Modeling of the subsystem of estimation of navigational parameters in automatic vehicle control systems

A Mrochko

ISMA University, 1 Lomonosova Str., Bld 6, LV-1019, Riga, Latvia

**Corresponding author's e-mail: aleksandrs.mrocko@isma.lv*

Abstract

One of the most actively developing spheres of applying modern information technologies is transport. Divisions of different services, departments and organisations are actively introducing and employing the system of *Automatic Vehicle Location* (AVL). The information systems of Automatic Vehicle Location solve the task of controlling and guiding transport means. Employing modern telecommunication technologies along with satellite navigation systems (SNS) facilitates and improves controlling of the mobile objects (MO). Modelling of the work and analysis of these systems' efficiency indicators sufficiently reduces the periods and costs of their testing and introducing in a particular region.

Keywords: information technologies satellite navigation systems mobile objects controlling AVL systems

1 Introduction

Active development of transport systems all over the world and a great increase of the variety of the provided services have led to the formation of an applied complex area of transport-dispatch information technologies, which basics are the following:

- satellite navigation systems that measure the main navigation parameters of the MO (coordinates, speed and direction of movement);
- modern telecommunication systems that transmit the necessary information to the dispatch center and other traffic participants;
- cartographic and special software solving the problem of accumulation, conversion, storage and submission of information on board the mobile object and dispatch center;
- onboard sensors of information and information mapping equipment.

The main consumers of these services are air, road, rail and marine transport means. The system of monitoring of mobile objects in real time improves the efficiency of cargo and passenger transportation, as well as ensures the safety of transport means, cargo, passengers and crew.

The efficiency of employing transport means greatly depends on the efficiency of their informational support. All this explains the increased number of papers considering the methods of estimating the indicators of the efficiency of Automatic Vehicle Location Systems. The main indicators of the efficiency of the mobile objects surveillance systems are characteristics of accuracy and reliability (integrity, availability and continuity of service). The increased requirements to these systems' characteristics are achieved by means of employing, particularly, the technologies of global satellite navigation systems (SNS), Currently we are watching the accumulation of the world experience in applying global radio navigation systems (GPS and GLONASS) and technologies based on these systems.

At the end of the XX century began testing and introducing the above systems in different regions of the world. The aim of the research was to provide accurate positioning information for all transport means located in the system working zone independently from the meteorological and topographic conditions and with the required rate of the data renewal. Some experience in building and testing such systems was received in realizing the CARD (CNS Applications Research and Development) project. The main research performed in the frame of this project is devoted to evaluating the characterristics of the data transmission line and developing the methods of estimating the efficiency indicators.

The first step in developing the methods of estimating the efficiency indicators should apparently be the analysis of errors of navigation – time determinations (NTD) of the SNS.

2 Analysis of errors of navigation – time determinations

The navigation task to be solved in the user's equipment (UE) SNS, in its simplest case, lies in defining space – time coordinates $P(t) = \left| x; y; z; W \right| T$. In the latest samples of UE there is adopted a two stage procedure of processing information. At the stage of primary procession they perform those measurements of navigation parameters (distance - D, speed of distance change - $\dot{D}$, etc.), which are only functionally connected with the state vector P(t). At the stage of secondary procession the received parameters are subjected to transformation based on navigation algorithms with the purpose of calculating vector P(t).

The accuracy of determining by the SNS user the location coordinates (x; y; z), the speed (W) and other parameters is influenced by many factors. They are connected with the peculiarities of primary and secondary navigation measurements, with the characteristics of the used signals and the media of propagation. To facilitate the influence of differrent factors on the NTD quality at the primary stage of processing they introduce UERE (User Equivalent Range Error) and speed of its change UERRE (User Equivalent Range Rate Error), conditioned by the non-correlated constituents of measurement errors. Secondary navigation definitions are easily characterized with help of geometric factors indicated as various DOP (Dilution of Precision).

Consider the main sources of these measurements' errors in application to the adopted in GPS and GLONASS long distance method NTD. The expression of the measured distance to the i-satellite D_i in this case will look as follows:

$$Di = D_{0i} + \delta D_{NS} + \delta D_{RL} + \delta D_{UE} ,$$

where D_{0i} – true value of the distance to the i-satellite; δD_{NS} – errors introduced in navigation satellites (NS) and control measuring set (CMS); δD_{RL} – errors introduced in the radio line "NS - user"; δD_{UE} – errors introduced by UE SNS.

Errors introduced in NS and CMS (δD_{NS}) are conditioned mainly by the insufficient frequency – temporal and ephemerae support of NS.

- *Errors of the frequency – temporal support* are caused mainly by insufficient procedures of verifying and storing the board time scale (board clock – BC) of NS. For typical caesium board frequency patterns the given errors between the correction moments may be approximated as follows:[1]

$$\sigma^2(t) = 2{,}5 \times 10^{-21}(t-t_c) + 5{,}76 \times 10^{-26}(t-t_c)^2,$$

where t – current time; t_c – time of correction BC.

CMS SNS is correcting the BC in such a way that $\sigma(t)$ of the BC shift would not exceed 10 nc. Besides, in the intervals between the apparatus corrections, the algorithmic correction of the BC of the given satellite is performed in UE. And here the unpredictable deviations of the BC of the given satellite in relation to the Time System Scale may reach 1nc (0.3 m) in one hour interval.

- *Errors of the ephemerae support* are caused by inaccurate definitions of orbit parameters NS in CMS and unpredictable shifts of NS in relation to the extra polar orbit. In SRNS GPS the average quadrant value of the ephemerae constituents UERE makes up about 1m [2].

- *Errors introduced in the radio line «NS – user» (δD_{RL})* are caused by insufficient knowledge of the ways of radio waves propagation in the Earth's atmosphere (refraction of the satellite signals in the ionosphere and the troposphere).

- *Troposphere errors.*

Judging by the experimental data for GPS additional delays of the NS signal in the troposphere may reach 8…80 nc [2]. For average meteorological conditions (temperature, pressure and air humidity) the value of this delay is defined by the expression:

$$\Delta t_{TR} \approx \frac{K_T}{\sin\beta} \cdot \int_0^{S_T} (n-1)dS,$$

where K_T – parameter characterising the condition of the troposphere; β - angle of the NS place; n – coefficient of the radio waves refraction; S_T – length of the troposphere line sector.

Troposphere models used in SNS allow to reduce the troposphere errors up to nanosecond units. In compensating the troposphere refraction, the periodicity of the user corrections is determined by the speed of the corresponding delays' change, which in normal circumstances does not exceed 10 m/h.

- *Ionosphere errors.*

The additional delay in the ionosphere Δt_{ion} of signal SNS GPS with frequency – f may be estimated as

$$\Delta t_{ion} = \frac{A}{f^2},$$

where A – the coefficient characterising the features of the propagation media.

The value of this delay changes widely depending on the Earth region where the mobile object is located, the time of the day, the season of the year, the Solar and the geomagnetic activities, etc., and makes up 5-500 ns [4]. The average value of Δt_{ion} for GPS makes up 5-10ns at night and 30…50ns in daytime for the angles of place β reaching 90^0. At $\beta < 15^0$ this delay increases by2-3 times.

- *Errors caused by the pass variety (multi pass).*

These errors are mainly dependent on the mutual location of the satellite, the receiving antenna and the reflecting objects. Experimental research has shown wide range of values of the long distance errors due to the rays variety, which makes up the best 0,5-2m (using special antennas) and up to 100m the worst in urban high buildings conditions. In most unfavourable conditions failure of surveillance can be incurred.

- *Errors introduced by UE SNS (δD_{UE})* are caused by errors in the surveillance at the moment of the satellite signal incoming. Typical error of UE makes up about 1,5-10m – for the standard GPS accuracy code.

The analysis of errors of primary navigation parameters estimation using SNS has shown that their summarising value ($2\sigma_D$) can reach 100 m and exceeds the limits of acceptable values adopted for most applications of AVL systems. Substantial reduction of NTD errors (by up to ten times) can be possible by using the differential SNS work mode. The basis of the differential method is relative stability of the considerable part of the SNS error in time and space. The main slightly varying errors of defining distance in SNS are [1]:

– errors in NS synchronisation;
– errors caused by the faulty ephemerae support of NS;
– non-compensated ionosphere errors.

Errors in NS synchronisation are constant in space and quite stable in the considered temporal intervals. Fluctuation of board clock NS by about 10^{-14} or 10^{-13} in the time of up to 15 min results in distance errors from 3 mm to 3 sm.

The error effect of the ephemerae information (ξ_i) can be characterised by the following model:

$$\xi_i < \frac{Ld_i}{D_i},$$

where d_i – error of the i - NS ephemerae (typical d_i value for GPS makes up 10 m); L, km - UE distance from the control point.

Calculations on this model show that space variation of distance measurement errors caused by insufficient ephemerae information is not substantial and at $d_i = 10$ m and $L < 200$ km does not exceed 10 sm and $L < 1000$ km - $\xi_i < 50$ sm. It is worth noting that with the data about the satellites constellation "aging" the errors of the ephemerae (d_i) also increase and, therefore, ξ_i increases.

Variation of ionosphere errors in time and space are characterised by the correlation function, which has times and space correlation radiuses at the level corresponding to several minute and thousands kilometres [4]. There is some of the experimental data of the temporal fluctuations of distance (D_i) errors caused by ionosphere [1], for example, in 1 min variation made up 0,1-0,2 m (σ), and in 6 min – 0,3-1,4m.

Differential work mode of SNS makes it possible to define and compensate the above errors. At the same time the main sources of errors in evaluating distances are noise constituents of UE, which in measuring curving delays make up metres and in measuring carrying delays –

centimetres and even millimetres. Another feasible source of errors is the property of multi pass.

• *Geometric factor in SNS*. Calculating the user's space – temporal coordinates is performed at the second stage of processing the NS signals. The ratio between the vector of errors in defining space – time coordinates P(t) $\delta_M = |\delta_x \delta_y \delta_z \delta_D|$ and the vector of errors in measuring distances $\delta_M = |\delta_{D1}\delta_{D2}\delta_{D3}\delta_{D4}|^T$ depends on the geometry of the corresponding location of NS and the user. Due to some special peculiarities of the NS and the user's space locations, the measure of decreasing the accuracy of navigational definitions in SRNS is the geometric coefficient GDOP (Geometric Dilution of Precision). The most important characteristic of SNS is the precision of place location, therefore for a surface mobile object the Horizontal Dilution of Precision (HDOP) is more often used

$$HDOP = \frac{\sqrt{\sigma_x^2 + \sigma_y^2}}{\sigma_D}.$$

The orbital configuration characteristics of satellites GPS provide, with the probability of 0.999 plus, the field of vision in a global working zone in any 24 hour interval of four satellites plus, the average value of HDOP making up 1.5 [2]. Increasing the number of visible satellites makes it possible to achieve a good gain in the accuracy of evaluating navigational parameters.

3 Reliability of navigation – time determinations

Besides accuracy properties of SRNS we should also regard reliability indicators of NTD as the indicators of efficiency of SRNS functioning. The main characteristic of reliability – system integrity – is defined as the ability to detect inadmissible system performance deterioration with the preset probability and time lag of informing the users thereabout [3].

The analysis of the factors, which influence the SNS integrity, makes it possible to divide them into two categories.

To the easily detected failures, there belong the following:

- signal fading from the NS;
- the distorted structure of the signal, which does not allow the user to come in synchronization with the satellite;
- presence of the sign in the navigation message of NS, which prohibits using its navigation information.

The user detects such situations without any additional equipment and without additional calculations.

To the difficultly detected failures, there belong:

- unpredictable shift of the board time scale (board clock) of the navigation satellite
- drift of frequency of the satellite reference generator;
- drift of the carrier frequency of the signal transmitted by the satellite;
- drift of the NS from the orbit;
- incorrect ephemerid information.

Such failures of navigation satellites lead to the errors of navigation-time determinations. Therefore, the problem of integrity control and development of algorithms and methods of detecting the above failures are of great practical interest. The analysis of SNS errors is the basis for the optimizing methods of estimating the efficiency of the AVL systems.

4 Methods of estimating the radio navigation parameters (RNP)

The maximum exactness of determining the location of the MO can be achieved by a ranging measuring method or by a range-difference measuring one. The navigation parameters (NP) in this case are, correspondingly, either distance **D** or the difference between distances ΔD. Their values are received on the basis of estimating the vector of the radio navigation parameters $\vec{\lambda} = \{\lambda_U, \lambda_\psi, \lambda_\omega, \lambda_\tau\}$ of the received signal $S(t, \vec{\lambda})$

$$S(t, \vec{\lambda}) = U(t-\tau) \cdot \cos[\varpi \cdot t + \psi(t)].$$

The elements of the parameter $\vec{\lambda}$ may be amplitude U(t), phase ψ(t), frequency ω and the time of delay of signal τ .

Under real conditions of receiving signal $S(t, \vec{\lambda})$ the estimation of parameter $\vec{\lambda}$ is performed against the background of noises n(t) and disturbances. The value of the radio navigation signal here is not known beforehand and changes randomly due to the noises.

Task setting. The receiving equipment of the radio navigation system (RNS) receives an additive mixture of a signal and a fluctuation noise

$$\xi(t) = S(t, \vec{\lambda}) + n(t),$$

where $\vec{\lambda}$ - the vector of the radio navigation parameters subjected to estimation; n(t) – fluctuation white noise with the parameters M[n(t)] = 0, k(τ) = N_0 δ(τ)/2.

It is suggested that parameters $\vec{\lambda}$ at the interval of observance [0, T] are constant and the priori density of possibilities may be either unknown, or known partially, or known completely. It is needed, by the accepted realization of fluctuation $\xi(t)$, to estimate optimally at the interval of observance [0, T] the values of parameters λ_α, where α = U, ψ, ω, τ.

As the result of solving the task there should be received the algorithms and the structural scheme of the optimal parameters' estimation and calculated their exactness characteristics.

The optimal estimation of parameters $\vec{\lambda}^*$ depends on the chosen function of losses $Q(\vec{\lambda}, \vec{\lambda}^*)$, which form is determined by the physical essence of the task [5, 6]. While estimating the radio navigation parameters we most often use the quadric function of losses

$$Q(\vec{\lambda}, \vec{\lambda}^*) = (\vec{\lambda} - \vec{\lambda}^*)^T \cdot (\vec{\lambda} - \vec{\lambda}^*).$$

Among the most frequently used methods for estimating NP, we should point out: the Bayesian method, the maximum Likelihood method and the method of Least squares.

Bayesian method. NP estimation is performed on the basis of the posteriori probability density with priori information about the values of the parameters estimated

$$W_{PS}(\vec{\lambda}) = c \cdot L(\vec{\lambda}) \cdot W_{PR}(\vec{\lambda}),$$

where $W_{PS}(\vec{\lambda})$, $W_{PR}(\vec{\lambda})$ - are correspondingly the posteriori and priori densities of possibilities of the parameters estimated; $L(\vec{\lambda})$ - likelihood function; c – constant coefficient. For the quadratic loss function, estimation by the given method corresponds to the mathematical expectation of the posteriori distribution [5]

$$\vec{\lambda}^* = \int_{-\infty}^{\infty} \vec{\lambda} \cdot W_{PS}(\vec{\lambda}) d\vec{\lambda} .$$

Maximum Likelihood method. It is used to receive estimation $\vec{\lambda}^*$ without priori information about the characteristics of the parameters estimated. The optimal estimation $\vec{\lambda}^* = \vec{\lambda}^*_{opt}$ is the root of the equation

$$\frac{d \ln L(\vec{\lambda})}{d\vec{\lambda}} = 0$$

The above method of estimating the navigation parameter $\vec{\lambda}$ allows us getting only optimal algorithms of processing the received fluctuation $\xi(t)$. It does not give quantitative characteristics of the algorithm's estimation exactness. For their determination we can use the method of consecutive approaches along the smaller parameter or Cramer-Rao Inequality [5]. For the smaller parameter we use the relation signal/noise.

Method of Least squares. When the radio navigation parameter $\vec{\lambda}$ is the linear function of the measurements $\xi(t)$ we can use for its estimation the method of least squares. We can use it, for example, for estimating the amplitude of the signal and the time of its delay at the impulse method of measurements of the navigation parameter.

The analytical record of the fluctuation $\xi(t)$ can be presented in the form

$$\vec{\xi}(t) = S\vec{\lambda} + \vec{V}, \quad (1)$$

where $\xi(t)$ - m-measure vector of measurements; $\vec{\lambda}$ - n-measure vector of the navigation parameters (m $\geq$ n); S – the known matrix of observances of size m×n; $\vec{V}$ - vector of the noises of measurements.

In the equation (1) the error of measurements $\vec{V}$ has zero mathematical expectation $M\{\vec{V}\} = 0$ and the covariance matrix $V = M\{\vec{V}, \vec{V}^T\}$. It is suggested that vectors $\vec{V}$ and $\vec{\lambda}$ are non-correlated $M\{\vec{\lambda}, \vec{V}\} = 0$, other priori information on the parameter $\vec{\lambda}$ and the process $\xi(t)$ lacking.

According to the considered method, we need to determine such value of the estimation $\vec{\lambda}^*$ at which the sum of the squares of faults $\tilde{\vec{\xi}} = \vec{\xi} - S\vec{\lambda}^*$ reaches its maximum

$$\left\| \tilde{\vec{\xi}}^2 \right\| = \tilde{\vec{\xi}}^T \tilde{\vec{\xi}} = \left(\vec{\xi} - S\vec{\lambda}^*\right)^T \left(\vec{\xi} - S\vec{\lambda}^*\right) \to \min . \quad (2)$$

Leveling to zero the quotient derivatives on the parameter $\vec{\lambda}$ in the expression (2) we get the optimal estimation of the parameter

$$\vec{\lambda}^* = \left(S^T S\right)^{-1} S^T \vec{\xi} = pS^T \vec{\xi} ,$$

where $p = \left(S^T S\right)^{-1}$.

Comparing the above methods by their exactness shows that for larger relations the estimations signal/noise received by the maximum likelihood method and the Bayesian method appear to be pretty close [5]. Though, the maximum likelihood method is more preferable for estimating HP in the case of no priori information regarding the estimated parameters. It is caused by its relative simplicity and asymptotically efficient estimation at increasing the signal/noise ratio (S/N ratio).

It should be noted that during long time of observances the distribution of the $\vec{\lambda}^*$ assessment is normal. In this case, the maximum likelihood method and the method of least squares are identical and dispose of the least fault of estimation determined by the lower border of *Cramer-Rao inequality*.

5 Estimation of the radio impulse radio navigation parameters

On the basis of the maximum likelihood method, we get the estimation of phase (φ^*) and the temporary state of radio impulse (τ^*)

$$S(t,\tau) = A_0 \cdot g(t-\tau) \cdot \cos(\omega t + \phi) .$$

The likelihood equation from which we deduct the algorithm of performance of the optimal phase measurer has the form [5]:

$$\frac{d}{d\phi} \ln F(\phi) = \frac{d}{d\phi} \left[\frac{2}{N} \int_0^T \xi(t) \cdot U(t-\tau) \cdot \cos(\omega t + \phi) dt \right]_{\phi^*} = 0 , \quad (3)$$

where F(φ) – functional of the likelihood, N – noise spectral density.

From (3) there goes

$$\int_0^T \xi(t) \cdot \sin(\omega t + \phi^*) dt = 0 . \quad (4)$$

The expression (4) is approximately modeled by the system phase automatic frequency.

The variance phase estimation, obtained by the method of successive approximations in the small parameter, equals

$$\sigma^2_{\varphi^*} = \frac{N}{2E} ,$$

where E – the signal energy.

Analogically, we can get the estimation of the temporary state of a known form impulse $U(t-\tau) = A_0 \cdot g(t-\tau)$ without the inter-impulse phase or frequency modulation. The likelihood functional for estimating parameter $\boldsymbol{\tau}$ is recorded in the form:

$$F(\tau) = \exp\left\{\frac{2A_0}{N}\int_0^T \xi(t-\tau)\cdot g(t-\tau)dt\right\}.$$

An equation for estimating $\boldsymbol{\tau}^*$ as

$$\int_0^T \xi(t)dt = \frac{1}{\beta^2 \cdot \dfrac{2E}{N}}.$$

The variance estimation of the temporal position is expressed as:

$$\sigma^2_{\tau^*} = \frac{1}{\beta^2 \cdot \dfrac{2E}{N}},$$

where β^2 - value that specifies the width of the spectrum of the envelope radio impulse.

For the real rectangular pulse limited by spectrum band Δf, the value $\beta = \dfrac{2\Delta f}{t_{imp}}$, and the variance parameter $\boldsymbol{\tau}^*$ estimation

$$\sigma^2_{\tau^*} = \frac{t_{imp}}{2\Delta f \cdot \dfrac{2E}{N}}. \tag{5}$$

In [5, 6] we get the expressions for the dispersion of the estimation of the temporary state of different form radio impulses. The analysis shows that at the fixed energy of the signal the exactness of estimation of the temporary state of the radio impulse increases with the increase of the signal specter width and with the decrease of the impulse length.

Also, the exactness of the estimation of the radio impulse temporary state can be increased by applying the inter-impulse modulation, for example, by pseudo random succession. In this case, if the energy of signal $\boldsymbol{E}$ remains constant, the width of the radio impulse specter increases and the dispersion of the estimation decreases correspondingly (5).

On the other hand, the exactness of estimation of the radio impulse temporary state can be increased by multiple repletion of measuring parameter $\boldsymbol{\tau}$ and further processing of the obtained selection by one of the above estimation methods. Multiple measurements can be obtained by applying the mode of the space recirculation of the signal.

Optimal estimation of the distance by using the method of least squares on selecting volume $\boldsymbol{\kappa}$ is given by the expression (6)

$$D_k^* = pS^T \vec{D}_k, \tag{6}$$

where p = $(S^T S)^{-1}$.

With the account of the fact that the distance measurements are scale ones and the size of the matrix is S^T k×1 estimation of the distance by the method of least squares coincides with the selection average

$$D_k^* = \frac{1}{k}\sum_{i=1}^{k} D_i$$

and is asymptomatically efficient with the increase of the number of recirculation cycles.

Errors of distance estimation at the **κ** cycles of recirculation are determined by the covariant matrix

$$V_K = (S^T S)^{-1} S^T V S (S^T S)^{-1},$$

where V – covariant matrix of the measurement noises ΔD_i.

In case of scale measurement

$$V_K = \sigma^2_{D_K^*} = \frac{\sigma^2_{\Delta D}}{k}. \tag{7}$$

Thus, the variance of the distance estimation decreases in reverse proportionality to the number of recirculation cycles. The lower border of the estimation variance is determined by the *Cramer-Rao inequality* [5, 6] and coincides, in this case, with the value (A.7).

It should be noted that the increase of the number of recirculation results in the decrease of the fluctuation measurement error, though it is accompanied with the increase of the dynamic error since the interval of averaging is increased. The analysis of the modeling results has confirmed our supposition about the existing optimal number of recirculations at which the errors of estimating the location of the MO would be minimal. Since the nature of the fluctuation and dynamic errors in measuring distance is different they can be considered independent and non-correlated ones. It gives possibility of analyzing the ways of decreasing the constituent errors independently for each other.

Analysis of sources of errors and estimation methods of navigation parameters sufficiently reduces the periods and costs of testing and introducing the *automatic vehicle control* systems in a particular region.

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Author

Alexander Mrochko, 1957, Kiev, Ukraine

Current position, grades: Dr. Sc. Ing., Ass.prof
University studies: Information Systems Management Institute
Scientific interest:
Publications:
Experience: