

Using grey-weighted Markov chain model to predict the quantum of highway passenger transport

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Abstract

The grey-weighted Markov model is a prediction model integrating the advantages of grey model and Markov chain model. It can be applied to predict the highway passenger transport quantum. Compared with grey model, the grey-weighted Markov chain model improved the precise of prediction, so the combined model was more appropriate for the prediction of highway passenger transport. Based on the original data of highway passenger transport quantum from 2001 to 2011, the passenger transport quantum in 2012 was predicted with grey-weighted Markov chain model.

Keywords: Grey model, weighted Markov chain, passenger transport quantum, prediction

1 Introduction

Grey system theory was brought forward and developed by Deng Julong in 1982 [1]. Grey model, i.e. GM, is a useful model to solve some problems with short time series, less statistical data, or incomplete information. But GM has a worse fitting and precision for the long term prediction or data series with great random wave (Deng 1989) [2]. Markov chain is a restricted class of stochastic process with finite or denumerable state-space. Their main property is that the joint distributions of the involved variables are fixed by the transition matrices of the process and the distribution of the initial variable. The transition matrices show the internal wave rule of state-space by which we can fix random errors of GM, therefore we can combine GM and Markov chain to predict value. Actually, the combined prediction models have widely appeared both in pure and applied mathematics, and have many applications in science and technology [3-8].

The highway passenger transport quantum is the number of passengers conveyed through the highway transportation system in some a period. The passenger transport quantum is an important data index reflecting the level of the transportation serves for national economy and people's living. It is also the index of programming the passenger transport quantum and studying the scale and the pace of the development of transportation. There are several methods to predict the passenger transport quantum such as the experts' experiences prediction method, the algorithm average method, the liner regression method, and BP neural network model etc, every method has advantages and disadvantages (Wang 2007) [9-10]. This paper mainly apply grey-weighted Markov chain model to predict the

highway passenger transport quantum. Compared with GM, the combined model improved the prediction precise.

2 Establishment of the model

2.1 GM (1, 1)

Step 1: Accumulate original series

$x^{(0)}(k)$ is the highway passenger transport quantum of the k th year, then original sequence is set:

$$X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(N)\}. \quad (1)$$

New sequence is created and defined as follows:

$$X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(N)\} \\ = \left\{ x^{(0)}(1), \sum_{i=1}^2 x^{(0)}(i), \dots, \sum_{i=1}^N x^{(0)}(i) \right\}. \quad (2)$$

Step 2: Establish the equation

GM (1,1) is a prediction model with an order and one variable. Its whitened equation is as follows:

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = u \quad (3)$$

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Where “ a ” is named evolution grey coefficient, “ u ” is named end genesis-control grey value, $U = \begin{bmatrix} a \\ u \end{bmatrix}$. \hat{U} is estimated by the method of GLS:

$$\hat{U} = (B^T B)^{-1} B^T Y, \tag{4}$$

where $B = \begin{pmatrix} -\frac{1}{2}(x^{(1)}(1) + x^{(1)}(2)) & 1 \\ -\frac{1}{2}(x^{(1)}(2) + x^{(1)}(3)) & 1 \\ \dots & \dots \\ -\frac{1}{2}(x^{(1)}(N-1) + x^{(1)}(N)) & 1 \end{pmatrix}$,

$$Y = [x_1^{(0)}(2), x_1^{(0)}(3), \dots, x_1^{(0)}(N)]^T.$$

We can obtain the estimation of “ a ” and “ u ” by \hat{U} , and then obtain the time-response Eq. which is as follows (Deng 1990):

$$\hat{x}^{(1)}(k+1) = \left[x^{(1)}(1) - \frac{\hat{u}}{\hat{a}} \right] e^{-\hat{a}k} + \frac{\hat{u}}{\hat{a}}, \quad k = 1, 2, \dots, \tag{5}$$

when $k = 1, 2, \dots, N-1$, $\hat{x}^{(1)}(k+1)$ is simulated value, when $k \geq N$, $\hat{x}^{(1)}(k+1)$ is predicted value.

Step 3: $x^{(0)}$ is deoxidized by $\hat{x}^{(1)}$

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k), \quad k = 1, 2, 3, \dots, \tag{6}$$

where $\hat{x}^{(0)}(1) = x^{(0)}(1)$, then $\hat{x}^{(0)}(k+1)$ is obtained:

$$\hat{x}^{(0)}(k+1) = (1 - e^a)(x^{(0)}(1) - \frac{\hat{u}}{\hat{a}})e^{-\hat{a}k}. \tag{7}$$

Step 4: The test of precision of GM (1,1)
Error is defined as:

$$E(k) = x^{(0)}(k) - \hat{x}^{(0)}(k), \quad k = 1, 2, \dots, N. \tag{8}$$

Relative error is defined as:

$$e(k) = \frac{E(k)}{x^{(0)}(k)} \times 100\%, \quad k = 1, 2, \dots, N. \tag{9}$$

Deviation of original series is defined as:

$$S_1^2 = \frac{\sum_{k=1}^N [x^{(0)}(k) - \bar{x}]^2}{N}, \tag{10}$$

where $\bar{x} = \frac{\sum_{k=1}^N x^{(0)}(k)}{N}$.

Deviation of error series is defined as:

$$S_2^2 = \frac{\sum_{k=2}^N [E(k) - \bar{E}]^2}{N-1}, \tag{11}$$

where $\bar{E} = \frac{\sum_{k=2}^N E(k)}{N-1}$.

Posterior ratio is calculated:

$$C = \frac{S_2}{S_1}. \tag{12}$$

The probability of error is calculated:

$$P = P\{|E(k) - \bar{E}| < 0.6745S_1\}. \tag{13}$$

The quality of prediction can be judged by table 1.

TABLE 1 The standard of the quality of GM (1, 1)

Grade	P	C
Good	$P > 0.95$	$C < 0.35$
Acceptable	$0.80 \leq P < 0.95$	$0.35 \leq C < 0.50$
Conceded acceptable	$0.70 \leq P < 0.80$	$0.50 \leq C < 0.65$
Unacceptable	$P \leq 0.70$	$C \geq 0.65$

2.2 GREY-WEIGHTED MARKOV CHAIN MODEL

Step 1 We establish the standard of the grade of relative error, i.e. set up the status bar of Markov chain according to the solution of GM(1,1).

Step 2 The grate of relative error sequence is ascertained by the standard.

Step 3 Autocorrelation coefficient r_k is calculated as follows:

$$r_k = \frac{\sum_{l=1}^{N-k} (x_l - \bar{x})(x_{l+k} - \bar{x})}{\sum_{l=1}^N (x_l - \bar{x})^2}. \tag{14}$$

r_k is the autocorrelation coefficient which step size is k , x_l is the l th relative error.

\bar{x} is the average of relative error, N is the length of highway passenger transport quantum sequence.

Step 4 Autocorrelation coefficient standardization

The weight of the Markov chain with step size k is defined as:

$$w_k = \frac{|r_k|}{\sum_{k=1}^m r_k}, \tag{15}$$

(m is the max order which the prediction model need).

Step 5 According to the transitive state-space of relative error series; we can obtain the transition probability matrix of different step sizes. Noting the original sequence and the corresponding transition probability matrix, we can predict the state possibility $p(k)$ of the relative error of the highway passenger transport quantum, where k is the step size, $k = 1, 2, \dots, m$.

We sum m weighted possibilities of the same state-space and regard the result as the possibility of the prediction, furthermore, the “ i ”, which corresponds $\max\{p(i), i \in E\}$ is the grade of relative error of prediction passenger transport quantum. We can fix the result of GM by the median of interval of relative error. Adding the final result to original sequence and repeat the

TABLE 2 The error and relative error of every year

Year	Actual value (ten thousand)	Simulated value (ten thousand)	Error (ten thousand)	Relative error (%)
2001	70497	70497.0000	0	0
2002	74626	56825.5172	17800.4828	23.8529
2003	75492	68203.3143	7288.6857	9.6549
2004	89388	81859.2124	7528.7876	8.4226
2005	98485	98249.3405	235.6595	0.2393
2006	109472	117921.1556	-8449.1556	-7.718
2007	123963	141531.7279	-17568.7279	-14.173
2008	168675	169869.6888	-1194.6888	-0.708
2009	234234	203881.5720	30352.428	12.9582
2010	248720	244703.4294	4016.5706	1.6149

According to table 2, the average of relative errors from 2001 to 2010 is 3.9033, the standard deviation is 11.60. The relative error is graded as follows:

The state-space of every year is ascertained according to the table 3:

above steps, we can predict the passenger transport quantum of next year.

3 Example analysis

The paper takes the highway passenger transport quantum of Shandong province from 2001 to 2010 for an example and uses grey-weighted Markov chain model to simulate or predict (the data source from Chinese Stat. Annual).

Step 1: With the program of GM(1,1), we get $a=-0.1825, u=38931.3719$, then get the simulation of the highway passenger transport quantum from 2001 to 2010 (table 3.1) and the prediction of the passenger transport quantum in 2011: $\hat{x}(2011) = 293698.7771$ (ten thousand).

The posterior ratio is calculated: $C = S_2/S_1 \approx 0.0225 < 0.35$, the probability of error is calculated: $P = P\{|E(k) - \bar{E}| < 0.6745S_1\} = 1 > 0.95$.

According to table 1, the quality of GM(1,1) is “Good”.

Step 2: We optimize the prediction by the mean of interval of relative error of weighted Markov chain.

Relative error is calculated as follows:

TABLE 3 The grade interval of relative error

Relative error state-space	The standard of grade	Relative error interval
1	$\bar{x} - 2.0s \leq x \leq \bar{x}$	$-19.29 \leq x \leq 3.90$
2	$\bar{x} < x \leq \bar{x} + 1.0s$	$3.90 < x \leq 15.50$
3	$\bar{x} + 1.0s < x \leq \bar{x} + 2.0s$	$15.50 < x \leq 27.10$

TABLE 4 The grade of relative error of every year

Year	Relative error	Grade	Year	Relative error	Grade
2001			2006	-7.718	1
2002	23.8529	3	2007	-14.173	1
2003	9.6549	2	2008	-0.708	1
2004	8.4226	2	2009	12.9582	2
2005	0.2393	1	2010	1.6149	1

Autocorrelation coefficients of different orders and weights of different step-sizes of Markov chain are calculated as follows:

TABLE 5 The autocorrelation coefficient and weight of different orders

State-space	1	2	3
r_k	1.4931	-0.0456	-1.0079
w_k	0.5863	0.0179	0.3958

From table 4, we can get the transition probability matrixes of different step sizes as follows:

$$p_1 = \begin{bmatrix} 3/4 & 1/4 & 0 \\ 2/3 & 1/3 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad p_2 = \begin{bmatrix} 3/4 & 1/4 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},$$

$$p_3 = \begin{bmatrix} 2/3 & 1/3 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

We can predict the state-space of relative error of highway passenger transport quantum in 2011 according to the data and the corresponding transition probability matrixes of 2008~2010. The results are as follows:

TABLE 6 The prediction of the range of relative error in 2011

Original year	State	Step size	Weight	1	2	3	Resource
2010	1	1	0.5863	0.75	0.25	0	p_1
2009	2	2	0.0179	1	0	0	p_2
2008	1	3	0.3958	2/3	1/3	0	p_3
Sum				0.4577	0.2772	0	

According to table 6, $\max\{p_i, i \in E\} = 0.4577$, the corresponding state space $i=1$, i.e. the relative error state-space is 1 in 2011. We fix the result of GM by the median of interval of relative error. Finally, we get the prediction of the highway passenger transport quantum of Shandong province in 2011: $\hat{x}(2011) = 272700$ (ten thousand).

The real highway passenger transport quantum of Shandong province in 2011 is 250469(ten thousand). It is obvious that the relative error of grey-weighted Markov combined model is smaller than the relative error of GM (1, 1), see table 7:

TABLE 7 Compare the precision of GM with Grey-weight Markov model

Year	Model	Actual value (ten thousand)	Simulated value (ten thousand)	Relative error (%)
2011	GM(1,1)	250469	293698	-17.26
	Grey-weight Markov	250469	272700	-8.88

In the same way, we can estimate other years, e.g. we compare the results of two models applied on the

prediction of passenger transport quantum in 2009 as follows:

TABLE 8 Compare the precision of GM with Grey-weight Markov model

Year	Model	Actual value (ten thousand)	Simulated value (ten thousand)	Relative error (%)
2009	GM(1,1)	234234	203882	12.96
	Grey-weight Markov	234234	225783	3.61

Therefore, we choose grey-weighted Markov model to predict the highway passenger transport quantum of Shandong province in 2012 as follows:

TABLE 9 The prediction of the highway passenger transport quantum in 2012

Year	Model	State	Predicted value (ten thousand)
2012	GM(1,1)		319387
	Grey-weight Markov	1	296553

$\hat{x}(2012) = 296553$ (ten thousand).

4 Conclusions

This paper compares the performances of GM (1, 1) and grey-weighted Markov chain method in passenger traffic quantum prediction. It is shown, that the latter model adjusts the error of GM, and then improves the prediction precision. However, the transition possibility matrix lies on the frequency, so the precision of grey-weighted Markov chain method is affected by the length of time series, we can obtain more accurate result with longer original time series.

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