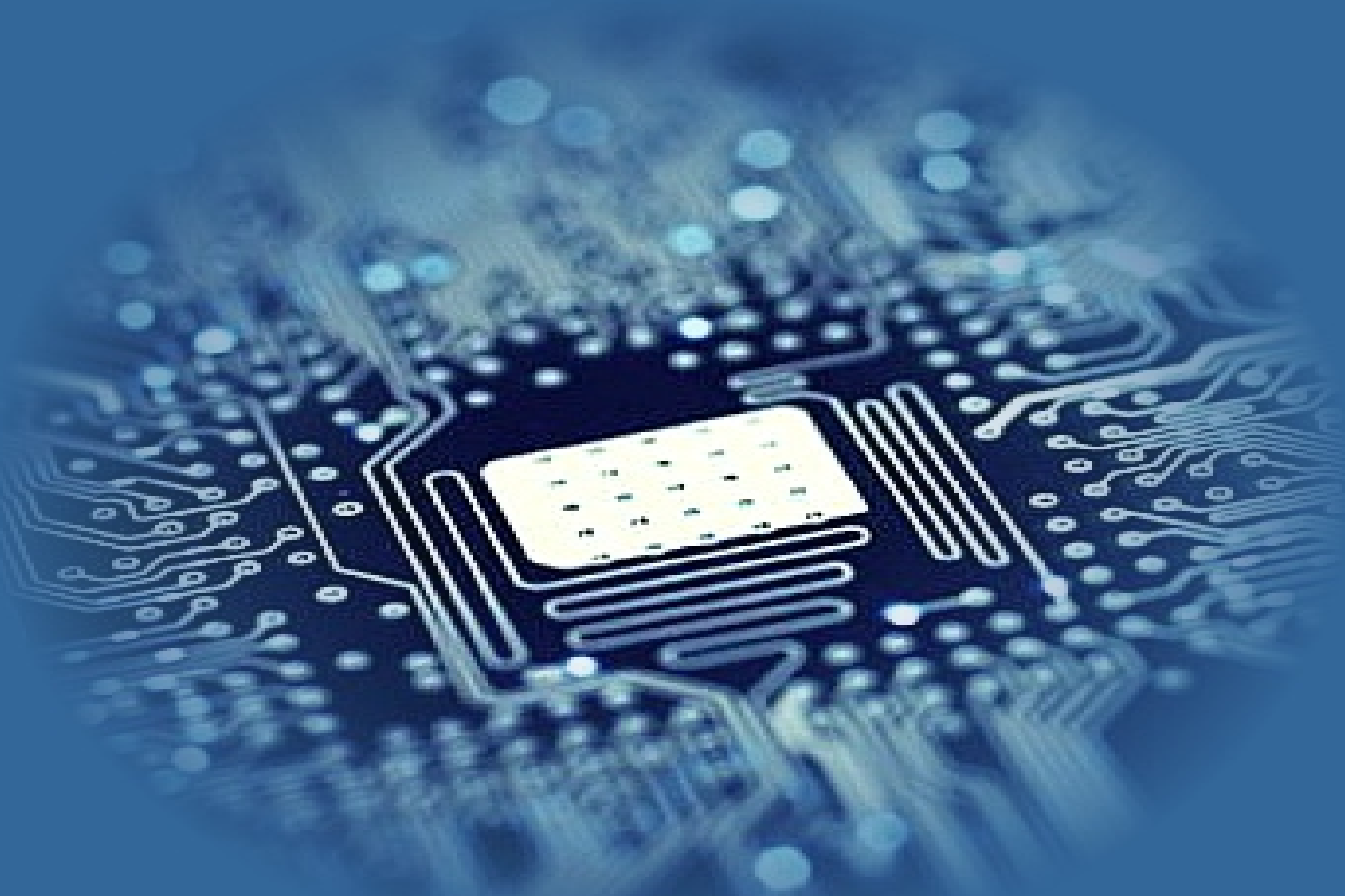

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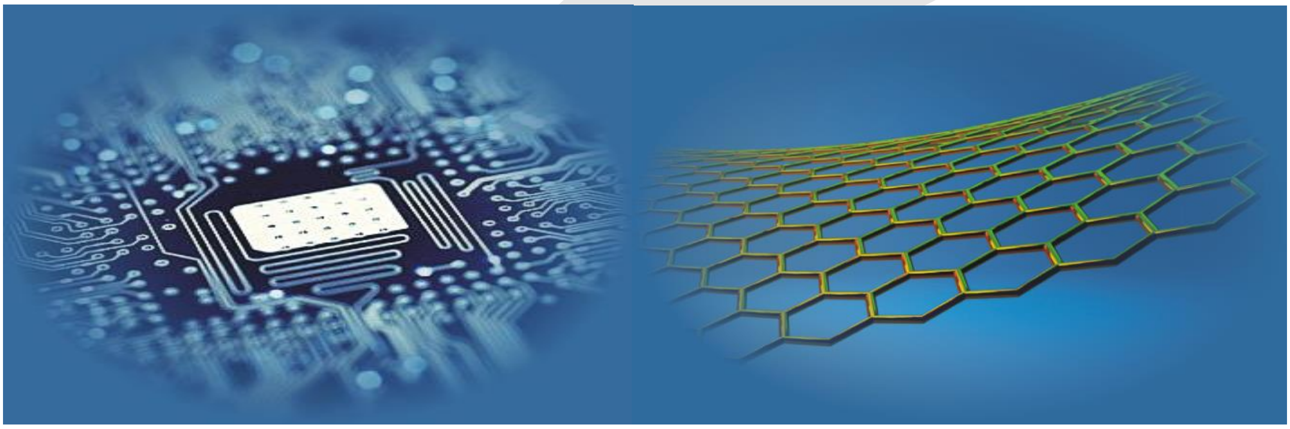

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Editors' Remarks

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Endless Time

by Rabindranath Tagore

Time is endless in thy hands, my lord.
There is none to count thy minutes.

Days and nights pass and ages bloom and fade like flowers.
Thou knowest how to wait.

Thy centuries follow each other perfecting a small wild flower.

We have no time to lose, and having no time we must scramble for a chance.
We are too poor to be late.

And thus it is that time goes by while I give it to every querulous man who claims it, and thine altar is empty of all offerings to the last

At the end of the day I hasten in fear lest thy gate be shut;
but I find that yet there is time.

Rabindranath Tagore (1861-1941)[♣]

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This 20th volume No.3 includes research papers on **Mathematical and Computer Modelling**.
Our journal policy is directed to fundamental and applied scientific researches, innovative technologies and industry, which is the fundamentals of the full-scale multi-disciplinary modelling and simulation. This edition is the continuation of our publishing activities. We hope our journal will be of interest for research community and professionals. We are open for collaboration both in the research field and publishing. We hope that the journal's contributors will consider collaboration with the Editorial Board as useful and constructive.

EDITORS

Yuri Shunin

Igor Kabashkin

[♣] **Rabindranath Tagore (7 May 1861 – 7 August 1941),** was a Bengali poet, novelist, musician, painter and playwright who reshaped Bengali literature and music. As author of Gitanjali with its "profoundly sensitive, fresh and beautiful verse", he was the first non-European and the only Indian to be awarded the Nobel Prize for Literature in 1913. His poetry in translation was viewed as spiritual, and this together with his mesmerizing persona gave him a prophet-like aura in the west. His "elegant prose and magical poetry" still remain largely unknown outside the confines of Bengal.

CONTENT

The detection system for greenhouse crop disease degree based on Android platform

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Abstract

A detecting way based on Android platform was proposed in order to detect greenhouse crop disease degree in real time. This way employed the camera in mobile phone to acquire crop disease leaf image in the greenhouse. Firstly, the detection system was built by the Eclipse based on the Android development environment. The iterative threshold segmentation algorithm was used to separate the crop disease leaf area from background. And the fuzzy C-means cluster algorithm was adopted to extract the disease spots. After analyzed the impact of different fuzzy weighted index m value, the value of m was selected 2 for the disease spots segmentation. After that the crop disease degree was determined based on the relevant standards and the total disease index of greenhouse was got based on disease index calculation standards. Finally, the calculating data could upload to the network server and was used management cloud achieved synchronous computer terminal query. The experimental results show that the detecting way could non-destructed measure the disease index of leaf diseases with non-destructive and exact in greenhouse.

Keywords:

Android greenhouse crop disease index
cloud management real time non-destructive

1 Introduction

The leaf is one of the most important parts of the plant during its growth process. But the disease of the leaf will bring a lot of production and economic loss. In order to reduce the agricultural workers loss, we must promptly acquire crop disease messages to control the crop disease. The visual method [1] and weighing method [2] were often used to detect crop disease degree as traditional methods. The visual method was simple and convenient, but it was used personal experience with low measuring precision and difference. Weighing method had fussy operation and its steps were complicated, it couldn't conducive to the actual disease detection. Leaf area analyzer could accurately measure the leaf area, but it was expensive and couldn't measured level of disease [3]. Therefore, it is extremely important to study the rapid and timely determination of crop leaf disease level [4-6]. In recent years, researchers had a lot of research in the field of disease degree detection at home and abroad. Daniel A. etc. used past experience and disease standards of assessment to grad single leaf image of two groups of crop powdery mildew disease. The results show that the grading of diseases of crop powdery mildew degree could be estimated reliably and could be used for important disease system evaluation and research [7]. Aleixos etc. used the RGB value of the image and the spectral image technology to carry on the flaw detection and the classification of oranges. The accuracy rate was higher [8]. Zhanliang Chen etc. used computer image processing technology and adopted Otsu method to extract leaf area and lesion area [9]. Youwen Tian etc. designed a classification system for crop leaf disease. The system was transplanted by the Linux operating system and ARM9 processor. It used USB external contact scanning device of farming leaf image acquisition, and used the threshold segmentation method and corresponding algorithm in the blade image processing. Finally, the classification results were shown on the LCD screen. It's realized the combination of embedded technology and scanning device staged processing of crop diseases [10]. In summary, although these methods could detect crop disease level better, the detection method was experimented in laboratory after the crop leaves was collected. This detecting ways were operation not only complicated and time-consuming, but also some detection methods had destructive and defected the crop growth.

At present, the Android system smart phones were of many features, such as low price, the use of a wide range and good portability. At the same times, Android system is open and freely [11]. Therefore, this paper proposes a detection system based on Android mobile phone platform to research and development or the greenhouse crop disease degree. The system will calculate the disease index of crop in the greenhouse, and the degree of disease grading. Finally the data and the results were uploaded to the server, and then the function of the computer terminal synchronous query was realized.

2 Experimental materials and methods

2.1 IMAGE ACQUISITION

The images of crop leaves such as cucumber, tomato, pepper and eggplant were collected from NO.22 greenhouse of North Mountain vegetable base in Shenyang Agricultural University. Sample images were collected at 3 different points in the shed, and 5 strains were collected at each point. The cucumber, tomato, pepper and eggplant diseases leaf image were collected 32, 43, 35, 40 photos respectively.

2.2 ANDROID OS

Android is based on Linux platform. From the perspective of software hierarchy, Android consists of applications, application framework, Android run-time, librarian and Linux kernel. Among them, all programs in application layer are written by JAVA language. Application framework layer allows the developers to visit all API interfaces of the core application. Android run-time includes two parts, which are Core libraries and Dalvik virtual machine. Application framework is supported by librarian. Linux kernel depends on the Linux2.6 kernel version [12].

2.3 HARDWARE AND SOFTWARE PLATFORM

Image detecting system for degree of crop leaf disease based on Android OS mobile phone is consists of hardware and software. The hardware part is a 4.8 inch mobile phone - Samsung I939d (Samsung Corp, South Korea). The software part is built on Android SDK 22.6.2 (Software development kit), Java (Java development kit), JDK 8, Eclipse 4.4 and ADT 23.0.3 (Android development tools).

2.4 THE ANALYSIS METHODS

2.4.1 Leaf image segmentation

There are many methods of image segmentation, such as edge detection method, region segmentation method and threshold segmentation method. Among them the threshold segmentation method was a very effective and simple method. Adopted threshold segmentation method could get very ideal results because the leaf was great differences in the color of foreground and background. The iterative threshold algorithm was adopted to do image segmentation in this paper. The basic idea is that a threshold is chosen as the initial estimate, and then the initial value is continuously improved until it meets the given criterion according to some strategy [13].

2.4.2 Disease spot extraction

Disease spots area and normal area could be seen as two categories segmentation on the disease leaves, therefore fuzzy C-means cluster algorithm method was used to extract disease leaf spots. The fuzzy C-means cluster algorithm （FCM）was first proposed by Dunn [14] and perfected by Bezdek [15]. At present, it is the most popular kinds of fuzzy clustering algorithm. The FCM algorithm is an iterative algorithm with iteration along the reducing direction of the objective function to determine the best category. The objective function is as follows:

$$J(U,V) = \sum_{i=1}^{n} \sum_{k=1}^{c} \mu_{ik}^{m} \left| P_i - V_k \right| \quad i = 1,2,\ldots,n, k = 1,2,\ldots,c., \tag{1}$$

where U is a fuzzy membership matrix and P is cluster sample set {P1,p2,p3,…}. Where V is cluster center set {v1,v2,v3,…}. c is the number of cluster categories，and n is the number of pixels. Where μ_{aik}^{m} is referring to the membership of k samples to the i class and m is input parameters express for all types of membership size. Where $|P_i-V_k|^2$ is refer to Euclidean metric between P_i and V_k.

2.4.3 Disease grading and method of disease index

According to the field efficacy experiment guidelines, the disease spot area accounted for whole leaf area classification method was adopted in crop leaf disease degree during determine [16]. Since the relationship between the leaf spots area and the number of the disease spot pixels is proportional, the relationship between the disease leaf and the number of the disease leaf pixels. The degree of crop leaf disease can be expressed by the ratio between the number of leaf spots pixels and the number of the disease leaf pixels.

The calculating formula of disease index is as follows:

$$f = \frac{\sum (a \times b)}{T \times 9} \times 100, \tag{2}$$

where f is disease index and a is number of disease leaves each level. And b is value of disease leaves each level. Where T is total number of diseases crop leaves.

2.4.4 Data upload and management

A web site was designed and developed for the system of detecting way for greenhouse crop disease degree. The user could upload data through the Android phone (Greenhouse number, Plants number, Leaf number, Time, Crop name, Disease name, Disease index) to the server. And user could log on the website to query upload data. The SAE server (Cloud computing platform) was used in test phase.

3 Results and analysis

3.1 LOAD IMAGE

Click on "Load image" button on the main interface and select the image stored in the smart-phone to get the sample information of leaf disease. The result was shown in Figure 1.

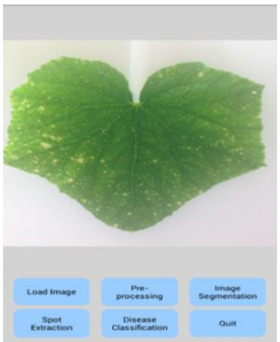


FIGURE 1 Load image effect

3.2 IMAGE PRE-PROCESSING

Click on "Pre-processing" button on the main interface. In this paper, *G* channel of original image was adopted to make enhancement filtering processing to realize function of image contrast enhancement. And this made the outline and edge image of the leaf and background clearer, and got the more obvious details. As a result, the segmentation step could be more convenient.

3.3 LEAF SEGMENTATION

Click on "Leaf segmentation" button to achieve the image leaf segmentation after image pre-processing. The initial threshold value was selected based on the section 2.4.1 in leaf image segmentation method. In RGB channels, the *R* channel was easier to do leaf segmentation because the edge between its leaf region and background region was obvious, while the *G* and *B* channel in leaf region and background region was very nearly hard to divide. The initial threshold was defined as 128 because leaf region and background region of the sample image is easy to separate at this value of 128.

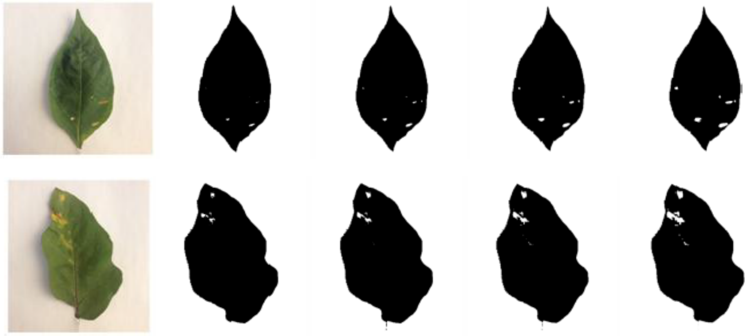
The final segmentation result was shown in Figure 2. It showed that the segmentation of the leaf region and the background region was well, the edge of the leaf was very clear, and the leaf region was reflected truly. Using the iterative threshold method could effectively separate leaf and background region so as to facilitate the subsequent data processing and grade the disease degree.

FIGURE 2 Segmentation results of leaf image

3.4 LEAF DISEASE SPOTS EXTRACTION

The FCM algorithm was used for leaf disease spots extraction to get better results. There were three parameters of FCM algorithm included feature vector of the sample, optimal cluster number *c* and fuzzy weighted index *m*.

1) The feature vector of the sample. The operation effect of cluster algorithm was associated with characteristics of the cluster samples [17]. Finally, the gray value of color components was chosen as the characteristics of the sample data through contrasting the three color space. Through the careful observation of crop leaf with disease spots, most colors of disease spots were found brown or white in this paper. So the RGB color information of the health leaf region and the disease spot region separately were acquired, and then their color features were analyzed on the *R*, *G* and *B* components of the health leaf region and the disease spot region. So the *R*, *G* and *B* three component value were chosen as the feature vector of the sample in view of some question of the application platform such as operation ability, operation time, and so on.
2) The optimal cluster number *c*. Our purpose was to separate the disease spot region from the crop leaf image, which means the image was separate into disease spot region and health leaf region. So the optimal cluster number *c* was set to 2.
3) The Fuzzy weighted index *m*. The fuzzy weighted index *m* was an important parameter in the process of fuzzy cluster, which determines the magnitude of the degree of fuzzy clustering. The larger *m* is, the fuzzier the classification is. When *m* was 1, the FCM algorithm was reduced to algorithm HCM [18]. In this paper, we obtained the fuzzy weighted index *m* for the best image segmentation, and the value of index *m* were selected 1.2, 1.5, 2 and 3 respectively. The results were shown in Fig 3. It can be seen from the Figure 3. The regional of disease spots were small when the value of index *m* took less than 2. It didn't conform to the actual situation. The regional of disease spots were closed to actual situation when the value of index *m* was selected 2. The regional of disease spots were increased and deviated when the value of index *m* took more than 2. So the value of fuzzy weighted index was selected 2 in this paper in consideration of the parameter *m* having the influence on the speed of segmentation.

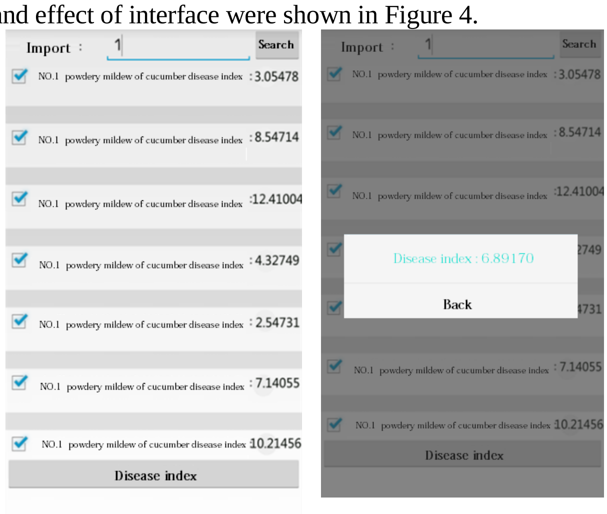


Leaf *m*=1.2 *m*=1.5 *m*=2 *m*=3

FIGURE 3 Segmentation results of different *m* value

3.5 DISEASE GRADING AND DISEASE INDEX CALCULATION

After all sample images were processed crop disease grades were gotten according to grading standards in chapter 2.4.3. Disease index of whole greenhouse crop was calculated using the formula (2). The results of the analysis and effect of interface were shown in Figure 4.



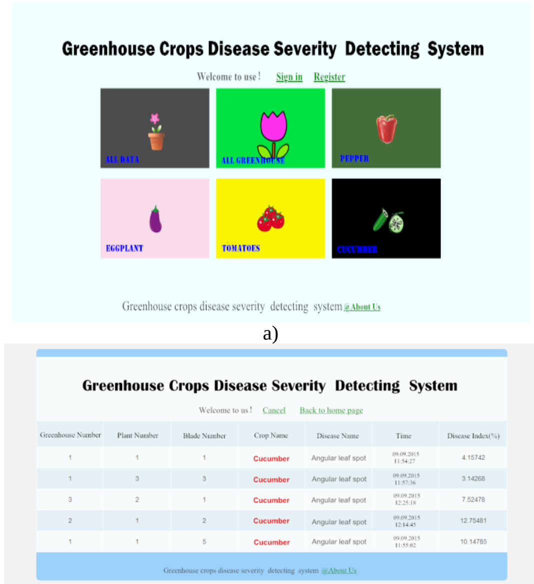
a) b)

FIGURE 4 Interface result of disease index calculation

The Adobe Photoshop CS5 was used to get the number of leaf pixels and the number of leaf spots pixels, and the ratio *k* was 6.7564. In summary, the ratio *k* was basically same in two different measurement methods. Although there were some errors, it had little effect on greenhouse crop leaf disease grade determination.

3.6 CLOUD DATA MANAGEMENT

The data of greenhouse crop disease degree could be uploaded to the server after the disease index was completed. The user could log in the greenhouse crop disease degree detection system through personal computer web site and registered user name. This website could receive the data of Android upload remotely and realized the remote view, save and statistics function of the computer. The computer detection system was shown in Figure 5.



a)

b)

FIGURE 5 Interface result

4 Conclusions

The greenhouse crop disease degree detection system was designed and developed based on Android mobile phone platform in this paper. The Android smart phone camera was used to collect leaf disease image. Then according to the color characteristics of healthy leaves and lesion isolated from the disease leaves. The leaf area and lesion area were spitted and extracted using the efficient of FCM algorithm. The pixels number of the leaf area and lesion area were compared, and then the disease level was judged by the standard classification criteria. The disease index of the whole greenhouse crop could be obtained according to disease index, and the corresponding control measures could be put up according to the disease index. A powerful cloud data management technology was used in the system. The mobile phone side of the data could be uploading to the server. The computer could be achieved on the data statistics and query functions. Experimental results show that the system of greenhouse crop disease index measured accurate, error was small, saved time and effort, and non-destructive leaf. So the greenhouse crop disease degree detection system could be providing convenient, fast and practical information for agricultural workers.

5 Acknowledgements

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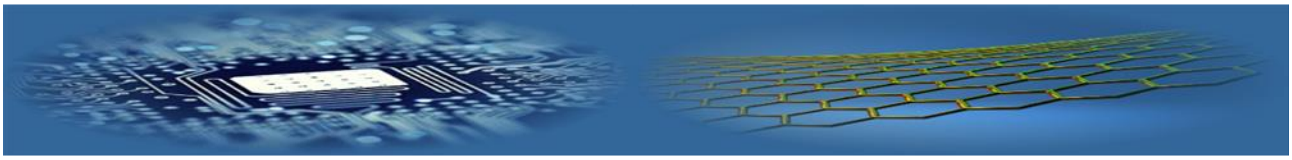

Relationships among convergence concepts of uncertain sequences

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Abstract

Uncertain sequence is a sequence of uncertain variables indexed by integers. In this paper, a new kind of sequence convergence that complete convergence was presented. Then, the relationships among complete convergence, convergence in p-distance, convergence in measure, convergence in distribution, convergence uniformly almost surely and convergence almost surely were investigated.

Keywords:

uncertain measure
uncertain variable
expectation
convergence

1 Introduction

In order to describe subjective uncertain phenomenon, Liu [6] founded uncertainty theory which based on uncertain measure that satisfies normality, duality, subadditivity, and product axiom in 2007, then refined by Liu [11] in 2010. Next, some properties of uncertain measure were studied by Gao [3]. Thereafter, Liu [6] introduced uncertain variable which is a measurable function from an uncertainty space to the set of real numbers. A sufficient and necessary condition of uncertainty distribution was proved by Peng and Imamura [16] in 2010. After introduced the definition of independence by Liu [9], Liu [11] presented the operational law of uncertain variables. Up to now, uncertainty theory has already applied to uncertain programming (Liu [8]), uncertain risk analysis (liu [13]), uncertain process (Liu [7], Liu [19]), and uncertain logic (Liu [14], Li and Liu [10]), etc. In addition, uncertain calculus which deals with differentiation and integration of uncertain processes was given by Liu [9]. Then uncertain differential equation was founded by Liu [7] and further researched by Yao and Chen [20] whose main contents include a concept of α -path to uncertain differential equation and a numerical method is designed for solving uncertain differential equations. Gao [4] and Chen [1] both researched uncertain inference which is a process of deriving consequences from human knowledge via uncertain set theory (Liu [12], Liu [15]).

Sequence convergence plays an important role in the fundamental theory of mathematics, therefore Liu [7] introduced some concepts of sequence convergence in uncertainty theory and discussed their relationships. Then another type of convergence named convergence uniformly almost surely was presented by You [18]. Xia [17] investigated the convergence of uncertain sequences which contains Cauchy convergence of uncertain sequences and the sufficient conditions of convergence almost surely. Thereafter, the dual convergence of uncertain sequence was investigated by Yuan, Zhu and Guo [21]. Guo, Zhu and Yuan [5] presented a necessary and sufficient condition of convergence in measure for uncertain sequences. In addition, the convergence concepts of complex uncertain sequence were proposed by Chen, Ning and Xiao [2] in 2016.

In this paper, we will give a new concept of convergence of uncertain sequences and discuss the relationships among the basic definitions of convergence. The rest of this paper is organized as follows. Uncertainty space and some basic contents and theorems of uncertainty theory will be introduced in Section 2. Then the relationships among complete convergence, convergence in p-distance, convergence in measure, convergence in distribution, convergence uniformly almost surely and convergence almost surely will be investigated in Section 3. At last, a brief summary is given.

2 Preliminary

In this section, we presented some definitions and theorems in uncertain environment which will be used in this paper.

Let Γ be a nonempty set, and let L be a $\sigma-$algebra over Γ. A number $M\{\Lambda\}$ indicates the level that each element $\Lambda \in L$ (which is called an event) will occur. Liu [6] proposed the set function M, which is called uncertain measures if it satisfies the following three axioms:

Axiom 1 (Normality) $M\{\Gamma\}=1$.

Axiom 2 (Self-Duality) $M\{\Lambda\}+M\{\Lambda^c\}=1$, for any event Λ.

Axiom 3 (Subadditivity) For every countable sequence of events $\{\Lambda_i\}$, we have

$$M\{\bigcup_{i=1}^{\infty}\Lambda_i\} \le \sum_{i=1}^{\infty} M\{\Lambda_i\}. \tag{1}$$

Next, the definition of uncertain space is introduced.

Definition 2.1 (Liu [6]) Let Γ be a nonempty set, let L be a $\sigma-$ algebra over Γ, and let M be an uncertain measure. Then the triple (Γ, L, M) is called an uncertainty space.

The properties of uncertain measure is recalled in the following theorem.

Theorem 2.1 (Monotonicity, Liu [6]) Uncertain measure M is a monotone increasing set function. That is, for any events $\Lambda_1 \subset \Lambda_2$, we have $M\{\Lambda_1\} \le M\{\Lambda_2\}$.

Definition 2.2 (Liu [6]) An uncertain variable is a function ξ from an uncertainty space (Γ, L, M) to the set of real numbers such that $\{\xi \in B\}$ is an event for any set B of real numbers.

Definition 2.3 (Liu [6]) Let ξ be an uncertain variable. Then the expected value of ξ is defined by

$$E[\xi] = \int_0^{+\infty} M\{\xi \ge x\}dx - \int_{-\infty}^0 M\{\xi \le x\}dx, \tag{2}$$

provided that at least one of the two integrals is finite.

Theorem 2.2 (Liu [6]) Let ξ be an uncertain variable with uncertainty distribution Φ. Then

$$E[\xi] = \int_0^{+\infty} (1-\Phi(x))dx - \int_{-\infty}^0 \Phi(x)dx. \tag{3}$$

Theorem 2.3 (Markov Inequality, Liu [6]) Let ξ be an uncertain variable. Then for any give numbers $t > 0$ and $p > 0$, we have

$$M\{|\xi| \ge t\} \le \frac{E[|\xi|^p]}{t^p}. \tag{4}$$

Then, we reviewed some convergence concepts of uncertain sequence and some theorems about them.

Definition 2.4 (Liu [6]) The uncertain sequence $\{\xi_i\}$ is said to be convergent a.s. to ξ if there exists an event Λ with $M\{\Lambda\} = 1$ such that

$$\lim_{i\to\infty} |\xi_i(\gamma) - \xi(\gamma)| = 0, \tag{5}$$

for event $\gamma \in \Lambda$. In that case we write $\xi_i \to \xi$, a.s.

Theorem 2.4 (You [18]) Let $\xi, \xi_1, \xi_2, \cdots$ be uncertain variables. Then $\{\xi_i\}$ convergent a.s. to ξ if and only if for any $\varepsilon > 0$, we have

$$M\left\{ \bigcap_{m=1}^{\infty}\bigcup_{i=m}^{\infty}(\gamma \in \Gamma \| \xi_i(\gamma) - \xi(\gamma)| \ge \varepsilon) \right\} = 0. \tag{6}$$

Definition 2.5 (Liu [6]) The uncertain sequence $\{\xi_i\}$ is said to be convergent in measure to ξ if

$$\lim_{i\to\infty} M\{|\xi_i - \xi| \ge \varepsilon\} = 0 \tag{7}$$

for every $\varepsilon > 0$.

Definition 2.6 (Liu [6]) Let $\Phi, \Phi_1, \Phi_2, \cdots$ be the uncertainty distributions of uncertain variables $\xi, \xi_1, \xi_2, \cdots$, respectively. We say the uncertain sequence $\{\xi_i\}$ converges in distribution to ξ if

$$\lim_{i\to\infty} \Phi_i(x) = \Phi(x) \tag{8}$$

for all x at which $\Phi(x)$ is continuous.

Definition 2.7 (You [18]) The uncertain sequence $\{\xi_i\}$ is said to be convergent uniformly a.s. to ξ if and only if

$$\lim_{i\to\infty} M\{\bigcup_{i=m}^{\infty}(\gamma \in \Gamma \| \xi_i(\gamma) - \xi(\gamma)| \ge \varepsilon)\} = 0. \tag{9}$$

Definition 2.8 (You and Yan [22]) The uncertain sequence $\{\xi_i\}$ is said to be convergent in p-distance to ξ if

$$\lim_{i\to\infty} d_p(\xi_i, \xi) = \lim_{i\to\infty}(E[|\xi_i - \xi|^p])^{\frac{1}{p+1}} = 0. \tag{10}$$

Theorem 2.5 (Liu [6]) If the uncertain sequence $\{\xi_i\}$ converges in measure to ξ, then $\{\xi_i\}$ converges in distribution to ξ.

Theorem 2.6 (You [18]) Suppose $\xi, \xi_1, \xi_2, \cdots$ are uncertain variables. If $\{\xi_i\}$ converges uniformly a.s. to ξ, then $\{\xi_i\}$ converges in measure to ξ.

3 Relationships among convergence concepts

A new definition of convergence of uncertain sequence is given in this section. Then, we discuss the relationships among these convergence concepts mentioned in Section 2.

Now we show the relationships among convergence in p-distance, convergence in measure, and convergence in distribution.

Theorem 3.1 Suppose that $\xi, \xi_1, \xi_2, \cdots$ are uncertain variables defined on uncertainty space (Γ, L, M). If $\{\xi_i\}$ converges in p-distance to ξ, then $\{\xi_i\}$ converges in measure to ξ.

Proof: If uncertain sequence $\{\xi_i\}$ converges in p-distance to ξ, then we have $\lim_{i\to\infty} E[|\xi_i - \xi|^p] = 0$. It follows from Theorem 2.3 that for any given number $\varepsilon > 0$, we have

$$M\{|\xi_i - \xi| \ge \varepsilon\} \le \frac{E[|\xi_i - \xi|^p]}{\varepsilon^p} \to 0, \tag{11}$$

as $i \to \infty$. Thus uncertain sequence $\{\xi_i\}$ converges in measure to ξ. The theorem is proved.

Example 3.1 Convergence in measure does not imply convergence in p-distance. For example, take an uncertainty space (Γ, L, M) to be $\{\gamma_1, \gamma_2, \cdots\}$ with $M\{\gamma_i\} = \frac{1}{2i}$. The uncertain variables are defined by

$$\xi_i(\gamma_j) = \begin{cases} 2i, & if \quad i = j \\ 0, & otherwise \end{cases}, \tag{12}$$

for $i = 1, 2, \cdots$ and $\xi \equiv 0$. For some small number $\varepsilon > 0$ and $i > 1$, we have

$$M\{|\xi_i - \xi| \geq \varepsilon\} = M\{\gamma \| \xi_i(\gamma) - \xi(\gamma)| \geq \varepsilon\} \\ = M\{\gamma_i\} = \frac{1}{2i} \to 0, \tag{13}$$

as $i \to \infty$. Thus, the sequence $\{\xi_i\}$ converges in measure to ξ.

However, for each $i > 1$, we have the uncertainty distribution of uncertain variable $|\xi_i - \xi|$ is

$$\Phi_i(x) = \begin{cases} 0, & if\ x < 0 \\ 1 - \frac{1}{2i}, & if\ 0 \leq x < 2i \\ 1, & if\ x \geq 2i. \end{cases} \tag{14}$$

Then according to Theorem 2.2, for each $i > 1$, we have

$$E[|\xi_i - \xi|] = \int_{2i}^{\infty} (1-1)dx + \int_0^{2i} (1 - (1 - \frac{1}{2i}))dx = 1 \tag{15}$$

It follows from Theorem 2.1 that

$$E[|\xi_i - \xi|^p] = \int_0^{\infty} M\{|\xi_i - \xi|^p \geq x\}dx \\ \geq \int_0^{\infty} M\{|\xi_i - \xi| \geq x\}dx \\ = E[|\xi_i - \xi|] = 1. \tag{16}$$

Therefore,

$$\lim_{i \to \infty} d_p(\xi_i, \xi) = \lim_{i \to \infty} (E[|\xi_i - \xi|^p])^{\frac{1}{p+1}} = 1 \neq 0 \tag{17}$$

That is, the uncertain sequence $\{\xi_i\}$ does not converges in p-distance to ξ.

Theorem 3.2 Let $\xi, \xi_1, \xi_2, \cdots$ be uncertain variables defined on uncertainty space (Γ, L, M). If uncertain sequence $\{\xi_i\}$ converges in p-distance to ξ, then $\{\xi_i\}$ converges in distribution to ξ.

Proof: By Theorem 2.5, we know that the uncertain sequence $\{\xi_i\}$ convergence in measure means it convergence in distribution. Then, it follows from Theorem 3.1 that $\{\xi_i\}$ converges in distribution to ξ.

Example 3.2 Convergence in distribution does not imply convergence in p-distance. Take an uncertainty space (Γ, L, M) to be $\{\gamma_1, \gamma_2\}$ with $M\{\gamma_1\} = M\{\gamma_2\} = \frac{1}{2}$.

The uncertain variable are defined by

$$\xi(\gamma) = \begin{cases} -a, & if\ \gamma = \gamma_1 \\ a, & if\ \gamma = \gamma_2 \end{cases}, \tag{18}$$

where a is a positive number. We also define $\xi_i = -\xi$ for $i = 1, 2, \cdots$. Thus ξ_i and ξ have the same uncertainty distribution

$$\Phi(x) = \begin{cases} 0, & if\ x < -a \\ \frac{1}{2}, & if\ -a \leq x \leq a \\ 1, & if\ x > a. \end{cases} \tag{19}$$

That is to say, uncertain sequence $\{\xi_i\}$ converges in distribution to ξ. Then we have $|\xi_i - \xi|_p = (2a)^p$, for $\gamma = \gamma_1, \gamma_2$ and its expected value is

$$E[|\xi_i - \xi|_p] = \int_0^{(2a)^p} 1dx = (2a)^p. \tag{20}$$

Therefore,

$$\lim_{i \to \infty} d_p(\xi_i, \xi) = \lim_{i \to \infty} (E[|\xi_i - \xi|^p])^{\frac{1}{p+1}} = (2a)^{\frac{p}{p+1}}. \tag{21}$$

Then $\{\xi_i\}$ does not converge in p-distance to ξ.

Next, we give a new convergence concept of uncertain sequence which is complete convergence.

Definition 3.1 Let $\xi, \xi_1, \xi_2, \cdots$ be uncertain variables defined on uncertainty space (Γ, L, M). Then $\{\xi_i\}$ is said to be completely convergent to ξ if

$$\lim_{i \to \infty} \sum_{k=i}^{\infty} M\{|\xi_k - \xi| \geq \varepsilon\} = 0, \tag{22}$$

for any $\varepsilon > 0$.

Theorem 3.3 Suppose $\xi, \xi_1, \xi_2, \cdots$ are uncertain variables defined on uncertainty space (Γ, L, M). If $\{\xi_i\}$ completely converges to ξ, then $\{\xi_i\}$ converges uniformly almost surely to ξ.

Proof: If uncertain sequence $\{\xi_i\}$ completely converges to ξ, it follows from Axiom 3 that

$$M\{\bigcup_{k=i}^{\infty} |\xi_k - \xi| \geq \varepsilon\} \leq \sum_{k=i}^{\infty} M\{|\xi_k - \xi| \geq \varepsilon\} \to 0 \tag{23}$$

as $i \to \infty$. Thus, uncertain sequence $\{\xi_i\}$ converges uniformly almost surely to ξ.

Theorem 3.4 Suppose $\xi, \xi_1, \xi_2, \cdots$ are uncertain variables defined on uncertainty space (Γ, L, M). If $\{\xi_i\}$ completely converges to ξ, then $\{\xi_i\}$ converges almost surely to ξ.

Proof: According to Definition 3.2, we have

$$\lim_{i \to \infty} \sum_{k=i}^{\infty} M\{|\xi_k - \xi| \geq \varepsilon\} = 0 \tag{24}$$

It follows from Axiom 3 that

$$M\{\bigcap_{i=1}^{\infty}\bigcup_{k=i}^{\infty}|\xi_k-\xi|\ge\varepsilon\}\le M\{\bigcup_{k=i}^{\infty}|\xi_k-\xi|\ge\varepsilon\} \le \sum_{k=i}^{\infty}M\{|\xi_k-\xi|\ge\varepsilon\}, \tag{25}$$

taking the limitation of $i\to\infty$ on both side of above inequality, we can get

$$M\{\bigcap_{i=1}^{\infty}\bigcup_{k=i}^{\infty}|\xi_k-\xi|\ge\varepsilon\}=0. \tag{26}$$

By Theorem 2.4, we can get the uncertain sequence $\{\xi_i\}$ converges almost surely to ξ.

Example 3.3 Convergence almost surely does not imply complete convergence. For example, take an uncertainty space (Γ, L, M) to be $\{\gamma_1,\gamma_2,\cdots\}$ with $M\{\gamma_i\}=\dfrac{2i}{4i+1}$ for $i=1,2,\cdots$. Then uncertain variables are defined by

$$\xi_i(\gamma_j)=\begin{cases}2i, & if\ j=i\\ 0, & otherwise\end{cases}, \tag{27}$$

for $i=1,2,\cdots$ and $\xi\equiv 0$. The uncertain sequence $\{\xi_i\}$ converges almost surely to ξ. However, for some small number $\varepsilon>0$, we have

$$M\{|\xi_i-\xi|\ge\varepsilon\}=\frac{2i}{4i+1}\to\frac{1}{2} \tag{28}$$

as $i\to\infty$.

Thus

$$\lim_{i\to\infty}\sum_{k=i}^{\infty}M\{|\xi_k-\xi|\ge\varepsilon\}\ne 0. \tag{29}$$

Therefore, the uncertain sequence $\{\xi_i\}$ does not completely converge to ξ.

Theorem 3.5 Suppose $\xi,\xi_1,\xi_2,\cdots$ are uncertain variables defined on uncertainty space (Γ, L, M). If $\{\xi_i\}$ completely converges to ξ, then $\{\xi_i\}$ converges in measure to ξ.

Proof: Since complete convergence means convergence uniformly almost surely. Then it follows from Theorem 2.6 that uncertain sequence $\{\xi_i\}$ converges in measure to ξ.

Example 3.4 In Example 3.1, uncertain sequence $\{\xi_i\}$ converges in measure to ξ. However, for each $i>1$,

$$\begin{aligned}&\lim_{i\to\infty}\sum_{k=i}^{\infty}M\{|\xi_k-\xi|\ge\varepsilon\}\\ &=\lim_{i\to\infty}\sum_{k=i}^{\infty}M\{\gamma\,|\xi_k(\gamma)-\xi(\gamma)|\ge\varepsilon\}\\ &=\lim_{i\to\infty}\sum_{k=i}^{\infty}\frac{1}{2k}\\ &=\frac{1}{2}\lim_{i\to\infty}\sum_{k=i}^{\infty}\frac{1}{k},\end{aligned} \tag{30}$$

as harmonic series $\sum_{i=1}^{\infty}\frac{1}{i}$ does not converge to 0 when $i\to\infty$, the uncertain sequence $\{\xi_i\}$ does not completely converge to ξ.

Theorem 3.6 Suppose $\xi,\xi_1,\xi_2,\cdots$ are uncertain variables defined on uncertainty space (Γ, L, M). If $\{\xi_i\}$ completely converges to ξ, then $\{\xi_i\}$ converges in distribution to ξ.

Proof: According to Theorem 3.5 and Theorem 2.5, we know that uncertain sequence $\{\xi_i\}$ converges in distribution to ξ.

Example 3.5 For Example 3.2, we know that uncertain sequence $\{\xi_i\}$ converges in distribution to ξ. However, $|\xi_i-\xi|=2a$, for $\gamma=\gamma_1,\gamma_2$, and for some given number $\varepsilon>0$, we have

$$M\{|\xi_i-\xi|\ge\varepsilon\}=1 \tag{31}$$

Then

$$\lim_{i\to\infty}\sum_{k=i}^{\infty}M\{|\xi_k-\xi|\ge\varepsilon\}\ne 0 \tag{32}$$

Thus, the uncertain sequence $\{\xi_i\}$ does not completely converge to ξ.

4 Conclusions

A new concept of convergence (complete convergence) for uncertain sequence was proposed in this paper. Then in the setting of uncertainty theory, we discussed the relationships among these different convergence concepts, which are complete convergence, convergence in p-distance, convergence in measure, convergence in distribution, convergence uniformly almost surely and convergence almost surely. The relationships among uncertain convergence concepts are shown in Figure 1.

$$
\begin{array}{ccccc}
\textit{Complete Convergence} & \rightarrow & \textit{Convergence Uniformly Almost Surely} & \rightarrow & \textit{Convergence Almost Surely} \\
 & & \downarrow & & \\
\textit{Convergence in } p-\textit{distance} & \rightarrow & \textit{Convergence in Measure} & \rightarrow & \textit{Convergence in Distribution}
\end{array}
$$

FIGURE 1 Relationships among Convergence Concepts

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Supervised images classification using metaheuristics

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Abstract

Image classification is a fundamental task in image processing because it is a crucial step toward image understanding. This paper exploits metaheuristics (Ant Colony Optimization and Electromagnetic Metaheuristic) to tackle the problem of supervised satellite image classification. Earlier studies have been used the Intra-Class Variance (ICV) for images classification but this function has a limits to solve classification problem. This study presents the introduction of the Davies-Bouldin Index (DBI) to the supervised images classification. This index is used in two stages: training step and classification step. In training step this index serve as criteria for controlling iterations. In the classification step this index help to classify each pixel in the image to their appropriate class using the class centers found during the training stage. The experimental results show that the introduction of the Davies-Boulin index is very effective for supervised images classification and help the community of researches to improve the classification accuracy of remotely sensed data. The utility of metaheuristics is also demonstrated for satellite image of Oran city.

Keywords:

image classification
metaheuristics
Davies-Bouldin index
ant colony optimization
electromagnetic metaheuristic.

1 Introduction

Remote sensing image classification is a preliminary step in computer vision applications. This processing is extensively used for agricultural planning and crop monitoring, providing a basis for decision-making [1]. The goal of this classification is to decompose an image into meaningful or spatially coherent regions sharing similar attributes [2]. A vast majority of classification techniques are supervised, which requires that the number of classes and the class distribution model to be known in advance. Unsupervised classification divides all pixels within an image into corresponding classes and proceeds with fewer interactions with the analyst. A lot of classification approaches have been proposed and applied to multispectral remote sensing images. 1) mathematical techniques: Support Vector Machine [3], Maximum Likelihood [4], K-means clustering [5], Bayesian Classifier [6] and Independent Components Analysis [7]. 2) bio-inspired techniques: different types of Neural Networks such as the Multilayer Perceptron [8], the Radial Basic Function Network [9] and the Kohonen Network [10]. Another example of algorithms inspired from biology: Genetic Algorithms [11, 12] and Artificial Immune Systems [13].

The aim of this research is to develop a repeatable, accurate meta-heuristic method to classify remote sensing imagery. This paper focuses mainly on integrating ACO [14] with different objective functions and comparing the different results. The utility of ACOs in solving problems that are large, multimodal and highly complex has been demonstrated in several areas [15-19].

It is known that the quality of the classification result using optimization technique depends wholly on the objective function performances. The ICV value has been used as objective function in several works [20 - 21]. This function requires that each class data approximately follow a normal distribution. To avoid this drawback, we have introduced DBI [22] to supervised classification. The DBI value depends both on the distance between class center and simples and on the distance between class's centers. This Index has been used for unsupervised remote sensing images classification in several works [12, 23, 24].

Finally, to evaluate the performance of DBI function on supervised remote sensing image classification, the ACO based classifier results were compared with those from another metaheuristic called Electromagnetic Metaheuristic (EM) [25].

This article is organized in the following ways: in the second section (methodology) we illustrate the ant colony optimization, coding and objective functions. The third section is devoted to the presentation of the study area and the experimental results. A conclusion and perspectives are presented in section four.

2 Methodology

2.1 ANT COLONY OPTIMIZATION

The ACO is a paradigm for designing metaheuristic algorithms for complex optimization problems. These algorithms can be regarded as a multi-agent system where each agent is functioning independently by very simple rules [26]. In this paper we are interesting to use API algorithm [27] for land cover classification. The API algorithm is based on natural behavior of *pachycondyla apicalis* ant. The API base algorithm is different from the basic ACO in terms of search strategy; in the process of foraging ants communicate with each other using visual landmarks rather than pheromone.

2.1.1. Pachycondyla apicalis behavior

This colony has been studied in Mexican tropical forest near Guatemalan border [28]. The behavior of such colony can be characterized as follows:

- The ants create their hunting sites around the nest within a radius of approximately 10m.
- The ants will intensify their search around some selected sites for prey capture.
- Each time a prey is found, it is brought back to the nest and the next ant's exit will focus on this profitable hunting site.
- When a hunting site impoverishes, the ant has a tendency to explore other hunting sites.
- When the nest is starting to be unhealthy, scant ants are searching from a new nest location.

2.1.2. Behavior modelling

The modeling behavior of the ant Pachycondyla apicalis is proposed by Monmarché et al. [26]. It corresponds to an algorithm called API, which is designed to solve optimization problems. The explored space by the ants is transformed to a search space noted by S (Fig.1, 2). The nest, ants and hunting sites are represented by points (location) on S.

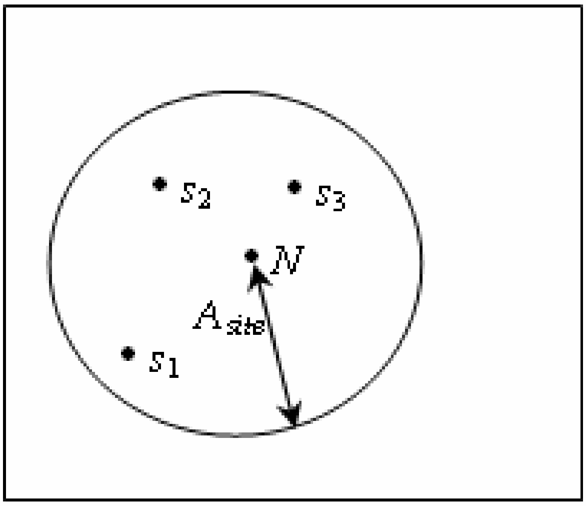


FIGURE 1 Creation of the hunting sites (s_1, s_2, s_3) around the nest N respecting amplitude A_{site}

Initially the nest (N) is uniformly generated in the search space (Figure 1) using equation (1). Afterward, each ant of the population (n ants) generates p hunting sites around the nest (Figure 1) by using equation 2 and by respecting an amplitude Asite. An ant explores a site of hunting using equation (2) by respecting an amplitude Alocale (Figure 2) during Plocale representing the number of successive failures in a hunting site. A failure means that an ant changed its location in the search space without improving the objective function.

The global goal of the population is to minimize a function during a number of iterations noted T. The ant tries to find s' in the neighborhood of s, such that f(s') is better than f(s). This is the modeling of the capture of the prey.

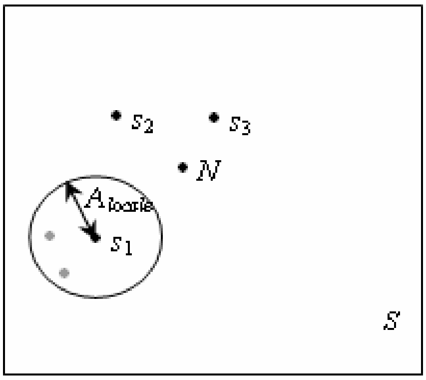


FIGURE 2 Exploring the hunting site s_1 respecting amplitude A_{locale}

The most important elements in the API algorithm are the exploration operators. The first operator (equation (1)) generates randomly a point s in the search space S: $S = [b_1, B_1][b_2, B_2]\ldots[b_l, B_l]$; l: is the dimension of the objective function. $s = (s_i)\ S\ ; i = 1, 2, \ldots, l$.

$$s_i = b_i + U[0,1](B_i - b_i) \tag{1}$$

$U[a,b]$: a uniformly random number in the interval $[a,b]$.

The second operator (equation 2) generates a point s' in the neighborhood of *s* by respecting an amplitude *A*. The amplitude *A* is equal to A_{site} during the creation of the hunting site and it's equal to A_{locale} during the exploration of the hunting site.

$$s_i' = s_i + AU[0,1](B_i - b_i) \tag{2}$$

2.1.3.API algorithm

Choose randomly the initial nest location
While (T<Number of iteration) do
For each ant
Create p hunting sites
While (patience<P_{locale}) do
Explore hunting site
End while
End for
If the nest must be moved, then move the nest to the best ant's coordinates
End while

2.2. CODING AND OBJECTIVE FUNCTION

There are several steps to establish an API classifier for supervised remote sensing image classification, including encoding ant's strings, definition of the objective function, and executing the API algorithm operators.

2.2.1. Ant coding

In ACO applications, the parameters of the search space are encoded in the form of string, so-called ants, representing a solution of problems. In this paper, an ant is encoded with units of positive integer numbers; each unit represents a class center. Take the following case as an example.

Assuming we have a satellite image with three bands, considering the four class' centers of the ant in the population is as shown in Figure 3.

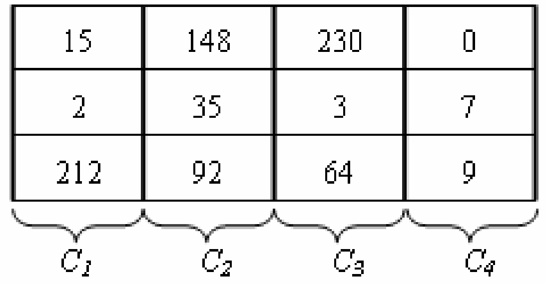


FIGURE 3 Structure of an ant in API algorithm

2.2.2. Objective function

Starting from initial solution (nest position) selected randomly, API algorithm preserve the appropriate solution based on an objective function which is associated with each ant that represents the degree of goodness of the solution encoded in it.

Two validity indices have been used in this paper, the first is the Intra-class Variance (ICV), and the second is the Davies-Bouldin Index (DBI).

Several classification validity indices have been developed to determine an optimal classification, for example the Separation Index, "SI", the Davies-Bouldin "DB" index and the Xie-Beni Index, "XBI" [22].

The within class variance value has been used in several researches [29, 20, 21].

$$IV = \sum_{i=1}^{k} \sum_{x \in X_k} \left(x_j - C_i \right). \tag{3}$$

The DBI has both a statistical and geometric rationale. The API algorithm adopts the DBI as objective function due to its suitability for remote sensing imagery [23, 24, 30, 31, 12]. The DBI can be calculated as follows:

$$\mu_{ki} = \begin{Bmatrix} 1; & \arg\min\limits_{1<j\le n} \left\| x_i , C_j \right\| = k \\ 0; & otherwise \end{Bmatrix}, \tag{4}$$

where:

x_i: The gray level of pixel *i*.

M_k: Number of elements in class *k*.

u_{ki}: Membership function of pixel *i* to class *k*.

X_k: All elements of class *k*.

Then the average (v_i) and standard deviation (S_i) of each class are calculated as follows:

$$v_k = \frac{\sum_{i=1}^{M_k} (\mu_{ki}) x_i}{\sum_{i=1}^{M_k} (\mu_{ki})} = \frac{\sum_{x_i \in X_k} x_i}{M_k}, \tag{5}$$

$$S_k = \left(\frac{1}{M_k} \sum_{x \in X_k} \left\| x - v_k \right\|^2 \right)^{1/2}. \tag{6}$$

Now calculate the distance between the averages' classes:

$$d_{kj} = \left\| v_k - v_j \right\|_t , \tag{7}$$

d_{kj} is the Minkowski distance of order t between the k^{th} center and the ith center.We set t=2.

Then the R_k value of k^{th} center is calculated by equation.

$$R_k = \max_{j, j \ne k} \left\{ \frac{S_k + S_j}{d_{kj}} \right\}. \tag{8}$$

The DB value is defined as the average R of all classes.

$$DB = \frac{1}{n} \sum_{k=1}^{n} R_k . \tag{9}$$

3 Experimental results

3.1 STUDY AREA

A satellite Landsat TM7 image of the Oran area, Algeria acquired on April 22, 2003 is used for the experiment of classification using API algorithm. The study area consists of 400×800 pixels with a ground resolution of 30 m (Fig. 4). This area is dominated by the following eight land use types: sea (C1), surf (C2), sand (C3), forest (C4), cereals (C5), burning (C6), fallow (C7) and urban (C8). Based on the field investigation and land-use maps, samples (pixels) are acquired. The sample data set is further divided into two groups, i.e., 1/3 as training data set and 2/3 as test data set.

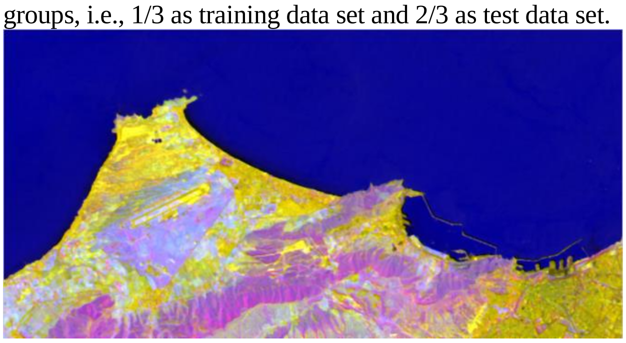


FIGURE 4 Oran TM image

3.2 TRAINING STEP

To achieve a good classification result, it is necessary to minimize the DBI value. A minimum DBI value is the averaged optimal ratio of the intra class scatter over the inter class separation. In our work the u_{ki} is not used because the training class samples are known a priori. In the following example we explain how we have used the DBI index and the ICV value as objective functions for supervised satellite images classification.

$A_1 = (1,\ 2,\ 3,\ 4), A_2 = (5,\ 6,\ 7,\ 8), A_3 = (9,\ 10,\ 11,\ 12).$

We add an element (integer number) in each class samples and subsequently we calculate the BDI value. This operation is summarized in Table 1.

TABLE 1 Search of the class centers using DBI

Test	Class 1	Class 2	Class 3	DBI value
1	$A_1 \cup \{3.5\}$	$A_2 \cup \{8\}$	$A_3 \cup \{12.5\}$	0.5802
2	$A_1 \cup \{3\}$	$A_2 \cup \{7.5\}$	$A_3 \cup \{12\}$	0.5352
3	$A_1 \cup \{2.5\}$	$A_2 \cup \{7\}$	$A_3 \cup \{11.5\}$	0.5051
4	$A_1 \cup \{2\}$	$A_2 \cup \{6.5\}$	$A_3 \cup \{11\}$	0.4926

In Table 1 we found that the minimum of DBI value is obtained when the elements 2.5, 4.5 and 6.5 are added to the class A_1, A_2 and A_3 respectively. It is also noted that the DBI value minimizes when the added values are close as in test 4. The DBI value becomes minimal when not only the added elements are the classes' centers, but also when they are distant between them.

TABLE 2 Search of the class centers using ICV value

Test	Class 1	Class 2	Class 3	ICV value
1	4	6	8	6
2	3.5	5.5	7.5	5
3	3	5	7	4
4	2.5	4.5	6.5	4
5	2	4	6	4

We propose to put the elements added in Table 1 as class' centers of A_1, A_2, A_3 and then we calculate the Euclidean distance (ED) between the classes' centers and the items of each class. This is summarized in Table 2.

In table 2 the minimum ICV value is repeated in several tests. Tests 3 and 5 have the same ICV value of test 4 (optimal class centers). We can say that the results are not guaranteed when using ICV as an objective function.

3.3 CLASSIFICATION STEP

Classifications using the Euclidean distance as objective function presents always limits because given items will not going allocate necessarily to the appropriate class. Figure 5 below shows a real example of our problem where the items will be classify to the small class because it is closer to the class center of this class; however it is more appropriate that it will be classify to the big class (Figure 5).

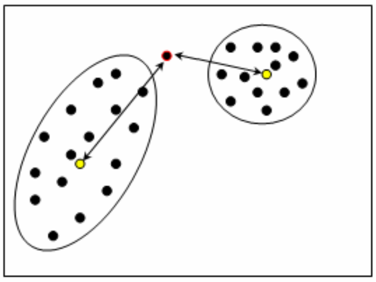


FIGURE 5 Classification problem

In this work we apply two objective functions: ICV and DBI for the classification stage. For the ICV function the principle is simple; simply allocate the pixel that we want to classify to the class where the class center is the closest. For the DBI function, we add the pixel to be classified to the training samples of a given class, and we add in the other class' samples the optimum class centers found during the training stage and subsequently we calculate the DB value. This procedure is repeated for each class. The pixel is thus allocated to the class where the DB value is minimal. Our principle of classification is summarized in Table 3.

Suppose we have the learning sample E_1, E_2, E_3 of three classes.

$$E_1 = \{1,\ 2,\ 3,\ 4\}; E_2 = \{5.5,\ 6,\ 6,\ 6.5\}, E_3 = \{9,\ 10,\ 11,\ 12\}$$

The classes centers found during learning using the DBI function are: 2, 6, 11. Our problem is to affect the item 8 in the appropriate class.

TABLE 3 classification using DBI

E_1	E_2	E_3	DB value
$E_1 \cup \{8\}$	$E_2 \cup \{6\}$	$E_3 \cup \{11\}$	0.9227
$E_1 \cup \{2\}$	$E_2 \cup \{8\}$	$E_3 \cup \{11\}$	0.4625
$E_1 \cup \{2\}$	$E_2 \cup \{6\}$	$E_3 \cup \{8\}$	0.4121

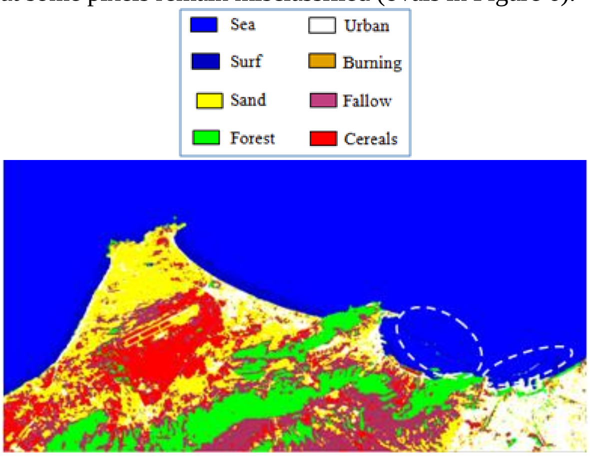
In Table 3 the minimum value of DB was obtained when we added the item 8 in the training sample of the class 3, thus this item will be allocated to Class 3. By contrast, if we use the ICV function, the item 8 will be allocated to the class 2 because it is so close to the class center of this class.

3.4 CLASSIFICATION OF THE STUDY AREA

The API algorithm has a number of parameters to be determined in advance. After several tests the algorithm parameters are fixed as the following: $n = 10$, $p = 40$; $A_{site} = 20$; $A_{locale} = 5$; $P_{locale} = 10$; $T = 5$.

We note API-DBI and API-ICV when using BDI and ICV as objective functions respectively.

In Figure 6 and 7 we can notice a small difference between the results of both approaches. The API-ICV method was able to distinguish the eight classes of the study area, but some pixels remain misclassified (ovals in Figure 6).



FIUGRE 6 Classification using API-ICV

The API-ICV approach produces confusion because some pixels of the urban class were classified in to the surf class (two ovals in Figure 6). Visually API-DBI method has produced a better classification. In Figure 7 the confusion has been reduced.

To make a quantitative comparison between the results, we examine the confusion matrix, the overall accuracy and the Kappa coefficient. Table 4 and 5 shows the confusion matrix of API-ICV and API-DBI. The first technique produced an overlapping between several classes.

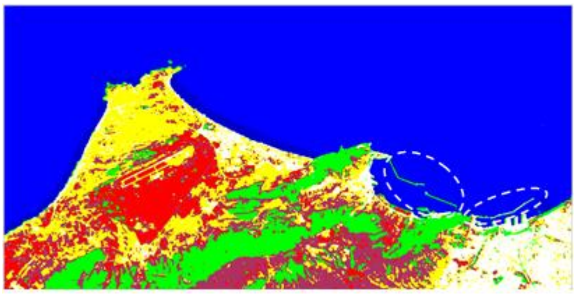


FIGURE 7 Classification using API-DBI

The most important overlapping made between sand class and cereals one, because the pixels of the sand class were recognized as pixels of the cereal class. This approach has also allocated pixels of the urban class to the forest class.

TABLE 4 Confusion matrix of API-ICV

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
C_1	**100**	0	0	0	0	0	0	0
C_2	2.5	**97.5**	0	0	0	0	0	0
C_3	0	0	**80**	0	12.5	0	0	7.5
C_4	0	0	0	**97.5**	0	2.5	0	0
C_5	2.5	0	2.5	0	**90**	0	0	5
C_6	0	0	0	2.5	0	**92.5**	5	0
C_7	0	0	0	5	0	0	**95**	0
C_8	0	10	0	0	2.5	0	0	**87.5**

The API-DBI method makes a good classification but there is a small confusion between cereals and sea classes and between sand and urban classes. Table 6 shows that the API-DBI technique produces a good overall accuracy, that is to say a high percentage of pixels well classify. We note an increase in the classification rate from 92,50% for API-ICV to 94,68% for API-DBI thus an improvement of 2.18%. It is recognized that the classification rate is not enough to know the performance of a given technique [32]. To measure this performance, we used furthermore the Kappa Coefficient. Table 6 shows that the Kappa coefficient increased from 0.9246 for API-ICV to 0.9466 API-DBI, so an improvement of 0.022.

TABLE 5 confusion matrix of API-DBI

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
C_1	**97.5**	0	0	2.5	0	0	0	0
C_2	2.5	**97.5**	0	0	0	0	0	0
C_3	0	0	**87.5**	0	7.5	0	0	5
C_4	0	0	0	**97.5**	0	2.5	0	0
C_5	2.5	0	2.5	0	**87.5**	0	0	7.5
C_6	0	0	0	5	0	**90**	5	0
C_7	0	0	0	5	0	0	**100**	0
C_8	0	0	0	0	0	0	0	**100**

To measure the quality of the new objective function we used a second metaheuristic inspired by the attraction and repulsion of electric charges [25]. This method called Electromagnetic Metaheuristic (EM), have been applied to various problems such as: production systems of the type Hybrid Flow Shop [33], scheduling projects [34], scheduling nurses [35] and solving nonlinear systems of equations [36]. We have used EM algorithm for classification of the study area. For more details about the EM algorithm refer to Birbil et al., [25]. We note EM-ICV when we use the ICV function, and we note EM-DBI when we using the DBI function. The EM algorithm parameters are set as following: $LsIter = 10$ (number of iterations in the local search), $m = 100$ (number of particles), $MaxIteration = 10$; $\delta = 0.05$;

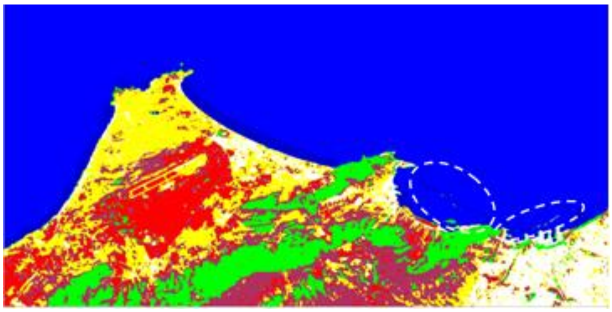


FIGURE 8 Classification using EM-ICV

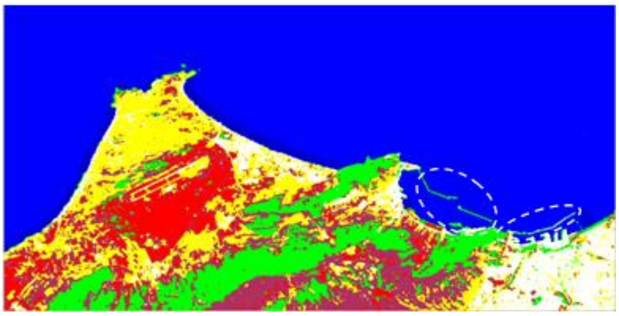


FIGURE 9 FIGURE Classification using EM-DBI

Visually the results obtained by EM algorithm are the same of those of the API algorithm (Figure 8, 9). The DBI confirm its performance on remote sensing images classify-cation. The EM-DBI classifies better than EM-ICV because in this approach pixels in the urban class are classified to the surf class. The EM-DBI produces a good classification; all classes are distinct without apparent great confusion.

Table 7, 8 shows the confusion matrix of the EM-ICV and EM-DBI approaches. The first method produces confusion between all classes except sea class. EM-DBI was able to improve the classification except a small confusion between the sand and cereals classes and between cereals and urban classes.

TABLE 7 confusion matrix of EM-ICV

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
C_1	**100**	0	0	0	0	0	0	0
C_2	2.5	**97.5**	0	0	0	0	0	0
C_3	0	0	**77.5**	0	15	0	0	7.5
C_4	0	0	0	**97.5**	0	2.5	0	0
C_5	2.5	0	2.5	0	**92.5**	0	0	2.5
C_6	0	0	0	2.5	0	**92.5**	5	0
C_7	0	0	0	2.5	0	0	**97.5**	0
C_8	0	10	0	0	2.5	0	0	**87.5**

TABLE 8 Confusion matrix of EM-DBI

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
C_1	**97.5**	0	0	0	0	0	0	0
C_2	2.5	**97.5**	0	0	0	0	0	0
C_3	0	0	**87.5**	0	7.5	0	0	5
C_4	0	0	0	**97.5**	0	2.5	0	0
C_5	2.5	0	2.5	0	**87.5**	0	0	7.5
C_6	0	0	0	5	0	**90**	5	0
C_7	0	0	0	0	0	0	**100**	0
C_8	0	0	0	0	0	0	0	**100**

We recorded an overall accuracy equal to 92.81% for EM-ICV and 94,68% for EM-DBI thus an increase of 1,87%. The Kappa coefficient was also improved from 0.9277 for EM-ICV to 0.9466 for EM-DBI.

4 Conclusion

The metaheuristic in fact complex multi-agent system in which agents with simple intelligence can complete complex tasks trough cooperation. Two metaheuristics (API, EM) have been applied to the classification of remote sensing image of Oran, Algeria. The comparison of classification result is carried out between DBI and ICV objective functions.

The overall accuracy on using ICV is 92.50% and 92.81% for API and EM respectively. The API-DBI and EM-DBI have an accuracy of 94.65 and 94.68 respectively. In this paper, DBI has been successfully introduced to supervised satellite image classification; however, there is still some limitation on using metaheuristics on remote sensing images classification problem. On the one hand, the DBI Objective function makes much longer time than ICV function during the training stage. On the other hand, the large number of parameters and the difficulty of their choices stand as major obstacles for the use of the metaheuristics (API, EM) for remote sensing images classification. As perspective, would not fix the parameters of the metaheuristics but to make it dynamic. Another perspective is to introduce the communication concept between ants during the exploration step. The last perspective is to use a robust local search algorithm.

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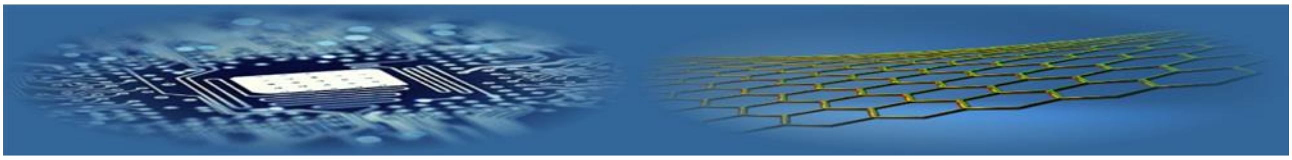

Powerdomains and modality, revisited with detailed proofs

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Abstract

We give direct detailed proofs for the connection between powerdomains and logic models which can be made about nondetermin- istic computations. In the proceeding of proofs, we prove some algebraic properties of them at the same time. Meanwhile, we take up some trick for constructing the finite branching tree, which can also be used into the other areas.

Keywords:

Power domain
Nondeterministic computation
Algebraic properties

1 Introduction

Powerdomains were originally proposed to model the semantics of nondeterministic programming languages [1, 2]. They can be taken as the domain analogues of powersets with elements which represent the sets of different courses a nondeterministic computation can follow. Winskel gives a simple connection between powerdomains and modal assertions that can be made about nondeterministic computations in [1]. He considers three kinds of powerdomains: the Smyth powerdomain, the Hoare powerdomain and the Plotkin powerdomain. Two kinds of modal operators are taken: ¤ for "inevitable" and § for "possible". It is shown in a precise sense how the Smyth powerdomain is built up from assertions about the inevitable behaviour of of a nondeterministic computation, the Hoare powerdomain is built up from assertions about the possible behaviour of of a nondeterministic computation, while the Plotkin powerdomain is built up from both kinds of assertions taken together. In [3], the detailed proofs are not given. It is also a little fuzzy to understand the sketch of proofs. We give here detailed direct proofs of thee results. On the way, we establish some algebraic properties of powerdomains and of nondeterministic computations. In particular, we spell out the construction of nondeterministic computations associated to the elements of the respective powerdomains.

We present the preliminaries on powerdomains and nondeterministic computations. To know more, see [4].

We first give some knowledge of domain theory. Let D be a partial order. A $directed\ set$ of $(D,\le)$ is a non-null subset S(µ D) such that $\forall s,t\in S\ \exists u\in S\ s.t.\ s\le u\ \&\ t\le u$. A $left\text{-}closed$ of (D;·) is a subset A(µ D) such that $\forall a,b\in D. a\le b\in A\Rightarrow a\in A$. An $ideal$ of $(D,\le)$ is a non-null subset $A(\subseteq D)$ such that A is a directed and left-closed set. A $complete\ partial\ order$($c.p.o.$ for short) is a partial order $(D,\le)$ which has a least element $\perp$ and all least upper bounds of directed subsets. A partial order $(D,\le)$ which has a least element $\perp$ and all least upper bounds of directed subsets. A $finite$ element of a $c.p.o.(D,\le)$ is an element $x\in D$ such that for any directed subset $S\subseteq D$ when $x\le\sqcup S$ there is an $s\in S$ such that $x\le s$. We write D^0 for the set of finite elements of D. Intuitively, the finite elements are that information which a computation can realize in finite time. The $c.p.o.$ $(D,\le)$ is $algebraic$ if and only if for all $x\in D$ we have$x=\sqcup\{e\le x\mid e\in D^0\}$. The $c.p.o.$ $(D,\le)$ is said to be countably algebraic or simply ω-algebraic if and only if D is algebraic and D^0 is countable.

In the sequel, $(D,\le)$ always stands for an ω-algebraic $c.p.o.$. Let $M[D]$ be the set of non-null finite subsets of D^0. There are three natural ways to preorder $M[D]$. We consider these three kinds of order: for A, B in $M[D]$, write

$$\begin{aligned} A\preccurlyeq_S B &\iff \forall b\in B\ \exists a\in A\ s.t.\ a\le b\\ A\preccurlyeq_H B &\iff \forall a\in A\ \exists b\in B\ s.t.\ a\le b\\ A\preccurlyeq_P B &\iff A\preccurlyeq_S B\ \&\ A\preccurlyeq_H B \end{aligned},$$

where $\preccurlyeq_S$, $\preccurlyeq_H$ and $\preccurlyeq_P$ are called $the\ Smyth\ order$, $the\ Hoare\ order$ and $the\ Plotkin\ order$ respectively.

There is a standard way to get an algebraic domain from a preorder with least element, often called $completion\ by\ ideals$ [5]. Let $(P,\le)$ be a preorder with a least element $\perp$. $I(P)$ is the set of ideals of P. It is easy to prove that $(I(P,\le),\subseteq)$ is an algebraic domain, with finite elements $\{q\in P\mid q\le p\}$ for $p\in P$. In this way, we can obtain three different powerdomains by completing by ideals the three preorder $\preccurlyeq_S$, $\preccurlyeq_H$, $\preccurlyeq_P$ on $M[D]$. We name them respectively *the Smythpowerdomain*, *the Hoare powerdomain*, and *the Plotkin powerdomain*.

$$\begin{aligned} (P_0[D],\le_0) &\triangleq (I(M[D],\preccurlyeq_S),\subseteq)\\ (P_1[D],\le_1) &\triangleq (I(M[D],\preccurlyeq_H),\subseteq)\\ (P_2[D],\le_2) &\triangleq (I(M[D],\preccurlyeq_P),\subseteq). \end{aligned}$$

Now, we define the notion of nondeterministic D-computation. A tree is $finitely\ branching$ if it has a finite number of branches at each fork.

A nondeterministic D-computation has the form $(T,\to,val)$ where $(T,\to)$ is a finitely branching tree and

val is a map to D^0 such that $\forall\ t, t' \in T,\ t \to t' \Longrightarrow val(t) \le val(t')$.

A $branch$ is a sequence $t_0, t_1, \cdots, t_n, \cdots$ where t_0 is the root node and $t_n \to t_{n+1}$ for each $n+1$ at which the sequence is defined. By a maximal branch of $(T, \to)$ we mean a branch which is either infinite, or finite of the form $t_0, t_1, \cdots, t_n$ with $t_n \not\to$.

2 The Smyth Powerdomain

We define a modal logic L_0 including D^0, for all $s \in L_0$, s is any formula built by the following syntax; $s ::= a \mid \Box s \mid s \vee s \quad (a \in D^0)$.

Let $(T, \to, val)$ be a nondeterministic D-computation. Define $\models_T$ to be the least relation included in $T \times L_0$ such that

$$\begin{array}{rll} t \models a & if & a \le val(t) \\ t \models (s \vee s') & if & t \models s \ \ or \ \ t \models s' \\ t \models \Box s & if & t \models s \ \ or \ \ (\forall t', t' \to t \Longrightarrow t' \models \Box s) \end{array}$$

Definition 1. Let T be a finitely branching tree. A coupe of T is a finite set C of the nodes of T such that

$$\begin{array}{c} ((t_1, t_2 \in C \ \&\ val(t_1) \le val(t_2)) \Rightarrow t_1 = t_2) \\ \text{and } (\forall s \in T \exists t \in C \ s.t.\ val(s) \le val(t) \\ or\ val(t) \le val(s)) \end{array}$$

Another way to define satisfaction for $\Box$-statements is as follows.

Lemma 1. *Let T be a finitely branching tree with the root t. We have,*

$$t \models \Box s \Leftrightarrow \exists C\ (C \text{ is a coupe of } T)\ s.t.\ (\forall t' \in C, t' \models s)$$

Proof. Suppose that $t \models \Box s$. To prove it, we are only allowed to use the following rules,

$$\frac{t \models s}{t \models \Box s} \quad (1) \qquad\qquad \frac{\forall t', t \to t', t' \models \Box s}{t \models \Box s} \quad (2)$$

The basic case is that from $t \models_T s$ we can get $t \models_T \Box s$. Let C be $\{t\}$. It is clear that C is a coupe, and $t \models s$. The other case is that if for all t' satisfying $t \to t'$, we have $t' \models \Box s$ then we can get $t \models \Box s$. By the assumption of induction, for any $t' \models \Box s$, there exists a coupe $C_{t'}$ such that $\forall t'' \in C_{t'}, t'' \models s$. Let C be the disjoin union of the coupes $C_{t'}$s. Then it is easy to prove that C is also a coupe of T and $\forall t'' \in C, t'' \models s$.

On the other hand, suppose that there is a coupe C of T such that $\forall t'' \in C, t'' \models s$. If $t \in C$, then $t \models \Box s$. If $t \notin C$, for any t' satisfying $t \to t'$.

Lemma 2. Let $T = (T, \longrightarrow, val)$ be a nondeterministic D-computation with root node t. Write $\models_T s$ for $t \models_T s$. Let $\mathbf{T}$ be the class of nondeterministic D-computations. Define $s \equiv s'$ iff $(\models_T s \Leftrightarrow \models_T s', \forall T \in \mathbf{T})$.

We have

(1) $s \vee (s' \vee s'') \equiv (s \vee s') \vee s''$;

(2) $s \vee s' \equiv s' \vee s$;

(3) $\Box(\Box s) \equiv \Box s$;

(4) $\Box(s \vee \Box s') \equiv \Box(s \vee s')$;

(5) $s \equiv s' \Longrightarrow \Box s \equiv \Box s'$;

(6) $s \equiv s' \Longrightarrow s \vee s'' \equiv s' \vee s''$;

Proof (1), (2), (5), (6) It is clear

(3) For any $T \in \mathbf{T}$, if $\models_T \Box s$, then $\models_T \Box(\Box s)$. On the other hand, we assume that $\models_T \Box(\Box s)$, then for every maximal branch φ in the subtree out of *t*, there is a node t' which satisfies $\Box s$. Let λ be the finite branch from t to t'. Define that $\varphi - \lambda$ as φ from which λ has been cut. It is clear that $\varphi - \lambda$ is a maximal branch in the subtree out of t'. So $\varphi - \lambda$ has a node t'' which satisfies *s*, that is, φ has the node t'' which satisfies *s*. By lemma 1, $\models_T \Box s$.

(4) For any $T \in \mathbf{T}$, if $\models_T \Box(s \vee s')$, then for every maximal branch in the subtree out of *t*, there is a node t' which satisfies $s \vee s'$, that is, $t' \models_T s$ or $t' \models_T s'$. It follows that $t' \models_T s$ or $t' \models_T \Box s'$, then $t' \models_T s \vee \Box s'$. So $\models_T \Box(s \vee \Box s')$. On the other hand, if $\models_T \Box(s \vee \Box s')$, then for every maximal branch φ in the subtree out of *t*, there is a node t' which satisfies $\models_T s \vee \Box s'$, that is, $t' \models_T s$ or $t' \models_T \Box s'$. (i) If $t' \models_T s$, then $t' \models_T s \vee s'$; (ii) If $t' \models_T \Box s'$, let λ be the branch from t to t', then $\varphi - \lambda$ is a maximal branch in the subtree out of t', so $\varphi - \lambda$ has a node t'' which satisfies s', hence $t'' \models_T s \vee s'$. Then φ has the node which satisfies $s \vee s'$. Therefore, $\models_T \Box(s \vee s')$.

Such properties make us can get the normal forms of the logic model L_0.

Lemma 3. $\Box s \in L_0$ is $\equiv -$ equivalent a normal form $\Box(a_0 \vee \cdots \vee a_n)$ for some $a_0, \cdots, a_n \in D^0$.

Proof. By definition of L_0, s is any formula built with the following syntax, $s ::= a|\Box s| s \vee s$.

It suffices to consider the following cases;

The basic case is that *s* = *a*. Then $\Box s = \Box a$, that is, $\Box s$ is $\equiv -$ equivalent the normal form $\Box a$. Another case is that $s = \Box s'$. Suppose that $\Box s'$ is $\equiv -$ equivalent a normal form $\Box(a_0 \vee \cdots \vee a_n)$. Then $\Box s = \Box(\Box s') \equiv \Box(\Box(a_0 \vee \cdots \vee a_n)) \equiv \Box(a_0 \vee \cdots \vee a_n)$ by the lemma 2. Another case is that $s = s' \vee s''$. Suppose that $\Box s'$ is $\equiv -$ equivalent a normal form $\Box(a_0 \vee \cdots \vee a_n)$ and s'' is $\equiv -$ equivalent a normal form $\Box(b_0 \vee \cdots \vee b_m)$. Then $\Box s = \Box(s' \vee s'') \equiv \Box(s' \vee \Box s'') \equiv \Box(\Box s'' \vee s') \equiv \Box(\Box s'' \vee \Box s') \equiv \Box(\Box(b_0 \vee \cdots \vee b_m) \vee \Box(a_0 \vee \cdots \vee a_n)) \equiv \Box(\Box(b_0 \vee \cdots \vee b_m) \vee (a_0 \vee \cdots \vee a_n)) \equiv \Box((a_0 \vee \cdots \vee a_n) \vee \Box(b_0 \vee \cdots \vee b_m)) \equiv \Box(a_0 \vee \cdots \vee a_n \vee b_0 \vee \cdots \vee b_m)$ by lemma 2.

We are interested in the statements which are inevitably true. Define the following set of assertions with nondeterministic computations.

Definition 2. *Let* $T = (T, \to, val)$ *be a nondeterministic* D-computation *with root node t. Define*

$$V_0(T) = \{\Box s \in L_0 \mid \models_T \Box s\}$$

Based on this assertions, we define an obvious preorder on nondeterministic computations

$$T \preccurlyeq_0 T' \Longleftrightarrow V_0(T) \subseteq V_0(T')$$

Quotienting the preorder $\preccurlyeq_0$ on nondeterministic computations by the equivalence $\simeq_0 \triangleq \preccurlyeq_0 \cap \preccurlyeq_0{}^{-1}$, we obtain the Smyth powerdomain by th eorem 1. Before we prove that, we give some algebraic properties of nondeterministic computations.

Let $V_0'(T) = \{\Box(a_0 \vee \cdots \vee a_n) \mid \models_T \Box(a_0 \vee \cdots \vee a_n), a_0, ..., a_n \in D^0\}$.

Lemma 4. Let T and T' be nondeterministic computation. t is the root of $T$and T' is the root of T', we have $V_0(T) \subseteq V_0(T') \Longleftrightarrow V_0'(T) \subseteq V_0'(T')$.

Proof. Firstly, we assume that $V_0(T) \subseteq V_0(T')$. Let $\Box(a_0 \vee \cdots \vee a_n) \in V_0'(T)$. Then $t \models_T \Box(a_0 \vee \cdots \vee a_n)$, because $\Box(a_0 \vee \cdots \vee a_n) \in V_0'(T) \in V_0(T)$, then $\Box(a_0 \vee \cdots \vee a_n) \in V_0(T')$ since $V_0(T) \subseteq V_0(T')$. Therefore $t' \models_{T'} \Box(a_0 \vee \cdots \vee a_n)$, and $\Box(a_0 \vee \cdots \vee a_n) \in V_0'(T')$.

On the other hand, we assume that $V_0'(T) \subseteq V_0'(T')$. Let $\Box s \in V_0(T)$. By lemma 3, there is a normal form $\Box(a_0 \vee \cdots \vee a_n) \equiv \Box s$, then $t \models_T \Box(a_0 \vee \cdots \vee a_n)$, then we have $t' \models_{T'} \Box(a_0 \vee \cdots \vee a_n)$ since $V_0{}'(T) \subseteq V_0{}'(T')$. Therefore, $t' \models_{T'} \Box s$.

Assume that $A = \{a_1, a_2, \cdots, a_n\}$, we write $t \models_T \Box A$ standing for $t \models_T \Box(a_0 \vee \cdots \vee a_n)$.

Lemma 5. Assume that $A = \{a_0, a_1, \cdots, a_n\}$, $B = \{b_0, b_1, \cdots, b_m\}$, and $B \preccurlyeq_S A$. If $t \models_T \Box A$, then $t \models_T \Box B$.

Proof. Because that $t \models_T \Box A$, then for every maximal branch φ in the subtree out of *t*, there is a node t' which satisfies $a_0 \vee \cdots \vee a_n$, that is, $\exists a_i \in A$, $s.t.\ t' \models_T a_i$. It follows that $a_i \leq val(t')$. So there is $b_j \in B$, $b_j \leq a_i \leq val(t')$ since $B \preccurlyeq_S A$, then $t' \models_T b_j$, and $t' \models_T (b_0 \vee \cdots \vee b_m)$. By lemma 1, $\models_T \Box B$.

The following theorem show how the Smyth powerdomain is built up from assertions about the inevitable behavior of a process. Winskel gave the sketch of the proof [], but it is fuzzy to understand. Here we give a kind of direct proof. The ideal is very simple, but there is a new trick to construct a finite branching tree by an element of Smyth powerdomain.

Theorem 1. *Let* $\mathbf{T}$ *be the class of nondeterministic D-computations. The Smyth powerdomain* $\mathbf{P_0}[\mathbf{D}]$ *is isomorphic to the quotient* $(\mathbf{T}/\simeq_0, \preccurlyeq_0/\simeq_0)$, *and to the order* $(\{V_0(T) | T \in \mathbf{T}\}, \subseteq)$.

Proof. By the lemma 4, it suffices to prove that $P_0[D]$ is isomorphic to $(\{V_0'(T) | T \in \mathbf{T}\}, \subseteq)$. Define $f: (\{V_0'(T) | T \in \mathbf{T}\}, \subseteq) \longrightarrow (\mathbf{P_0}[\mathbf{D}], \subseteq)$ as follows; for any $T \in \mathbf{T}$, $f(V_0'(T)) = \{\{a_0, a_1. \cdots, a_n\} \mid \Box(a_0 \vee \cdots \vee a_n) \in V_0'(T)\} \triangleq I(T)$.

Firstly, we prove that $I(T) \in (\mathbf{P_0}[\mathbf{D}], \subseteq)$. It is clear that $\{\bot\} \in I(T)$. so $I(T) \neq \emptyset$. Assume that $B \triangleq \{b_0, b_1, \cdots, b_p\} \preccurlyeq_S A \triangleq \{a_0, a_1, \cdots, a_q\}$, and $A \in I(T)$. By the lemma 5, $t \models_T \Box B$. Then $B \in I(T)$. On the other hand, let $A = \{a_0, ..., a_n\}$, $B = \{b_0, ..., b_m\} \in I(T)$, that is, $\models_T \Box(a_0 \vee \cdots \vee a_n)$ and $\models_T \Box(b_0 \vee \cdots \vee b_m)$. Then every maximal branch φ in T has a node t' which satisfies $a_0 \vee \cdots \vee a_n$ and a node t'' which satisfies $b_0 \vee \cdots \vee b_m$. So, just like lemma 1, we can construct two finite subtrees T_A and T_B of T where the leaves of T_A satisfy $a_0 \vee \cdots \vee a_n$ and the leaves of T_B satisfy $b_0 \vee \cdots \vee b_m$. Let C be a set as follows; for every maximal branch of T, if the leave t_A of T_A is less than the leave t_B of T_B, then $val(t_B) \in C$, otherwise, $val(t_A) \in C$. Because T_A and T_B are finite, C is a finite set. $C \in I(T)$is clear. For any $val(t) \in C$, $t \models_T a_0 \vee \cdots \vee a_n$ and $t \models_T b_0 \vee \cdots \vee b_m$, so there exist a_i, b_j so that $t \models_T a_i$, $t \models_T b_j$, that is, $a_i \leq val(t)$ and $b_j \leq val(t)$. Hence C is a upper bound w.r.t $\preccurlyeq_S$ of A and B.

Secondly, if $V_0'(T_1) = V_0'(T_2)$, then the fact that $f(V_0'(T_1)) = f(V_0'(T_2))$ is clear.

Thirdly, the fact that f is one-one is clear.

Next, we prove f is onto.

Assume that $\mathcal{A} \in \mathbf{P_0}[\mathbf{D}]$. Since D^0 is countable, then we can know that $\mathcal{A}$ is countable and nonempty ($\{\bot\} \in \mathcal{A}$). So we can assume $\mathcal{A} = \{B_0', \cdots, B_n', \cdots\}$. Let A_0 be B_0'; let A_1 be an upper bound $w.r.t. \preccurlyeq_S$ of A_0 and B_1'; $\cdots$; let A_n be an upper bound $w.r.t. \preccurlyeq_S$ of A_{n-1} and B_n'; $\cdots$. Hence, we get a $\omega-$chain $A_0 \preccurlyeq_S A_1 \preccurlyeq_S \cdots \preccurlyeq_S A_n \preccurlyeq_S \cdots$. Define a function $\varphi_n : A_{n+1} \longrightarrow A_n$ where for any $a \in A_{n+1}$, $\varphi_n(a) \leq a$.

When $m > n$ denote $\varphi_n(\varphi_{n+1}(\cdots(\varphi_{m-1}\varphi_m)\cdots)): A_{m+1} \longrightarrow A_n$ as φ_{nm}

Let $A_n' = \bigcap_{m>n} \varphi_{nm}(A_{m+1})$, $n = 0, 1, 2, \cdots$

We construct T as follows;

let $\bot$ be the root of *T*.

A_n' is the set of nodes of *T* at the height n+1

$a_{nk} \longrightarrow a_{n+1k'}$ iff $a_{nk} = \varphi_n(a_{n+1k'})$

So $(T, \rightarrow, id)$ is a nondeterministic D-computation

$(T, \rightarrow, id)$ has the following properties;

(1) $A_n' \preccurlyeq_S A_{n+1}'$.

For any $a_{n+1}' \in A_{n+1}'$, that is, for any $m > n+1, a_{n+1}' \in \varphi_{n+1m}(A_{m+1})$, there is an $a_{m+1} \in A_{m+1}$ such that $a_{n+1}' = \varphi_{n+1m}(a_{m+1})$. Then $\varphi_n(a_{n+1}') = \varphi_n(\varphi_{n+1m}(a_{m+1})) = \varphi_{nm}(a_{m+1}) \in \varphi_{nm}(A_{m+1}) \subseteq \varphi_n\varphi_{n+1}(A_{n+2})$. So, $\varphi_n(a_{n+1}') \in \bigcap_{m>n} \varphi_{nm}(A_{m+1})$, that is, $\varphi_n(a_{n+1}') \in A_n'$ and $\varphi_n(a_{n+1}') \leq a_{n+1}'$. Hence, $A_n' \preccurlyeq_S A_{n+1}'$.

(2) For any n, $A_n' \in \mathcal{A}$

$A_n' = \bigcap_{m>n} \varphi_{nm}(A_{m+1})$. If for any m, there is an $m' > m$ such that $\varphi_{nm'}(A_{m'+1}) \neq \varphi_{nm}(A_{m+1})$, that is, $\varphi_{nm'}(A_{m'+1}) \subset \varphi_{nm}(A_{m+1})$. Let $k_1 > n+1$ be a least number so that $\varphi_{nk_1}(A_{k_1+1}) \subset \varphi_{nm}(A_{m+1})$.

Let $C_1 = \varphi_{nm}(A_{m+1}) - \varphi_{nk_1}(A_{k_1+1}) \neq \emptyset$; Let $k_2 > k_1$ be a least number so that $\varphi_{nk_2}(A_{k_2+1}) \subset \varphi_{nk_1}(A_{k_1+1})$. Let $C_2 = \varphi_{nk_1}(A_{k_1+1}) - \varphi_{nk_2}(A_{k_2+1}) \neq \emptyset$;

$$\vdots$$

Let $C = \bigcup_{i \geq n} C_i$. So C is an infinite set which contradicts the fact that $C \subseteq \varphi_n(A_{n+1})$ is a finite set. Hence, there exists an m, for any $m' > m$, $\varphi_{nm'}(A_{m'+1}) = \varphi_{nm}(A_{m+1})$. Then for any n, there is an m so that $A_n' = \varphi_{nm}(A_{m+1})$. Because $\varphi_{nm}(A_{m+1}) \preccurlyeq_S A_{m+1}$, then $A_n' \in \mathcal{A}$.

(3) For any node a_{nk} of T at the height $n+1$, there is a node $a_{n+1k'}$ of T at the height $n+2$ such that $a_{nk} \longrightarrow a_{n+1k'}$.

According to property (2), there is an $m > n+1$ such that $A_n' = \varphi_{nm}(A_{m+1})$ and $A_{n+1}' = \varphi_{n+1m}(A_{m+1})$. Because $a_{nk} \in A_n'$, there is an $a_{m+1} \in A_{m+1}$ such that $a_{nk} = \varphi_{nm}(a_{m+1}) = \varphi_n(\varphi_{n+1m}(a_{m+1}))$.

Let $a_{n+1k'} = \varphi_{n+1m}(a_{m+1}) \in A_{n+1}'$. So $a_{n+1k'}$ is node in the *T* at the height n+2. We have $a_{nk} = \varphi_n(a_{n+1k'})$, that is, $a_{nk} \longrightarrow a_{n+1k'}$.

(4) For any A_n', $t \models_T \Box A_n'$

According to property (3), for any maximal branch ϕ in the subtree out of t, there is an element of A_n' is one of the node of ϕ. By the lemma 1, $t \models_T \Box A_n'$

Now we prove that $f(V_0'(T)) = \mathcal{A}$

For any $\Box(a_0 \vee \cdots \vee a_n) \in V_0'(T)$, because $\models_T \Box(a_0$

$\vee \cdots \vee a_n$), then for any maximal branch in the T, there is a node which satisfies $a_0 \vee \cdots \vee a_n$. According to the proof of the lemma 1, there is a finite subtree T' whose leaves satisfy $a_0 \vee \cdots \vee a_n$. Assume that the height of the highest leave is m. According to property (3), the set of leaves of T' is less than A'_m w.r.t. $\preccurlyeq_S$, then $\{a_0, \cdots, a_n\} \preccurlyeq_S A'_m$. From $A'_m \in \mathcal{A}$, we get $\{a_0, ..., a_n\} \in \mathcal{A}$. On the other hand, for any $A = \{a_0, ..., a_n\} \in \mathcal{A}$, by the construction of $\omega-$chain, there is a A_n such that $A \preccurlyeq_S A_n$. Since $A_n \preccurlyeq_S A'_n$, then $A \preccurlyeq_S A'_n$. From the property (4) and the lemma 4, $t \models_T \square A$. So we can prove that $A \in f(V'_0(T))$.

Finally, the fact that $V_0'(T) \subseteq V_0'(T') \Longleftrightarrow f(V_0'(T)) \subseteq f(V_0'(T'))$is clear.

3 Hoare Powerdomain

To get Hoare powerdomain, we look at assertions built using the logic model which standard 'possibly' operator. In fact, Hoare powerdomain has an even simpler construction.

Lemma 6. Let $\mathcal{L}(D^0)$ consist of the non-null, left-closed subsets of D^0, then $(\mathcal{L}(D^0), \subseteq)$ is isomorphic to $(P_1[D], \subseteq)$, the Hoare powerdomain.

Proof. Let F be a function from $\mathcal{L}(D^0)$ to $P_1[D]$ as follows.

Firstly, $F(X) \neq \emptyset$ since X is non-null. For any $A, B \in M[D]$, if $A \preccurlyeq_H B$ and $B \in F(X)$, then for any $a \in A$, there is a $b \in B \subseteq X$ such that $a \leq b$, so $a \in X$ by definition of X. Therefore $A \in F(X)$. For any $A, B \in F(X)$, then $A \preccurlyeq_H A \cup B$ and $B \preccurlyeq_H A \cup B$ and $A \cup B \in F(X)$. Secondly, if $X_1 = X_2$, then $F(X_1) = F(X_2)$. Therefore, F is well-defined.

Let $F^\star$ be a function from $P_1[D]$ to $\mathcal{L}(D^0)$ as follows, $F^\star(\mathcal{A}) = \cup\{A \mid A \in \mathcal{A}\}$

Firstly, $F^\star(\mathcal{A}) \in \mathcal{L}(D^0)$. In fact, since $\{\perp\} \in \mathcal{A}, F^\star(\mathcal{A}) \neq \emptyset$. For any a, b, $a \leq b \in F^\star(\mathcal{A})$, then there is an A in $\mathcal{A}$ such that $b \in A$. So $\{a\} \preccurlyeq_H \{b\} \preccurlyeq_H A$. then we have $\{a\} \in \mathcal{A}$, hence, $a \in F^\star(\mathcal{A})$. Secondly, if $\mathcal{A}_1 = \mathcal{A}_2$, then $F^\star(\mathcal{A}_1) = F^\star(\mathcal{A}_2)$. Therefore, $F^\star$ is also well-defined.

Next, we prove that the follow results.

(1) $FF^\star(\mathcal{A}) = \mathcal{A}, \forall \mathcal{A} \in P_1[D]$;

It follows that $FF^\star(\mathcal{A}) = \{X \mid X \subseteq F^\star(\mathcal{A}), X \ is \ finite\} = \{X \mid X \subseteq \cup\{A \mid A \in \mathcal{A}\}, X \ is \ finite\} = \{A \mid A \in \mathcal{A}\} = \mathcal{A}$

(2) $F^\star F(X) = X, \forall X \in \mathcal{L}(D^0)$

It follows that $F^\star F(X) = \cup\{A \mid A \in F(X)\} = \cup\{A \mid A \in \{A \mid A \subseteq X, A \ is \ finite\}\} = X$

Now, we define this logic model and the satisfaction relation.

Let $(T, \rightarrow, val)$ be a nondeterminstic D-computation. Define $\models_T$ to be the least relation included in $T \times L_1$ such that $t \models a \ \ if \ \ a \leq val(t)$, $t \models \Diamond s \ \ if \ \ t \models s \ \ or \ \ (\exists t', t' \rightarrow t \Longrightarrow t' \models \Diamond s)$.

Here we are interested in those possible statements.

Definition 3. *Let* $T = (T, \rightarrow, val)$ *be a nondeterministic* D-computation *with root node* t. *Define* $V_0(T) = \{\Diamond s \in L_1 \mid \models_T \Diamond s\}$.

Based on this assertions, we define an obvious preorder on nondeterministic computations, $T \preccurlyeq_1 T' \Longleftrightarrow V_1(T) \subseteq V_1(T')$

Quotienting the preorder $\preccurlyeq_1$ on nondeterministic computations by the equivalence $\simeq_1 \triangleq \preccurlyeq_1 \cap \preccurlyeq_1{}^{-1}$, we obtain the Hoare powerdomain by theorem 2.

Lemma 7. For all $s \in L_1$, $\Diamond s \in L_1$ is $\equiv$-equivalent a normal form $\Diamond a$, for $a \in D^0$.

Proof. By definition of L_1, s is any formula built with the following syntax,

$$s ::= a \mid \Diamond s$$

We proceed by induction on s

(1) The basic case is that $s = a$. Then $\Diamond s = \Diamond a$, that is, $\Diamond s$ is $\equiv$-equivalent the normal form $\Diamond a$.

(2) The other case is that $s = \Diamond s'$. Suppose that $\Diamond s'$ is $\equiv$-equivalent a normal form $\Diamond a$. Then $\Diamond s \equiv \Diamond(\Diamond s') \equiv \Diamond s' \equiv \Diamond a$.

Let $V'_1(T) = \{\Diamond a \mid \models_T \Diamond a, a \in D^0\}$.

Lemma 8. Let T and T' be nondeterministic D-computations. The node t is the root of T, and the node t' is the root of T', we have $V_1(T) \subseteq V_1(T') \Leftrightarrow V'_1(T) \subseteq V'_1(T')$.

Proof. We assume that $V_1(T) \subseteq V_1(T')$. For any $\Diamond a \in V'_1(T)$, we have $\models_T \Diamond a$. Then $\Diamond a \in V_1(T) \subseteq V_1(T')$. So $\models_{T'} \Diamond a$. Therefore, $\Diamond a \in V'_1(T')$. On the other hand, we assume that $V'_1(T) \subseteq V'_1(T')$. For any $\Diamond s \in V_1(T)$, we have $\models_T \Diamond s$, then there is an $a \in D^0$ such that $\Diamond s \equiv \Diamond a$ by the lemma 7, that is, $\models_T \Diamond a$, so $\Diamond a \in V'_1(T) \subseteq V'_1(T')$. Then we have $\models_{T'} \Diamond a$. Therefore, $\models_{T'} \Diamond s$, that is, $\Diamond s \in V_1(T')$.

Theorem 2. Let **T** be the class of nondeterministic D-computation. The Hoare powerdomain $P_1[D]$ is isomorphic to the quotient $(T/\simeq_1, \preccurlyeq_1/\simeq_1)$, and to the order $(\{V_1(T) \mid T \in \mathbf{T}\}, \subseteq)$.

Proof. By lemma 6, and lemma 8, we just need to prove that $(\mathcal{L}(D^0), \subseteq)$ is isomorphic to $(\{V'_1(T) \mid T \in \mathbf{T}\}, \subseteq)$. Define $f : (\{V'_1(T) \mid T \in \mathbf{T}\}, \subseteq) \longrightarrow (\mathcal{L}(D^0), \subseteq)$

For any $T \in \mathbf{T}$, $f(V'_1(T)) = \{a \in D^0 \mid \models_T \Diamond a\} \triangleq I$. Firstly, we prove that $I \in \mathcal{L}(D^0)$. It is clear that $\perp \in I$, so $I \neq \emptyset$. Assume that $a, b \in D^0$, $a \leq b$ and $b \in I$, then we have $\models_T \Diamond b$, that is, there is a finite branch from the root t to t' which t' satisfies b. So $b \leq val(t')$. Since $a \leq b \leq val(t')$. t' also satisfies a. Hence $\models_T \Diamond a$, $a \in I$. Secondly, if $V'_1(T) = V'_1(T')$, then $f(V'_1(T)) = f(V'_1(T'))$.

Thirdly, the fact that f is one-one is clear.

Next we prove that f is onto.

Assume that $X \in \mathcal{L}(D^0)$. Since $\perp \in X$, we assume that $X = \{\perp, a_0, a_1, \cdots, a_n, \cdots\}$ We construct T as follows;

The fact that $f(V'_1(T)) = X$ is clear.

Finally, the fact that $V_1'(T) \subseteq V_1'(T') \Longleftrightarrow f(V_1'(T)) \subseteq f(V_1'(T'))$ is clear.

4 Plotkin Powerdomain

From the two sections above, we can find the same way to obtain the plotkin powerdomain. In fact, it is obtained by considering information about both the inevitable and possible behaviour of a computation.

We define a logic modal L_2 including D^0, for all $s \in L_2$, s is any formula built with the following syntax;

$$s ::= a \mid \Box s \mid s \vee s \vee \Diamond s \;(a \in D^0)$$

Let $(T, \to, val)$ be a nondeterminstic D-computation. Define $\models_T$ to be the least relation included in $T \times L_2$ such that

$$\begin{array}{c} t \models a \quad if \quad a \leq val(t) \\ t \models (s \vee s') \quad if \quad t \models s \quad or \quad t \models s' \\ t \models \Box s \quad if \quad t \models s \quad or \quad (\forall t', t' \to t \Longrightarrow t' \models \Box s) \\ t \models \Diamond s \quad if \quad t \models s \quad or \quad (\exists t', t' \to t \Longrightarrow t' \models \Diamond s) \end{array}$$

In our proofs, the following properties are also needed.

Lemma 9. *$t \models_T \Diamond s$ if and only if there is a finite branch from t to t' with the node t' satisfying s.*

Proof. Suppose that $t \models_T \Diamond s$. To prove it, by the definition of $\models_T$, we are only allowed to use the following rules.

The basic case is that from $t \models_T s$ we can get $t \models_T \Diamond s$. Then of course, there is a finite branch from t to t and the node t satisfies s. The other case is that if there is a t' such that $t \to t'$ and we have $t' \models_T \Diamond s$, then we can get $t \models_T \Diamond s$. By induction we may assume that for t', there is a finite branch ϕ from t' to t'' with the node t'' satisfying s. Let ϕ' be a branch which $t \to t'$ has been added to ϕ. Then ϕ' is a finite branch from t to t'' with the node t'' satisfying s. On the other hand, suppose that there is a finite branch ϕ from t to t' with the node t' satisfying s. Let the height of the branch ϕ be n. For any node t_h of the branch ϕ, let h be the height from t_h to t' in ϕ. When $h = 0$, $t_h = t'$, then from $t' \models_T s$, we have $t_h \models_T \Diamond s$ by rule (1). Assume when $h < n$, $t_h \models_T \Diamond s$, then when $h = n$, we have $t_n \models_T \Diamond s$ by the a rule (2). Hence, $t \models_T \Diamond s$.

Lemma 10. Let $T = (T, \to, val)$ be a nondeterministic D-computation with root node t. Write $\models_T s$ for $t \models_T s$. Let $\mathbf{T}$ be the class of nondeterministic D-computations. Define $s \equiv s' \;\; iff \;\; (\models_T s \Leftrightarrow \models_T s', \forall T \in \mathbf{T})$.

We have

(1) $s \equiv s'$ iff $(\models_T s \Leftrightarrow \models_T s', \forall T \in \mathbf{T})$;

(2) $s \vee s' \equiv s' \vee s$;

(3) $\Box(\Box s) \equiv \Box s$;

(4) $\Box(s \vee \Box s') \equiv \Box(s \vee s')$;

(5) $\Diamond(s \vee s') \equiv \Diamond s \vee \Diamond s'$;

(6) $\Diamond(\Diamond s) \equiv \Diamond s \equiv \Box(\Diamond s) \equiv \Diamond(\Box s)$;

(7) $\Box(s \vee (\Diamond s')) \equiv (\Box s) \vee (\Diamond s')$;

(8) $s \equiv s' \Longrightarrow \Box s \equiv \Box s'$;

(9) $s \equiv s' \Longrightarrow \Diamond s \equiv \Diamond s'$;

(10) $s \equiv s' \Longrightarrow s \vee s'' \equiv s' \vee s''$

Proof. The proofs of (1), (2), (3), (4), (8), (10) are seen at lemma 2.

(5) For any $T \in \mathbf{T}$, if $t \models_T \Diamond(s \vee s')$, then there is finite branch from t to t' with the node t' satisfying $s \vee s'$. If t' satisfies s, then $t \models_T \Diamond s$, of course, $t \models_T \Diamond s \vee \Diamond s'$. The case is similar if t' satisfies s'. On the other hand, if $t \models_T \Diamond s \vee \Diamond s'$, then $t \models_T \Diamond s$ or $t \models_T \Diamond s'$. If $t \models_T \Diamond s$, then there is a finite branch from t to t' with the node t' satisfying s, so $t' \models_T s \vee s'$. The case is similar if $t \models_T \Diamond s'$. Hence, $t \models_T \Diamond(s \vee s')$.

(6) For any $T \in \mathbf{T}$, if $t \models_T \Diamond(\Diamond s)$, then there is a finite branch ϕ from t to t' with the node t' satisfying $\Diamond s$. So there is another finite branch ϕ' from t' to t'' with the node t'' satisfying s. Define that $\phi + \phi'$ as ϕ to which ϕ' has been added. It is clear that $\phi + \phi'$ is a finite branch from t to t'' with the node t'' satisfying s. Then $t \models_T \Diamond s$.

For any $T \in \mathbf{T}$, if $t \models_T \Diamond s$, then $t \models_T \Box(\Diamond s)$.

For any $T \in \mathbf{T}$, if $t \models_T \Box(\Diamond s)$, then $t' \models_T \Box s$. Hence, $t \models_T \Diamond(\Box s)$.

For any $T \in \mathbf{T}$, if $t \models_T \Diamond(\Box s)$, then there is a finite branch from t to t' with the node t' satisfying $\Box s$. So any maximal branch in the subtree out of t' has a node which satisfies s, of course, there is a finite branch from t' with a node satisfying s, that is, $t' \models_T \Diamond s$. Hence, $t \models_T \Diamond(\Diamond s)$.

So far, we have $\Diamond(\Diamond s) \equiv \Diamond s \equiv \Diamond(\Box s)$.

(7) For any $T \in \mathbf{T}$, if $t \models_T \Box(s \vee (\Diamond s'))$, then for any maximal branch ϕ in the subtree out of t there is a node t_ϕ which satisfies $s \vee (\Diamond s')$. If there is a maximal branch ϕ such that $t_\phi \models_T \Diamond s'$, then there is a finite branch from t_ϕ to t' with the t' satisfying s. Hence, there is a finite branch from t to t' via t_ϕ with t' satisfying s, namely, $t \models_T \Diamond s$, of course, $t \models_T (\Box s) \vee (\Diamond s')$. If for any maximal branch ϕ, $t_\phi \models_T s$, then $t \models_T \Box s$, of course, $t \models_T (\Box s) \vee (\Diamond s')$.

On the other hand, if $t \models_T (\Box s) \vee (\Diamond s')$, then $t \models_T \Box s$ or $t \models_T \Diamond s'$. If $t \models_T \Box s$, then any maximal branch in the subtree out of t has a node which satisfies s, of coure satisfies $s \vee (\Diamond s')$. Hence, $t \models_T \Box(s \vee (\Diamond s'))$. If $t \models_T \Diamond s'$, then $t \models_T s \vee (\Diamond s')$, hence, $t \models_T \Box(s \vee (\Diamond s'))$.

(9) It is clear.

In the logic model L_2, the normal forms are following;

Lemma 11. *For all $s \in L_2$, $\Box s \in L_2$ is $\equiv -$ equivalent a normal form $\Box(a_0 \vee \cdots \vee a_n) \vee \Diamond b_0 \vee \cdots \vee \Diamond b_m$ for some $a_0, \cdots, a_n, b_0, \cdots, b_m \in D^0$.*

Proof. By definition of L_2, s is any formula built with the following syntax, $s ::= a | \Box s | \Diamond s | s \vee s$.

We proceed by induction on s

(1) The basic case is that $s = a$. Then $\Box s = \Box a$, that is, $\Box s$ is $\equiv -$ equivalent the normal form $\Box a$.

(2) Another case is that $s = \Box s'$. Assume that $\Box s'$ is $\equiv -$ equivalent a normal form $\Box(a_0 \vee \cdots \vee a_n) \vee \Diamond b_0 \vee \cdots \vee \Diamond b_m$. Then $\Box s = \Box(\Box s') \equiv \Box s' \equiv \Box(a_0 \vee \cdots \vee a_n) \vee \Diamond b_0 \vee \cdots \vee \Diamond b_m$ by the lemma 10.

(3) Another case is that $s = s' \vee s''$. Suppose that $\Box s'$ is $\equiv -$ equivalent a normal form $\Box(a_0 \vee \cdots \vee a_n) \vee \Diamond b_0 \vee \cdots \vee \Diamond b_m$ and s'' is $\equiv -$ equivalent a normal form $\Box(c_0 \vee \cdots \vee c_p) \vee \Diamond d_0 \vee \cdots \vee \Diamond d_q$. Then $\Box s = \Box(s' \vee s'') \equiv \Box(s' \vee \Box s'') \equiv \Box(\Box s'' \vee s') \equiv \Box(\Box s'' \vee \Box s') \equiv \Box(\Box(c_0 \vee \cdots \vee c_p) \vee \Diamond d_0 \vee \cdots \vee \Diamond d_q \vee \Box(a_0 \vee \cdots \vee a_n) \vee \Diamond b_0 \vee \cdots \vee \Diamond b_m) \equiv \Box(\Box(c_0 \vee \cdots \vee c_p) \vee \Box(a_0 \vee \cdots \vee a_n) \vee \Diamond d_0 \vee \cdots \vee \Diamond d_q \vee \Diamond b_0 \vee \cdots \vee \Diamond b_m) \equiv \Box(\Box(c_0 \vee \cdots \vee c_p) \vee \Box(a_0 \vee \cdots \vee a_n) \vee \Diamond d_0 \vee \cdots \vee \Diamond d_q \vee \Diamond b_0 \vee \cdots \vee \Diamond b_m) \vee \Box(a_0 \vee \cdots \vee a_n)) \vee \Diamond d_0 \vee \cdots \vee \Diamond d_q \vee \Diamond b_0 \vee \cdots \vee \Diamond b_m \equiv \Box(\Box(c_0 \vee \cdots \vee c_p) \vee (a_0 \vee \cdots \vee a_n)) \vee \Diamond d_0 \vee \cdots \vee \Diamond d_q \vee \Diamond b_0 \vee \cdots \vee \Diamond b_m \equiv \Box(a_0 \vee \cdots \vee a_n \vee$

$\Box(c_0 \vee \cdots \vee c_p)) \vee \Diamond d_0 \vee \cdots \vee \Diamond d_q \vee \Diamond b_0 \vee \cdots \vee \Diamond b_m$ $\equiv \Box(a_0 \vee \cdots \vee a_n \vee c_0 \vee \cdots \vee c_p) \vee \Diamond d_0 \vee \cdots \vee \Diamond d_q \vee$ $\Diamond b_0 \vee \cdots \vee \Diamond b_m$by the lemma 10.

(4) Another case is that $s = \Diamond s'$. Suppose that s' is $\equiv$-equivalent a normal form $\Box(a_0 \vee \cdots \vee a_n) \vee \Diamond b_0 \vee \cdots \vee \Diamond b_m$. Then $\Box s = \Box(\Diamond s') \equiv \Diamond(\Box s') \equiv \Diamond(\Box(a_0 \vee \cdots \vee a_n) \vee \Diamond b_0 \vee \cdots \vee \Diamond b_m) \equiv \Diamond(\Box(a_0 \vee \cdots \vee a_n)) \vee \Diamond(\Diamond b_0 \vee \cdots \vee \Diamond b_m) \equiv \Diamond(a_0 \vee \cdots \vee a_n) \vee \Diamond b_0 \vee \cdots \vee \Diamond b_m$ $\equiv \Diamond a_0 \vee \cdots \vee \Diamond a_n \vee \Diamond b_0 \vee \cdots \Diamond b_m$ by the lemma 10.

The same as Smyth powerdomain, here we are only interested in that information $\Box((\Diamond a) \vee b)$.

Definition 4. Let $T = (T, \to, val)$ be a nondeterministic D-computation with root node t. Define $V_2(T) = \{\Box s \in L_2 \mid \models_T \Box s\}$

Based on this assertions, we define an obvious preorder on nondeterministic computations, $T \preccurlyeq_2 T' \iff V_2(T) \subseteq V_2(T')$

Quotienting the preorder $\preccurlyeq_2$ on nondeterministic computations by the equivalence $\simeq_2 \triangleq \preccurlyeq_2 \cap \preccurlyeq_2{}^{-1}$, we obtain the Plotkin powerdomain in theorem 3.

Let $V_2'(T) = \{\Box(a_0 \vee a_1 \vee \cdots \vee a_n\} \mid \models_T \Box(a_0 \vee \cdots \vee a_n), t \models_T \Diamond a_i, i = 0, ..., n\}$.

Lemma 12. Let T and T' be nondeterministic D-computations. The node t is the root of T, and the node t' is the root of T' . We have $V_2(T) \subseteq V_2(T') \Leftrightarrow V_2'(T) \subseteq V_2'(T')$

Proof. We assume that $V_2(T) \subseteq V_2(T')$. For any $\Box(a_0 \vee \cdots \vee a_n) \in V_2'(T)$, we have $t \models_T \Box(a_0 \vee \cdots \vee a_n)$, and $t \models_T \Box(\Diamond a_i)$ (Since $\Box(\Diamond a_i) \equiv \Diamond a_i$), $i = 0, \cdots, n$. Then we have $\Box(a_0 \vee \cdots \vee a_n) \in V_2(T)$ and $\Box(\Diamond a_i) \in V_2(T)$. Since $V_2(T) \subseteq V_2(T')$, we have $t' \models_{T'} \Box(a_0 \vee \cdots \vee a_n)$ and $t' \models_{T'} \Box(\Diamond a_i)$, $i = 0, \cdots, n$. Then $t' \models_{T'} \Diamond a_i$ (Also since $\Diamond a_i \equiv \Box(\Diamond a_i)$), $i = 0, \cdots, n$. So $\Box(a_0 \vee \cdots \vee a_n) \in V_2'(T')$.

On the other hand, we assume that $V_2'(T) \subseteq V_2'(T')$. Let $\Box s \in V_2(T)$, we have $t \models_T \Box s$. By the lemma 11, $\Box s \equiv \Box(a_0 \vee \cdots \vee a_k) \vee \Diamond a_{k+1} \vee \cdots \vee \Diamond a_n$, where $a_0, \cdots, a_n \in D^0$. So $t \models_T \Box(a_0 \vee \cdots \vee a_k) \vee \Diamond a_{k+1} \vee \cdots \vee \Diamond a_n$. If $t \models_T \Box(a_0 \vee \cdots \vee a_k)$, there is a finite subtree T' where the leaves satisfy $a_0 \vee \cdots \vee a_k$. Let $\{b_1, b_2, \cdots, b_l\} \subseteq \{a_0, \cdots, a_k\}$, where for any$b_j, j = 1, \cdots, l$, there is some leave of T' satisfy b_j . Then $t \models_t \Box(b_1 \vee \cdots \vee b_l)$ and $t \models_T \Diamond b_j, j = 1, \cdots, b_l$. So $\Box(b_1 \vee b_2 \vee \cdots \vee b_l) \in V_2'(T)$, Since $V_2'(T) \subseteq V_2'(T')$, $t' \models_{T'} \Box(b_1 \vee b_2 \vee \cdots \vee b_l)$ and $t' \models_{T'} \Diamond b_j, j = i, \cdots, l$, of course, $t' \models_{T'} \Box(a_0 \vee \cdots \vee a_k) \vee \Diamond a_{k+1} \vee \cdots \vee \Diamond a_n$.

If $t \models_T \Diamond a_i$, there is a finite branch from t to s_j with the node s_j satisfying a_i. Let h be the height from t to s_j. Let T' be a subtree of T where the nodes of T' are the nodes of T which height is less than and equal to h. Assume that the set of leaves of T' is $\{s_1, s_2, \cdots, s_j, \cdots, s_n\}$, then it is easy to prove that $t \models_T \Box(val(s_1) \vee \cdots val(s_j) \vee \cdots \vee val(s_n))$, and $t \models_T \Diamond val(s_i), i = 1, \cdots, n$, So $\Box(val(s_1) \vee \cdots \vee val(s_n)) \in V_2'(T)$, since $V_2'(T) \subseteq V_2'(T')$, $t' \models_{T'} \Box(val(s_1) \vee \cdots val(s_j) \vee \cdots \vee val(s_n))$ and $t' \models_{T'} \Diamond val(s_i), i = 1, \cdots, n$. Specially, $t' \models_{T'} \Diamond val(s_j)$, So, $t' \models_{T'} \Diamond a_i$. Therefore, $t' \models_{T'} \Box(a_0 \vee \cdots \vee a_k) \vee \Diamond a_{k+1} \vee \cdots \vee \Diamond a_n$. Hence $V_2(T) \subseteq V_2(T')$.

Lemma 13. Let $B = \{b_0, ..., b_m\}$, $A = \{a_0, ..., a_n\}$. $B \preccurlyeq_P A$. If $t \models_T \Box A$ and $t \models_T \Diamond a_i$, for any $a_i \in A$, then $t \models_T \Box B$ and $t \models_T \Diamond b_j$, for any $b_j \in B$.

Proof. Because $t \models_T \Box(a_0 \vee \cdots \vee a_n)$, and $t \models_T \Diamond a_i, i = 0, ..., n$. So for every maximal branch ϕ in the subtree out of t , there is a node t' which satisfies $a_0 \vee \cdots \vee a_n$, that is, there is a $a_i \in A$ such that $t' \models_T a_i$. It follows $a_i \leq val(t')$. Because $B \preccurlyeq_S A$, then there is a $b_j \in B$ such that $b_j \leq a_i \leq val(t')$, that is, $t' \models_T b_j$. Therefore $t \models_T \Box(b_0 \vee \cdots \vee b_m)$. On the other hand, for any $b_j \in B$, because $B \preccurlyeq_H A$, there is $a_i \in A$ such that $b_j \leq a_i$. Because $t \models_T \Diamond a_i$, there is a finite branch from t to t'' where t'' satisfies a_i, that is , $a_i \leq val(t'')$. Hence, $b_j \leq a_i \leq val(t'')$, that is, $t'' \models_T b_j$. So we have $t \models_T \Diamond b_j, j = 0, .., m$.

Theorem 3. Let $\mathbf{T}$ be the class of nondeterministic D-computations. The Plotkin powerdomain $\mathbf{P_2}[\mathbf{D}]$ is isomorphic to the quotient $(\mathbf{T}/\simeq_2, \preccurlyeq_2/\simeq_2)$ and to the order $(\{V_2(T) | T \in \mathbf{T}\}, \subseteq)$.

Proof. By lemma 12, it suffices to prove that $P_2[D]$ is isomorphic to $(\{V_2'(T) | T \in \mathbf{T}\}, \subseteq)$ Define $f : (\{V_2'(T) | T \in \mathbf{T}\}, \subseteq) \longrightarrow (\mathbf{P_2}[\mathbf{D}], \subseteq)$ as follows;

for any $T \in \mathbf{T}$, $f(V_2'(T)) = \{\{a_0, a_1, \cdots, a_n\} \mid t \models_T \Box(a_0 \vee \cdots \vee a_n), t \models_T \Diamond a_i, i = 0, ..., n\} \triangleq I(T)$.

Firstly, we prove that $I(T) \in (\mathbf{P_2}[\mathbf{D}], \subseteq)$.

It is clear that $\{\perp\} \in I(T)$, so $I(T) \neq \emptyset$. Assume that $B \triangleq \{b_0, \cdots, b_m\} \preccurlyeq_P A \triangleq \{a_0, \cdots, a_n\}$, and $A \in I(T)$. Since $A \in I(T)$, from the lemma 13, we have $B \in I(T)$. Assume that $A \triangleq \{a_0, \cdots, a_n\}, B \triangleq \{b_0, \cdots, b_m\}$ and $A, B \in I(T)$. So from $t \models_T \Box(a_0 \vee \cdots \vee a_n)$ and $t \models_T \Box(b_0 \vee \cdots \vee b_m)$, we can construct two finite subtrees T_A and T_B of T where the leaves of T_A satisfy $a_0 \vee \cdots \vee a_n$ and the leaves of T_B satisfy $b_0 \vee \cdots b_m$. From $t \models_T \Diamond a_i$, and $t \models_T \Diamond b_j$ $(i = 0, ..., n, j = 0, ..., m)$, we have at most $n + m$ finite branches where each leave satisfies some a_i or some b_j respectively. Let C be a set of $val(t)$ where, for every branch of T, t is the highest node of the leave t_A of T_A, the leave t_B of T_B, the leave which satisfies some a_i(if it exists), and the leave which satisfies some b_j (if it exists). Because T_A and T_B are finite, then C is a finite set, and $C \in I(T)$ is clear. For any $val(t_c) \in C$, there are some a_i, b_j such that $t_c \models_T a_i$, $t_c \models_T b_j$, that is, $a_i \leq val(t_c)$and $b_j \leq val(t_c)$. Hence, $A \preccurlyeq_S C, B \preccurlyeq_S C$. On the other hand, for any $a_i, i = 0, ..., n$, because $t \models_T \Diamond a_i$, there is a finite branch from t to t_{a_i} where $t_{a_i} \models_T a_i$, from the definition of C , there is a $val(t) \in C$ such that $val(t_{a_i}) \leq val(t_c)$, so $a_i \leq val(t_{a_i}) \leq val(t_c)$. Hence, $A \preccurlyeq_H C$. Similarly, we can prove that $B \preccurlyeq_H C$. Therefore,

C is an upper bound of A and B $w.r.t\preccurlyeq_P$.

Secondly, if $V_2'(T_1) = V_2'(T_2)$, then $f(V_2'(T_1)) = f(V_2'(T_2))$.

Thirdly, the fact that f is one-one is clear.

Next, we prove that f is onto.

Assume that $\mathcal{A} \in P_2[D]$, since D^0 is countable, A is countable and nonempty($\{\perp\} \in \mathcal{A}$). So we can assume that $\mathcal{A} = \{B_0, B_1, \cdots, B_n, ...\}$. Let A_1 be B_0; let A_2 be an upper bound of A_1 and B_1 $w.r.t.\preccurlyeq_P$; $\cdots$; let A_n be an upper bound of A_{n-1} and B_{n-1} $w.r.t\preccurlyeq_P, \cdots$. Hence, we get a ω-chain $A_1 \preccurlyeq_P A_2 \preccurlyeq_P \cdots \preccurlyeq_P A_n \preccurlyeq_P \cdots$.

Let $A_0' = A_0$; Let $A_1' = A_0 \cup A_1$; $\cdots$;

Let $A_1' = A_0 \cup A_1$; $\cdots$

The set $\{A_0', A_1', ..., A_n', ...\}$ has the following properties;

(1) $A_n' \preccurlyeq_P A_{n+1}'$.

Proof. $A_{n+1}' = A_n' \cup A_{n+1}$. We have $A_n' \preccurlyeq_H A_{n+1}'$ since $A_n' \subseteq A_{n+1}'$. For any $a_{n+1}' \in A_{n+1}'$, that is, $a_{n+1}' \in A_n'$ or $a_{n+1}' \in A_{n+1}$. If $a_{n+1}' \in A_n'$, then there is the $a_{n+1}' \in A_n'$ such that $a_{n+1}' \le a_{n+1}'$; if $a_{n+1}' \in A_{n+1}$, since $A_n \preccurlyeq_P A_{n+1}$, there is an $a_n \in A_n \subseteq A_n'$ such that $a_n \le a_{n+1}'$. So $A_n' \preccurlyeq_S A_{n+1}'$. Hence, $A_n' \preccurlyeq_P A_{n+1}'$.

$A_n' \preccurlyeq_P A_n$

Proof. We show this claim by induction. When $n = 0$, $A_0' = A_0$ then the fact that $A_0 \preccurlyeq_P A_0$ is clear. Suppose that when $k < n$, $A_k' \preccurlyeq_P A_k$. Then when $k = n$, $A_n' = A_{n-1}' \cup A_n$. We have $A_n' \preccurlyeq_S A_n$ since $A_n \subseteq A_n'$. On the other hand, let $a_n' \in A_n'$, that is, $a_n' \in A_{n-1}'$or $a_n' \in A_n$. If $a_n' \in A_{n-1}'$, since $A_{n-1}' \preccurlyeq_P A_{n-1}$, $A_{n-1} \preccurlyeq_P A_n$, then there is an $a_n \in A_n$ such that $a_n' \le a_n$; if $a_n' \in A_n$, then of course, there is the $a_n' \in A_n$ such that $a_n' \le a_n'$. So $A_n' \preccurlyeq_H A_n$. Hence, $A_n' \preccurlyeq_P A_n$.

$A_n \preccurlyeq_P A_{n+1}'$.

Proof. We show this claim by induction. When $n = 0$, the fact that $A_0 \preccurlyeq_H A_1'$ is clear since $A_0 \subseteq A_0 \cup A_1 = A_1'$. Let $a_1' \in A_1'$, that is, $a_1' \in A_0$ or $a_1' \in A_1$. If $a_1' \in A_0$, there is the $a_1' \in A_0$ such that $a_1' \le a_1'$; if $a_1' \in A_1$, since $A_0 \preccurlyeq_P A_1$, then there is an $a_0 \in A_0$ such that $a_0 \le a_1'$. So $A_0 \preccurlyeq_S A_1'$. Hence, $A_0 \preccurlyeq_P A_1'$. Suppose that when $k < n$, $A_k \preccurlyeq_P A_{k+1}'$. Then when $k = n$, $A_{n+1}' = A_n' \cup A_{n+1}$. We have $A_n \preccurlyeq_H A_{n+1}'$ since $A_n \subseteq A_n' \subseteq A_{n+1}'$. Let $a_{n+1}' \in A_{n+1}'$, that is, $a_{n+1}' \in A_n'$ or $a_{n+1}' \in A_{n+1}$. If $a_{n+1}' \in A_n'$, since $A_{n-1} \preccurlyeq_P A_n'$, then there is an $a_{n-1} \in A_{n-1} \subseteq A_{n-1}' \subseteq A_n$ such that $a_{n-1} \le a_{n+1}'$; if $a_{n+1}' \in A_{n+1}$, since $A_n \preccurlyeq_P A_{n+1}$, then there is an $a_n \in A_n$ such that $a_n \le a_{n+1}'$. So $A_n \preccurlyeq_S A_{n+1}'$. Hence, $A_n \preccurlyeq_P A_n'$.

If $A_{n+1}' - A_n' \neq \emptyset$, then $A_n' \preccurlyeq_S A_{n+1}' \preccurlyeq_S A_{n+1}' - A_n'$ since $A_{n+1}' - A_n' \subseteq A_{n+1}'$.

Define a function $\varphi_n : A_{n+1}'$-$A_n' \longrightarrow A_n'$, where for any $a \in A_{n+1}'$-A_n', $\varphi_n(a) \le a$.

We construct T as follows;

Let $\perp$ be the root of T;

A_n' is the set of nodes of T at the height $n+1$;

$\perp \longrightarrow a_{0i}$, for any $a_{0i} \in A_0'$;

$a_{nk} \longrightarrow a_{n+1k'}$ iff $a_{nk} = \varphi(a_{n+1k'})$ or $a_{nk} = a_{n+1k'}$.

So $(T, \rightarrow, id)$ is a nondeterministic D-computation.

It is easy to check that $(T, \rightarrow, id)$ has the following properties:

(i) For any *n*, $A_n' \in \mathcal{A}$.

Since $A_n' \preccurlyeq_P A_n$, and $A_n \in \mathcal{A}$, then $A_n' \in \mathcal{A}$.

(ii) For any node a_{nk} of T at the height $n+1$, there is a node $a_{n+1k'}$ of T at the height $n+2$ such that $a_{nk} \longrightarrow a_{n+1k'}$.

Because $A_{n+1}' = A_n' \cup A_{n+1}$, that is, for any $a_{nk} \in A_n'$, $a_{nk} \in A_{n+1}'$. By the construction of, $a_{nk} \longrightarrow a_{nk}$.

(iii) For any n, $\Box A_n' \in V_2'(T)$.

According to the property (ii), for any maximal branch ϕ in the subtree out of$t$, there is an element of A_n' is one of the node of ϕ. Then $t \models_T \Box A_n'$. $t \models_T \Diamond a_i$ is clear, for any $a_i \in A_n'$. Hence $A_n' \in V_2'(T)$.

Next we prove that $f(V_2'(T)) = \mathcal{A}$.

Let $\Box(a_0 \vee \cdots \vee a_n) \in V_2'(T)$, then $t \models_T \Box(a_0 \vee \cdots \vee a_n)$, and $t \models_T \Diamond a_i$, $i = 0, ..., n$, then there is a finite subtree T' whose leaves satisfy $a_0 \vee \cdots \vee a_n$. And there is at least a leave of T' which satisfies a_i, $i = 0, ..., n$. Assume that the height of the highest leave of T' is m. According to property (ii), the set of leaves of T' is less than A_m' w.r.t. $\preccurlyeq_P$, then $\{a_0, \cdots, a_n\} \preccurlyeq_P A_m'$. From $A_m' \in \mathcal{A}$, we have $\{a_0, \cdots, a_n\} \in \mathcal{A}$. On the other hand, for any $A = \{a_0, \cdots, a_n\} \in \mathcal{A}$, by the construction of ω-chain, there is an A_n such that $A \preccurlyeq_2 A_n$. From the property (3), we have $A_n \preccurlyeq_2 A_{n+1}'$, so, by the lemma 13, $A \in f(V_2'(T))$ since $\Box A_{n+1}' \in V_2'(T)$. Finally, the fact that $V_2'(T) \subseteq V_2'(T') \Leftrightarrow f(V_2'(T)) \subseteq f(V_2'(T'))$ is clear.

5 Conclusions

We give direct detailed proofs for the connection between powerdomains and logic models which can be made about nondeterministic computations. We believe there must be the other proofs. The cause that we chose this kind of proofs is that the ideals of proofs are simple but clear. In the proceeding of proofs, we prove some algebraic properties of them at the same time. Meanwhile, we take up some trick for constructing the finite branching tree, which can also be used into the other areas.

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Improvement combined metrics routing of IP-telephony

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Abstract

The introduction of modern technologies in the production process is a pledge of improving the quality and efficiency. The introduction of IP-telephony - is no exception. The purpose of this article is to analyze basic modern routing protocols, identification of deficiencies. The task - to propose ways to improve algorithms of traffic routing optimization. This article briefly described mechanism of action of static routing. More are detailed modern dynamic routing protocols. As examples are presented RIP, OSFP, IGRP, EIGRP protocols, as most implemented in modern routing devices. It was a comparative analysis, revealed the advantages and disadvantages of the algorithms given routing protocols, was shown comparative table of described protocols with basic characteristics. Based on analyzed data were revealed existing challenges of routing protocols such as lack of consideration of an unlimited number of criteria and non-obviousness of impact of priority criteria to choosing route . It was suggested the most optimal solution implementation of the algorithm routing protocol in case of IP telephony, which simultaneously takes into account any amount of criteria and allows the administrator to intuitively distribute the impact those or other criteria of channel that to choose route of traffic through the node. Was analyzed example of the work of proposed algorithm, the conclusions are made.

Keywords:

IP-telephony
packet switching
routing
multi-criteria optimization

1 Introduction

The first steps in the development of IP telephony has made an Israeli company VocalTec in 1995 [1]. The company has developed the first version of the program, which allowed to make calls via Internet, regardless of the distance between the subscribers and call duration. With the implementation of a VOIP, communication is simplified, reduces the cost and improves the quality of voice transmission. In the past generation telephone networks is used static routing, his based on general international telecommunication numbering plan. Routing takes place by a previously prepared table of numbers and directions- according to the recommendation E.164 of the Telecommunication Standardization Committee [2].

Each subscriber's number has corresponding a path throught certain ATS. This routing scheme is extremely difficult and costly to be scaled and optimized.

VOIP or IP-telephony, does not have a hard peg to routing ATS. Here applies packet switching. Here used IP-addressing with the advantages of this technology - packets can go through any available router to anywhere in the world, without taking up the entire communication channel, but with deficiencies - problems timely delivery of packages and their loss.

Companies, using their own ATS confronted with the task of optimizing traffic, because voice information is susceptible to delays and the percentage of lost packets has an important role in assessing quality of communication and subscriber satisfaction. In the event of packet loss by more than 5%, speech will become as a set of indecipherable sounds.

Packets are routed in the network according to certain algorithms. Therefore, the key point is choosing the right algorithm for routing packets, that provides the minimum of delay and loss, so the best voice quality, while not overburdening entire channel. There is not have its own protocol for VoIP, with taking into account the specifics of VoIP communication. Existing protocols have a number of advantages and disadvantages, but often one lack of overlaps all the advantages. Therefore, the task of improving the routing protocol is an urgent task in general and in particular for VOIP.

2 Formulation of the problem

The problem of today's networks with packet switching - it's optimizing the algorithm for packet routing protocols. Since IP-telephony also is technology with packet-switched, a task to select the best packet routing algorithm affects to it.

Also, IP-telephony as technology for transmitting media data in real time, in particular voice, quite sensitive to delay and packet loss, and this requires even more careful approach to the choice of routing protocol packets. Modification of existing protocols to VOIP specifics can solve this problem.

3 Theory aspect

3.1 STATIC ROUTING ALGORITHMS

Routing using static algorithms do not change with changes in topology and network status. Such algorithms will not be considered, because do not correspond to modern realities of building a corporate network [7-9].

3.2 DYNAMIC ROUTING

Adaptive algorithms require periodic measurement of channel characteristics, constant research of route topology and timely rebuilding routes for provide the most secure and timely delivery of packets.

For the analysis of the most popular protocols and for proposals to improve them, analysis was conducted and constructed a comparative table of the main characteristics of.

TABLE 1 Comparative table of routing protocols

Algorithm	RIP [3]	OSPF [4]	IGRP [5]	EIGRP [6]
Type of algorithm	distance vector	status channels of communication	distance vector	combined
Max number of routers in the network	15	65534	255	255
Load distribution	no possibility	equal distribution between channels with similar metrics	distribution by priority criteria metrics	distribution by priority criteria metrics
The number of channel characteristics in the overall route metric	one characteristic	three characteristics	combined metric	combined metric
Update routing information	dispatched entire table	transmits only changes	dispatched entire table	transmits only changes
Technical availability	open	open	only Cisco Systems	only Cisco Systems

4 Ways to improve the quality

As a basis we take the routing protocol OSFP. Change only the algorithm for calculating its metrics, through the introduction of the combined metrics, which takes into account all the characteristics of the channel, some of which must be maximized, and others - are minimized.

Multi-criteria optimization task is incorrect, since private quality criteria conflict with each other. Regularization of ill-posed problem of multi criteria optimization we perform by scalar convolution particular criteria of quality for nonlinear compromise scheme. In order to introduce the possibility administrator redirect traffic on certain criteria, we introduce weights for each private characteristic of channel – $\alpha_i \geq 0$ and $\sum_{i=1}^{n} \alpha_i = 1$. Then we get:

$$L = \sum_{i=1}^{n} \frac{\alpha_i}{1 - \frac{I_i}{I_{i\max}}}, \tag{1}$$

where (1) is weight of the edge based on the priority coefficients

$$\min L = \sum_{j=1}^{r} \sum_{i=1}^{n} \frac{\alpha_i}{1 - \frac{I_i}{I_{i\max}}}, \tag{2}$$

$$\alpha + \beta = \chi,$$

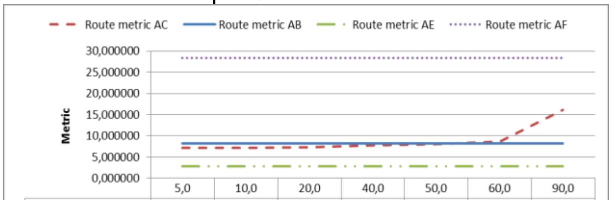
where (2) is all shortest path.

Where L - the weight of the arc, n - the number of partial criteria, r - number of ribs on the fast track, I_i - particular quality criterion of specific edges of the graph, $I_{i\max}$ maximum permissible value, which is given by technical characteristics of the channel.

5 The discussion of the results

The proposed formula for calculating metrics allows you to increase the number of defining criteria, that will optimize the routing for existing equipment and existing requirements [9-10].

The inclusion of priority criteria will enable system administrators to simply and clearly manage the redirection of traffic. Thus, when the load increases to channel, flow can be routed to another path.

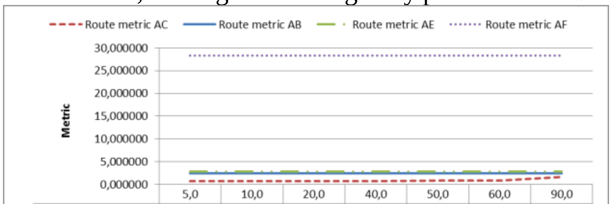


	5,0	10,0	20,0	40,0	50,0	60,0	90,0
Route metric AC	7,144312	7,202792	7,341681	7,758347	8,091681	8,591681	16,091681
Route metric AB	8,220333	8,220333	8,220333	8,220333	8,220333	8,220333	8,220333
Route metric AE	2,776667	2,776667	2,776667	2,776667	2,776667	2,776667	2,776667
Route metric AF	28,350000	28,350000	28,350000	28,350000	28,350000	28,350000	28,350000

Functioning capacity

FIGURE 1 Metrics of node A channel without taking into account the priority criteria

Route AC, with increasing load on it, acquires almost worst metrics, although it was originally preferred route.



	5,0	10,0	20,0	40,0	50,0	60,0	90,0
Route metric AC	0,714431	0,720279	0,734168	0,775835	0,809168	0,859168	1,609168
Route metric AB	2,466100	2,466100	2,466100	2,466100	2,466100	2,466100	2,466100
Route metric AE	2,776667	2,776667	2,776667	2,776667	2,776667	2,776667	2,776667
Route metric AF	28,350000	28,350000	28,350000	28,350000	28,350000	28,350000	28,350000

Functioning capacity

FIGURE 2 Metrics of node channel A with priority criteria

Due to the influence of priority criterion metric of route AC is smaller than any other metric, even when heavily loaded channel.

6 Conclusion

The analysis showed that the choice of dynamic routing protocol depends on the size and requirements of a specific corporate network, depends from the installed equipment and depends of need for a detailed configuration of routing traffic by system administrator.

The organization of VoIP in the enterprise network, EIGRP can be selected if all routers are presented by Cisco (but it's expensive equipment) and network administrator can to configure routing without the appearance of a loop (which is not trivial task). Or use the OSPF protocol, as the most universal and having a sufficient number of advantages, but with a one-criterion metric. But the best solution is to use an advanced algorithm above, which based on the OSPF protocol, using multicriteria metrics and the obvious setting via the priority criteria.

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The Development of Risk Assessment System for Accidental Oil Spill in the Northern Caspian Sea

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Abstract

This article describes the development of a multifunctional geoinformation system RANDOM (Risk Assessment of Nature Detriment due to Oil spill Migration), realizing a multiprocessor calculation of probabilistic risk models to assess the negative impact of the oil spill on the biota of the North Caspian. The urgency of the problems associated with the development of oil fields in a very vulnerable shallow part of the Caspian Sea, where a major accident could have disastrous consequences. This article describes the development process from design to implementation to testing. The system is designed on the basis of service-oriented architecture (SOA), which allows for easy, flexible integration of services, and access them via the Internet. Through the use of SOA, the system can be expanded and upgraded. In this approach, the services may be located on physically different servers. Described in detail the process of parallel processing of large data set, shows the comparative tests on performance calculations. Tests have shown the benefit of using a supercomputer, it enables us to obtain a risk assessment for an adequate time. This system is designed for professionals in the field of ecology and mathematical modeling and subsoil oil fields on the continental shelf of the seas and oceans. RANDOM system as the final result of the decision of risk assessment tasks includes a series of calculation modules based on the methods of probability theory, computational mathematics, hydrodynamics, oil chemistry, marine biology, mathematical modeling and geoinformatics.

Keywords:

GIS
RANDOM
Risk
Oil Spill
Caspian Sea

1 Introduction

The rapid growth of the activity of oil and gas operations in the Kazakhstan sector of the Caspian Sea in recent years increases the urgency of the environmental safety of the Caspian Sea. The uniqueness and isolation of the Caspian Sea may result in the event of a major oil accident to a large-scale ecological catastrophe of the whole region, and the damage could exceed the damage from a similar accident in the Gulf of Mexico. The most dangerous for the coastal zone of anthropogenic impact is the accidental spill of oil, particularly high risk of such accidents in areas where fish oil or its transportation is conducted. Multiple oil spill could lead to a deterioration of the environmental situation not only at the spill site but in the surrounding areas. In this type of coast and local climatic conditions determine the behavior of the oil spill and the extent of its impact on the surrounding area. Therefore, risk of oil contamination zone maps are an information system allowing operatively to determine priorities for spill response, to model and predict the process related to oil spills, as well as to evaluate possible damage on the coast as a result of oil spills. Emergency oil spill in the North Caspian Sea can cause catastrophic damage to flora and fauna of the sea. As a result, the intensification of oil operations on the shelf of the Northern Caspian Sea brings to the task of risk assessment plan for the defeat of biota with accidental oil spills [1-4].

To automate the process of calculating the subsequent spills and environmental risk assessment was designed and implemented a 4-tier service-oriented computer-aided calculation of risk mapping with GIS elements. Simulation of a large number of oil spills with different parameters is performed using a high-performance cluster. The developed system is used as a management tool for resource assessment, oil spill response, planning and damage assessment.

2 Systems development

2.1 ENGINEERING DESIGN

The main functions of the system RANDOM are interaction procedures with a risk assessment system and produce presentable results in a spatial and temporal map for the following services:

1. Service Meteo - provides the user with the processed data ECMWF weather forecast maps as a predictive meteorological elements in vector format for 120 hours.
2. Service Hydro - allows to perform the calculation of the forecast of the Caspian marine currents, formed under the influence of wind, temperature and other weather forecast data for 120 hours.

3. Service Oil Migration – allows to calculate the forecast for the next 120 hours spread detected oil spill taking into account the physico-chemical transformations of oil.
4. Service Risk Oil - provides the possibility of building an oil pollution risk maps of the sea above the maximum permissible concentration (MPC) for the given parameters of the accident (place, length, power, oil properties, and others.)
5. Service Risk Biota - provides an opportunity to build risk maps damage to flora and fauna of the sea with oil spills.

On an abstract level logical view of the system architecture can be viewed as a set of interacting components, grouped into layers. Design involves charting series. Among them occupies an important place component diagram (Figure 1). As physical components can serve files, libraries, modules, executables, packages, etc. The components are linked through dependencies, when connected to the required interface of one component with another available component interface. Thus is illustrated the relationship of client-source between all pairs of components.

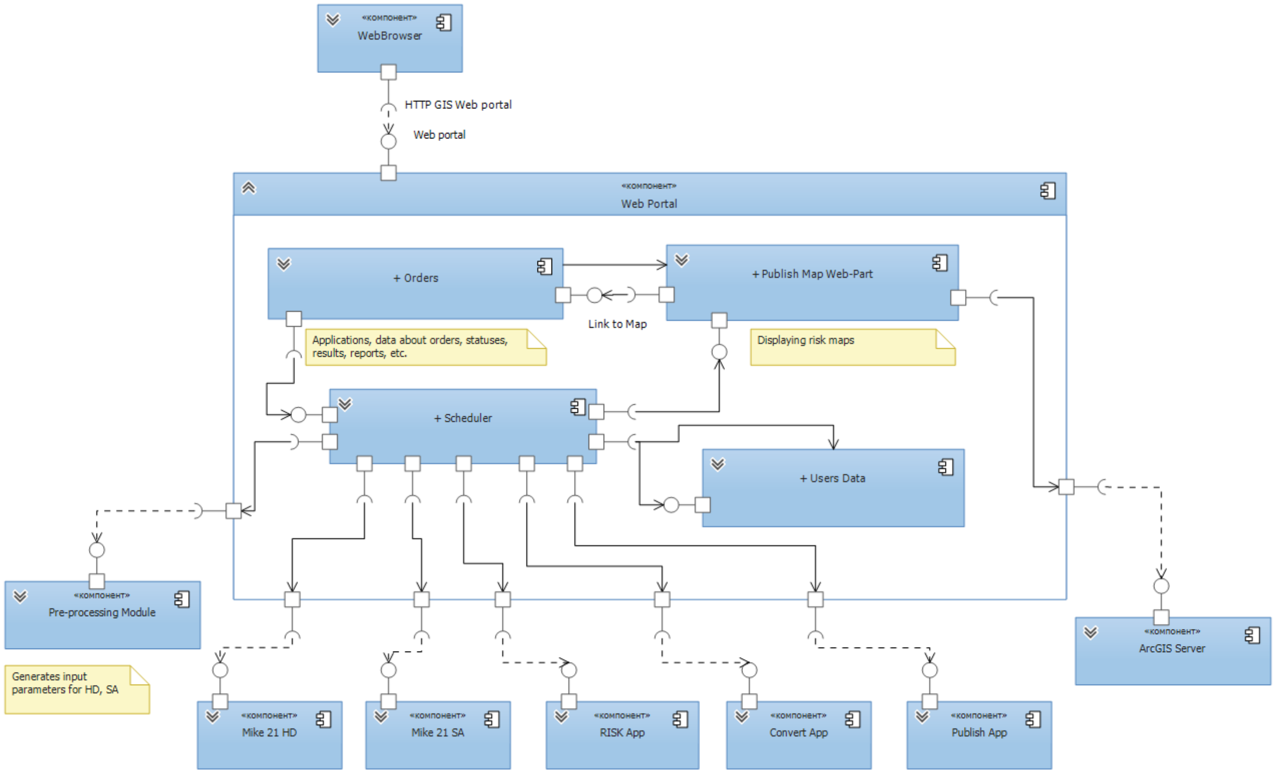

FIGURE 1 Component diagram

Web Portal component is a web server that will be deployed RANDOM system developed on the Microsoft ASP.NET platform. Users access to the system through a web browser. When accessing to the system, the user will need to authenticate to the web server. The web server is responsible for processing all user queries.

Orders component provides storage and the structural representation of users' requests.

Component Publish Map Web-Part provides display bands on the map, also provides tools for processing geospatial data.

Component Users Data storage provides risk maps and accompanying reports.

Scheduler component is responsible for the order of startup applications for payment. Produces the coordination of the calculations, depending on the type of service selected, parameter passing, and creating subtasks.

ArcGIS Server component provides storage cards as a service to display in the Publish Map Web-Part.

Pre-processing Module Component allows the generation of configuration files required for the calculation of wind, hydrodynamics and oil spill.

Component Mike 21 HD [5] provides the calculation of hydrodynamics.

Component Mike 21 SA [6] calculates an oil spill.

RiskApp component calculates the probability risk of oil pollution of the sea and the destruction of biota in the Northern Caspian.

ConvertApp component converts the digital data received from Mike 21 and RiskApp classified in the respecttive colors of the map layers.

PublishApp component is responsible for the automated collection of converted data in the map service, and further publication to ArcGIS Server by using ArcMap services.

At the final stage of design to integrate RANDOM with applications MIKE 21 HD, MIKE 21 OS, Risk App, Convert App, Publish App, were designed web services Preprocessing, Proxy HD, Proxy SA, HPCRiskModel, MapConverter and Task Scheduler.

1. Task Scheduler - an application that is responsible for the procedure call web services depending on the requested service and control and monitoring of running tasks.
2. Web Service Preprocessing, provides a configuretion file for the module and Proxy SA Proxy HD

(calculate spill and hydrodynamics).

3. On the Web Service Proxy HD provides interaction with the scheduler functional part of the automated calculation of hydrodynamics at the predetermined period. The functional part is implemented using MIKE 21 HD applications.
4. On the Web Service Proxy SA provides interaction with the scheduler functional part of the automated calculation of the spill in the given period. The functional part of the application is implemented MIKE 21 SA.
5. The Web service HPCRiskModel provides interaction scheduler module for calculating the risk of marine pollution and the risk of biota. The functional part of the application module is implemented Risk App.
6. The Web service MapConverter provides interaction scheduler with automated conversion of files obtained from the risk unit in vector layers and publishing them to ArcGIS Server. The functional part of the converter implemented application Convert App, and the publication module using the Publish App application [7].

2.2 THE ARCHITECTURE

Selecting a Service Oriented Architecture (SOA) is connected with the fact that the technology in the construction of corporate automation and information systems specifically designed for the integration of differently-platform applications that provide business processes as required in connection with inclusion in the set of independent software.

In this section detailed structure of the system in the form RANDOM 4-level service-oriented architecture based on the W3C Web Services standard. The choice of this approach is based on the need to integrate differently-platform applications, as well as the need for their reuse. The modular approach to software development provides for the expansion of software processes into separate services, where each service has the functionality. This achieves flexibility.

Figure 2 is a service-oriented system architecture. This figure shows the 4-level system: the client, the interface, the level of applications and the level of data storage.

Client layer provides access to the system. Accessing the system can execute both from PC and mobile devices.

Presentation layer consists of a web server on which the platform is Microsoft SharePoint Server 2013 is deployed for the demo version of the portal and portal RANDOM working on Microsoft ASP.NET, published on the Web server IIS. Selecting the Microsoft ASP.NET platform was due to the fact that it provides tools to automate business processes and supports the principle of service-oriented architecture.

The application layer is a set of web services that represent management services over the software installed on the computer cluster and performing calculations and spill risk analysis. This level is the functional core of the system. At this level will be used computing cluster, which provides fast parallel processing of data it. In computing cluster installed the following software: Mike 21 SA, Mike 21 HD, Risk Biota, Meteo. This level closed to the user and the interaction with the system and management of software packages is done via web services: Task Controller, Preprocessing, HPCRiskModel and MapConverter.

Storage layer comprises a database for storing information [8].

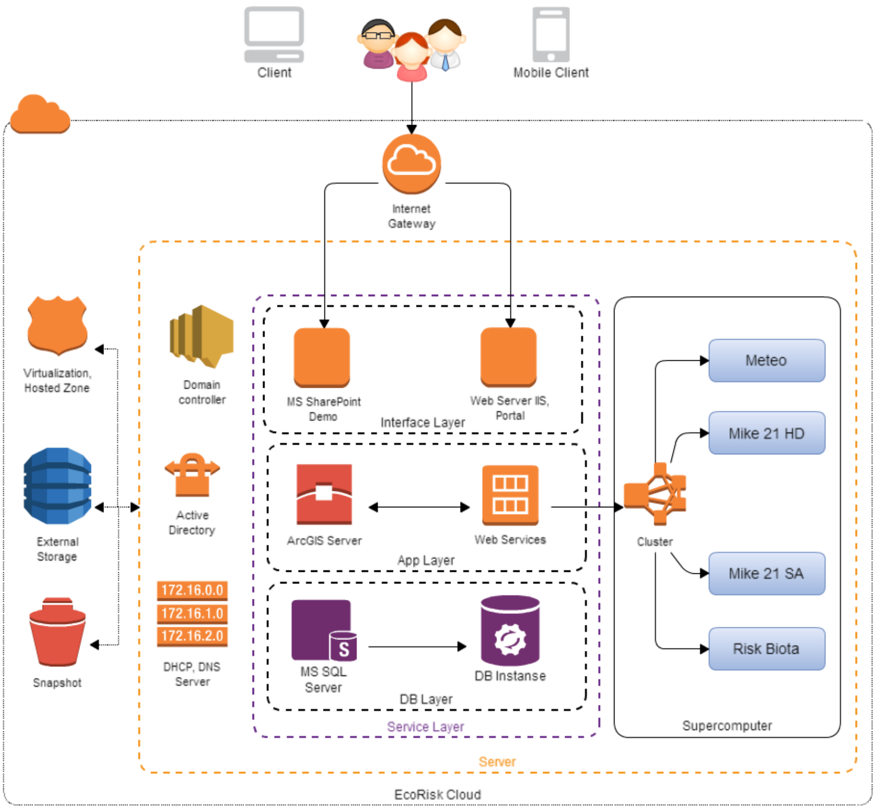

FIGURE 2 General scheme of the system architecture random

2.3 THE SCHEME CALCULATIONS

The system RANDOM implements two computational branches: one is related to the generation of short-term forecast, the second - with the statistics and risk. The first branch is implemented through a series of phases: forecast, marine forecast hydrodynamics and marine pollution forecast with the oil spill. Those the latest forecast is an eventual result of the first leg, so it runs at the same time testing the calculations of all kinds of short-term forecast based on data from the European Centre for Medium Range Weather Forecasts (ECMWF). The nett result of the settlement of the second branch are risk maps, using the statistics in the form of wind fields, pressure, water temperature and air for 36 years from the database ERA Interim (Figure 3).

To implement the risk measurement system required the development of software for automated calculation of fluid flow and the oil spill. Thus, the system allowed the maximum eliminate the human factor for the duration of the complete cycle of calculations. Before calculations of hydrodynamics and oil spill is required to produce the preliminary operations on the input data of the wind. For this purpose the package pre- and post- processing programs, which interpolates each wind forecast file to calculate the Caspian hydrodynamics. Also, the subsequent processing and saving the file to calculate the wind oil spill. For these operations, spent considerable time for a set of statistics are needed repeatedly calculated oil spills, so the task parallelization and automation of these processes.

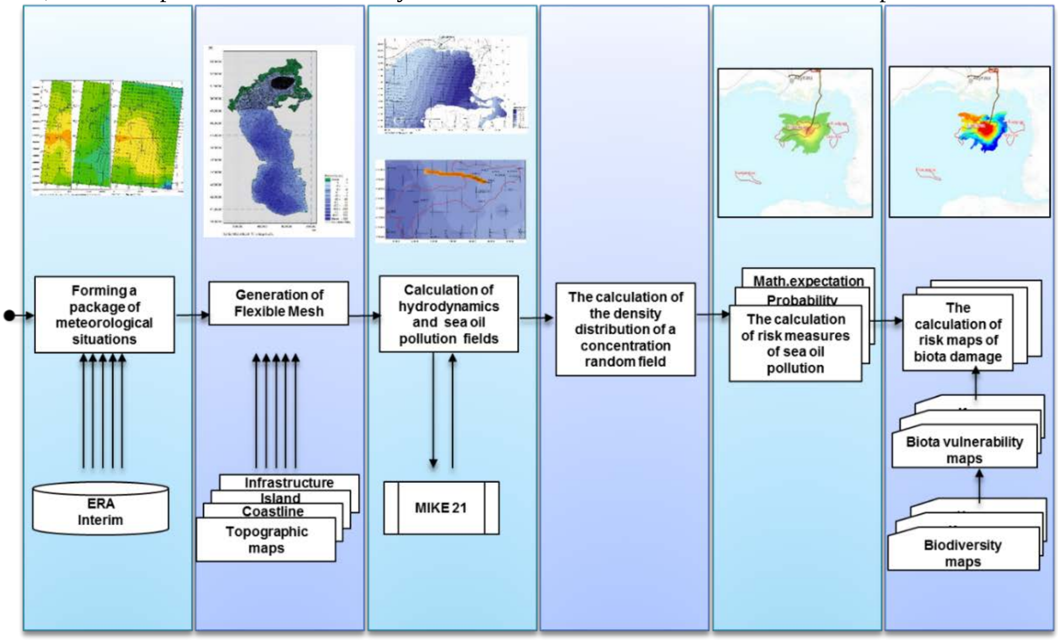

FIGURE 3 Scheme of the calculation of risk maps

3 Computation

3.1 SINGLE COMPUTATION

Service "Prediction of Oil Spill" is a service for the calculation of the spread of the oil spill, working in real time and provides the user on the sea surface maps the spread of the oil slick from the sources specified by the user, taking into account the physico-chemical properties of oil entered them. The calculations are carried out on the model MIKE 21 Oil Spill, which takes into account the basic processes of transfer and physical-chemical transformations of oil (emulsification, precipitation, evaporation, dispersion, dissolution, biodegradation, etc.). Results are available in vector format and would be used in planning for oil spill response, placing booms, protection of coastal infrastructure and others.

For the development of the service modeling the spread of the oil spill following the procedures have been implemented:

1) Pre-processing of the necessary meteorological data;
2) Calculation of the sea hydrodynamics model MIKE-21 HD for the selected date;
3) The calculation of the spread of oil pollution on the model MIKE-21 OS;
4) Post-processing results and publication RANDOM system;

Figure 4 illustrates an example of imaging oil spill modeling results RANDOM system. There were performed more than 40 runs with bug fixes.

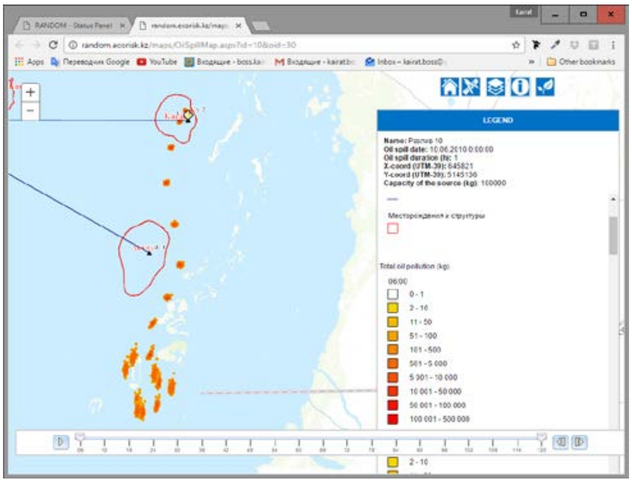

FIGURE 4 Vizualization of an oil spill simulation results on the field in Kayran 10.06.2010 RANDOM system

3.2 PARALLEL COMPUTATION

This service is carried out construction of sea oil pollution risk maps. Map zoned on the degree of probability of oil pollution. The user can set the parameters of the accident (the coordinates of the source, the accident time, power spill, oil properties) or choose from the attached background information.

Technology risk mapping of oil contamination includes the following steps:

1) Pre-treatment package of historical meteorological data, including the user selected each day of the month for the period from 1979 to the present;
2) Sea hydrodynamic simulation model for the MIKE-21 HD for each day of the selected month;
3) The calculation of the spread of oil pollution on the model MIKE-21 OS for each day of the selected month;
4) The calculation of the risk of oil pollution;
5) Post-processing results and publication RANDOM system;

The results are an execute in the form of a card containing a legend and the visualization of the scale necessary signatures. Then, map the results with the use of ArcGIS Server is published in RANDOM system (see Figure 5).

This service provides mapping risky damage to marine biological communities in the propagation of oil spills. As part of the service performed the most complex calculations. This takes into account the probability of sea pollution in the vicinity of the accident, and the sensitivity of the population living there to this contamination and especially seasonal migration of species.

Technology risk mapping destruction of marine biota includes the following steps:

1. Pre-treatment package of historical meteorological data, including the user selected each day of the month for the period from 1979 till the present;
2. Simulation of the sea hydrodynamics on the model MIKE-21 HD for each day of the selected month;
3. There is a growing spread of oil pollution on the model MIKE-21 OS for each day of the selected month;
4. Building biodiversity maps and sensitivity of biota [9];
5. The calculation of risk destruction of biota;
6. Post-processing results and publication RANDOM system;

Figure 6 illustrates the results of risk mapping destruction of biota, rendered in RANDOM system.

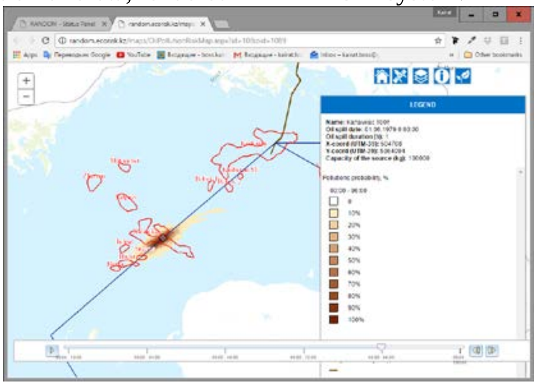

FIGURE 5 Visualization an oil marine pollution risk maps at a point in time 96 hours

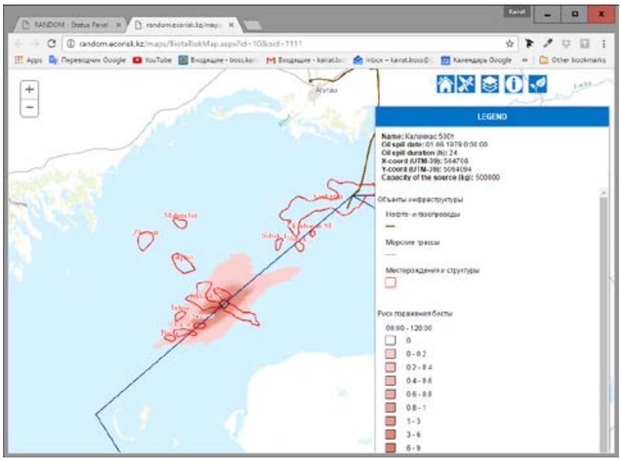

FIGURE 6 Risk map destruction of biota in the accident from the source to Kalamkas-sea after 240 hours

3.3 COMPARISON

To calculate the risk maps of marine pollution and the destruction of biota is necessary to calculate crowd an oil spills to obtain representative placement of the sample at various meteorological situation. It was implemented an automated system for multiple calculations of hydrodynamics and oil spill without the intermediate participation specialist. Below is a comparative analysis of travel time for the calculation of risk maps using a supercomputer and without it. In Figure 7 we can see that for the calculation of the oil spill took about 7 minutes considering the use of Fujitsu BX920S1 calculation server with 2 Intel Xeon 5550 processors and 16 gigabytes of RAM.

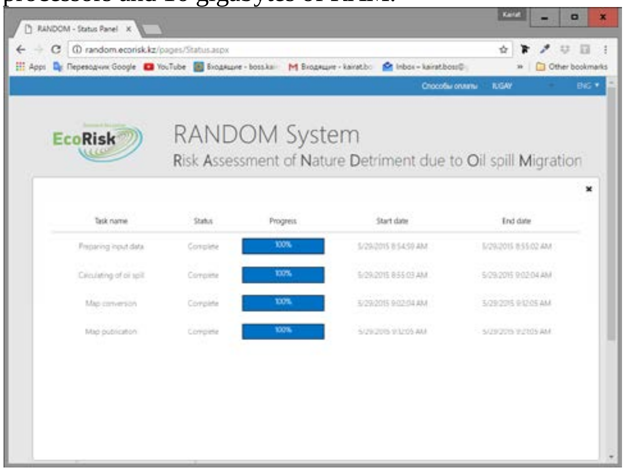

FIGURE 7 Time expended to calculate in a single case of oil spill

To obtain the necessary sample meteorological situation various meteorological parameters were downloaded for the period from 1979 to 2014. For the calculation of risk maps for the selected month is necessary to calculate the oil spill for each day of the month for 36 years. Thus it is necessary to count 36 years x 30 days = 1080 different cases of oil spill. Total estimated amount required to obtain the result T = 1080 x 7 minutes = 7560 minutes = 126 hours = 5 days 6 hours. So we decided to use the power of a supercomputer and parallelized calculation 4 compute node. How we see in Figure 8 for the calculation of the oil spill took 27 hours 6 min or 1686 min. It is 4.48 times faster than the calculation would be on the same node (see Figure 9). You could use even more components, but we are limited by the terms of the license software MIKE 21, which enables the parallel run only on 32 cores.

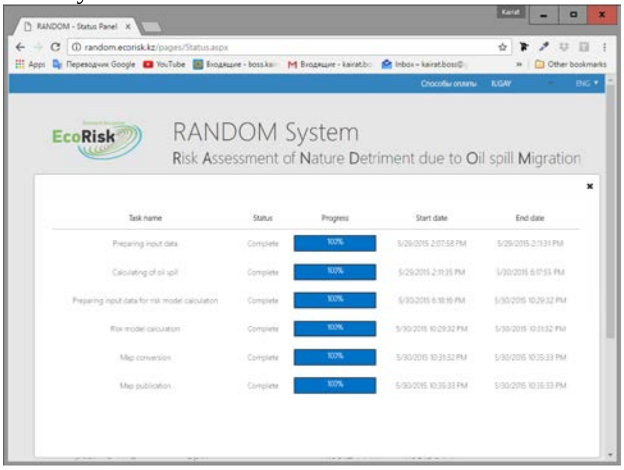

FIGURE 8 Time spent for calculation of marine pollution risk maps

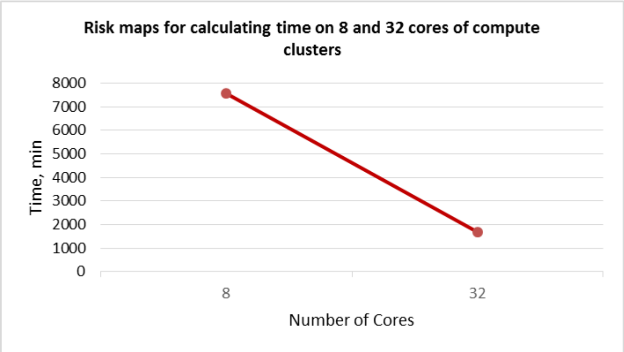

FIGURE 9 Schedule time spent before and after parallelization

To calculate the required oil spill previously calculated hydrodynamics file the time of receipt of which is how we see the figure 10 of about 25 minutes. Hydrodynamics is used to calculate the oil spill, so we decided to pre-calculate it for the entire set meteosituation that we have. Total turn out to count all the hydrodynamics need T = 25 min x 365 days x 36 years = 328,500 minutes = 5475 hours = 228 days for 3 hours. = More than 7 months of continuous calculation. Using 4 node we considered it for 2 months. In all it took 14.5 Tbyte of disk storage.

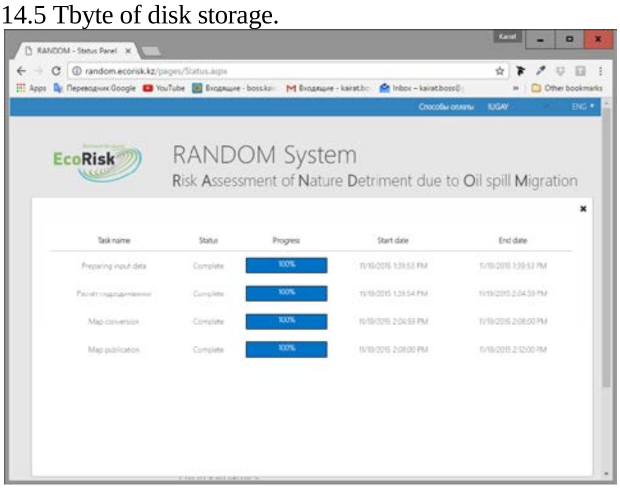

FIGURE 10 Time expended to calculate the hydrodynamic

4 Conclusions

We have presented in the paper the service-oriented GIS system RANDOM for risk mapping of oil spills integrated with high performance cluster. The design and integration methodology of the system are based on service-oriented architecture that allows provide an easy, flexible integration of any service into any desktop or mobile client. We have designed and build 4-tier SOA on the basis of W3C Web service standard. The process of multiple modeling of oil spills has been automated on the high performance cluster. The Risk model for risk assessment is implemented as an application. We have developed the portal with user-friendly interface and sequence of user order processing. The following results were obtained within the framework of this work:

– a risk model of destruction of biota at the man-made accidents. A mathematical model for the description of the biota in the form of map algorithms biodiversity and vulnerability and performed it for the implementation of the Northern Caspian. The developed method has been used for risk analysis in accidental oil spills in the Caspian Sea oil fields.
– a service-oriented web portal to evaluate the environmental risk of biota at the oil spill in the North Caspian Sea. The system is automated and integrated with high-performance computing cluster for the calculation of high-tech research problems. Fully implemented and integrated into all blocks RANDOM project a single system, including the portal, interfaces, geodatabase design models, and others.
– as a result on the basis of the developed technique based series of maps showing the risk of exposure to the accident at the fauna of the North Caspian Sea. Skill testing RANDOM logical structure of the system for two main branches, one of which is associated with the development of short-term forecast, the second - with the statistics and risk. At the same time identified and corrected software errors and interface errors.
– carry out a test the test model of the transport and transformation of the oil spill on the sea MIKE 21 OS, which is the main design module for modeling of oil marine pollution forecasts and risks.
– a comparative analysis of the runtime calculations using computational cluster, which confirms the need to use for this type of research.

Acknowledgment

This work was supported by the Technology Commercialization Project (Senior Scientist Group Grant «Development of technology of risk mapping defeat of marine biota at emergency oil spills on the Caspian shelf» 2014-2015). Also, work was supported by the Grant funding of projects (Grant No. 1049/GF4 “Modeling of contaminant transport in Ili-Balkhash basin by using supercomputer” 2015-2017 and Grant No. 1029/GF4 “The simulation of oil displacement process from core samples to analyze the efficiency of oil recovery by using high-performance computing” 2015-2017).

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AUTHORS' INDEX

MATHEMATICAL AND COMPUTER MODELLING

The detection system for greenhouse crop disease degree based on Android platform

Penghui Zheng, Youwen Tian, Ruiyao Shi

*Computer Modelling & New Technologies 2016 **20**(3) 7-11*

A detecting way based on Android platform was proposed in order to detect greenhouse crop disease degree in real time. This way employed the camera in mobile phone to acquire crop disease leaf image in the greenhouse. Firstly, the detection system was built by the Eclipse based on the Android development environment. The iterative threshold segmentation algorithm was used to separate the crop disease leaf area from background. And the fuzzy C-means cluster algorithm was adopted to extract the disease spots. After analyzed the impact of different fuzzy weighted index m value, the value of m was selected 2 for the disease spots segmentation. After that the crop disease degree was determined based on the relevant standards and the total disease index of greenhouse was got based on disease index calculation standards. Finally, the calculating data could upload to the network server and was used management cloud achieved synchronous computer terminal query. The experimental results show that the detecting way could non-destructed measure the disease index of leaf diseases with non-destructive and exact in greenhouse.

Keywords: Android greenhouse crop disease index, cloud management real time non-destructive

Relationships among convergence concepts of uncertain sequences

Cuilian You, Lijuan Yan

*Computer Modelling & New Technologies 2016 **20**(3) 12-16*

Uncertain sequence is a sequence of uncertain variables indexed by integers. In this paper, a new kind of sequence convergence that complete convergence was presented. Then, the relationships among complete convergence, convergence in p-distance, convergence in measure, convergence in distribution, convergence uniformly almost surely and convergence almost surely were investigated.

Keywords: uncertain measure, uncertain variable, expectation, convergence

Supervised images classification using metaheuristics

Amir Mokhtar Hannane, Hadria Fizazi

*Computer Modelling & New Technologies 2016 **20**(3) 17-23*

Image classification is a fundamental task in image processing because it is a crucial step toward image understanding. This paper exploits metaheuristics (Ant Colony Optimization and Electromagnetic Metaheuristic) to tackle the problem of supervised satellite image classification. Earlier studies have been used the Intra-Class Variance (ICV) for images classification but this function has a limits to solve classification problem. This study presents the introduction of the Davies-Bouldin Index (DBI) to the supervised images classification. This index is used in two stages: training step and classification step. In training step this index serve as criteria for controlling iterations. In the classification step this index help to classify each pixel in the image to their appropriate class using the class centers found during the training stage. The experimental results show that the introduction of the Davies-Boulin index is very effective for supervised images classification and help the community of researches to improve the classification accuracy of remotely sensed data. The utility of metaheuristics is also demonstrated for satellite image of Oran city.

Keywords: image classification metaheuristics, Davies-Bouldin index, ant colony optimization, electromagnetic metaheuristic

Powerdomains and modality, revisited with detailed proofs

Xiang Zhou

*Computer Modelling & New Technologies 2016 **20**(3) 24-31*

We give direct detailed proofs for the connection between powerdomains and logic models which can be made about nondeterministic computations. In the proceeding of proofs, we prove some algebraic properties of them at the same time. Meanwhile, we take up some trick for constructing the finite branching tree, which can also be used into the other areas.

Keywords: Power domain, Nondeterministic, computation, Algebraic properties

Improvement combined metrics routing of IP-telephony

N Vihrov, V Nikiforov, J Polugina, S Sokolov, A Nyrkov, V Gaskarov, A Zhilenkov

*Computer Modelling & New Technologies 2016 **20**(3) 32-34*

The introduction of modern technologies in the production process is a pledge of improving the quality and efficiency. The introduction of IP-telephony - is no exception. The purpose of this article is to analyze basic modern routing protocols, identification of deficiencies. The task - to propose ways to improve algorithms of traffic routing optimization. This article briefly described mechanism of action of static routing. More are detailed modern dynamic routing protocols. As examples are presented RIP, OSFP, IGRP, EIGRP protocols, as most implemented in modern routing devices. It was a comparative analysis, revealed the advantages and disadvantages of the algorithms given routing protocols, was shown comparative table of described protocols with basic characteristics. Based on analyzed data were revealed existing challenges of routing protocols such as lack of consideration of an unlimited number of criteria and non-obviousness of impact of priority criteria to choosing route. It was suggested the most optimal solution implementation of the algorithm routing protocol in case of IP telephony, which simultaneously takes into account any amount of criteria and allows the administrator to intuitively distribute the impact those or other criteria of channel that to choose route of traffic through the node. Was analyzed example of the work of proposed algorithm, the conclusions are made.

Keywords: IP-telephony, packet switching, routing, multi-criteria optimization

The Development of Risk Assessment System for Accidental Oil Spill in the Northern Caspian Sea

Kairat Bostanbekov, Daniyar Nurseitov

Computer Modelling & New Technologies 2016 ***20****(3) 35-41*

This article describes the development of a multifunctional geoinformation system RANDOM (Risk Assessment of Nature Detriment due to Oil spill Migration), realizing a multiprocessor calculation of probabilistic risk models to assess the negative impact of the oil spill on the biota of the North Caspian. The urgency of the problems associated with the development of oil fields in a very vulnerable shallow part of the Caspian Sea, where a major accident could have disastrous consequences. This article describes the development process from design to implementation to testing. The system is designed on the basis of service-oriented architecture (SOA), which allows for easy, flexible integration of services, and access them via the Internet. Through the use of SOA, the system can be expanded and upgraded. In this approach, the services may be located on physically different servers. Described in detail the process of parallel processing of large data set, shows the comparative tests on performance calculations. Tests have shown the benefit of using a supercomputer, it enables us to obtain a risk assessment for an adequate time. This system is designed for professionals in the field of ecology and mathematical modelling and subsoil oil fields on the continental shelf of the seas and oceans. RANDOM system as the final result of the decision of risk assessment tasks includes a series of calculation modules based on the methods of probability theory, computational mathematics, hydrodynamics, oil chemistry, marine biology, mathematical modelling and geoinformatics.

Keywords: GIS, RANDOM, Risk, Oil Spill, Caspian Sea

