Using cubature Kalman filter to estimate the vehicle state

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Received 1 March 2014, www.cmnt.lv

Abstract

The vehicle state is of significant to examine and control vehicle performance. But some vehicle states such as vehicle velocity and side slip angle which are vital to active safety application of vehicle can not be measured directly and must be estimated instead. In this paper, a Cubature Kalman Filter (CKF) based algorithm for estimation vehicle velocity, yaw rate and side slip angle using steering wheel angle, longitudinal acceleration and lateral sensors is proposed. The estimator is designed based on a three-degree-of-freedom (3DOF) vehicle model. Effectiveness of the estimation is examined by comparing the outputs of the estimator with the responses of the vehicle model in CarSim under double lane change and slalom conditions.

Keywords: cubature Kalman filter, vehicle state, 3DOF, CarSim

1 Introduction

A variety of active vehicle safety applications are being developed in modern cars to reduce driver burden and road accidents. Traction control system (TCS) and electronic stability program (ESP) are two popular active safety applications in vehicles. TCS concerned with controlling longitudinal motion of the vehicle and ESP concerned with controlling lateral motion of the vehicle. Traction control system works by controlling slip ratio of the four vehicle wheels. Although vehicle speed is required to calculate the slip ratio of the wheel in TCS, the absolute vehicle speed can not be accurately measured by wheel speed because of wheel slip. ESP works by controlling yaw rate and side slip angle of the vehicle. Side slip angle can not be measured directly. Due to these factors, vehicle speed and side slip angle are not directly measured on production cars and must be estimated instead.

Although there are other nonlinear observer [1-5] based study about vehicle state estimation, the main research activities in the field concentrate on the application of Kalman filter theory, which is the most powerful tool for multi-sensor data fusion problems [6]. In [7-9], Kalman filter is used to estimate yaw rate, lateral acceleration and tire slip angle with linear vehicle model. Since Kalman filter is based on linear stochastic differential equations, it can only be used in the linear system estimation. As a nonlinear filter, extended Kalman filter (EKF) extend the use of Kalman filtering through a linearisation procedure. Ray proposes an extended Kalman filter (EKF) based method for estimating vehicle speed, braking forces, wheel slip and side-slip angle [10]. A nonlinear extended adaptive.

Kalman filter is proposed for the estimation of vehicle handling dynamic states in [11]. In [12-13], dual EKF is used for vehicle state and parameter estimation. The EKF works well in many application, but may suffer from large estimate errors when system have strong nonlinearities, and also suffer from the computation burden of the Jacobians [14]. Unscented Kalman filter (UKF) is used to vehicle state estimation because it overcomes these hurdles [15]. The UKF reduces computational costs compared to EKF and needn't linearize the system and measurement equations as required by the EKF.

Recently, a cubature Kalman filter is proposed by Arasaratnam and Haykin, which improves the performance over UKF [16]. Since nonlinear filtering can be reducing to a problem of how to compute integral, cubature Kalman Filter introduce a third-degree spherical-radial cubature rule to achieve the cubature points which are used to approximate the multi-dimensional integral [14]. CKF has been proposed and used in many application, such as positioning [17-18] and attitude estimation [19]. For nonlinear system with additive Gaussian noise, cubature Kalman filer (CKF) can achieve more accurately than the UKF with similar computational complexity [20].

In this paper, we propose a CKF based estimator with a 3DOF vehicle is to estimate vehicle velocity, yaw rate and side slip angle. The inputs of the estimator are steering wheel angle, longitudinal acceleration and lateral acceleration with additive noise. Effectiveness of the estimation is examined by co-simulation between the software CarSim and Matlab-Simulink under double lane change and slalom conditions.

The rest of paper is structured as follow: The 3DOF vehicle model are described in Section 2. CKF based estimator is presented in Section 3. Our experiments and results are introduced in Section 4. Finally the main conclusion and future works are summarized in Section 5.

2 Vehicle model

The proposed method is based on a nonlinear 3DOF vehicle model, which is shown in Figure 1.

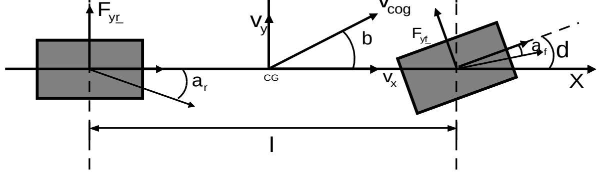

FIGURE 1 Nonlinear 3DOF vehicle model

Segel present a vehicle model with 3DOF in order to describe lateral movements including both roll motion and yaw motion [21]. By reducing the roll motion, a two-degrees of freedom linear bicycle model is obtained [22]. The linear two degrees of freedom related to vehicle body are yaw rate (r) and side slip (β). The motion of yaw rate is described as:

$$\dot{r} = \frac{a^2C_f + b^2C_r}{I_z v_x} r + \frac{aC_f - bC_r}{I_z}\beta - \frac{aC_f}{I_z}\delta, \tag{1}$$

where a is the distance from the front axel to the centre of gravity (CG), b is the distance from the rear axel to CG, C_f is the effective cornering stiffness of the front axel, C_r is the effective cornering stiffness of the rear axel, I_z is the vehicle moment of inertia about Z axis and δ is the steering wheel angle.

The motion of side slip is described as:

$$\dot{\beta} = (\frac{aC_f - bC_r}{mv_x^2} - 1)r + \frac{C_f + C_r}{mv_x}\beta - \frac{C_f}{mv_x}\delta, \tag{2}$$

where m is vehicle mass and v_x is the longitudinal velocity. In order to estimate the longitudinal velocity of the vehicle, longitudinal motion is required.

The longitudinal motion is described as:

$$\dot{v}_x = r\beta v_x + a_x. \tag{3}$$

Equations (1), (2) and (3) form the nonlinear three degrees of freedom of vehicle model. In this paper, $\begin{bmatrix}\gamma & \beta & v_x\end{bmatrix}^T$ is the state vector of the proposed estimator, and a_y is the measurement. The measurement equation is written as:

$$a_y = (\frac{aC_f - bC_r}{mv_x} - 1)r + \frac{C_f + C_r}{m}\beta - \frac{C_f}{m}\delta. \tag{4}$$

3 CKF state estimation

3.1 CUBATURE KALMAN FILTER

Kalman filter is a special case of the Bayesian filter, which assuming that the dynamic system is linear and both the dynamic noise and measurement noise are statistically independent processes [23]. Considering a nonlinear discrete-time system of the form

$$x(k) = f(x(k-1), u(k)) + w(k-1)), \tag{5}$$

$$y(k) = h(x(k-1), u(k-1), v(k)). \tag{6}$$

where $x(k)$ is a N-dimensional state vector, the output $y(k)$ is a M-dimensional vector, $u(k)$is the known control input, $w(k-1)$ and $v(k)$ are independent process and measurement Gaussian noise sequences with zero means and covariance Q and R respectively. The heart of the Bayesian filter is to compute multi-dimensional weighted integral of the form

$$I(f) = \int_{R^n} f(X)\omega(X)dX. \tag{7}$$

Since it's difficult to obtain the solution of the above integral, the challenge is to compute the integral numerically by finding a set of cubature point ω_i and ξ_i that approximates the integral $I(f)$ by a weight sum of function evaluations

$$I(f) \approx \sum_{i=1}^{m} \omega_i f(\xi_i). \tag{8}$$

Cubature Kalman Filter introduce a third-degree spherical-radial cubature rule to achieve the cubature point as:

$$\xi_i = \sqrt{n}\,[1]_i, \tag{9}$$

$$\omega_i = \frac{1}{2n} \qquad i = 1, 2, ..., 2n. \tag{10}$$

The entire algorithm is presented as follows:

1. Time update

Evaluate the cubature points

$$S(k-1) = chol\{P(k-1)\}, \tag{11}$$

$$\hat{X}_i(k-1) = S(k-1)\xi_i + X(k-1). \tag{12}$$

where $P(k-1)$ is associated covariance matrix, $chol\{\}$ denotes a Cholesky decomposition of a matrix.

Evaluate the propagated cubature points

$$X_i^*(k-1) = f(\hat{X}_i(k-1), U(k)). \tag{13}$$

Estimate the predicated state

$$\hat{X}(k) = \sum_{i=1}^{2n} \omega_i X_i^*(k-1). \tag{14}$$

Estimate the predicated error covariance

$$\hat{P}(k) = \sum_{i=1}^{2n} \omega_i X_i^*(k-1) X_i^*(k-1)^T - \hat{X}(k)\hat{X}(k)^T + Q. \tag{15}$$

2 Measurement update

Evaluate the cubature points

$$\hat{S}(k) = chol\{\hat{P}(k)\}, \tag{16}$$

$$\hat{X}_i(k) = \hat{S}(k)\xi_i + \hat{X}(k). \tag{17}$$

Evaluate the propagated cubature points

$$Y_i^*(k) = h(\hat{X}_i(k), U(k)). \tag{18}$$

Estimate the predicated measurement

$$\hat{Y}(k) = \sum_{i=1}^{m} \omega_i Y_i^*(k). \tag{19}$$

Estimate the innovation covariance matrix

$$P_{yy}(k) = \sum_{i=1}^{m} \omega_i Y_i^*(k) Y_i^*(k)^T - y(k)y(k)^T + R. \tag{20}$$

Estimate the cross-covariance matrix

$$P_{xy}(k) = \sum_{i=1}^{m} \omega_i \hat{X}_i(k) Y_i^*(k)^T - \hat{X}(k)\hat{Y}(k)^T. \tag{21}$$

Estimate the Kalman gain

$$K(k) = P_{xy}(k)P_{yy}^{-1}(k)\cdot \quad (22)$$

Estimate the updated state

$$X(k) = \hat{X}(k) + K(k)(Y(k) - \hat{Y}(k))\cdot \quad (23)$$

Estimate the corresponding error covariance

$$P(k) = \hat{P}(k) + K(k)P_{yy}(k)K(k)^T\cdot \quad (24)$$

The proposed sate estimation method is based on CKF, the block diagram of the method is shown in Figure 2. As can be seen in Figure 2, the state estimator is designed to estimate the vehicle state by using steering wheel angle, lateral and longitudinal acceleration signals.

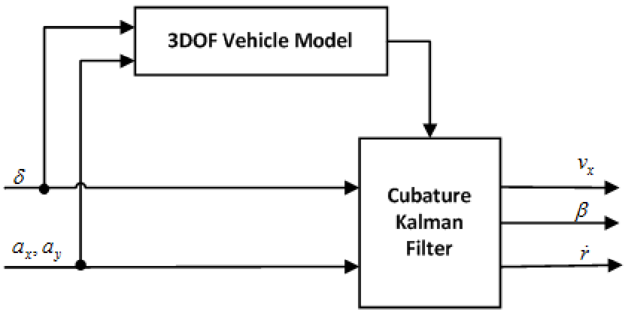

FIGURE 2 Estimation block diagram

For the state estimator, the state vector is written as

$$x(k) = \begin{bmatrix} \gamma & \beta & v_x \end{bmatrix}^T. \quad (25)$$

The measurement is written as

$$y(k) = a_y. \quad (26)$$

The known input is written as

$$u = \begin{bmatrix} \delta & a_x \end{bmatrix}^T. \quad (27)$$

The state vector equation of the proposed estimator can be written as:

$$\begin{cases} r(k) = (\frac{a^2k_f+b^2k_r}{I_z v_x}r(k-1) + \frac{ak_1-bk_2}{I_z}\beta(k-1) \\ \quad -\frac{ak_1}{I_z}\delta(k-1))\Delta t + r(k-1) \\ \beta(k) = (\frac{ak_f-bk_r}{mv_x^2}r(k-1) + \frac{k_f+k_r}{mv_x}\beta(k-1) \\ \quad -\frac{k_f}{mv_x}\delta(k-1))\Delta t + \beta(k-1) \\ v_x(k) = (r(k-1)\beta(k-1)v_x(k-1) + a_x(k))\Delta t \\ \quad +v_x(k-1) \end{cases}, \quad (28)$$

where $\Delta t = t_{k+1} - t_k$ is the sampling interval.

The measurement matrix is described as:

$$H = \begin{bmatrix} (ak_f - bk_r)/(mv_x) \\ (k_f + k_r)/m \\ -(ak_f - bk_r)/(mv_x^2) \end{bmatrix}. \quad (29)$$

4 Experiments

Two simulation cases under double lane change and slalom conditions are conducted based on Matlab/Simulink and CarSim. CarSim is a multi-DOF nonlinear simulation software for vehicle dynamics control and integration, and detailed mathematical models for simulating automotive vehicle dynamics have been in use for decades [24]. Since CarSim can work with Simulink, we build estimation model in Simulink and test it with the full nonlinear CarSim vehicle model. The Simulink Model for the proposed estimation is shown as Figure 3. The known parameters of the vehicle model are listed in Table 1.

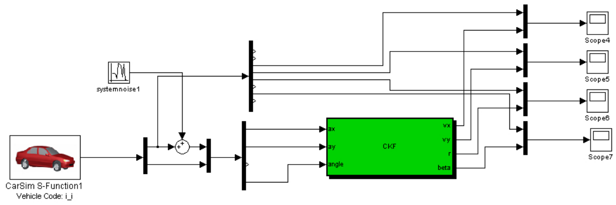

FIGURE 3 The Simulink model

TABLE 1 Specification of the vehicle model

Parameter	Symbol	Unit	Value
Vehicle mass	m	kg	1650
Vehicle moment of inertia about Z axis	I_z	kg m^2	3234
Distance from front axel to CG	a	m	1.4
Distance from rear axel to CG	b	m	1.65
Effective cornering stiffness of the front axel	C_f	N/rad	- 97000
Effective cornering stiffness of the rear axel	C_r	N/rad	- 120000

The process noise covariance of CKF is $Q = I_{3\times3}$, and measurement noise covariance is $R = [10000]$. The sampling interval is $\Delta t = 0.001s$.

4.1 DOUBLE LANE CHANGE TEST

The initialization of the State vector of the double lane change simulation case is $x(0) = [0, 0, 80]^T$. Simulation results are shown in Figure 4, Figure 5, Figure 6, Figure 7, Figure 8 and Figure 9. For the double lane change test, Figure 4, Figure 5 and Figure 6 are respectively the vehicle sensor signal of steering wheel angle, longitudinal acceleration and lateral acceleration.

As can can be seen from Figure 4, Figure 5 and Figure 6, all simulated sensor signals for CKF contain white noise which simulates the sensor noise in the real world. Figure 7, Figure 8 and Figure 9 are respectively the estimation of longitudinal velocity, side slip angle and yaw rate. As can be seen from Figure 7, Figure 8 and Figure 9, the estimated value of longitudinal velocity, side slip angle and yaw rate capture the trends in the data from CarSim. The additive noise of the sensor signal is filter by the CKF well.

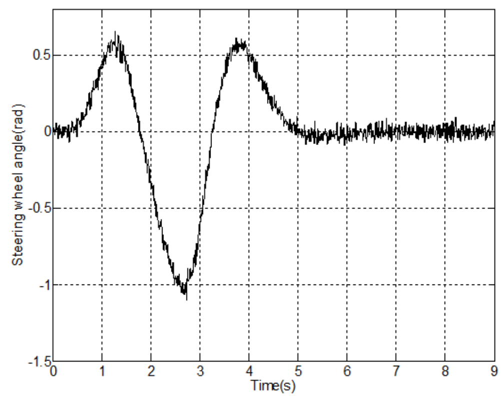

FIGURE 4 Steering angle with noise

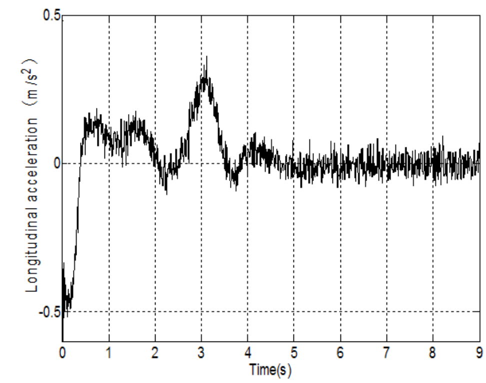

FIGURE 5 Longitudinal acceleration with noise

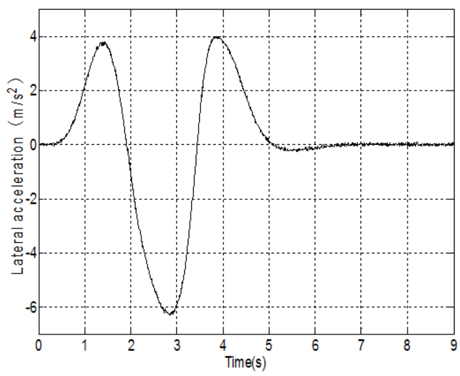

FIGURE 6 Lateral acceleration with noise

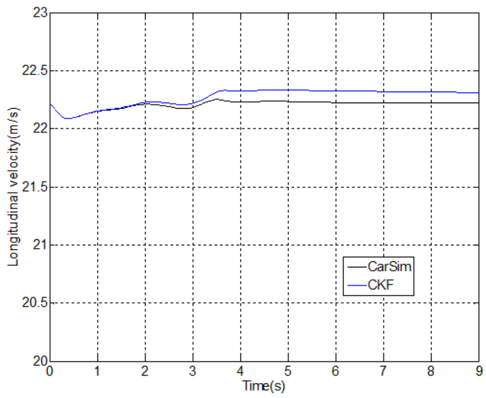

FIGURE 7 Estimation of longitudinal velocity

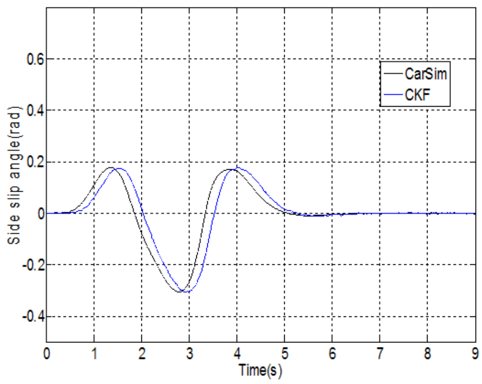

FIGURE 8 Estimation of side slip angle

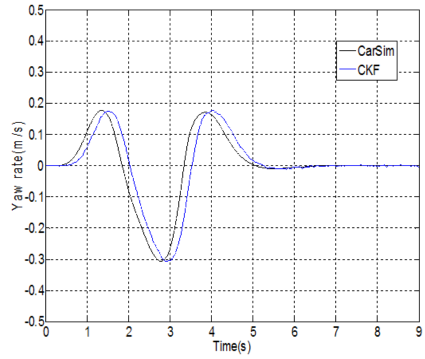

FIGURE 9 Estimation of yaw rate

4.2 SLALOM TEST

The initialisation of the State vector of the slalom simulation case is $x(0)=[0,0,50]^T$. Simulation results are shown in Figure 10, Figure 11, Figure 12, Figure 13, Figure 14 and Figure 15.

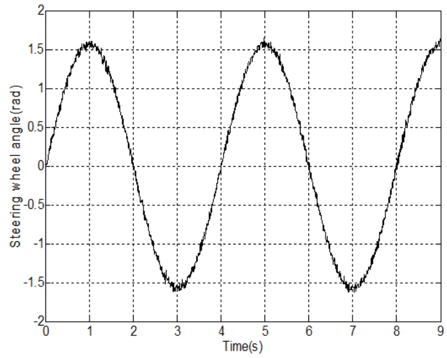

FIGURE 10 Steering angle with noise

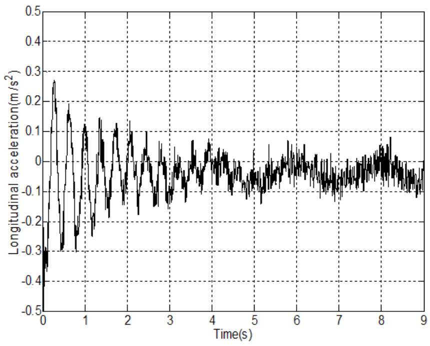

FIGURE 11 Longitudinal acceleration with noise

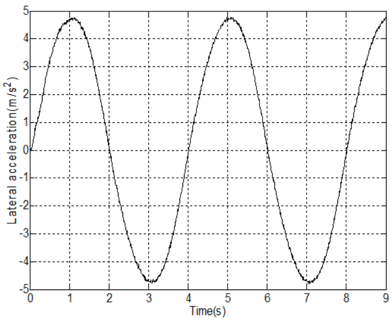

FIGURE 12 Lateral acceleration with noise

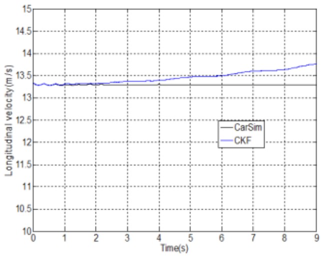

FIGURE 13 Estimation of longitudinal velocity

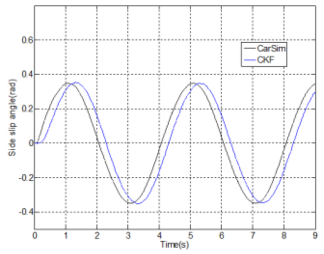

FIGURE 14 Estimation of side slip angle

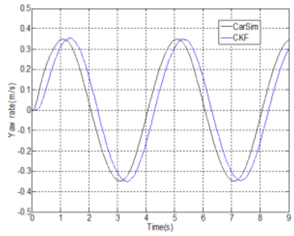

FIGURE 15 Estimation of yaw rate

For the slalom test, Figure 10, Figure 11 and Figure 12 are respectively the vehicle sensor signal of steering wheel angle, longitudinal acceleration and lateral acceleration. Figure 13, Figure 14 and Figure 15 are respectively the estimation of longitudinal velocity, side slip angle and yaw rate. As can can be seen from Figure 13, Figure 14 and Figure 15, the estimated value of longitudinal velocity, side slip angle and yaw rate capture the trends in the data from CarSim. The additive noise of the sensor signal is filter by the CKF well.

5 Conclusions

In this paper, some works are proposed to estimate vehicle speed, side slip angle and yaw rate of the vehicle. Firstly, a nonlinear 3DOF vehicle model is presented. Secondly, the estimator based on CKF is designed. Finally, the estimation is examined by comparing the outputs of the estimator with the responses of the vehicle model in CarSim under double lane change and slalom conditions. Experimental results of the simulation show the effectiveness of the proposed method.

Acknowledgements

This research has been supported by the National Nature Science Foundation of China under Grant No. 61201010.

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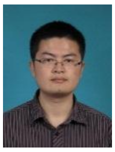	**Jinxi Chen, 1988, Fujian, China** **Current position, grades:** Engineer of FAW-VALKSWAGEN, Master of Science **University studies:** University of Electronic Science and Technology of China, in the School of Automation Engineering (2001-2005) **Scientific interest:** Signal processing, Vehicle control **Publications:** 3 papers **Experience:** He has completed 2 scientific research projects