A novel method to calculate relative permeability of fluids based on fractal theory and core NMR experiment

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Abstract

An analytical relative permeability model based on fractal theory and NMR principle is described. The relationship between NMR transversal relaxation time (NMR T_2) and resistivity index is deduced by fractal theory and capillary bundle model. The reciprocity theory of percolation field and electricity field is used to get the relationship between resistivity index and relative permeability. The 'Brooks-Corey/Burdine' equation was adopted to construct the relationship between relative permeability and NMR transversal relaxation time. By introducing the NMR parameters, the relative permeability model is improved since that it quantified the influence of pore structure. The results show that for water (wetting phase in water-gas system), the predicted permeability is exactly consistent with the experiment data, whereas for gas (the non-wetting phase in water-gas system), the fitted results is to some degree deviated from the experiment data and need more extensive research.

Keywords: relative permeability, fractal theory, NMR transversal relaxation time, reciprocity, pore structure

1 Introduction

The relative permeability of multiphase in porous rock is important to fluid recognition, reservoir numerical simulation, productivity evaluation and design of oil/gas field development plan. Purcell (1949) [1] derived an equation between relative permeability and capillary pressure based on a simplified working hypothesis. Burdine (1953) [2] got formulas for calculating relative permeability from pore size distribution data by introducing tortuosity factors and basic laws of fluid flow in porous media, which is a revised version of Purcell's equation since it allowed the impact of irreducible fluid saturation. Corey (1954) [3] investigated the interrelation between gas and oil relative permeabilities then proposed two equations for relative permeability calculation by approximation. Pirson (1958) [4], Brooks et al (1966) [5] also researched the influential factors of relative permeability and got analytical expressions related to capillary pressure and saturation. He, et al (2000) [6], Zhou et al (2007) [7] calculated fractal dimension of pore space from capillary pressure data then established gas/water relative permeability model by fractal theory. O.A.Olafuyi et al (2008) [8] investigated the relationship between capillary pressure and relative permeability of small cores. Li et al (2008, 2010 and 2011) [9-11], conducted systematically research on the interrelationships among relative permeability, capillary pressure and resistivity index then obtained analytical expressions for those parameters. From literatures review we know that most relative permeability models presented ever were deduced from capillary pressure or by simple fitting methods. Many researches noticed that the relative permeability has a great relation of intrinsic pore structure, but the quantitative expressions were not well proposed yet. Nuclear Magnet Resonance (NMR) is a well used technology to study the pore structure of rock-pore system, by which one can obtain pore throat distribution, total porosity, irreducible fluid saturation, absolute permeability etc. In this paper, the author started to investigate the fractal property of porous rock and construct the relationships among fluid saturation, capillary pressure and resistivity index. Then the reciprocity between percolation field and electricity field was discussed and simple capillary bundle model was used to connect the relative permeability and resistivity index. At last, relative permeability model, which related to fluid saturation, NMR transversal relaxation time (NMR T_2) based on 'Brooks-Corey/Burdine' equation was established. The model was discussed and analysed by core experiments.

2 Fractal theory and conductivity property rock-pore system

Researches demonstrated that pore space of sediment rock has fractal feature and can be depicted by power law function. (A.J.Katz et al, 1985 [12]). Li et al (2011) [11] got the relationship between water saturation and resistivity by fractal theory, which can be expressed as:

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$$\frac{1}{R_t} \propto S_w^{\frac{1}{f(3-D_f)}}, \tag{1}$$

where S_w is the water saturation, R_t is the resistivity of rock at a water saturation of S_w, D_f is fractal dimension of pore space and f is the parameter, which affected by the thickness of water film and surface tension.

Accordingly, with the theory of fractal and capillary bundle model, the relationship between water saturation and capillary pressure can be expressed as:

$$S_w \propto P_c^{-(3-D_f)}, \tag{2}$$

where P_c is the capillary pressure at a water saturation of S_w.

Combing Equations (1), (2) and Archie's equation, we can easily obtain:

$$P_c = P_0 I^f, \tag{3}$$

where P_0 is the capillary pressure at a water saturation of 1 and I is the resistivity index.

The NMR transversal relaxation time (NMR T_2) can be expressed as (George Coates, et al, 2000 [13]):

$$\frac{1}{T_2} = \frac{1}{T_{2b}} + \frac{1}{T_{2s}} + \frac{1}{T_{2d}}, \tag{4}$$

where T_2 is the total transversal relaxation time, T_{2b} is the bulk relaxation time, T_{2s} is the surface relaxation time and T_{2d} is the diffusion relaxation time. Practically, the bulk relaxation part and the diffusion relaxation part are often omitted, so the total transversal relaxation time can be summarized as:

$$\frac{1}{T_2} = \frac{1}{T_{2b}} = \rho_2\left(\frac{S}{V}\right), \tag{5}$$

where ρ_2 is the transversal relaxation ratio, S/V is the ratio of surface to bulk for pore space.

From the capillary theory, for the simplified capillary tube model, one can solve Laplace equation of capillary pressure and get:

$$P_c = \frac{2\sigma\cos\theta}{r}, \tag{6}$$

where σ is surface tension, θ is the contact angle of wetting and non-wetting phase, r is the radius of capillary tube.

For the simple sphere or cylindrical pore structure, the ratio of surface to bulk(S/V) has a linear relationship with pore radius. Combining Equation (5) and (6), the relationship between capillary and NMR T_2 can be expressed as:

$$P_c = C\frac{1}{T_2}, \tag{7}$$

where C is the transforming parameter.

In fact, the pore space is far more complicated than simple sphere or cylindrical-like shape, so the relationship between T_2 and P_c is unknown. But it is definitely that the capillary is a function of NMR T_2:

$$P_c = g\left(\frac{1}{T_2}\right), \tag{8}$$

where g is a function with no specific form.

He et al (2005) [14] got an analytical relationship between capillary pressure and NMR T_2 by large amount of experiments:

$$P_c = m\left(\frac{1}{T_2}\right)^n, \tag{9}$$

where m and n are transforming parameters.

Combing Equation (3) and (9), the relationship between resistivity index and NMR T_2 can be expressed as:

$$I = a\left(\frac{1}{T_2}\right)^b, \tag{10}$$

where a and b are transforming parameters.

Equation (10) is important because it connected the conductivity property with pore structure, which is the bridge for the NMR T_2 and relative permeability discussed below.

3 Reciprocity theory and calculation of relative permeability

The conduction of electricity and the percolation of fluid are similar from the view of mathematical and physical theory. Since the similarities of boundary condition and initial condition, the solving methods of both problems are alike to some extent. Fatt (1956) [15] conducted extensive comparison between the Poiseuille's equation and Ohm's law. King (1989) [16], Li (2000) [17], B.Shimekit, H.Mukhtar et al (2010) [18] also researched the interrelationship between electricity transportation properties and fluid percolation features. With Poiseuille's equation, the flow of capillary bundle-like porous rock (for single phase) can be expressed as:

$$Q = \frac{\pi r^4}{8\mu l}\Delta P, \tag{11}$$

where Q is flow of single phase, r is the radius of capillary bundle, μ is the viscosity of fluid, l is the length of capillary bundle and ΔP is the pressure difference between inlet and outlet.

By Ohm's law, the electric current of capillary bundle can be expressed as:

$$E_c = G\Delta V, \tag{12}$$

where E_c is the current, G is the conductance and ΔV is the potential difference.

With the reciprocity of percolation field and electricity field, combining Equations (11) and (12), the conductance of the capillary bundle can be expressed as (Fatt, 1956) [15]:

$$G = \frac{\pi r^4}{8\mu l}. \tag{13}$$

The flow of single phase in capillary bundle can be expressed by Darcy law:

$$Q = \frac{AK}{\mu l}\Delta P, \tag{14}$$

where A is the area of cross section of the capillary bundle, K is the absolute permeability of single phase.

Assuming the water and gas in the capillary bundle, combining Equations (11), (13) and (14), the conductance of rock fully saturated with water is:

$$G_o = \frac{AK}{\mu l}. \tag{15}$$

By introducing the relative permeability, the conductance of rock partly saturated with water can be expressed as:

$$G_t = \frac{AKK_{rw}}{\mu l}, \tag{16}$$

where K_{rw} is the relative permeability of water.

From Archie's equation, the resistivity index can be expressed as:

$$I = \frac{R_t}{R_o}, \tag{17}$$

where R_o is the resistivity of core fully saturated with water.

Combining Equations (15), (16) and (17), we obtained:

$$I = \frac{R_t}{R_o} = \frac{G_o}{G_t} = \frac{1}{K_{rw}}. \tag{18}$$

From equations above we know that if rock is fully saturated with water, both the resistivity index and the relative permeability of water are 1, if rock is fully saturated with gas, the relative permeability of water equals to 1 and the resistivity equals to infinite value. But from experiments we know that the relative permeability of water becomes zero when water saturation reduced to irreducible state, so Equation (18) should be revised to suit the actual condition, we called it saturation revision here. After revision, Equation (18) can be expressed as:

$$K_{rw} = \frac{1}{I}\frac{S_w - S_{wir}}{1 - S_{wir}}, \tag{19}$$

where S_w is water saturation and S_{wir} is irreducible water saturation.

Combining Equation (10) and (19), the relative permeability of water can be expressed as:

$$K_{rw} = aT_2^{\,b}\frac{S_w - S_{wir}}{1 - S_{wir}} = ce^{dT_2}\frac{S_w - S_{wir}}{1 - S_{wir}}, \tag{20}$$

where c and d are fitting parameters.

Equation (20) is the new model to calculate the relative permeability of water.

Burdine et al (1953) [2] proposed a new method to calculate relative permeability:

$$K_{rw} = \left(\frac{S_w - S_{wir}}{1 - S_{wir}}\right)^2 \left(\int_0^{S_w} dS_w / P_c^2\right) / \int_0^1 dS_w / P_c^2, \tag{21}$$

$$K_{rnw} = \left(\frac{S_{nw} - S_{wir}}{1 - S_{nwr} - S_{wir}}\right)^2 \left(\int_{S_w}^1 dS_w / P_c^2\right) / \int_0^1 dS_w / P_c^2, \tag{22}$$

$$S_w + S_{nw} = 1, \tag{23}$$

where K_{rw} is the relative permeability of wetting phase, K_{rnw} is the relative permeability of non-wetting phase, S_w is the saturation of wetting phase, S_{wir} is the irreducible saturation of wetting phase, S_{nw} is the saturation of non-wetting phase and S_{nwr} is the irreducible saturation of non-wetting phase. For simplicity, we often assume it is 0.

Olafuyi et al (2008) [8] proposed the relationship between capillary pressure and saturation of wetting phase by fractal theory:

$$P_c = P_o\left(\frac{S_w - S_{wir}}{1 - S_{wir}}\right)^{-1/\lambda}, \tag{24}$$

where P_o is the capillary pressure of core fully saturated with wetting phase, λ is a parameter which relates to fractal feature of pore space.

Substituting Equations (24) to Equations (21) and (22), we can get:

$$K_{rw} = \left(\frac{S_w - S_{wir}}{1 - S_{wir}}\right)^{2/\lambda + 3}, \tag{25}$$

$$K_{rnw} = \left(1 - \frac{S_w - S_{wir}}{1 - S_{wir}}\right)^2 \left[1 - \left(\frac{S_w - S_{wir}}{1 - S_{wir}}\right)^{2/\lambda + 1}\right]. \tag{26}$$

Equations (25) and (26) are the known 'Brooks-Corey/Burdine' relative permeability equations. In this study, only gas and water are existed in rock, so water is the wetting phase and gas is the non-wetting phase and S_w denotes the water saturation, S_{wir} denotes the irreducible water saturation, S_{nw} denotes the gas saturation, K_{rw}

denotes the relative permeability of water, K_{rnw} denotes the relative permeability of gas from Equation (21) to Equation (26), which has the same meanings from Equation (1) to Equation (20).

Combining Equations (20) and (25) we can obtain:

$$\left(\frac{S_w-S_{wir}}{1-S_{wir}}\right)^{2/\lambda}=ce^{dT_2}\left(\frac{S_w-S_{wir}}{1-S_{wir}}\right)^{-2}. \qquad (27)$$

Substituting Equation (27) to Equation (26) we can obtain the relative permeability of gas:

$$K_{rnw}=\left(1-\frac{S_w-S_{wir}}{1-S_{wir}}\right)^2\left[1-ce^{dT_2}\left(\frac{S_w-S_{wir}}{1-S_{wir}}\right)^{-1}\right]. \qquad (28)$$

Equation (20) and equation (28) are the new models presented to calculate the relative permeability of water and gas. From the formulas and parameters we know that the new models related not only the fluid saturation, but also the irreducible fluid saturation and NMR T_2, which are more suitable since it reveals that percolation process is affected by pore structure and the relationship between the relative permeability and pore structure is quantified by NMR T_2.

4 Experiments and discussions

Four shaly sand samples were selected for experiments in order to testify the model proposed. The basic parameters were shown in Table 1. The porosity ranges from 11.49% to 17.53% and the absolute permeability ranges from 1.44mD to 15.99mD.Nacl with a concentration of 8000ppm and a viscosity of 0.7328mPa.s was used for the saturated solution. Nitrogen with a viscosity of 0.0174mPa.s was used as the injecting phase (non-wetting phase). The echo time and waiting time for NMR test were 0.3ms and 6s respectively. The scanning number and echo number were 64 and 2048 respectively. Cores were centrifuged for 30 minutes with a velocity of 8000rpm.SVD decomposition method was used to fitting the NMR T_2 spectrum and 64 points of inversion was adopted. Irreducible water saturation was obtained by relative permeability and centrifuge methods. All the tests were conducted under a temperature of 19°C.

TABLE 1 Basic petrophysical parameters of core samples

Sample ID	Nitrogen porosity (%)	NMR porosity (%)	Nitrogen permeability (mD)	Swi by relative permeability method (%)	Swi by centrifuge (%)
1	11.49	11.13	1.44	68.27	63.94
2	13.72	13.94	3.18	75.02	71.53
3	17.53	17.25	15.99	51.39	53.26
4	16.51	16.88	7.24	39.11	36.75

Figure 1 shows NMR T_2 spectrum of cores with a saturation of 1. Except for No.4 sample, others bear single peak or irregular double peaks in NMR T_2 spectrum. The percentage of immobile part is to some extent larger than that of mobile part, which reveals that the percentage of micro porosity is higher than the macro porosity. Figure 2 depicts the comparison between NMR porosity and nitrogen porosity. Both of methods have similar results, the average absolute error is 0.31% and the average relative error is 2.14%. The irreducible water saturations obtained by gas-water relative permeability method and centrifuge methods were shown in table.1, both methods have exactly the same result, the average absolute error is 3.02% and the average relative error is 5.4%. Discussions above show that the quality of data obtained in our study are fine.

Since the measured number of relative permeability is not equal to that of NMR T_2 spectrum, interpolation was conducted to adjust the number of NMR T_2 and relative permeability to the same for curve fitting. Log-log method was adopted to get 100 points of water saturation and the corresponding NMR T_2 and relative permeability by our experiences. Noticing that the upper limit of water saturation is 1 and the lower limit of water saturation is its irreducible water saturation. Figure 3 shows comparison between interpolated relative permeability and measured data. The interpolated data were in agreement with the experiment data.

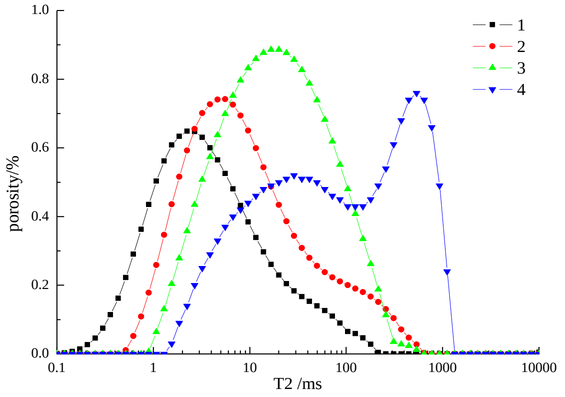


FIGURE 1 NMR T2 spectrum of samples fully saturated with salinity

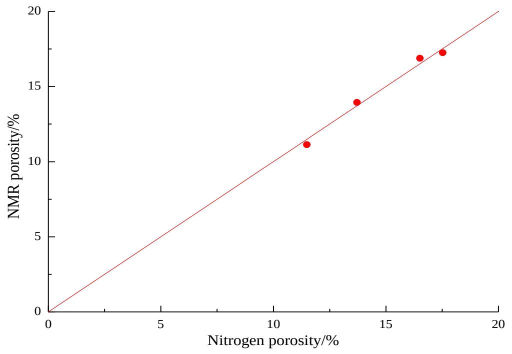
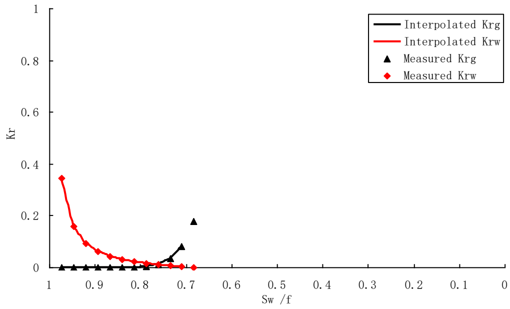
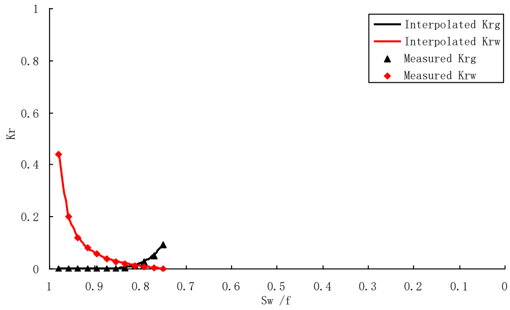


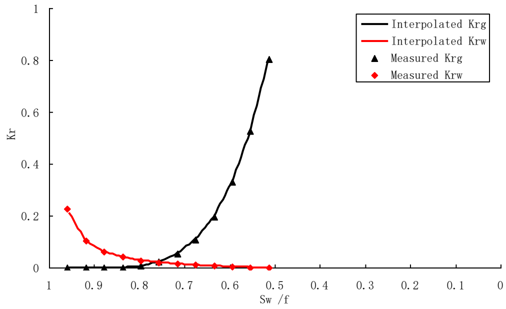
FIGURE 2 Comparison between NMR porosity and Helium porosity



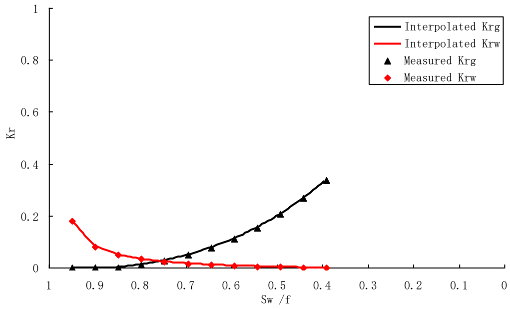
a) Core sample No.1



b) Core sample No.2



c) Core sample No.3

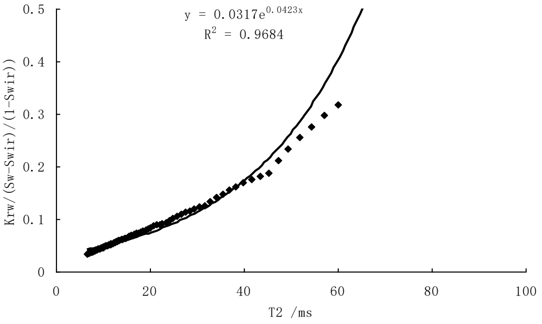


d) Core sample No.4

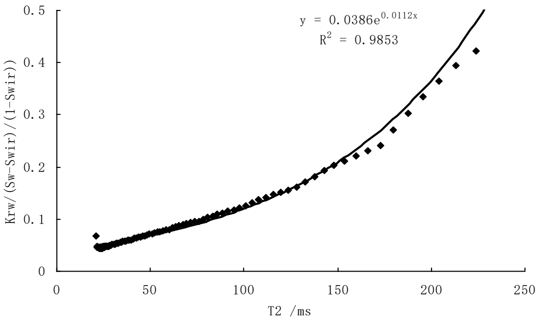
FIGURE 3 Comparison between interpolated water/gas relative permeability and measured data

NMR T_2 and gas/water relative permeability under the same water saturation were obtained after interpolation. The next procedure was the fit of relative permeability. We fitted the relative permeability of water by Equation (20) for every sample firstly, then fitted the relative permeability of gas by Equation (28) with parameters of '*c*' and '*d*' fitted from Equation (20) for every sample.

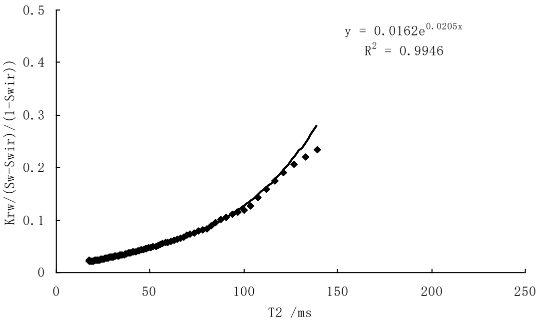
Figure 4 shows the relation between $K_{rw}(1-S_{wir})/(S_w-S_{wir})$ and T_2. The black dots are data after interpolation and the black line are fitting curves using equation (20).The fitting results showed that the both has a excellent relationship of power function. The average multiple correlation coefficient is 0.975.



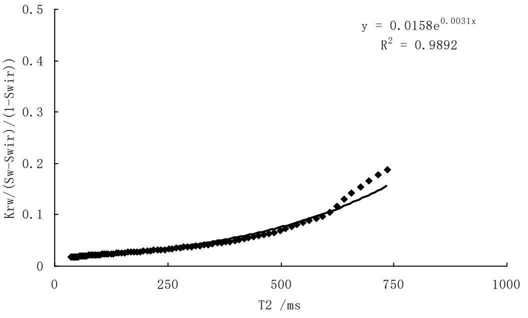
a) Core sample No.1



b) Core sample No.2



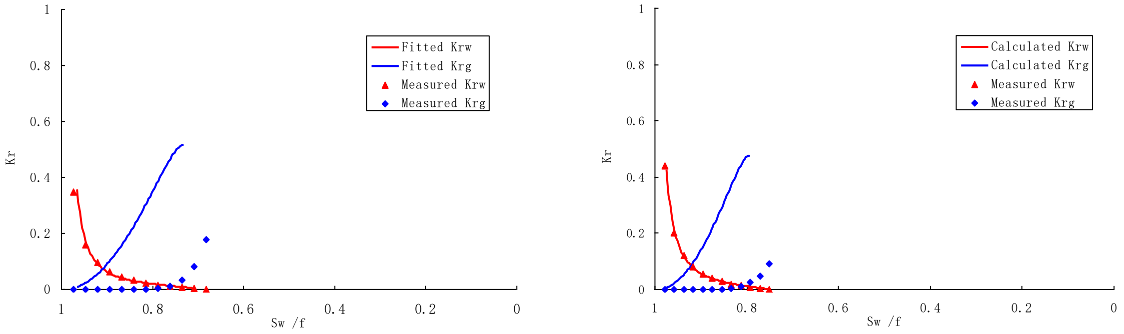
c) Core sample No.3



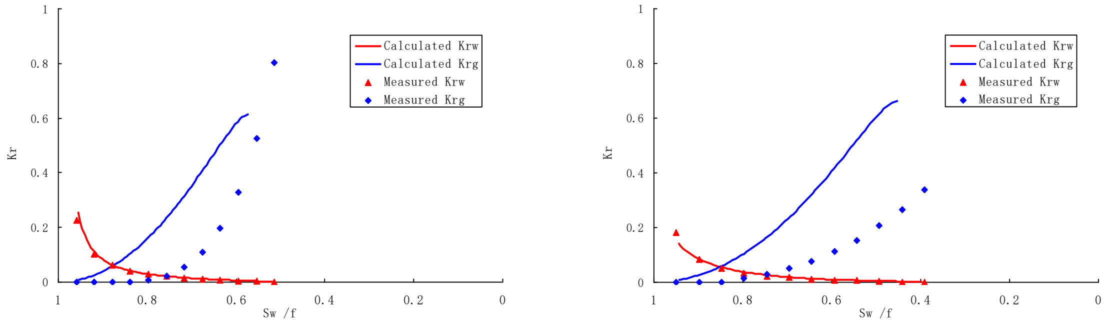
d) Core sample No.4

FIGURE 4 Interrelationship between $K_{rw}(1-S_{wir})/(S_w-S_{wir})$ and T2

Figure 5 shows the comparison between fitted water/gas relative permeability and measured water/gas relative permeability. It can be seen from the results that for water (wetting phase), the fitted relative permeability is in accordance with the measured data, the average relative error is 2.45%; but for gas (non-wetting phase), the fitted relative permeability is deviated from the measured data, which may caused by the simplicity of irreducible gas(non-wetting phase) saturation. If we want to get more exact result of the relative permeability of gas (non-wetting phase), the irreducible gas (non-wetting phase) saturation should be considered.



a) Core sample No.1 b) Core sample No.2



c) Core sample No.3 d) Core sample No.4

FIGURE 5 Comparison between fitted relative permeability and measured data

5 Conclusions

With the fractal theory and the reciprocity of percolation field and electricity field, the relationship between relative permeability and NMR T_2 was constructed and testified by experiment; on the basis of this study we make the following conclusions:

1) The power function relationship between resistivity index and NMR T_2 was established by fractal feature of pore space and principles of NMR and capillary pressure, which made the interrelationship of conductivity property and pore structure more quantitative.

2) The boundary conditions, initial conditions and solving methods for percolation field and electricity field are to large extent the same; with the reciprocity of the two fields, the relationship between relative permeability and resistivity index was constructed, which made the conductivity property and percolation property more quantitative.

3) The model proposed for the calculation of relative permeability of water (wetting phase) is fine and exact; but for gas (non-wetting phase), the predicted relative permeability is not coherent with the measured data since the model did not consider the influence of irreducible gas saturation (non-wetting phase). If one need to predict the relative permeability of non-wetting phase exactly, the determination of irreducible saturation is inevitable.

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