Application of federated particle filter to SINS-GPS/BDS integrated navigation system

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Abstract

When integrated navigation information is filtered, there may be non-linear sub-filter. The paper proposes federated particle filter. It is based on the framework of federated Kalman filter and uses the method of particle filter to process non-linear sub-filter, which enhances adaptability of federated Kalman filtering model. The paper applies federated particle filter to SINS-GPS/BDS integrated navigation system to establish filtering model. The simulation is made to verify the effectiveness of federated particle filter.

Keywords: federated Kalman filter, particle filter, integrated navigation

1 Introduction

In recent years, multi-sensor integrated navigation system with inertial navigation as core has developed rapidly. For example, based on inertial navigation system, GPS and BDS are introduced. Compared with single system, the advantage of integrated navigation is that navigation information of various sub-systems can be integrated [1]. Each system observing the same information sources makes redundancy of measured value increase, which enhances reliability and stability of navigation system. In order to develop the advantages of various navigation systems and enhance stability and accuracy of the system, the high-accuracy and adaptable data integration technique must be applied.

The common data integration methods for navigation system are Kalman filter and some improved algorithms [2]. There are two ways applying Kalman filter technique to data integration for navigation system, centralized Kalman filter and decentralized Kalman filter. For centralized Kalman filter, information fusion is full and filtering accuracy is high. But it includes error conditions of all sub-systems, which not only makes state dimension high and makes calculation great, but also is not good for real-time filter and is bad for fault detection and isolation. Federated Kalman filter is a decentralized Kalman filter. It uses the principle of information distribution. And it can achieve good compromise under different performance requirements. It has the advantages of parallel data process, flexible design, simple calculation and good fault tolerance, which makes it widely applied to integrated navigation system.

The conventional FKF uses the filtering framework of Kalman Filter. It requires that the senior filter and sub0-filters should meet linear gauss assumptions. But it is very difficult to meet the requirement in navigation system. When there is non-linear element in the system, the conventional KF needs to linearize non-linear element, and the structure of filter needs to be redesigned, which has an influence on filter performance of system.

In order to solve the problem that conventional Kalman filter appears non-linear sub-filter, the paper proposes a new FPF algorithm. The algorithm uses particle filter method to process non-linear sub-filters, and integrates the information of sub-filters in senior filter, which not only achieves good filter effect, but also improves the performance of integration navigation system. The paper takes integration navigation multi-integration problem as an example, and establishes federated particle filter model. And the simulation is made to verify the effectiveness of the algorithm.

2 Design of federated particle filter

2.1 PARTICLE FILTER

Particle filter is a non-linear filtering algorithm. It uses a group of discrete weighted particles to simulate posterior probability. It completes the filtering process by predicting state, updating weight and resampling [3].

The random-state space model is: $\begin{cases} x_{\theta+1} = f(x_\theta) + \omega_\theta \\ y_\theta = h(x_\theta) + v_\theta \end{cases}$,

x_θ and y_θ are state and measurement information of the system. Map $f(\bullet)$ and $g(\bullet)$ are transference model function and measurement model function of system state. ω_θ and v_θ are process noise and measurement noise.

The ultimate goal of filter is to be based on observation sequence $y_{1:\theta} = \{y_i, i = 1,\ldots,\theta\}$ to estimate posterior probability distribution function $p(x_\theta / y_\theta)$ recursively. From Bayesian framework and Monte Carlo

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implementation method, we can summarize the algorithm flow of particle filter.

1) Initializing particles and weights. When $\theta = 0$, the particles are extracted from prior density functions to constitute sample set: $\left\{x_0^i \sim p(x_0), d_0^i = 1/N; i = 1,\ldots,N\right\}$

2) Importance sampling. When $\theta \geq 1$,

a) Extracting N samples $\left\{x_\theta^i; i = 1,\ldots,N\right\}$ from importance density functions.

b) Computing weight of each particle, $d_\theta^i = d_{\theta-1}^i p(y_\theta / x_\theta^i)$.

c) Normalizing weights. $\tilde{d}_\theta^i = d_\theta^i / \sum_{i=1}^{N} d_\theta^i$

3) Resample. Computing the number of effective samples $N_{eff} = round\left(1/\sum_{i=1}^{N}(\tilde{d}_\theta^i)^2\right)$, where round (•) is the rounding operation. When $N_{eff} < N_{thr}$, the particles are resampled, and *N-th* is the given threshold, which can get the new sample set: $\left\{x_\theta^i \sim N(x_\theta^i; \hat{x}_\theta^i, P_\theta^i), i = 1,\ldots, N\right\}$.

4) Output. State and variance is estimated by using the following Equation:
$$\begin{cases} \hat{x}_\theta = E(x_\theta / y_\theta) \approx \sum_{i=1}^{N} \tilde{d}_\theta^i x_\theta^i \\ \hat{P}_\theta = \sum_{i=1}^{N} \tilde{d}_\theta^i (\hat{x}_\theta - x_\theta^i)(\hat{x}_\theta - x_\theta^i)^T \end{cases},$$
$\theta = \theta + 1$, and returning to the second step.

2.2 DESIGN OF FEDERATED KALMAN FILTER

The architecture of federated Kalman filter is shown in Figure 1. Compared with conventional federated Kalman filter, the sub-filter of it is nonlinear filter consisting of particle sub-filter. As the common reference system, the output information of it and GPS, SINS composes sub-filter. The local estimation $\hat{X}_i$ and covariance matrix K_i of each sub-filter is loaded into senior filter. It is integrated with the estimated value of senior filter, which can get the optimal estimation. From the algorithm principle and flow of particle filter, we can see that particle filtering process achieves posterior distribution of state parameter, which can get the mean and variance information of the state parameter [4]. So the nonlinear sub-filter can be integrated into the framework of federated Kalman filter.

Federated Kalman filter only makes global filter on common state parameter of each system [5]. Applying the method of information distribution makes estimate status of local filters processed by irrelevant ways, which simplifies the flow of filtering algorithm. By combining particle filter and conventional federated Kalman filter, the paper gives the algorithm flow of federated Kalman filter.

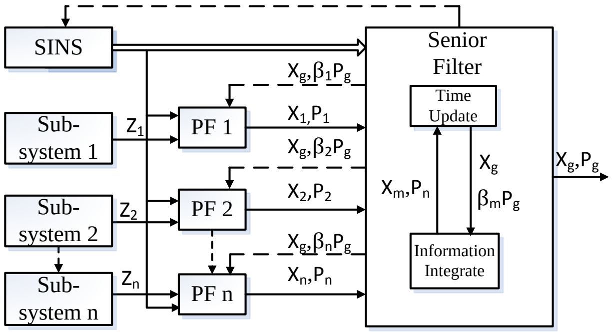

FIGURE 1 Structure of federated Kalman filter

1) Initializing particles and weights. When $\theta = 0$, the particles are extracted from prior density functions to constitute sample set: $\left\{x_{0j}^i \sim p(x_0), d_0^i = 1/N; i = 1,\ldots,N, j = 1,\ldots,M\right\}$, where N is the number of particles, and M is the number of sub-filters.

2) Information distribution process. According to the following formula, federated Kalman filter distributes the information of initial value of the combined system to local filters: $\begin{cases} \hat{X}_i = \hat{X}_g \\ K_i = \gamma_i^{-1} K_g \\ R_i = \gamma_i^{-1} R_g \end{cases}$, where γ_i is information distribution coefficient, and meets $\sum_{i=1}^{N} \gamma_i + \gamma_m = 1$. Estimation error covariance matrix K meets $\sum_{i=1}^{N} K_i^{-1} + K_m^{-1} = K_g^{-1}$. System noise covariance matrix R meets $\sum_{i=1}^{N} R_i^{-1} + R_m^{-1} = R_g^{-1}$.

3) According to state equation, each sub-filter makes filter. When $\theta \geq 1$,

a) Extracting N samples $\left\{x_\theta^i, i = 1,\ldots,N\right\}$ from importance density function: $q(x_\theta^i \mid x_{\theta-1}^i, y_\theta) = N(x_\theta^i; \hat{x}_\theta^i, K_\theta^i)$.

b) Computing weight of each particle, $d_\theta^i = d_{\theta-1}^i p(y_\theta \mid x_\theta^i)$.

c) Normalizing weights. $\tilde{d}_\theta^i = d_\theta^i / \sum_{i=1}^{N} d_\theta^i$.

d) Computing the number of effective particles: $N_{eff} = round\left(1/\sum_{i=1}^{N}(\tilde{d}_\theta^i)^2\right)$.

e) Resample. If $N_{thr} \leq N_{eff} \leq N$ (N is the given threshold), particle filter algorithm is used for estimation [6]. If $N_{eff} \leq N_{thr}$ it indicates that particle degeneracy is serious, and the particles need to be resampled, which can get the new sample set $\left\{x_\theta^i \sim N(x_\theta^i; \hat{x}_\theta^i, K_\theta^i) i = 1,\ldots,N\right\}$.

f) Status update. The particles and weights are used to compute output state estimation and variance, as follows:

$$\begin{cases} \hat{x}_\theta = \sum_{i=1}^{N} \tilde{d}_\theta^i x_\theta^i \\ \hat{K}_\theta = \sum_{i=1}^{N} \tilde{d}_\theta^i (\hat{x}_\theta - x_\theta^i)(\hat{x}_\theta - x_\theta^i)^T \end{cases}.$$

4) Integrating global information. After getting local estimation and estimation of senior filter of each sub-filter, the following formula is used for integration, which can get global state filter and estimator of variance.

$$\begin{cases} \hat{x}_g(\theta) = K_g(\theta)\sum_{i=1}^{N}(K_i^{-1}(\theta)\hat{x}_i(\theta)) \\ K_g(\theta) = (\sum_{i=1}^{N} K_i^{-1}(\theta))^{-1} \end{cases}.$$

5) After obtaining global state and estimator of variance, the formula in the second step is used to allocate and reset local filters according to information distribution principles.

6) $\theta = \theta + 1$ and it returns to the third step for repeating the above steps.

3 INS-GPS/BDS integration navigation system filter model

The structure diagram of federated particle filter of SINS-GPS/BDS integration navigation system is shown in Figure 2. SINS/GPS and SINS/BDS composes nonlinear PE sub-filter by the combination mode of position and velocity.

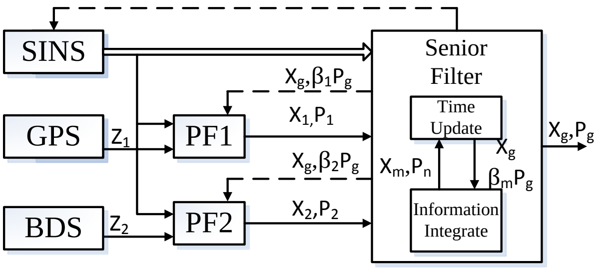

FIGURE 2 Structure of integration navigation filter

3.1 STATE EQUATION

The state variable of the selected system is: $X(t) = [\varphi_e, \varphi_n, \varphi_u, \delta v_{ie}, \delta v_{in}, \delta v_{iu}, \delta L_i, \delta\lambda_i, \delta h_i, \varepsilon_{bx}, \varepsilon_{by}, \varepsilon_{bz}, \nabla_{bx}, \nabla_{by}, \nabla_{bz}, \delta t_u, \delta t_{ru}]^T$.

The sensor error model and inertial navigation equation is used to establish state equation. ϕ_e, ϕ_n, ϕ_u means, that attitude angle error, $\delta v_{ie}, \delta v_{in}, \delta v_{iu}$ means velocity error, $\delta L_i, \delta\lambda_i, \delta h_i$ represents position error, $\varepsilon_{bx}, \varepsilon_{by}, \varepsilon_{bz}$ means constant drift of gyroscope, $\nabla_{bx}, \nabla_{by}, \nabla_{bz}$ means accelerometer zero offset, δtu is equivalent distance error caused by clock error, and δtru is equivalent distance error caused by clock error frequency. The noise of state equation is: $W = [\omega_{rbx}, \omega_{rby}, \omega_{rbz}, \omega_{abx}, \omega_{aby}, \omega_{abz}]^T$.

The variable in the formula means the noise of gyroscope and accelerometer on coordinate system of carrier [7]. According to the description, the state equation of the system is $\dot{X}(t) = F(t)X(t) + G(t)W$ in which F(t) is state-transition matrix, X(t) is state parameter, G(t) is noise driving matrix and W is system noise matrix.

3.2 MEASUREMENT EQUATION

The measurement equation of SINS/GPS sub-filter is the same to that of SINS/BDS sub-filter model. The paper takes SINS/BDS as an example for analysis. SINS/BDS integration navigation uses the combination mode of pseudo-range and pseudo-range rate [8].

In earth coordinate system $Ox_e y_e z_e$, pseudo-range calculated by inertial navigation location is: $\rho_{Ii} = \sqrt{(x_I - x_{si})^2 + (y_I - y_{si})^2 + (z_I - z_{si})^2}$, where x_I, y_I, z_I is the location of $Ox_e y_e z_e$ system output by SINS, which is achieved by calculating longitude, latitude and height output by SINS. x_{si}, y_{si}, z_{si} is the location of the *i* satellite [9].

The pseudo-range measured by the *i* satellite for BDS is: $\rho_{Bi} = R_i + \delta t_u + v_{\rho i}$.

From the equation above, we can get pseudorange measurement value is: $\delta\rho_i = \rho_{Ii} - \rho_{Bi}$.

The measurement equation of pseudo range difference is: $Z_\rho(t) = [\delta\rho_1 .. \delta\rho_n]^T = H_\rho(t)X(t) + V_\rho(t)$, where $V_\rho(t)$ is the pseudo range measurement noise of system [10] and *n* is the number of visible stars.

4 Simulation of federated particle filter algorithm

The structure of SINS-GPS/BDS integrated navigation and federated particle filter is used to simulate the system. And the conventional federated Kalman filtering method is used to simulate the system. The simulation condition requires that the output frequency of inertial navigation is 50Hz, the output frequency of GPS and BDS is 1Hz, zero bias stability of gyroscope is 0.01°/h, and zero bias stability of accelerometer is the flight path consists of acceleration, climb and level flight. The initial position of plane is 39 degrees north latitude and 116 degrees eastern longitude. The height is 500m. The initial velocity is 350m/s, the number of particles is 2000 and the simulation time is 1200s.

Figure 3 is the velocity error of conventional federated Kalman filtering method and federated particle filtering method. Figure 4 is position error of two methods. We can see that the filtering effect of federated particle filter is evidently better than that of conventional federated Kalman filter. The reason for which is that conventional federated Kalman filter generally uses expanded Kalman filter to process nonlinear subfilter, and expanded Kalman filter uses the method of Taylor expansion to process nonlinear link, which ignores higher order term, linearizes nonlinear model and introduces linear error. But particle filter directly processes nonlinear model without introducing new error, so the accuracy of federated particle filter is better than that of conventional federated Kalman filter for processing integrated navigation system with nonlinear links.

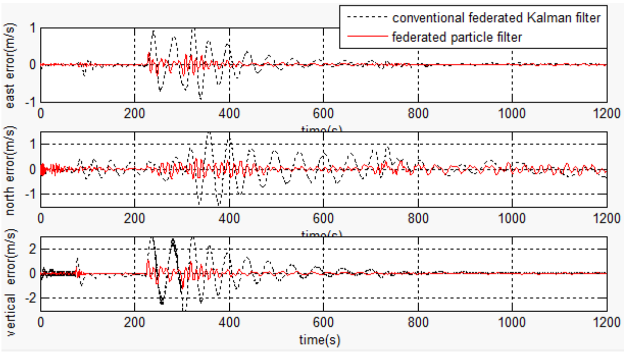

FIGURE 3 Velocity error curve of federated particle filter and conventional federated Kalman filter

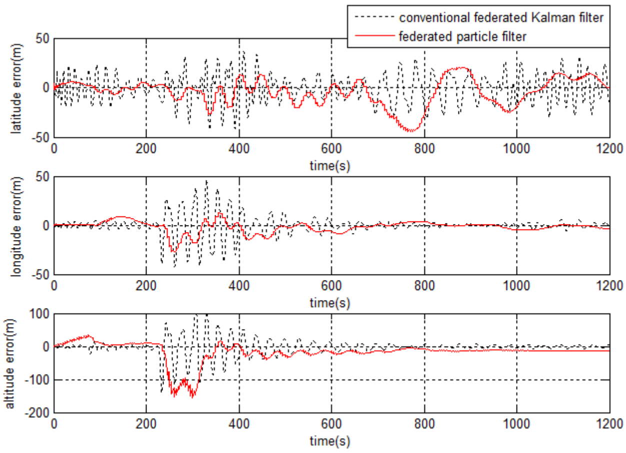

FIGURE 4 Position error of conventional federated Kalman filter and federated particle filter

5 Conclusions

In integrated navigation multi-information data processing system, federated Kalman filter has the characteristics of flexible design and good error resilience, which can solve multi-information data integration of integrated navigation system. The paper introduces particle filter into the structure of federated Kalman filter, which expands conventional federated Kalman filter to solve nonlinear system state and parameter estimation under complicated environment. Federated particle filter is applied to combination navigation, which provides a reference method for processing nonlinear problems in integrated navigation multi-information data processing system. The simulation verifies the effectiveness of algorithm.

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