A computer aided kineto-static analysis method for spatial robot mechanism based on vector bond graph

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Abstract

In order to increase the reliability and efficiency of the kineto-static analysis of complex robot systems, the corresponding vector bond graph procedure is proposed. From the algebraic relations of input and output vectors in the basic fields, junction structure and Euler-junction structure of system vector bond graph model, the unified formulae of driving moment (or force) and constraint forces at joints are derived, which are easily derived on a computer in a complete form. For the algebraic difficulties brought by differential causality and nonlinear junction structure in system automatic modelling and kineto-static analysis, the effective bond graph augment method is proposed. Based on the kinematic constraint relations, the vector bond graph model of the spatial robot mechanism with five degrees of freedom can be made. As a result, the automatic modelling and kineto-static analysis of complex robot system on a computer is realized, its validity is illustrated.

Keywords: robot mechanism, kineto-static analysis, vector bond graph, causality, joint constrain

1 Introduction

The kineto-static analysis is very important for the control, static and dynamic strength check of robot system. For complex robot systems, e.g. the spatial robot systems containing different constraint joints, determining driving moment (or force) and the constraint forces at joints is a very tedious and error-prone task on account of the nonlinearities and couplings involved. The Newton-Euler technique and Lagrange technique are two of the well known methods used for the dynamic analysis of a robot system [1, 2]. These techniques however, are only suitable for a single energy domain systems, e.g. mechanical systems, and cannot be used to tackle systems that simultaneously include various physical domains in a unified manner.

The bond graph technique developed since the 1960's has potential applications in analysing such complex systems and has been used successfully in many areas [3, 4]. It is a pictorial representation of the dynamics of the system and clearly depicts the interaction between elements, it can also model multi-energy domains, for example, the actuator systems, which may be electrical, electro-magnetic, pneumatic, hydraulic or mechanical. But for spatial multibody systems such as spatial robot mechanism with different constraint joints, the kinematic and geometric constraints between bodies result in differential causality loop, and the nonlinear velocity relationship between the mass centre and an arbitrary point on a body leads to the nonlinear junction structure. The bond graph procedures mentioned above were found to be very difficult algebraically in automatic modelling and kineto-static analysis of system on a computer. To solve this problem, the Lagrange multiplier approach and Karnopp-Margolis approach can be employed to model multibody systems based on scalar bond graph concept [5, 6].

For spatial multibody systems, the scalar bond graph technique is found to be complex and difficult. To address this problem, the vector bond graph techniques were proposed [7-9]. In vector bond graphs, single power bonds are replaced by multi-power bonds, this makes it posses more concise presentation manner and be more suitable for modelling spatial multibody systems. But some problems should be studied further, such as modelling spatial robot mechanism with different constraint joints by vector bond graphs, augmenting the vector bond model to avoid differential causality, developing the generic algorithm for automatic kineto-static analysis of spatial robot mechanism. To solve above problems, a more efficient and practical computer aided kineto-static analysis procedure for spatial robot mechanism based on vector bond graph is proposed here.

2 The vector bond graph model of spatial cylindrical joint

The diagram of spatial cylindrical joint is shown in Figure 1. This joint allows only a straight displacement and one direction rotation between its joined body B_α and B_β, fixing the remaining two translational and two rotational degrees of freedom. Therefore, only two generalized coordinates are free to change. Joint point P and Q are fixed on rigid body B_α and B_β respectively, vector h_α is used to describe the relative motion of the two rigid bodies,

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$h_\alpha = r_\alpha^P - r_\beta^Q$. Where r_α^P and r_β^Q represent the position vector of joint point P and Q in global coordinates respectively, $r_\alpha^P = \begin{bmatrix} x_\alpha^P & y_\alpha^P & z_\alpha^P \end{bmatrix}^{\mathrm{T}}$, $r_\beta^Q = \begin{bmatrix} x_\beta^Q & y_\beta^Q & z_\beta^Q \end{bmatrix}^{\mathrm{T}}$. d_β^1 and d_β^2 are two unit vectors fixed on rigid body B_β, which are all orthogonal to slide axis, and orthogonal to each other. d_α is the unit vector fixed on rigid body B_α along slide axis, $d_\alpha^{'}$, $d_\beta^{'1}$ and $d_\beta^{'2}$ are the corresponding vectors in body frame. From the kinematic constraint condition of spatial cylindrical joint [1], we have

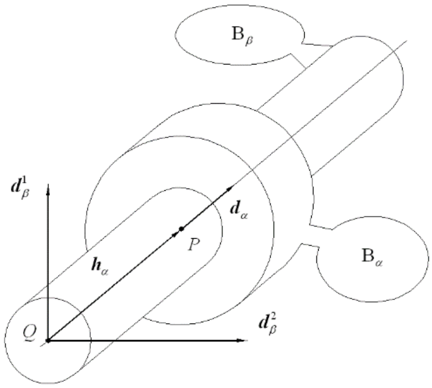

FIGURE 1 The diagram of spatial cylindrical joint

$$\underline{\Phi}^{(r_2)}(d_\alpha, d_\beta^1, d_\beta^2) = \begin{bmatrix} d_\beta^{1T} d_\alpha \\ d_\beta^{2T} d_\alpha \end{bmatrix} = \begin{bmatrix} d_\beta^{'1T} A^{\beta T} A^\alpha d_\alpha^{'} \\ d_\beta^{'2T} A^{\beta T} A^\alpha d_\alpha^{'} \end{bmatrix} = 0, \qquad (1)$$

$$\underline{\Phi}^{(d_2)}(h, d_\beta^1, d_\beta^2) = \begin{bmatrix} d_\beta^{1T} h_\alpha \\ d_\beta^{2T} h_\alpha \end{bmatrix} = \begin{bmatrix} d_\beta^{'1T} A^{\beta T} \left(r_\beta^Q - r_\alpha^P\right) \\ d_\beta^{'2T} A^{\beta T} \left(r_\beta^Q - r_\alpha^P\right) \end{bmatrix} = 0, \quad (2)$$

where A^α and A^β are the direction cosine matrices of body B_α and body B_β respectively.

The corresponding velocity and angular velocity constraint equations can be written as:

$$\underline{\Phi}^{(r_2)}(d_\alpha, d_\beta^1, d_\beta^2) = \begin{bmatrix} d_\beta^{2T} \left(\omega_\alpha - \omega_\beta\right) \\ d_\beta^{1T} \left(\omega_\alpha - \omega_\beta\right) \end{bmatrix} = \begin{bmatrix} d_\beta^{'2T} A^{\beta T} \left(\omega_\alpha - \omega_\beta\right) \\ d_\beta^{'1T} A^{\beta T} \left(\omega_\alpha - \omega_\beta\right) \end{bmatrix} = 0, \qquad (3)$$

$$\underline{\Phi}^{(d_2)}(h, d_\beta^1, d_\beta^2) = \begin{bmatrix} d_\beta^{'2T} A^{\beta T} \left(\omega_\alpha - \omega_\beta\right) \\ d_\beta^{'1T} A^{\beta T} \left(\omega_\alpha - \omega_\beta\right) \end{bmatrix} = 0, \qquad (4)$$

where ω_α and ω_β represent the angular velocity vectors of the rigid body B_α and B_β determined in global coordinates, ω_α^b and ω_β^b are the corresponding angular velocity vectors of the rigid body determined in body frame.

$$\tilde{d}_\beta^i = \begin{bmatrix} 0 & -d_{\beta z}^i & d_{\beta y}^i \\ d_{\beta z}^i & 0 & -d_{\beta x}^i \\ -d_{\beta y}^i & d_{\beta x}^i & 0 \end{bmatrix}, (i=1,2).$$

The velocity and angular velocity constraint equations shown as Equations (3) and (4) can be presented by vector bond model shown in Figure 2.

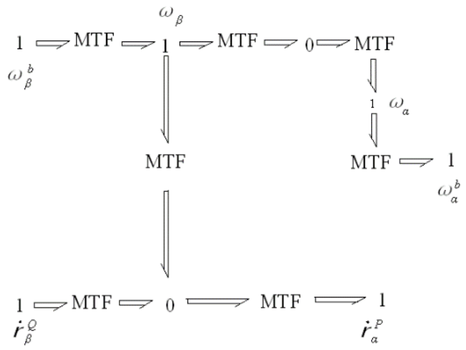

FIGURE 2 The vector bond graph model of spatial cylindrical joint

3 The unified formulae of driving moment and constraint forces for spatial robot systems

The basic fields and junction structure of system bond graph is shown in Figure 3 [3], where Euler-junction structure (EJS) [9, 10] is added. X_{i_1} represents energy vector variable of independent storage energy field corresponding to independent motion, X_{i_2} represents energy vector variable of independent storage energy field corresponding to dependent motion, Z_{i_1} and Z_{i_2} are the corresponding coenergy vector variables. D_{in} and D_{out} represent input and output vector variables in resistive field, U and V represent input and output vector variables of source field respectively, $U = \begin{bmatrix} U_1 & U_2 & U_3 \end{bmatrix}^{\mathrm{T}}$, $V = \begin{bmatrix} V_1 & V_2 & V_3 \end{bmatrix}^{\mathrm{T}}$. Where U_1 is driving moment (or force) vector, U_2 is the constraint force vector of joint and U_3 is known source vector. E_{in} and E_{out} are the input and output vector variables in Euler-junction structure(EJS).

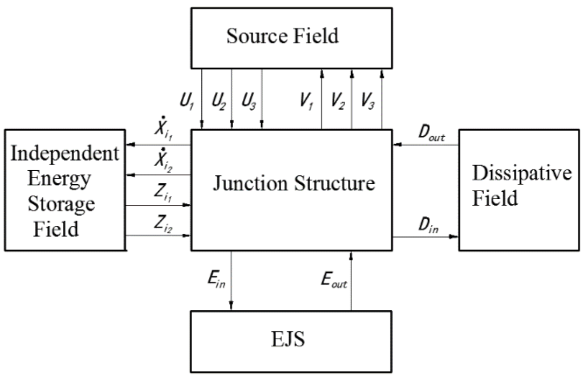

FIGURE 3 The basic field and junction structure of system

For independent energy storage field, we have:

$$Z_{i_1} = F_{i_1} X_{i_1} , \tag{5}$$

$$Z_{i_2} = F_{i_2} X_{i_2} , \tag{6}$$

where F_{i_1} and F_{i_2} are $m_1 \times m_1$ and $m_2 \times m_2$ matrices respectively.

For resistive field, we have:

$$D_{out} = RD_{in} , \tag{7}$$

where R is $L \times L$ matrix.

For Euler-junction structure (EJS), we have

$$E_{out} = R_E E_{in} , \tag{8}$$

where R_E is $L_E \times L_E$ matrix [9-11].

The corresponding junction structure equations can be written as:

$$\begin{aligned}\dot{X}_{i_1} &= J_{i_1 i_1} Z_{i_1} + J_{i_1 i_2} Z_{i_2} + J_{i_1 L} D_{out} + \\ &J_{i_1 u_1} U_1 + J_{i_1 u_2} U_2 + J_{i_1 u_3} U_3 + J_{i_1 E} E_{out} ,\end{aligned} \tag{9}$$

$$\begin{aligned}\dot{X}_{i_2} &= J_{i_2 i_1} Z_{i_1} + J_{i_2 i_2} Z_{i_2} + J_{i_2 L} D_{out} + J_{i_2 u_1} U_1 + \\ &J_{i_2 u_2} U_2 + J_{i_2 u_3} U_3 + J_{i_2 E} E_{out} ,\end{aligned} \tag{10}$$

$$\begin{aligned}D_{in} &= J_{Li_1} Z_{i_1} + J_{Li_2} Z_{i_2} + J_{LL} D_{out} + J_{Lu_1} U_1 + \\ &J_{Lu_2} U_2 + J_{Lu_3} Z_3 + J_{LE} D_{out} ,\end{aligned} \tag{11}$$

$$\begin{aligned}E_{in} &= J_{Ei_1} Z_{i_1} + J_{Ei_2} Z_{i_2} + J_{EL} D_{out} + J_{Eu_1} U_1 + \\ &J_{Eu_2} U_2 + J_{Eu_3} U_3 + J_{EE} E_{out} .\end{aligned} \tag{12}$$

From the flow summation of 0-junctions corresponding to m_2 constraint force vectors in system vector bond graph model, we have:

$$0 = J_{Ci_1} Z_{i_1} + J_{Ci_2} Z_{i_2} + J_{CL} F_{out} + J_{Cu_3} U_3 + J_{CE} E_{out} . \tag{13}$$

By the algebraic manipulation from Equations (5)-(13), the system driving moment and constraint force equations can be written as:

If $J_{CL} = 0$, $J_{CE} = 0$:

$$\left.\begin{aligned}&U_1 = S_{u_1 u_1}^{-1} (S_{u_1 i_1} X_{i_1} + S_{u_1 i_2} X_{i_2} + S_{u_1 u_3} U_3 + \\ &T_{i_1 u_1}^{\mathrm{T}} \dot{X}_{i_1} + T_{i_1 u_1}^{\mathrm{T}} T_{i_1 u_2} H_4^{-1} J_{cu_3} \dot{U}_3) \qquad (a)\\ &U_2 = (-H_4)^{-1} (H_1 X_{i_1} + H_2 X_{i_2} + H_3 U_1 + \\ &H_5 U_3 + J_{cu_3} \dot{U}_3) \qquad (b)\end{aligned}\right\} , \tag{14}$$

where:

$$A_1 = [I_2 - J_{EL} R (I_1 - J_{LL} R)^{-1} J_{LE} R_E - J_{EE} R_E]^{-1} ,$$

$$A_2 = J_{Ei_1} F_{i_1} + J_{EL} R (I_1 - J_{LL} R)^{-1} J_{Li_1} F_{i_1} ,$$

$$A_3 = J_{Ei_2} F_{i_2} + J_{EL} R (I_1 - J_{LL} R)^{-1} J_{Li_2} F_{i_2} ,$$

$$A_4 = J_{Lu_1} + J_{EL} R (I_1 - J_{LL} R)^{-1} J_{Lu_1} ,$$

$$A_5 = J_{Eu_2} + J_{EL} R (I_1 - J_{LL} R)^{-1} J_{Lu_2} ,$$

$$A_6 = J_{Eu_3} + J_{EL} R (I_1 - J_{LL} R)^{-1} J_{Lu_3} ,$$

$$B_1 = (I_1 - J_{LL} R)^{-1} (J_{Li_1} F_{i_1} + J_{LE} R_E A_1 A_2) ,$$

$$B_2 = (I_1 - J_{LL} R)^{-1} (J_{Li_2} F_{i_2} + J_{LE} R_E A_1 A_3) ,$$

$$B_3 = (I_1 - J_{LL} R)^{-1} (J_{Lu_1} + J_{LE} R_E A_1 A_4) ,$$

$$B_4 = (I_1 - J_{LL} R)^{-1} (J_{Lu_2} + J_{LE} R_E A_1 A_5) ,$$

$$B_5 = (I_1 - J_{LL} R)^{-1} (J_{Lu_3} + J_{LE} R_E A_1 A_6) ,$$

$$T_{i_1 i_1} = J_{i_1 i_1} F_{i_1} + J_{i_1 L} R B_1 + J_{i_1 E} R_E A_1 A_2 ,$$

$$T_{i_1 i_2} = J_{i_1 i_2} F_{i_2} + J_{i_1 L} R B_2 + J_{i_1 E} R_E A_1 A_3 ,$$

$$T_{i_1 u_1} = J_{i_1 L} R B_3 + J_{i_1 u_1} + J_{i_1 E} T_E A_1 A_4 ,$$

$$T_{i_1 u_2} = J_{i_1 L} R B_4 + J_{i_1 u_2} + J_{i_1 E} T_E A_1 A_5 ,$$

$$T_{i_1 u_3} = J_{i_1 L} R B_5 + J_{i_1 u_3} + J_{i_1 E} T_E A_1 A_6 ,$$

$$H_1 = \dot{J}_{Ci_1} F_{i_1} + J_{Ci_1} F_{i_1} T_{i_1 i_1} + J_{Ci_2} F_{i_2} T_{i_2 i_1} ,$$

$$H_2 = \dot{J}_{Ci_2} F_{i_2} + J_{Ci_1} F_{i_1} T_{i_1 i_2} + J_{Ci_2} F_{i_2} T_{i_2 i_2} ,$$

$$H_3 = J_{Ci_1} F_{i_1} T_{i_1 u_1} + J_{Ci_2} F_{i_2} T_{i_2 u_1} ,$$

$$H_4 = J_{Ci_1} F_{i_1} T_{i_1 u_2} + J_{Ci_2} F_{i_2} T_{i_2 u_2} ,$$

$$H_5 = \dot{J}_{Cu_3} + J_{Ci_1} F_{i_1} T_{i_1 u_3} + J_{Ci_2} F_{i_2} T_{i_2 u_3} ,$$

$$S_{u_1 u_1} = T_{i_1 u_1}^{\mathrm{T}} [T_{i_1 u_1} + T_{i_1 u_2} (-H_4)^{-1} H_3] ,$$

$$S_{u_1 i_1} = T_{i_1 u_1}^{\mathrm{T}} (T_{i_1 u_2} H_4^{-1} H_1 - T_{i_1 i_1}) ,$$

$$S_{u_1 i_2} = T_{i_1 u_1}^{\mathrm{T}} (T_{i_1 u_2} H_4^{-1} H_2 - T_{i_1 i_2}) ,$$

$$S_{u_1 u_3} = T_{i_1 u_1}^{\mathrm{T}} (T_{i_1 u_2} H_4^{-1} H_5 - T_{i_1 u_3}) ,$$

If $J_{CL} \neq 0$ or $J_{CE} \neq 0$:

$$\left.\begin{aligned}&U_1 = D_{u_1 u_1}^{-1} (D_{u_1 i_1} X_{i_1} + D_{u_1 i_2} X_{i_2} + D_{u_1 u_3} U_3 + \\ &T_{i_1 u_1}^{\mathrm{T}} \dot{X}_{i_1}) \qquad (a)\\ &U_2 = (-T_{Cu_2})^{-1} (T_{Ci_1} X_{i_1} + T_{Ci_2} X_{i_2} + T_{Cu_1} U_1 + \\ &T_{Cu_3} U_3) \qquad (b)\end{aligned}\right\} , \tag{15}$$

where:

$$T_{Ci_1} = J_{Ci_1} F_{i_1} + J_{CL} R B_1 + J_{CE} R_E A_1 A_2 ,$$

$$T_{Ci_2} = J_{Ci_2} F_{i_2} + J_{CL} R B_2 + J_{CE} T_E A_1 A_3 ,$$

$$T_{Cu_1} = J_{CL} R B_3 + J_{CE} T_E A_1 A_4 ,$$

$$T_{Cu_2} = J_{CL} R B_4 + J_{CE} R_E A_1 A_5 ,$$

$$T_{Cu_3} = J_{CL} R B_5 + J_{Cu_3} + J_{CE} R_E A_1 A_6 ,$$

$$D_{u_1 u_1} = T_{i_1 u_1}^{\mathrm{T}} [T_{i_1 u_1} + T_{i_1 u_2} (-T_{Cu_2})^{-1} T_{Cu_1}] ,$$

$$D_{u_1 i_1} = T_{i_1 u_1}^{\mathrm{T}} (T_{i_1 u_2} T_{Cu_2}^{-1} T_{Ci_1} - T_{i_1 i_1}) ,$$

$$D_{u_1 i_2} = T_{i_1 u_1}^{\mathrm{T}} (T_{i_1 u_2} T_{Cu_2}^{-1} T_{Ci_2} - T_{i_1 i_2}) ,$$

$$D_{u_1 u_3} = T_{i_1 u_1}^{\mathrm{T}} (T_{i_1 u_2} T_{Cu_2}^{-1} T_{Cu_3} - T_{i_1 u_3}) .$$

Giving the system independent moving state variable vector X_{i_1} and its derivative $\dot{X}_{i_1}$, the corresponding system driving moment (or force) vector U_1 and constraint force vector U_2 can be determined from Equations (14) or (15) directly.

4 Example System

A robot mechanism with five degrees of freedom is shown in Figure 4, global coordinates $O_0X_0Y_0Z_0$ is located at the point O_0, and the body frame $C_iX_iY_iZ_i$ (i =1,2,3,4) is located at the centre of mass. T_1, T_2 and T_3 are the driving moments, F_1 and F_2 are the driving forces along Z_1 axis and Y_2 axis respectively. The structure parameters of the robot mechanism are shown in Table 1, $c = 0.05$m, L=0.50m. The system input motion are as following, $Z_{C_1} = \cos(\pi t)$, $\theta_1 = \sin(\pi t)$, $Y_{C_2} = \cos(2\pi t)$, $\theta_2 = \sin(2\pi t)$, $\theta_3 = \sin(3\pi t)$.

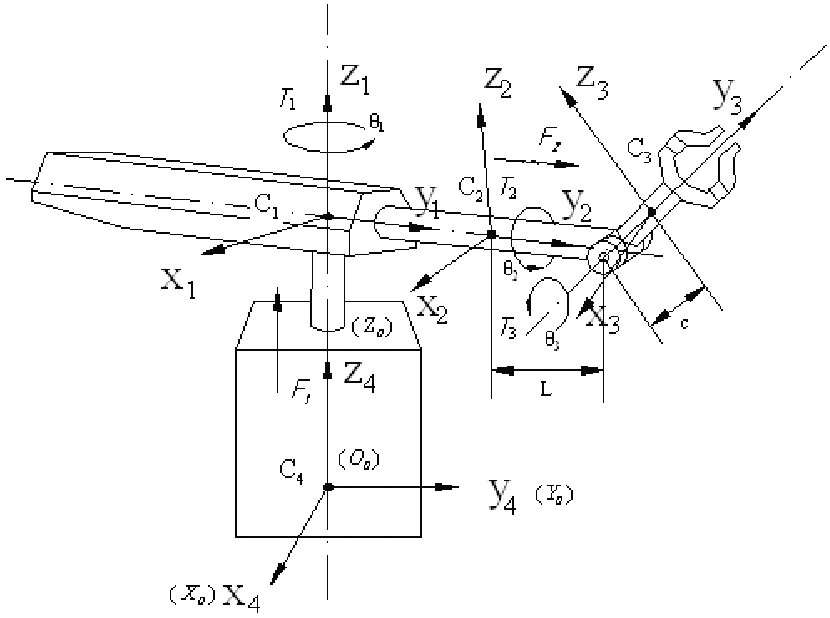

FIGURE 4 The robot system with five degrees of freedom

TABLE 1 The structural parameters of robot mechanism

Body *i*	**Mass（kg）**	**Moment of inertia（Kg·m²）** *X*	*Y*	*Z*
1	250	90	10	90
2	150	13	0.75	13
3	100	4	1	4.3

The components for this example are four rigid bodies, which are joined by two cylindrical joints and one revolute joint, shown as Figure 5. For revolute joint, the constraint limits the relative translation of the two bodies B_α and B_β along three directions, and limits the relative rotation of the two bodies B_α and B_β along two directions, leaving only one rotation degree of freedom free. From the kinematic constraint condition, its vector bond graph can be obtained [11]. By the procedures mentioned above, the vector bond graph model of cylindrical joints can be made. By assembling the vector bond graph models of a single space moving rigid body [9, 11], the revolute joint, and the cylindrical joints, the overall robot system vector bond graph model can be obtained and shown as Figure 6, where part I represents the cylindrical joint between body 1 and body 2, and part II is the revolute joint between body 2 and body 3.

Here, the constraint force vectors of joints can be considered as unknown source vectors, such as Se_3, Se_7, Se_{12} , Se_{14} in Figure 6 and added to the corresponding 0-junctions to eliminate differential causality. As a result, all differential causalities in this system vector bond graph can be eliminated, thus the procedure presented here can be used directly.

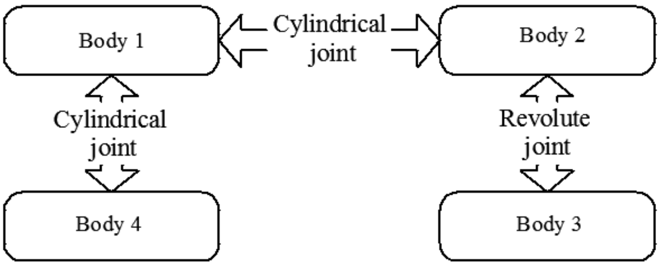

FIGURE 5 The jointing structure diagram of robot system

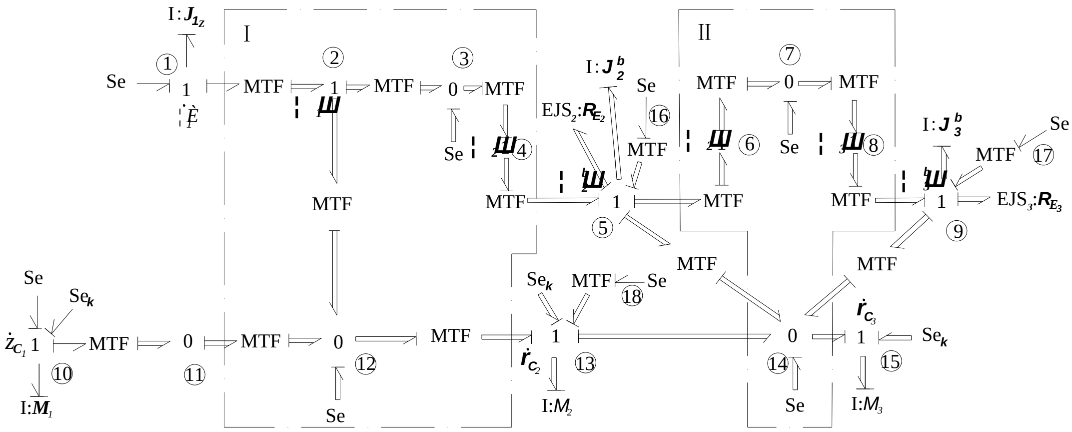

FIGURE 6 The vector bond graph model of robot mechanism system

$J_{1Z} = [\mathrm{I}_{Z_1}]$, $J_2^b = \mathrm{diag}(\mathrm{I}_{X_2} \quad \mathrm{I}_{Y_2} \quad \mathrm{I}_{Z_2})$, $J_3^{\mathrm{b}} = \mathrm{diag}(\mathrm{I}_{X_3} \quad \mathrm{I}_{Y_3} \quad \mathrm{I}_{Z_3})$. $\dot{r}_{C_2}$, $\dot{r}_{C_3}$ are the mass centre velocity vector of body 2 and body 3 in global coordinates, ω_1^b, ω_2^b and ω_3^b are the angular velocity vector of body 1, body 2 and body 3 in body frame respectively, $\omega_1^b = \dot{\theta}_1$. ω_1, ω_2 and ω_3 are the angular velocity vector of body 1, body 2 and body 3 in global coordinates. The mass of body i is m_{C_i}, $M_1 = m_{C_1}$, $M_2 = \mathrm{diag}(m_{C_2} \quad m_{C_2} \quad m_{C_2})$, $M_3 = \mathrm{diag}(m_{C_3} \quad m_{C_3} \quad m_{C_3})$.

Inputting the physical parameters of the robot mechanism, the coefficient matrices of Equations (5)-(13), known source vector U_3, system independent moving state variable vector X_{i_1}, and its derivative $\dot{X}_{i_1}$ into the program associated with the procedure presented here based on MATLAB [12], the system driving moment (or force) and constraint force equations in the form of Equation (15) can be derived on a computer. The corresponding driving moment (or force) and constraint forces can be determined. Some of results are shown in Figures 7-10.

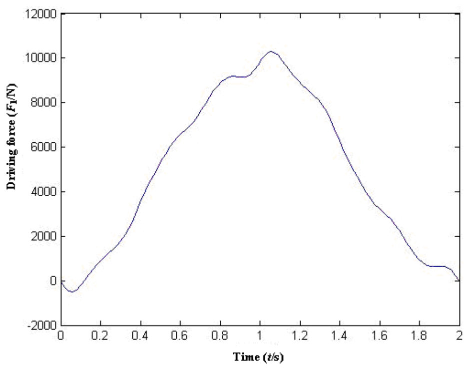

FIGURE 7 The driving force of body 1

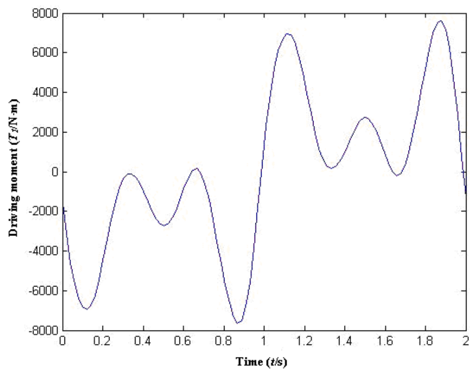

FIGURE 8 The driving moment of body 1

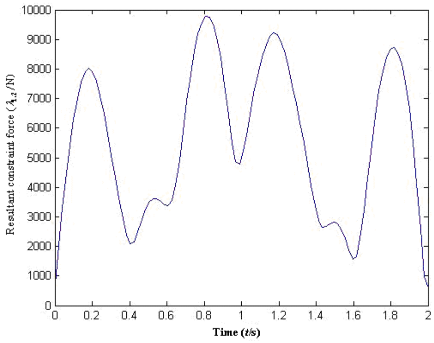

FIGURE 9 Resultant constraint force between body 1 and body 2

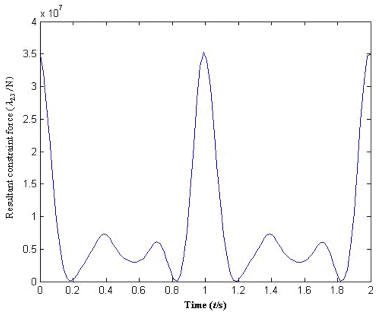

FIGURE 10 Resultant constraint force between body 2 and body 3

5 Conclusions

The vector bond graph procedure presented here is very suitable for dealing with computer aided kineto-static analysis of complex robot systems with the coupling of multi-energy domains. Compared with traditional scalar bond graph method, this vector bond graph procedure is more suitable for complex spatial robot mechanism because of its more compact and concise representation manner. The differential causalities in the vector bond graph model of spatial robot mechanisms can be avoided by the bond graph augment method proposed here, thus the algebraic difficulties in system automatic modelling and kineto-static analysis can be overcome. In the case of considering EJS, the unified formulae of system driving moment and constraint force equations are derived, which are easily derived on a computer in a complete form. These lead to a more efficient and practical automated procedure for kineto-static analysis of complex robot systems over a multi-energy domains in a unified manner. The validity of the procedure is illustrated by successful application to the kineto-static analysis of spatial robot systems with five degrees of freedom.

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