Design of grey PID controller for DC Servo Motor

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Abstract

Concerning the uncertainties may be existed in DC motor servo system and the control quality that the external disturbance may influence the control algorithm of traditional PID, a kind of PID control algorithm based on grey prediction theory was proposed. With the grey theory's processing ability on the unknown information data, the algorithm established the grey model for the uncertainties, and real-time compensated the system's unmodelled feature and disturbance signal, thus improving the control precision. The simulation result can show that the proposed PID control algorithm based on grey prediction theory can effectively predict and compensate the unmodelled feature and disturbance signal in DC motor servo system and improve the control precision of the controller, thus providing the theoretical basis for the industrial application of PID control algorithm based on grey prediction theory.

Keywords: PID control, DC motor, Noise, Disturbance, Grey prediction

1 Introduction

With the feature of simple structure, small size, lightweight, reliable operation, easy maintenance, good servo performance, fast response speed and good stability, the DC motor is widely applied in the fields of servo system, factory automation and defence industry [1]. However, the measurement noise commonly existed in the industrial field, the structural disturbance caused by the abstract and simplification when establishing the model and the parameter disturbance caused by the parameter change in the actual work of the servo will seriously influence the working performance of the servo system [2]. The existence of above noise and disturbance makes the traditional PID control strategy fail to meet the control demand of DC motor servo system [3]. In order to solve the influence of above measurement noise and disturbance on the system control feature, the measurement noise and disturbance need to be predicted and compensated effectively. Grey prediction is a good choice to predict the above measurement noise and disturbance. It was firstly proposed by Professor Deng Julong from Huazhong University of Science and Technology [4]. Through the small amount of incomplete information, the grey differential prediction model was established to conduct the fuzzy and long description on the development law of the things. Introducing the grey prediction in servo control system can effectively predict the above measurement noise and disturbance, and conduct the corresponding compensation scheme according to the prediction result.

Concerning the uncertainties may be existed in DC motor servo system and the control quality that the external disturbance may influence the control algorithm of traditional PID, the author proposed a kind of PID control algorithm combined the grey prediction theory and traditional PID control algorithm and applied in the DC motor servo system. With the grey theory's processing ability on the unknown information data, the algorithm established the grey model for the uncertainties, and real-time compensated the system's unmodelled feature and disturbance signal, thus improving the control precision.

The main structure of the paper is as follows: Section 2 established the mathematical modelling of DC motor; Section 3 described the theoretical basis for the grey prediction; the design of grey PID controller was described in Section 4; Section 5 was the numerical analysis and conclusion discussion.

2 Mathematical modelling of DC motor

In modern industry, DC motor is the implementation terminal with the most widely application in servo system. With the research object of brushless electromagnetic DC motor, the paper established the mathematical model [5]. The working principle of DC motor controlled by armature is as shown in Figure 1 [5].

According to Figure 1, the following equation can be obtained:

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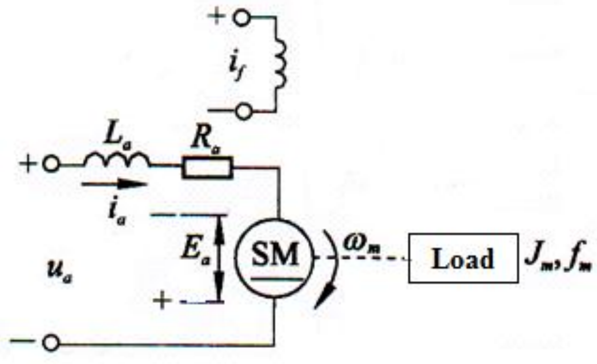

FIGURE 1 Working principle of DC motor

(1) Armature circuit voltage balance equation:

$$u_a(t) = L_a \frac{di_a(t)}{dt} + R_a i_a(t) + E_a . \quad (1)$$

In the formula, E_a is the counter electromotive force of the armature. It is produced when rotating the electric digital. The size is proportional to the excitation flux and revolving speed, and the direction is opposite to the armature voltage $u_a(t)$.

(2) Electromagnetic torque equation:

$$M_m(t) = C_m i_a(t) . \quad (2)$$

In the formula, C_m (N.m/A) is the moment coefficient of the motor. $M_m(t)$ (N.M) is the electromagnetic torque produced by the armature current.

(3) Torque balance equation of motor shaft:

$$J_m \frac{d\omega_m(t)}{dt} + f_m \omega_m(t) = M_m(t) - M_c(t) . \quad (3)$$

In the formula, f_m (N. m/rad/s) is the viscous friction coefficient converted by the electromotor and load in the motor shaft. $J_m s$ (kg.m.s2) is the rotational inertia converted by the electromotor and load in the motor shaft.

Erasing the intermediate variables $i_a(t)$, E_a and $M_m(t)$ in (1)-(3), $\omega_m(t)$ can be obtained as the output quantity, and the DC motor differential equation with the input quantity of $u_a(t)$ is:

$$L_a J_m \frac{d^2\omega_m(t)}{dt^2} + (L_a f_m + R_a J_m)\frac{d\omega_m(t)}{dt} + (R_a f_m + C_m C_e)\omega_m(t) = C_m u_a(t) - L_a \frac{dM_c(t)}{dt} - R_a M_c(t) . \quad (4)$$

For the armature circuit inductance L_a is smaller in the engineer, it can be neglected; therefore, the above formula can be simplified as:

$$T_m \frac{d\omega_m(t)}{dt} + \omega_m(t) = K_1 U_a(t) - K_2 M_c(t) . \quad (5)$$

In the formula, $T_m = \frac{R_a J_m}{R_a f_m + C_m C_e}$, $K_1 = \frac{C_m}{R_a f_m + C_m C_e}$, $K_2 = \frac{R_a}{R_a f_m + C_m C_e}$.

The motor's transfer function from the control voltage to the angular displacement can be obtained after the LAPLACE conversion:

$$G(s) = \frac{\theta(s)}{U(s)} = \frac{1/C_e}{s(T_m s + 1)} . \quad (6)$$

In the formula, $1/C_e$ is the velocity constant, and T_m is the mechanical time constant.

3 Foundation for grey prediction theory

Grey theory was firstly proposed by Professor Deng Julong from Huazhong University of Science and Technology [4]. Through the small amount of incomplete information, the grey differential prediction model was established, thus whitening the grey quantity obtained in the system to improve the control effect and robustness of the controller.

3.1 ESTABLISHMENG OF GM (0, N) MODEL [6]

Supposing $x_1 = \{x_1(1), x_1(2), x_1(3), \cdots, x_1(n)\}$ is the dependent variable, $x_i = \{x_i(1), x_i(2), x_i(3), \cdots, x_i(n)\}$ is the independent variable, $i = \{1, 2, 3, \cdots, N\}$ is the time series data, the step to establish the time series data grey prediction model GM (0, N) with the above small amount of information is:

(1) Initialization time series data

The time series for the dependent variable and independent variable after the initialization is [7]: $x_i^{(0)} = x_i(k)/x_i(1)$

In the formula, $x_i(1)$ is the first value of the variable sequence.

(2) Establish the GM(0, N) model

Supposing $x_i^{(1)}$ is the sequence obtained after one AGO of $x_i^{(0)}$; The prediction model of GM(0,N) can be obtained as [8]: $x_1^{(1)}(k) = \sum_{i=2}^{N} b_i x_i^{(i)}(k) + a$

Parameter $\hat{b} = [b_2, b_3, \cdots, b_N, a]$ can obtain the parameter $\hat{b}$ through the least square method:

$$\hat{b} = (B^T B)^{-1} B^T Y . \quad (7)$$

In the formula, $B=\begin{bmatrix} x_2^{(1)}(2) & \cdots & x_N^{(1)}(2) & 1 \\ \vdots & & & \vdots \\ x_2^{(1)}(n) & \cdots & x_N^{(1)}(n) & 1 \end{bmatrix}$,

$$Y=\begin{bmatrix} x_1^{(1)}(2) \\ x_1^{(1)}(3) \\ \vdots \\ x_1^{(1)}(n) \end{bmatrix}.$$

The solved approximate time response is:

$$x_1^{(1)}(k)=\sum_{i=2}^{N} b_i x_i^{(i)}(k)+a. \quad (8)$$

Reducing (7) can obtain:

$$\hat{x}_1^{(0)}(k+1)=\hat{x}_1^{(1)}(k+1)-\hat{x}_i^{(1)}(k),\ k=1,2,3\cdots,n-1. \quad (9)$$

In the formula $\hat{x}_1^{(0)}(1)=\hat{x}_1^{(1)}(1)$.

3.2 GM(0, N) MODEL PREDICTION PROCESS

GM(0, N) model prediction process is as shown in the following FIGURE 2 [9],

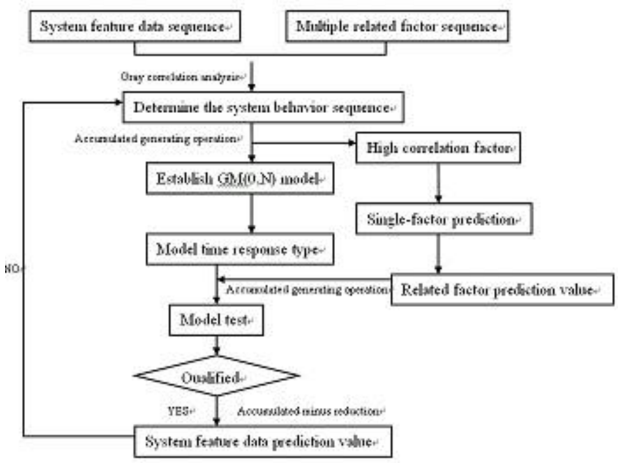

FIGURE 2 Flowchart

4 Design of grey PID controller

4.1 PARAMETER ESTIMATION

If the following nonlinear uncertain system is the research object,

$$\dot{x}=Ax(t)+bu(t)+bD(x,t). \quad (10)$$

$bD(x,t)$ is the uncertain part to meet the system matching condition, and it contains the parameter uncertainty and external interference mentioned previously. Supposing

$$D(x,t)=V_1x_1+V_2x_2+\cdots V_nx_n+f(t). \quad (11)$$

When we use the traditional PID control algorithm, the uncertainty existed in the system will influence the system's control performance. In order to reduce the influence brought by the uncertainty, improve the control effect and system robustness, the corresponding GM (0,N) model can be established with previous grey prediction theory, the coarse value of the parameter V in the uncertain part model can be estimated, and the uncertain part $D(x,t)$ can be correspondingly compensated, thus improving the control effectiveness and system robustness. $D(x)$ in $D(x,t)$ cannot be directly measured through our measurement means, therefore, the data obtained through our measurement can be obtained after the grey prediction, and the discrete value is:

$$D(x,k)=\frac{1}{b}(\dot{x}(t)-Ax(t)-bu_p(t)). \quad (12)$$

In the formula, $u_p(t)$ is the control volume of traditional PID.

The specific algorithm for the estimation of coarse value of model parameter V is as follows [9]:

Step 1, Establish the original discrete sequence

$$\begin{aligned} D^{(0)} &= \left(D(1)D(2)\ldots \mathrm{D}(N)\right) \\ f^{(0)} &= \left(f(1)f(2)\ldots \mathrm{f}(N)\right) \\ x_1^{(0)} &= \left(x_1(1)x_1(2)\ldots x_1(N)\right) \\ x_2^{(0)} &= \left(x_2(1)x_2(2)\ldots x_2(N)\right) \\ &\ldots \\ x_n^{(0)} &= \left(x_n(1)x_n(2)\ldots x_n(N)\right) \end{aligned}. \quad (13)$$

Step 2, Sequence accumulation establishing model

$$x^{(1)} \triangleq \sum_{m=1}^{k} x^{(0)}(m). \quad (14)$$

Supposing $D^{(1)}$, $f^{(1)}$, $x_i^{(1)}$ (i=1,2,…,n) is the accumulating generation sequence of $D^{(0)}$, $f^{(0)}$, x_i (i=1,2,…,n). The grey model in the uncertain part $D(x,t)$ is the accumulating generation sequence.

$$D^{(1)}(x,t)=V_1x_1^{(1)}+V_2x_2^{(1)}+\ldots V_nx_n^{(1)}+f^{(1)}. \quad (15)$$

Concerning the slowly time-varying interference part, it can be regarded as

$$\begin{aligned}&f^{(1)}(1)=f(1)\triangleq f\\&f^{(1)}(2)=2f(1)\triangleq 2f\\&\ldots\\&f^{(1)}(N)=Nf\end{aligned}\quad. \tag{16}$$

Remember the parameter list as

$$\hat V^T=\left(\hat V_1\hat V_2\ldots\hat V_n\hat f\right)^T. \tag{17}$$

Step 3, Calculation

$$B=\begin{bmatrix}x_1^{(1)}(2)\ldots x_n^{(1)}(2)\ldots 1\\ x_1^{(1)}(3)\ldots x_n^{(1)}(3)\ldots 2\\ \ldots\\ x_1^{(1)}(N)\ldots x_n^{(1)}(N)\ldots N-1\end{bmatrix}. \tag{18}$$

In the formula, $B^T B$ must be reversible. If it is irreversible, N shall be increased or decreased appropriately until $B^T B$ becoming reversible.

Step 4, According to the state $B^T B$ and Formula (15), calculate $D^{(0)}(k)$ discrete sequence,

Step 5, Calculate $D^{(0)}(k)$ accumulating discrete sequence

$$D_N^{(1)}=\left(D^{(1)}(1)D^{(1)}(2)\ldots D^{(1)}(N)\right). \tag{19}$$

Step 6, Calculate the uncertain parameter estimating value

$$\begin{aligned}&\hat V=\left(B_b^T B_b\right)^{-1}B_b^T D_N^{(1)}\\&\hat V=\left(\hat V_1\hat V_2\ldots\hat V_n\hat f\right)^T\end{aligned}\quad. \tag{20}$$

4.2 GREY PID CONTROLLER

Step 1, Firstly adopt PID control, and the control formula is

$$\begin{aligned}&u_p=u(kT)=K_p e(kT)\\&+K_t\sum_{j=0}^{k}e(jT)T+K_D\frac{e(kT)-e((k-1)T)}{T}\end{aligned}. \tag{21}$$

Step 2, Estimate the parameter V with grey theory. Based on the above control law, estimate the parameter V with compensation control u, the estimator stops working, the grey PID control algorithm is u+u, thus inputting the clutch control after the compensation. Wherein,

$$\begin{aligned}u_e&=-\left(\sum_{i=1}^{n}\hat V_i x_i+\hat f\right)\hat D(x,t)\\&=\sum_{i=1}^{n}(V_i-V_i)x_i\left(f(t)-\hat f\right)\end{aligned}. \tag{22}$$

5 Numerical simulation

In order to initially verify the PID control algorithm based on grey prediction theory's location tracking efficiency in the DC motor, the system is simulated with MATLAB simulating software. According to the selected related parameter of DC motor in Formula (6), the motor's transfer function is:

$$G(s)=\frac{\theta(s)}{U(s)}=\frac{1/C_e}{s(T_m s+1)}=\frac{86}{s(0.01s+1)}. \tag{23}$$

In the simulation process, the specific simulation parameter is as follows: V1=[5 5], f=5, kp=30, kd=5.0. The simulation result is as shown in FIGURE 3-8, Figure 3-5 are the location tracking under the traditional PID control, error and controller output situation respectively, Figure 6-8 are location tracking under the grey prediction PID control, error and controller output situation respectively.

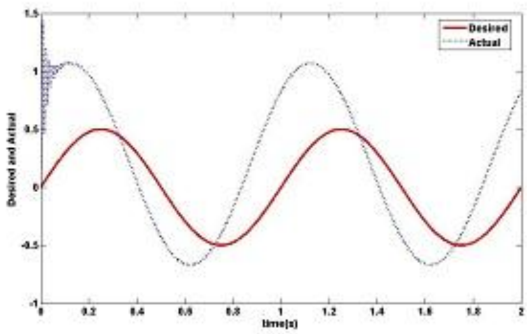

FIGURE 3 Traditional PID control location tracking

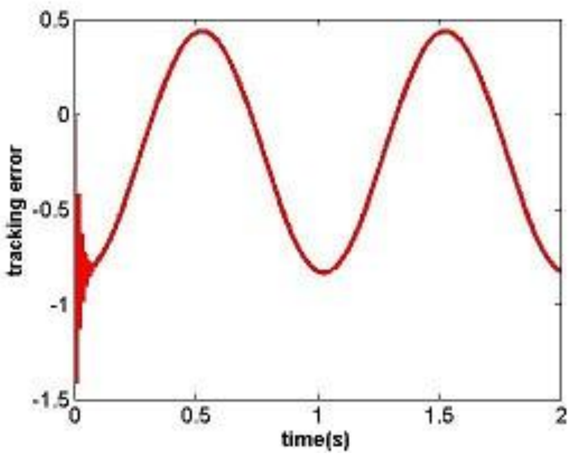

FIGURE 4 Traditional PID control location tracking error

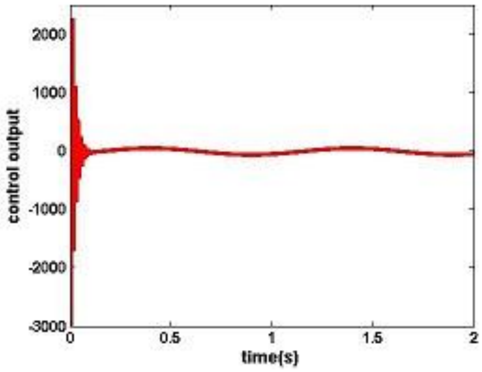

FIGURE 5 Traditional PID control output

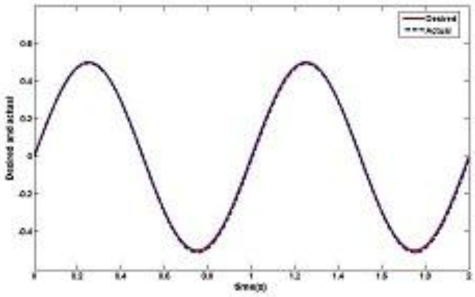

FIGURE 6 Grey PID control location tracking

According to the above simulation result, it can be seen that under the same simulation environment and parameter condition, the PID control algorithm based on grey prediction theory showed a good tracking performance in the location tracking process in DC motor. Compared with the traditional PID control algorithm, PID control algorithm based on grey prediction theory can effectively use the grey theory's processing capability on the unknown information data. The uncertain amount (parameter uncertainty and disturbance) in the system is established the grey model and effectively predict and compensate the unmodelled feature and disturbance signal in DC motor servo system, thus improving the control accuracy of the controller.

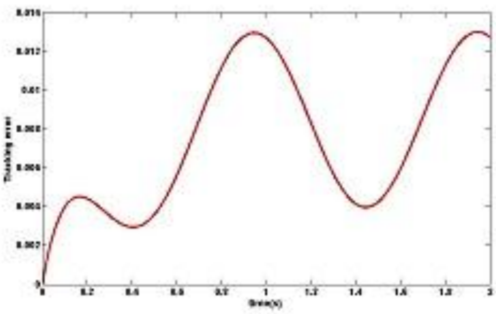

FIGURE 7 Grey PID control location tracking error

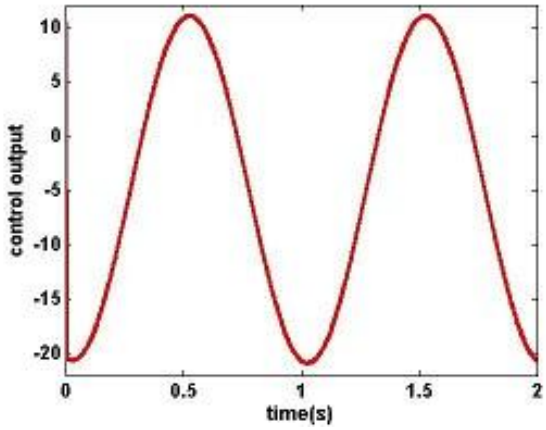

FIGURE 8 Grey PID control output

6 Conclusions

Concerning the uncertainties may be existed in DC motor servo system and the control quality that the external disturbance may influence the control algorithm of traditional PID, a kind of PID control algorithm combined the grey prediction theory and traditional PID control algorithm was proposed and applied in DC motor servo system. With the grey theory's processing ability on the unknown information data, the proposed control algorithm established the grey model for the uncertainties, and real-time compensated the system's unmodelled feature and disturbance signal. The simulation result can show that under the same simulation environment and parameter condition, compared with the traditional PID control algorithm, the proposed PID control algorithm based on grey prediction theory showed a good tracking performance in the location tracking process of DC motor. It can effectively use grey theory's processing capacity on the unknown information data. The uncertain amount (parameter uncertainty and disturbance) in the system is established the grey model and effectively predict and compensate the unmodelled feature and disturbance signal in DC motor servo system, thus improving the control accuracy of the controller.

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