

# Research on Strategies of Pricing and Coordinating in Reverse Supply Chain for Duopoly Recovery Competition

**Zhou Zhanfeng\***

*School of Mechatronic Engineering, XI'AN Technological University, China*

*Received 1 November 2014, www.cmnt.lv*

---

## Abstract

By using the Game Theory, this paper studied the pricing problem in multi-level reverse supply with a single manufacturer, a single retailer and duopoly recycler. Pricing model was established in non-cooperative game and cooperative game separately and the optimal pricing strategy was got. Considering that there are many procedures in multi-level reverse supply and the preference for the contract of each enterprise maybe distinct, this paper built a combined contract with price discount contract and revenue sharing contract. At last an example was given and the result shows that the combined contract is effective to coordinate reverse supply chain

*Keywords:* Game Theory; Pricing model; Contract

---

## 1 Introduction

Traditional forward Supply Chain includes supplier, manufacturer and vendors, the supplier provides raw materials for manufacturer and manufacturer sells the products to the vendors<sup>[1]</sup>. Reverse supply chain is an effective way to achieve resources recycling and sustainable development and a series of process of recycling used goods from consumers to classify, test, and disassemble until final disposal<sup>[2]</sup>. The pricing problem in reverse supply chain not only affects the profits of each enterprises, but also directly determines the total supply chain efficiency, therefore caused the wide attention of many scholars in the world. Many theories has been applied to research about the pricing problem such as Game Theory, Economic Theory and Optimization, but the object of the current study is mainly aimed at the two-level reverse supply chain with a single-supplier and single-manufacturer, the structure of supply chain is simple and difficult to apply in practice<sup>[3]</sup>. Though a few scholars have studied the pricing problem in three level of reverse supply chain with a single-recycler, single-manufacturer and single-manufacturer, but don't consider that the competitive relation between the recycler maybe affect the pricing of reverse supply chain. Because of this, the objective of this work is to resolve the pricing and benefit coordination problems in three-level reverse supply chain with a single-manufacturer, single-manufacturer and duopoly recycler when the recycler compete

## 2 Problems description and assumptions

In order to establish the model of pricing problem and solve, some assumptions must be made about reverse supply chain, as follows:

Recycler, Manufacturer and Seller all make decisions under entirely symmetric information and neutral risk, the

Recycler, Manufacturer and Seller are all perfectly rational and the aim of decisions is to maximize their profits.

When recycling the different types and quality of waste products, the recovery costs are same and all can be used to remanufacture. All remanufactured products can be sold.

The recycling market is duopoly and the marginal logistics cost for recycling are same.

Recycler must make a profit by recycling waste products.

Manufacturer must make a profit by remanufacturing waste products.

Seller must make a profit by selling remanufactured products.

The recycling waste products number of recycler is not only related to its own recycling price but also the competitor and is a linear increasing function of its own recycling price and linear decreasing function of competitor's recycling price.

Remanufactured products and new products make no difference in quality and can be sold by the same price in the market.

The market demand is linear with respect to the selling price.

## 3 Pricing strategy in non-cooperative game

In non-cooperative game, manufacturer, seller and recycler competes perfectly with each other, manufacturer, seller and recycler are all independent policymaker during the process of competition, the policy goals is to maximize their profits, but due to the different position in competition, then the stage and sequence of decision are also diverse. Generally, the manufacturer is leader, seller and recycler is all follower, manufacturer makes decision firstly, and then seller and recycler make policy according to the decisions of manufacturer, so it belongs to the typi-

---

\* *Corresponding author's* e-mail: nwpuzhouzhanfeng@163.com

cal Stackelberg Game problem. A Sequential Non-cooperative Stackelberg Game is established in the paper.

3.1. EXPLANATIONS OF THE SYMBOLS IN MODELS

- $P_{r1}$  : Recycling price of the first recycler.
- $P_{r2}$  : Recycling price of the second recycler.
- $C_r$  : Unit marginal recycling cost of recycler including the logistics cost and testing cost.
- $\omega_r$  : Recycling price of the manufacturer.
- $\omega_s$  : Remanufactured products' wholesale price of manufacturer.
- $C_n$  : Unit marginal production cost of new products.
- $C_m$  : Unit marginal production cost of remanufactured products.
- $C_s$  : Sales cost of unit product
- $P$  : Selling price of new products and remanufactured products.
- $Q_{r1}$  : Recycling quantity of the first recycler.
- $Q_{r2}$  : Recycling quantity of the second recycler.
- $Q_d$  : Total demand for new products and remanufactured products
- $\pi_{r1}$  : Profit of the first recycler.
- $\pi_{r2}$  : Profit of the second recycler.
- $\pi_m$  : Profit of the manufacturer.
- $\pi_s$  : Profit of the seller.
- $\pi$  : Total profit of the reverse supply chain.

3.2. ESTABLISHMENT OF MODEL

When constructing the non-cooperative model, firstly recycling quantity of the recycler and total demand for products must be determined as in Eq. (1) and Eq. (2) <sup>[4]</sup>:

$$Q_{r1} = Q_{r0} + \alpha \times P_{r1} - \beta \times P_{r2}, \tag{1}$$

$$Q_{r2} = Q_{r0} + \alpha \times P_{r2} - \beta \times P_{r1}, \tag{2}$$

In Eq. (1) and Eq. (2),  $Q_{r0}$  indicates the value of basic recycling quantity independent on the recycling price, it reflects the environmental awareness of the consumer,  $\alpha$  indicates the elasticity coefficient of recycling price, it reflects the sensitivity degree of consumer to recycling price.  $\beta$  indicates the substitute coefficient of recycling price, it reflects the sensitivity degree of consumer to the comparison of recycling price. According to the assumption in section 1, total demand for products can be determined as in Eq. (3).

$$Q_d = Q_{d0} - \gamma \times P, \tag{3}$$

In this relation,  $Q_{d0}$  indicates the max demand for products,  $\gamma$  indicates the elasticity coefficient of sale price, it reflects the sensitivity degree of consumer to sale price.

The profit of duopoly recycler can be indicated as in Eq. (4) and Eq. (5).

$$\pi_{r1} = (\omega_r - P_{r1} - C_r) \times (Q_{r0} + \alpha \times P_{r1} - \beta \times P_{r2}), \tag{4}$$

$$\pi_{r2} = (\omega_r - P_{r2} - C_r) \times (Q_{r0} + \alpha \times P_{r2} - \beta \times P_{r1}), \tag{5}$$

The profit of manufacturer can be indicated as in Eq. (6).

$$\pi_m = (\omega_s - C_n) \times (Q_{d0} - \gamma \times P) - (\omega_r + C_m - C_n) \times [2Q_{r0} + (\alpha - \beta) \times (P_{r1} + P_{r2})], \tag{6}$$

The profit of manufacturer can be indicated as in Eq. (7).

$$\pi_s = (P - \omega_s - C_s) \times (Q_{d0} - \gamma \times P), \tag{7}$$

According to the assumption in section 1, some conditions should be satisfied as in Eq. (8) to Eq. (15)

$$\omega_r - P_{r1} - C_r > 0, \tag{8}$$

$$\omega_r - P_{r2} - C_r > 0, \tag{9}$$

$$\omega_s + C_n - C_m - \omega_r > 0, \tag{10}$$

$$P - \omega_s - C_s > 0, \tag{11}$$

$$C_m < C_n, \tag{12}$$

$$Q_{r1} + Q_{r2} < Q_d, \tag{13}$$

$$\alpha > \beta > 0, \tag{14}$$

$$\gamma > 0, \tag{15}$$

The policy goal in non-cooperative is to maximize the profit of each member in reverse supply chain, that is how to determine the variable values of  $P_{r1}, P_{r2}, \omega_r, \omega_s$  and  $P$  to make the values of  $\pi_{r1}, \pi_{r2}, \pi_m, \pi_s$  maximize.

3.3. SOLVER OF MODEL

In non-cooperative game, Manufacturer and Recycler, Seller and Manufacturer formed a Stackelberg Game relationship separately, manufacturer firstly determines  $\omega_r$  and  $\omega_s$ , Recycler and Seller respond to develop the pricing strategies separately after the pricing information about manufacturer is got. So, in order to acquire the optimal solution of the model, the recycler and seller's reaction function in the second stage of Stackelberg Game must be established, that is how to determine the optimal recycling price and sale price to make the profit of the recycler and seller maximize. Therefore, the derivation of  $P_{r1}, P_{r2}$  and  $P$  was calculated as in Equation (16) to Equation (18).

$$\frac{\partial \pi_{r1}}{\partial P_{r1}} = -Q_{r0} + \alpha \times \omega_r - 2\alpha \times P_{r1} - \alpha \times C_r + \beta \times P_{r2}, \tag{16}$$

$$\frac{\partial \pi_{r2}}{\partial P_{r2}} = -Q_{r0} + \alpha \times \omega_r - 2\alpha \times P_{r2} - \alpha \times C_r + \beta \times P_{r1}, \tag{17}$$

$$\frac{\partial \pi_s}{\partial P} = Q_{d0} - 2\gamma \times P + \gamma \times \omega_s + \gamma \times C_s, \tag{18}$$

In Eq. (16) to Eq. (18), making the derivation equal zero, the optimal values can be got as in Equation (19) to Eq. (21)

$$P_{r1} = \frac{-Q_{r0} + \alpha \times \omega_r - \alpha \times C_r}{2\alpha - \beta}, \tag{19}$$

$$P_{r2} = \frac{-Q_{r0} + \alpha \times \omega_r - \alpha \times C_r}{2\alpha - \beta}, \tag{20}$$

$$P = \frac{Q_{d0} + \gamma \times \omega_s + \gamma \times C_s}{2\gamma}, \tag{21}$$

Calculating the derivation of  $\omega_r, \omega_s$  by substituting the value of  $P_{r1}, P_{r2}$  and  $P$ , as in Eq. (22) and Eq. (23).

$$\frac{\partial \pi_m}{\partial \omega_s} = \frac{Q_{d0} - 2\gamma \times \omega_s - \gamma \times C_s + \gamma \times C_n}{2}, \tag{22}$$

$$\frac{\partial \pi_m}{\partial \omega_r} = \frac{-2\alpha \times Q_{r0} + 2\alpha(\alpha - \beta)(C_n - C_m + C_r) - 4\alpha(\alpha - \beta) \times \omega_r}{(2\alpha - \beta)}, \tag{23}$$

In the above formula, making the derivation equal zero, the optimal pricing strategy of manufacturer can be got in Eq. (24).

$$(\omega_r^*, \omega_s^*) = \left( \frac{-Q_{r0} + (\alpha - \beta) \times (C_n - C_m + C_r)}{2(\alpha - \beta)}, \frac{Q_{d0} - \gamma \times C_s + \gamma \times C_n}{2\gamma} \right), \tag{24}$$

So, the optimal pricing strategy of recycler and seller can also be got as follows in Equation (25) and Eq. (26).

$$P_{r1}^* = P_{r2}^* = \frac{(-3\alpha + 2\beta) \times Q_{r0} + \alpha(\alpha - \beta) \times (C_n - C_m - C_r)}{2(2\alpha - \beta)(\alpha - \beta)}, \tag{25}$$

$$P^* = \frac{3Q_{d0} + \gamma \times C_n + \gamma \times C_s}{4\gamma}, \tag{26}$$

According to the optimal pricing strategy, each optimal profit can be got as in Eq. (27) to Eq. (30).

$$\pi_{r1}^* = \pi_{r2}^* = \alpha \left[ \frac{Q_{r0} + (\alpha - \beta) \times (C_n - C_m - C_r)}{2(2\alpha - \beta)} \right]^2, \tag{27}$$

$$\pi_m^* = \frac{(Q_{d0} - \gamma \times C_n - \gamma \times C_s)^2}{8\gamma} + \left[ \frac{\alpha}{2(\alpha - \beta)(2\alpha - \beta)} \right] \times [Q_{r0} + (\alpha - \beta) \times (C_n - C_m - C_r)]^2, \tag{28}$$

$$\pi_s^* = \frac{(Q_{d0} - \gamma \times C_n - \gamma \times C_s)^2}{16\gamma}, \tag{29}$$

$$\pi^* = \frac{3(Q_{d0} - \gamma \times C_n - \gamma \times C_s)^2}{16} + \frac{\alpha(3\alpha - 2\beta)}{2(\alpha - \beta)(2\alpha - \beta)^2} \times [Q_{r0} + (\alpha - \beta) \times (C_n - C_m - C_r)]^2, \tag{30}$$

#### 4 Pricing strategy in cooperative game

In Cooperative Game, the manufacturer, recycler and seller collaborates each other and form a community of interest, they make collective decisions to develop strategies based on sharing all information fully, the policy goals is to maximize the profits of the whole reverse supply chain.

##### 4.1. EXPLANATIONS OF THE SYMBOLS IN MODELS

- $P_{r1}''$ : Recycling price of the first recycler.
  - $P_{r2}''$ : Recycling price of the second recycler.
  - $P'$ : Selling price of new products and remanufactured products.
  - $\pi''$ : Total profit of the reverse supply chain.
- The rest unspecified symbol is the same as section 3.

##### 4.2. ESTABLISHMENT OF MODEL

The total profit of the reverse supply chain in Cooperative Game can be indicated as follows in Eq. (31).

$$\pi'' = (P' - C_n - C_s) \times (Q_{d0} - \gamma \times P'') - \alpha(P_{r1}''^2 + P_{r2}''^2) + 2\beta P_{r1}'' P_{r2}'' + 2Q_{r0}(C_n - C_m - C_r) + [-Q_{r0} + (\alpha - \beta)(C_n - C_m - C_r)] \times (P_{r1}'' + P_{r2}'') \tag{31}$$

The decision goal in Cooperative Game is to make the total profit of the reverse supply chain that is how to determine the value of  $P_{r1}'', P_{r2}''$  and  $P'$  to maximize the value of  $\pi''$  in Eq. (31).

4.3. SOLVER OF MODEL

In the above formula, the derivation of  $P_{r1}''$ ,  $P_{r2}''$  was calculated as in Eq. (32) and Eq. (33).

$$\frac{\partial \pi''}{\partial P_{r1}} = -2\alpha P_{r1}'' + 2\beta P_{r2}'' - Q_{r0} + (\alpha - \beta)(C_n - C_m - C_r) = 0 \tag{32}$$

$$\frac{\partial \pi''}{\partial P_{r2}} = 2\beta P_{r1}'' - 2\alpha P_{r2}'' - Q_{r0} + (\alpha - \beta)(C_n - C_m - C_r) = 0 \tag{33}$$

According to Eq. (32) and Eq. (33), the optimal recycling price in Cooperative Game can be got as in Eq. (34).

$$(P_{r1}'')^* = (P_{r2}'')^* = \frac{-Q_{r0} + (\alpha - \beta)(C_n - C_m - C_r)}{2(\alpha - \beta)} \tag{34}$$

The sale price can also be got by resolving the derivation of  $P''$  as in Eq. (35).

$$(P'')^* = \frac{Q_{d0} + \gamma(C_n + C_s)}{2\gamma} \tag{35}$$

According to Eq. (34) and Eq. (35), the total optimal profit can be got as in Eq. (36).

$$(\pi'')^* = \frac{[Q_{d0} - \gamma(C_n + C_s)]^2}{4\gamma} + \frac{[Q_{r0} + (\alpha - \beta)(C_n - C_m - C_r)]^2}{2(\alpha - \beta)} \tag{36}$$

5 Coordination strategy of reverse supply chain

Traditional coordination strategy includes contracts-return contract, price discount contract, and revenue sharing contract and so on<sup>[5]</sup>. But for three-level reverse supply chain with a single-manufacturer, single-manufacturer and duopoly recycler, there are to transaction process, the transaction between manufacturer and recycler and transaction between manufacturer and seller. For the different contract, the operation process and the complexity of management is also different, the preference for the contract of each enterprise maybe distinct. So, various contracts must be combined in order to meet the different need of each enterprise<sup>[6]</sup>. In view of this, some assumptions in this section are made as follows:

A price discount contract was established between recycler and manufacture, recycler must recycle the waste products according to the price in Cooperative Game and manufacturer compensates for the recycler by raise the price when recycling the waste products from recycler. Meanwhile a revenue sharing contract was reached between manufacturer and seller, manufacturer sells the new products and remanufactured products to seller and the sales revenue of seller must be returned to the manufacturer in certain proportion in order to make up the loss of manufacturer. Supposing the proportion of sales revenue that seller acquired is indicated by  $\lambda$ , the profits of manufacturer and seller can be expressed as in Eq. (37), (38).

$$\pi_m = (1 - \lambda) \times P \times (Q_{d0} - \gamma \times P) + (\omega_s - C_n) \times (Q_{d0} - \gamma \times P) - (\omega_r + C_m - C_n) \times [2Q_{r0} + (\alpha - \beta) \times (P_{r1} + P_{r2})] \tag{37}$$

$$\pi_s = \lambda \times P \times (Q_{d0} - \gamma \times P) - (\omega_s + C_s) \times (Q_{d0} - \gamma \times P) \tag{38}$$

The optimal sale price can be got by calculating the derivation of  $P$  in the above formula, and making the derivation to zero as in Eq. (39).

$$P^* = \frac{\lambda Q_{d0} + \gamma \omega_s + \gamma C_s}{2\lambda\gamma} \tag{39}$$

In order to coordinate the reverse supply chain, the optimal sale price must meet the Eq. (40).

$$P^* = \frac{\lambda Q_{d0} + \gamma \omega_s + \gamma C_s}{2\lambda\gamma} = \frac{Q_{d0} + \gamma C_n + \gamma C_s}{2\gamma} \tag{40}$$

According to the value of  $P$  in the Eq. (40), the value of wholesale price can be got in Eq. (41).

$$\omega_s = \lambda C_n - (1 - \lambda) C_s \tag{41}$$

Because the waste products were recycled at the price in Cooperative Game according to the price discount contract, so  $\omega_r$  can be calculated by Equation (42).

$$\frac{-Q_{r0} + \alpha \times \omega_r - \alpha \times C_r}{2\alpha - \beta} = \frac{-Q_{r0} + (\alpha - \beta)(C_n - C_m - C_r)}{2(\alpha - \beta)} \tag{42}$$

From Eq. (42), the optimal recycling price of the manufacturer can be got as in Eq. (43).

$$\omega_r = \frac{-\beta Q_{r0}}{2\alpha(\alpha - \beta)} + \frac{(2\alpha - \beta)(C_n - C_m) + \beta C_r}{2\alpha} \tag{43}$$

then the profit of recycler, manufacturer and seller in combined contract can be indicated as in Eq.(44) to Eq.(46).

$$\frac{\pi_{r1}}{\pi_{r2}} = \frac{[Q_{r0} + (\alpha - \beta) \times (C_n - C_m - C_r)]^2}{4\alpha} \tag{44}$$

$$\pi_m = \frac{(1 - \lambda)(Q_{d0} - \gamma \times C_n - \gamma \times C_s)^2}{4\gamma} + \frac{\beta [Q_{r0} + (\alpha - \beta) \times (C_n - C_m - C_r)]^2}{2\alpha(\alpha - \beta)} \tag{45}$$

$$\pi_s = \frac{\lambda(Q_{d0} - \gamma \times C_n - \gamma \times C_s)^2}{4\gamma} \tag{46}$$

In order to coordinate the reverse by combined contract, some requirements must be met as in Eq. (47) to Eq. (50).

$$\pi_{r1} \geq (\pi_{r1}')^* \tag{47}$$

$$\pi_{r2} \geq (\pi_{r2}')^* \tag{48}$$

$$\pi_m \geq \pi_m^* \tag{49}$$

$$\pi_s \geq \pi_s^* \tag{50}$$

From Eq. (47) to Eq. (50), the requirements of combined contract can be got as in Eq. (51) and Eq. (52).

$$\alpha \geq \frac{4}{3} \beta, \tag{51}$$

$$\frac{1}{4} \leq \lambda \leq \frac{1}{2} - \frac{2\gamma(\alpha - \beta)[Q_{r0} + (\alpha - \beta)(C_n - C_m - C_r)]^2}{\alpha(2\alpha - \beta)(Q_{d0} - \gamma C_n - \lambda C_s)^2}, \tag{52}$$

**6 Example and analysis**

There are some parameters about reverse supply chain as follows:

$$\alpha = 10, \beta = 1, \gamma = 1, C_n = 80, C_m = 50, C_r = 20, C_s = 10, Q_{r0} = 10, Q_{d0} = 300$$

Firstly the value of  $\lambda$  should be determined according to the requirements as in Eq. (53).

$$0.25 \leq \lambda \leq 0.4785, \tag{53}$$

Let the value of  $\lambda$  equal to 0.3, the value of  $\omega_r$  and  $\omega_s$  can be got by calculating as follows:

$$\omega_r = 26.5, \omega_s = 17$$

By calculating, recycling price in Non-cooperative Game and Cooperative game is shown as Table 1.

TABLE 1 Optimal price in different kind of Game

Optimal price	Non-cooperative Game	Cooperative game	Combined contract
$P_r$	1.81	4.44	4.44
$\omega_r$	24.44		26.5
$\omega_s$	185		17
P	247.5	195	195

By calculating, the optimal profit in Non-cooperative Game, Cooperative game and Combined Contract is shown as Table 2.

TABLE 2 Optimal profits in different kind of Game

Optimal profit	Non-cooperative Game	Cooperative game	Combined contract
Profit of the first recycler	69.25		250
Profit of the second recycler	69.25		250

**References**

[1] Ting Li 2007 Journal of Logistics Technology (26), 80-83.  
 [2] Binshi Xu 2009 Journal of Internal Combustion Engine Parts (4), 3-8.  
 [3] Haibo Luo, Bangyi Li, Yuanyuan Liu 2009 Journal of Value Engineering (4), 4-7.  
 [4] Juhong Gao, Chenhe Jin 2010 Journal of Logistics Engineering and Managemen (32), 76-79.

Profit of the manufacturer	5804.90		7773.05
Profit of the seller	2756.25		3307.5
Profit of the reverse supply chain	8699.65	11580.55	11580.55

**7 Conclusions**


Waste products recovery is the first step in reverse supply chain, reasonable price in each stage of reverse supply chain will directly affect every enterprise's profit and the smooth implementation of reverse supply chain. Many scholars have studied the pricing problems in simple reverse supply chain with a single manufacture and a single recycler or a single seller, this paper studied the pricing problems in three-level reverse supply chain with a single-manufacturer, single-manufacturer and duopoly recycler, but recycling price problem in practice maybe more complex because that there are many recyclers, manufacturers and sellers in reverse supply chain. So, further research on complex multi-level reverse supply chain with many recyclers, manufactures and retailers is very necessary.

Research shows that Non-cooperative Game in reverse supply chain will reduce the total profit and lead to the loss of interest, thus benefit coordinating mechanism must be applied into reverse supply chain. Traditional single contract can't meet the different preference of each member in reverse supply chain and various contracts should be combined. This paper established a kind of combined contract with price discount contract and revenue sharing contract and the effectiveness is proved by an example. For complex multi-level reverse supply with many recyclers, manufactures and retailers, combination of more contracts will be the next research direction.

**Acknowledgments**

This research was financially supported by the XI'AN Technological University Principal Fund (605-01001306) and Foundation of Shan' Xi Educational Committee (12JK0072).

**Author**



**Zhou zhanfeng, 1980.9.16, Bengbu City, Anhui Province**  
**Current position, grades:** Lecturer  
**University studies:** Industry Engineering  
**Scientific interest:** Management Information System, Computer Integrated Manuafcturing System, Advanced Manufacturing System  
**Publications:** Published a number of papers in domestic and foreign publications and seven papers was included by EI.  
**Experience:** Successively presided and participated in many projects such as XI'AN Technological University Principal Fund, Shan' xi Educational Committee Fund, Xi'an Municipal Science and Technology Bureau and so on.