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The Arithmetic of Passions

With Numeration Moralists begin
Upon the Passions, putting Quotients in;
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And in their Definitions are exact;
As for Subtracting, take but one from three,
Add it to four, and it makes five to be:
Thus the odd Numbers to the even joined,
Will make the Passions rise within the mind.

Margaret Cavendish[1]

This 12th volume No.1 deals with various questions of **applied statistics**, **computer modelling and applied economics**, and with problems that are really actually for this day. We also continue our activities in the field of **solid state physics**. In particular, actual papers on various statistical problems, computer modelling and economics from Israel, Lithuania, Ukraine, India and Latvia are presented in the journal.

This means that our journal policy is directed on the fundamental and applied sciences researches, which is the basement of a full-scale modelling in practice.

This edition is the continuation of our publishing activities. We hope our journal will be interesting for research community, and we are open for collaboration both in research and publishing.

EDITORS

Yu.N. Shunin

I.V. Kabashkin

[1] **Margaret Cavendish** was a better poet than a natural philosopher. Poetry was more popular than science in the 17th century when Margaret lived. She believed that poetry "is the finest work that Nature hath made...playing so well upon the Brain as it strikes the strings of the heart with delight."

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STATISTICAL PROBABILITY MODELS FOR TRANSPORT MEANS FLEET

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Some algorithms to forecast repair time of damaged vehicle, time losses for lack of drivers and various accidental hindrances-during goods handling are proposed using methods of mathematical statistics and probability theory.

Statistical information on transport means fleet is renewed and replenished in the course of time. With the growth of information amounts the costs of its storage increase as well. Therefore the relevant algorithms for obtaining required statistical assessments with the least statistical information are presented in the article. It is deduced that in the modelling of transport networks and freights as well as the flows of transport means in them, it is analytically proper to describe random factors by the non-parametric assessment.

Keywords: *transport means, repair time, statistical probability algorithms*

1. Introduction

Among the essential factors hindering the normal haulage of goods there may be indicated insufficient reliability of transport means (particularly related to increase of exploitation time) and their damages caused by various reasons. The main point of this factor is that in entirely normal conditions during the process of haulage, separate specimen of transport means come out of order – break down – at accidental time intervals. These transport means may be repaired during the time τ_p, which will be considered a random quantity with a relevant distribution law. This article analyses how to evaluate this quantity, to define the losses of useful time of transport means; it will also deal with the road transport means – vehicles – (however, these models may be applied to other transport means as well).

2. Evaluation of the Probability Characteristics of Vehicles Coming Out of Order

The operation process of a transport means $Y(t)$ is a combination of two non-intersecting sets: A – an operating transport means and B – a damaged transport means.

Thus, in this way, the number of operation hours of the j^{th} type transport means in the period T, has to be analysed as a random quantity, composed of the sum of productive operation segments:

$$S_j = \sum_{i=1}^{N+1} S_{ji} ,$$

where N is the number of intervals in the j^{th} type transport operation; and the sums of idle time:

$$x_j = \sum_{i=1}^{N} x_{ji} ,$$

i.e. $t_j = S_j + x_j$, where t_j is the number of the j^{th} type transport means operation hours during period T.

Let us assume that at the beginning of operation the working transport means is $\varphi(0) \in A$. Consequently, the operation process of the transport means will consist of status changes A_1, B_1, A_2, B_2, ..., A_i, B_i. This is a stationary Markov's process with two possible statuses, because every other process in it is determined only by the present status, irrespective of all the former processes.

This Markov's process may be entirely described by the matrix of transitional probabilities.

$$\begin{Vmatrix} P_{11}^{*} & P_{10} \\ P_{01}^{*} & P_{00}^{*} \end{Vmatrix} = \begin{Vmatrix} 1-\lambda\Delta t & \lambda\Delta t \\ \mu\Delta t & 1-\nu\Delta t \end{Vmatrix},$$

where P_{ij}^{*} is the probability of transition during a time interval from j^{th} status into the j^{th} status, as $i=0$, $j=0$; μ – is the intensity of transport means restoration; λ – is the intensity of transport means coming out of order.

Let us assume, that time intervals φ_i, correspond to the statuses A_i, $i = 1, 2, \ldots$ and time intervals q_j correspond to the statuses B_i. The laws of distribution of these time intervals will be as follows:

$$\left.\begin{aligned} P\{\varphi_i < x\} &= F(x); \\ P\{q_i < x\} &= G(x). \end{aligned}\right\}$$

Over the period $\alpha(0, t_j)$ a transport means was able to operate during the time period $\alpha(t_j)$, and unable to operate over the time period $\beta(t_j)$. It is obvious that:

$$\alpha(t_j) + \beta(t_j) = t_j \ .$$

Therefore it may be written as follows:

$$\left.\begin{aligned} P\left\{\beta(t_j) < x\right\} &= P(t_j, x); \\ P\left\{\alpha(t_j) < x\right\} &= 1 - P(t_j, x). \end{aligned}\right\}$$

Consequently, we may write:

$$P(t_j, x) = \sum_{i=0}^{\infty} G_i^{*}(x)\left[F_i^{*}(t_j - x) - F_{i+1}(t_j - x)\right],$$

where $G_i^{*}(x)$ – the j^{th} contraction of distribution function $G(x)$; $F_i^{*}(x)$ – the j^{th} contraction of distribution function $F(x)$.

This expression allows us to discover the law of distribution of general time of the non-operational status of the transport means, in existing different flows of failures and restorations.

As the investigations show, a single restoration time of separate transport means divides according to Weibull's and indicative laws, uninterrupted operation time corresponds to the logarithmic-normal and indicative laws.

The analysis of a great number of histograms showed that the following variants are possible: 1) uninterrupted operation time distributed according to the logarithmic-normal law; the restoration time – according to Weibull's law; 2) uninterrupted operation time distributed according to the indicative law; the restoration time – according to Weibull's law; 3) uninterrupted operation time distributed according to the logarithmic-normal law; the restoration time – in accordance with the indicative law; 4) uninterrupted operation and restoration time distributed according to the indicative law [2].

Knowing the evaluation of the j^{th} type of transport means general repair time probability characteristics, we may use them in planning the work of a transport enterprise. Therefore, it should be known how many transport means will have to be repaired during the period t, and this will be analysed in the [1].

The author created: 1) Probability model for estimation of average number of vehicle repair per time period; 2) stochastic model for the determination of desirable vehicle productivity; 3) Probability model dedicated for the sort-term forecasts of vehicle productive time losses; 4) the algorithm for forecasting a transport vehicle repair time [1–4].

For using the aforementioned models it is necessary to have sufficient statistical information on the fleet of vehicles. The handling of this information will be analysed hereinafter.

3. Mathematical Models in Accumulation of Statistical Information

Statistical information on the transport means fleet is renewed and replenished in due course. With the growth of the amounts of the information the costs of its storage are increasing too. Therefore, our aim is to obtain necessary statistical assessments with the minimal amount of statistical information.

The assessment of distribution of random factors by the Bayesian method. Presumably we have a sample X, made of N meanings x_1, …, x_N. Considering the meanings x as random quantities we shall assess distribution function of the random quantity X. For an analytical description of the distribution X, there is necessary a relevant prior information on the distribution type. Let us presume that it is known, i.e. there is known the parametric probabilities' density $f(X/\theta)$,

here $\theta = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_r \end{pmatrix}$ – vector of the parameters describing the distribution of the random quantity x; the r – is the number of parameters and presumably known the prior distribution of the vector's parameters $\theta f_{apr}(\theta)$, the latter may be defined by an experimental assessment. Then the X distribution may be assessed with the help of the Bayesian formula [5].

At the beginning we shall obtain the aposterioric density of distribution of the vector's parameters. Considering that observations in the sample x_1, …, x_N are independent, according to the Bayesian formula it is as follows

$$f(\theta / x_1, ..., x_N) = \frac{\prod_{i=1}^{N} f(x_i/\theta) f_{apr.}(\theta)}{\int_{\Omega_\theta} \prod_{i=1}^{N} f(x_i/\theta) f_{apr.}(\theta) d\theta}, \tag{1}$$

here the integration is in r – measurable area Ω_θ by changing the parameter θ. The aposterioric distribution of X is obtained by integrating

$$f(X / x_1, ..., x_N) = \int_{\Omega_\theta} f(X/\theta) f(\theta / x_1, ..., x_N) d\theta. \tag{2}$$

By putting the (1) into (2) we obtain

$$f(X / x_1, ..., x_N) = \frac{\int_{\Omega_\theta} f(X/\theta) \prod_{i=1}^{N} f(x_i/\theta) f_{apr.}(\theta) d\theta}{\int_{\Omega_\theta} \prod_{i=1}^{N} f(x_i/\theta) f_{apr.}(\theta) d\theta}.$$

After the new sample of the phenomena x_{n+1}, …, x_{n+m} is obtained and it is necessary to specify the distribution $f(X)$ by using new data, then instead of the prior distribution $f_{apr}(\theta)$ we shall use the former prior one $f(\theta / x_1, \ldots, x_N)$. Let us notice that such method does not require the storage of all the meanings of a sample; only the meanings of the aposterioric distribution $f(\theta / x_1, \ldots, x_N)$ are subject to storage, namely – only a certain number of coefficients characterising it. At the beginning of the data accumulation process we may not have the f_{apr} on the whole, then the Bayesian postulate is applied and the prior distribution is considered uniform [5].

The presented formulae would lose their practical importance if they would not have concrete distribution types $f(X/\theta)$ and $f_{apr}(\theta)$. Besides, it is preferable that the analytical type of the prior and aposterioric density would be uniform, because then the calculation algorithms and programmes become simpler.

Normal distribution. Let us analyse the case when random qualities are distributed according to the normal law, the density of which is

$$f(X/\mu,\sigma)=\frac{1}{\sqrt{2\pi\sigma}}\exp\left[-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}\right]. \tag{3}$$

In the given case the parameters' vector $\theta=\begin{pmatrix}\mu\\ \sigma\end{pmatrix}$, and

$$f(x_1,...,x_N/\mu,\sigma)=C_0\sigma^{-N}\exp\left(-\frac{1}{2}\frac{S(N-1)}{\sigma^2}\right)\times\ \exp\left(-\frac{1}{2}\frac{N(\bar{x}-\mu^2)}{\sigma^2}\right), \tag{4}$$

here $\bar{x}=\frac{1}{N}\sum_{i=1}^{N}x_i$; $S=\frac{1}{N-1}\sum_{i=1}^{N}(x_i-\bar{x})^2$ – the sample medium and dispersion accordingly $C_0=(2\pi)^{-N/2}$.

The prior distribution of parameters μ and σ is selected so that its density would be analogous to the density of the conditional density

$$f(\mu,\sigma)=C_1\sigma^{-n}\exp\left[-\frac{1}{2}\frac{a(n-1)+n(\mu+\mu_0)^2}{\sigma^2}\right]. \tag{5}$$

The coefficient C_1 depends on n and may be obtained from the rationing condition. The coefficient C_1 is not calculated here, because it does not depend on μ and σ. After putting the (5) into (1), and after a corresponding rearrangement, we shall obtain

$$f(\sigma,\mu/x_1,...,x_N)=\frac{1}{C_3}\sigma^{-N*},\ \exp\left[-\frac{1}{2}\frac{(N^*-1)a^*+N^*(\mu-\mu^*)}{\sigma^2}\right], \tag{6}$$

here C_3 is obtained from the rationing condition; $N^*=N+n$;

$$a^*=S\frac{N-1}{N+n-1}+a\frac{n-1}{N+n-1}+\frac{Nn(\mu_0-\bar{x})^2}{(N+n)[(N+n)-1]}; \tag{7}$$

$$\mu^*=\bar{x}\frac{N}{N+n}+\mu_0\frac{n}{N+n}. \tag{8}$$

Now let us calculate the coefficient C_3.

Having noticed, that

$$\frac{1}{\sqrt{2\pi}\sigma/\sqrt{N^*}}\int_{-\infty}^{\infty}\exp\left(-\frac{1}{2}\frac{(\mu-\mu^*)^2}{\sigma^2/N^*}\right)d\mu=1,$$

after the relevant rearrangements

$$C_3 = \frac{1}{2}\sqrt{\frac{2\pi}{N^*}}\left[\frac{a^*\left(N^*-1\right)}{2}\right]^{\left(N^*-2\right)/2}\Gamma\left(\frac{N^*-2}{2}\right).$$

Thus the aposterioric distribution of the parameters σ and μ

$$f\left(\sigma,\mu / x_1,\ldots,x_N\right) = \frac{\sigma^{-N^*}\exp\left[-\frac{1}{2}\frac{\left(N^*-1\right)a^* + N^*\left(\mu-\mu^*\right)^2}{\sigma^2}\right]}{\sqrt{\frac{\pi}{2N^*}}\left[\frac{a^*}{2}\left(N^*-1\right)\right]^{\left(N^*-2\right)/2}\Gamma\left(\frac{N^*-2}{2}\right)}. \tag{9}$$

The aposterioric density of the x distribution

$$f\left(X / x_1,\ldots,x_N\right) = \frac{\Gamma\left(\frac{N^*-1}{2}\right)}{\sqrt{2\pi}\Gamma\left(\frac{N^*-2}{2}\right)\sqrt{\frac{a^*\left(N^*-1\right)}{2}}}\left[1+\frac{N^*\left(X-\mu^*\right)^2}{\left(N^{*2}-1\right)a^*}\right]^{-\left(N^*-1\right)/2}. \tag{10}$$

The aposterioric density may be approximated by the normal one (3) by putting instead of the σ and μ more probable meanings obtained from (9). By differentiation we obtain the following more expected meanings of the parameters $\mu=\mu^*$,

$\hat{\sigma}^2 = a^*$. Then the aposterioric density is approximated by the phenomenon

$$f\left(X / \hat{\mu},\hat{\sigma}\right) = \frac{1}{\sqrt{2\pi}\hat{\sigma}}\exp\left[-\frac{1}{2}\frac{\left(X-\hat{\mu}\right)^2}{\hat{\sigma}^2}\right]. \tag{11}$$

In the presence of large N (11), the (10) approximates the density with great precision. As the (10) and (11) include only parameters $N^* = N+n$, a^* (or $\hat{\sigma}$), μ^* (or $\hat{\mu}$), then every aposterioric distribution may be very simply used as an aprioric one. For this purpose only the parameters are recalculated

$$N_i = N_{i-1} + N,$$

$$\hat{\mu}_i = \mu_{i-1}\frac{N_{i-1}}{N_{i-1}+N} + \frac{N}{N_{i-1}+N}\bar{x},$$

$$\hat{\sigma}^2 = \hat{\sigma}^2_{i-1}\frac{N_{i-1}-1}{N+N_{i-1}-1} + S\frac{N}{N+N_{i-1}-1} + \frac{NN_i\left(\mu_i-\bar{x}\right)^2}{N+N_i\left(N+N_{i-1}\right)}.$$

Consequently suffice it to remember only three coefficients: N_{i-1}, μ_{i-1} and σ_{i-1}. To demonstrate under which given N it is possible to use the simple (11) instead of the complicated formula (10) let us analyse the following example.

The sample, having 100 random quantity meanings, characterising the quantity alteration of freight consignment brought to the terminal, was divided into 20 portions with 5 realisations each. The meanings of the parameters of the aprioric distribution are: $\mu_0 = 300$, $\sigma_0 = 10$, $N_0 = 1$. According to the first sample we calculated x and s^*, and further according to (7) and (8) a^* and μ^*. According to these parameters we have calculated the meanings of the aposterioric density by the precise formula (10) and the approximate formula (11). Further on we use the meanings a^* and μ^* as the parameters of aprioric distribution and we repeat the calculation procedure for the second, the third, etc., samples. The diagrams of the distribution density are presented on the Figure 1, where the continuous line shows the distribution obtained according to the (10), and the dotted line – those according to the (11).

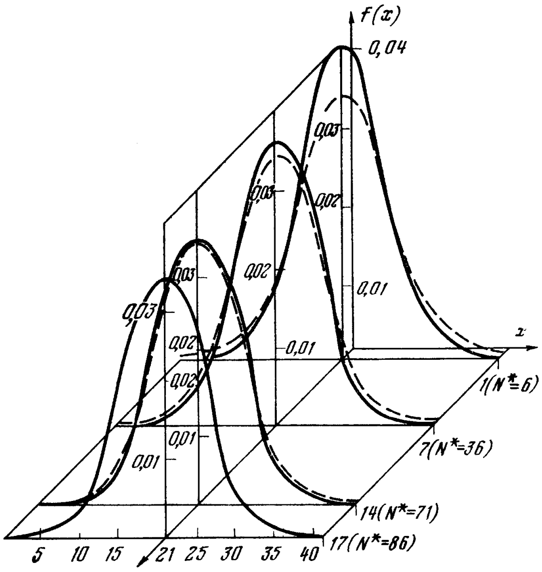

Figure 1. The diagrams of the distribution density

In the seventeenth interval the curves of the distribution density practically coincide completely. Already in the presence of $N = 70 - 85$ both density curves practically coincide and when the sample exceeds 100 it is possible to use much more simple (normal) density.

Logarithmic-normal distribution. As experiments have proved, by using logarithmic normal distribution

$$f(x) = \frac{1}{x\beta\sqrt{2\pi}} \exp\left(-\frac{\ln(x-a)^2}{2\beta^2} \right)$$

it is possible to introduce simplification without losing precision (sufficient for practice). Namely, if indicated $y = \ln x$, then random quantity y will be distributed according to the normal density

$$f(y) = \frac{1}{\sqrt{2\pi}\beta} \exp\left(-\frac{(y-a)^2}{2\beta^2} \right). \tag{12}$$

There may occur difficulties in the interpretation of the results with (12), however during the calculation the simplicity of the formula (12) is very clear. For example, we need to calculate probability limits for x. Since y – monotharic function x, it is possible to define probability limits y: y_1 and y_2 (probability limits x: $x_1 = e^{y_1}$ and $x_1^{y_1}$).

The parameters a and β are correspondingly the medium and dispersion of the rearranged random quantity $y = \ln x$ and they are defined

$$a = \frac{1}{N}\sum_{i=1}^{N} y_i = \frac{1}{N}\sum_{i=1}^{n} \ln x_i \,, \tag{13}$$

$$\beta^2 = \frac{1}{N-1}\left(\sum_{i=1}^{n} (\ln x_i)^2 - a^2 \right). \tag{14}$$

Thus, the Bayesian model of normal distribution may be used for logarithmic normal distribution as well, after its relevant rearrangement.

Finally it should be mentioned that stochastic models have to reflect the main regularities of the investigated object. The degree of adequacy in the given case depends on how precisely in the models is assessed the interdependency between incoming and outgoing parameters, interactive system and environment, ability to correct by a model the decisions and finally – the application of stochastic methods in obtaining the optimal behaviour scheme of the investigated system.

Conclusions

When the estimations of the probability characteristics of the transport means general operation time are known, then we may use them in planning transport enterprise work. Therefore, it is necessary to know how many transport means will have to be repaired within a relevant time period. For this purpose the probability model is proposed, which estimates an average number of transport means repairs within a time period.

While planning the operation of the transport means the volume of works to do is not always known beforehand, i.e. in the course of time the volume of works changes. Therefore, a stochastic model for defining the necessary productivity of transport means is proposed.

Statistical information on transport means is renewed and replenished in the course of time. With the growth of information amounts the costs of its storage increase too, therefore it is necessary to obtain the required statistical assessment with the least information amounts. For this purpose the mathematical models necessary for accumulation of statistics are created and verified. There have been created models for the following cases, when the density of distribution of random factors is assessed by: a) the Bayesian analysis; b) the normal distribution; c) the logarithmic-normal distribution; d) the non-parametric distribution; e) the Weibull's distribution. For all these cases there are created the algorithms for accumulation of minimum statistical data.

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STANDARDIZED FORMS OF KULLBACK-LEIBLER INFORMATION BASED STATISTICS FOR NORMALITY AND EXPONENTIALITY

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Our aim in this article is to present clear forms of well-known Kullback-Leibler information based tests. We focus on tests for normality and exponentiality, common tasks in applied studies. The proposed approach standardized the Kullback-Leibler information based tests. We show that the proposed tests are asymptotically consistent and convenient to practical applications. We conduct broad Monte Carlo analysis for judging our conclusions and the suggested tests. Thus, we can infer that the proposed tests for normality and exponentiality are powerful procedures that can be applied to real data studies.

Keywords: *Empirical distribution, entropy, goodness-of-fit tests, normality tests, exponentiality tests, Kullback-Leibler information*

1. Introduction

In many cases related to reliability studies, and engineering and management sciences, it is required to detect whether the assumption of a particular distribution is statistically justified. Testing distribution assumptions in general and for normality and exponentiality in particular has been one of major areas of continuing statistical research-both theoretically and practically.

The problem of testing a composite hypothesis of normality with the mean and / or variance unknown is perhaps the most common goodness-of-fit problem in current statistical practice. For example, an examination of residuals in order to ascertain the normality of the error variable is a routine part of regression analyses and the analyses of linear models in general. The literature on testing for normality in this context is vast. In particularly, the tests proposed by [Pearson, 1930] and [Shapiro; Wilk, 1965] are well-known, as well as empirical distribution function tests such as the Kolmogorov & Smirnov and Anderson & Darling. These empirical distribution function tests originally were proposed in canonical form and lately were variously adapted for testing composite hypotheses of normality [Stephens, 1974]. A more recent tests due to [Vasicek, 1976] and [Arizono; Ohta, 1989], based upon the entropy characterization of normality and Kullback-Leibler information (an extended concept of entropy), appear to compare reasonable with previous existing tests for normality in power and convenience. Their method does not require an estimation of parameters, transformations to uniformity as in empirical distribution functions tests or use of tables as in Shapiro & Wilk test. However, their method involves choosing the best integer parameter m which depends on the sample size n and hence their tests can not be simple applied to real data studies. Dong and Giles (2007) based their test for normality on an empirical likelihood approach, relatively recently developed by Owen (Owen, 2001). This distribution-free method for conducting estimation and hypothesis testing that still incorporates the notions of the likelihood function and the likelihood ratio.

Side by side with testing for normality, various tests for exponentiality have been proposed in the literature. Results on their properties can be found in [Ebrahami et al., 1992] and references therein. The practical applications of testing for exponentiality follow from the fact that many current results in life testing are based on the assumption that the life of a product is described by an exponential distribution. Theoretical justification for this distribution as a failure model is well established in the relevant literature. [Ebrahami et al., 1992] proposed a test of fit for exponentality based on Kullback-Leibler information. Their procedure is applicable when the exponential parameter is or is not specified under the null hypothesis. They presented also Monte Carlo power comparisons of the proposed test with other tests for exponentiality. The results show that their test is a very efficient decision rule provided that optimal values of m, subject to alternative distribution and the sample size n, are applied to their statistic.

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In practice, since the alternative distribution is completely unknown, we risk choosing m that leads to a Kullback-Leibler information based test having the power that is lower than that of other known tests for exponentiality.

In this paper we suggest new modifications of Kullback-Leibler information based tests for composite hypotheses of normality and exponentiality. The proposed method presents in clear form and standardizes these tests. We show that the suggested tests are asymptotic power one (i.e. consistent) and convenient to practical applications. Our broad Monte Carlo investigations results confirm the preference of the proposed method from a power perspective. The paper is organized as follows. In Section 2 we state formally the considered problem and describe the Kullback-Leibler information based tests for normality and exponentiality. In Section 3 the proposed test statistics are formulated and their main properties are presented. In Section 4 we provide Monte Carlo simulation results. We conclude In Section 5.

2. Tests for Normality and Exponentiality Based on Kullback-Leibler Information

Given a random sample $X_1,\dots,X_n$ from a population with a density function f and a finite variance, consider the problem of testing for

$$H_0 : f = f_0 \tag{1}$$

versus

$$H_1 : f = f_1 ,$$

where, under the alternative hypothesis, f_1 is completely unknown, whereas, under the null hypothesis, $f_0(x) = f_0(x;\boldsymbol{\theta})$ is known up to the vector of parameters $\boldsymbol{\theta} = (\theta_1,...,\theta_d)$ (here, $d \geq 1$ defines a dimension of the vector $\boldsymbol{\theta}$). We focus on the cases where $f_0(x;\boldsymbol{\theta})$ is a density function of normal or exponential distributions. The vector of parameters $\boldsymbol{\theta}$ may be particularly or completely unknown.

The Kullback-Leibler information defined by [Kullback; Leibler, 1951] as

$$I(f, f_0) = \int_{-\infty}^{+\infty} f(x)\log\left(\frac{f(x)}{f_0(x)}\right)dx \tag{2}$$

is the distance between the observed distribution F with the density function $f(x)$ and the model distribution F_0 with the density function $f_0(x)$, where $I(f, f_0) \geq 0$ and the equal sign is valid if $f(x) = f_0(x)$. The smaller $I(f, f_0)$ means that the observed distribution F is closer to the model distribution. [Arizono; Ohta, 1989] showed that

$$I(f, f_0) = -H(f) - \int_{-\infty}^{+\infty} f(x)\log(f_0(x))dx , \tag{3}$$

where

$$H(f) = E(-\log(f(X_1))) = -\int_{-\infty}^{+\infty} f(x)\log(f(x))dx = \int_0^1 \log\left(\frac{d}{dp}F^{-1}(p)\right)dp \tag{4}$$

is the entropy of a distribution F with a density function $f(x)$ and cumulative distribution function $F(x)$. An estimate of (4) can be constructed by replacing the distribution function $F(x)$ by the empirical distribution function $F_n(x) = \frac{1}{n}\sum_{i=1}^{n} I(X_n \leq x)$, where $I(x)$ is the indicator function. The derivation of $F^{-1}(p)$ is estimated by $\frac{n}{2m}\left(X_{(i+m)} - X_{(i+m)}\right)$ for $(i-1)/n < p \leq i/n$, $i = 1,..., n$, where $X_{(j)} = X_{(1)}$, if $j \leq 1$, and $X_{(j)} = X_{(n)}$, if $j \geq n$, $X_{(1)} \leq X_{(2)} \leq \ldots \leq X_{(n)}$ are the order statistics based on the observations $X_1,\dots,X_n$. Then, an estimate H_{mn} of $H(f)$ is produced as

$$H_{mn} = \frac{1}{n}\sum_{j=1}^{n} \log\left(\frac{n}{2m}\left(X_{(j+m)} - X_{(j+m)}\right)\right), \tag{5}$$

where $X_{(j)} = X_{(1)}$, if $j \leq 1$, and $X_{(j)} = X_{(n)}$, if $j \geq n$ [Vasicek, 1976]. Thus, when the null hypothesis is that the observations come from the normal distribution with the expectation μ and the variance σ^2:

$f_0(x;\mu,\sigma^2)=\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{1}{2\sigma^2}(x-\mu)^2\right)$, the entropy based test for composite hypothesis of normality is: to reject H_0 if

$$E_{mn}=\frac{1}{s}\exp(H_{mn})=\frac{n}{2ms}\left(\prod_{i=1}^{n}\left(X_{(i+m)}-X_{(i+m)}\right)\right)^{1/n}<C\,, \tag{6}$$

where $\overline{X}=\frac{1}{n}\sum_{i=1}^{n}X_i$, $s^2=\frac{1}{n}\sum_{i=1}^{n}\left(X_i-\overline{X}\right)^2$, C is a test-threshold.

By the similar way, from (3), by estimating $\int_{-\infty}^{+\infty}f(y)\log(f_0(y))dy$, [Arizono; Ohta, 1989] proposed the following Kullback-Leibler information based test for the simple hypotheses of normality (that is, when under H_0, the expectation μ and the variance σ^2 are known): to reject H_0 if

$$KL_{mn}=\frac{n\left(\prod_{j=1}^{n}\left(\frac{X_{(j+m)}-X_{(j-m)}}{\sigma}\right)\right)^{1/n}}{2m\exp\left(\frac{1}{2n}\sum_{j=1}^{n}\left(\frac{X_j-\mu}{\sigma}\right)^2\right)}<C\,, \tag{7}$$

where C is a test-threshold. It can be shown that under the null hypothesis, $KL_{mn}\xrightarrow{P}\sqrt{2\pi}$ as $n\to\infty$, $m\to\infty$, $m/n\to 0$. In addition, $0\le KL_{mn}\le\sqrt{2\pi}$. Therefore, the test (7) is asymptotically consistent as $n\to\infty$, $m\to\infty$, $m/n\to 0$. When the KL_{mn} statistic is applied to the composite hypothesis of normality, we substitute the estimate of the mean ($\hat{\mu}=\overline{X}$) and the variance ($\hat{\sigma}^2=s^2$) into (7). Thus, we obtain the following Kullback-Leibler information based test for the composite hypotheses of normality: to reject H_0 if:

$$KL_{mn}=\frac{n}{2ms\sqrt{e}}\left(\prod_{j=1}^{n}\left(X_{(j+m)}-X_{(j-m)}\right)\right)^{1/n}=\frac{E_{mn}}{\sqrt{e}}<C\,, \tag{8}$$

where C is a test-threshold. Since under the null hypothesis, $KL_{mn}\xrightarrow{P}\sqrt{2\pi}$ as $n\to\infty$, $m\to\infty$, $m/n\to 0$, the test (8) is asymptotically consistent as $n\to\infty$, $m\to\infty$, $m/n\to 0$. As the distribution KL_{mn} for given values of n and m has not been obtained analytically, [Arizono; Ohta, 1989] evaluated the critical points of the KL_{mn} statistic for the significance level $\alpha=0.05$ and different values of n, m, by using Monte Carlo simulations. The power of the test (7) was also estimated against several alternatives. The similar Monte Carlo power comparisons of the test (8) with other tests for normality were presented in [Vasicek, 1976]. These simulation results point out values of m (subject to n) that provide high levels of the power of the tests (7) and (8). However, the power of the tests (7) and (8) strongly depends on values of m. For example, when $n=50$ the test-statistic KL_{mn} with $m=1$ used to compare unfavourably with other tests for normality.

When the null hypothesis is that the observations come from the exponential distribution with the parameter λ, that is, $f_0(x;\lambda)=\lambda\exp(-\lambda x)I(x\ge 0)$, [Ebrahami et al., 1992] presented the Kullback-Leibler information (3) in the following form:

$$I(f,f_0)=-H(f)-\log(\lambda)+\lambda\int_{-\infty}^{+\infty}xf(x)dx=-H(f)-\log(\lambda)+1\,. \tag{9}$$

Then, by estimating of $H(f)$ and the parameter λ, they proposed the following Kullback-Leibler information based test for composite hypothesis of exponentiality (that is, when the parameter λ is unknown): to reject H_0 if:

$KL^*_{mn} = \dfrac{\exp(H_{mn})}{\exp(\log(\overline{X})+1)} < C$, where H_{mn} is defined by (5), C is a test-threshold. We rewrite this test in the equivalent form: to reject H_0 if:

$$KL^*_{mn} = \frac{n\left(\prod_{j=1}^{n}\left(X_{(j+m)} - X_{(j-m)}\right)\right)^{1/n}}{2me\overline{X}} < C, \tag{10}$$

where C is a test-threshold. For the situation where the exponential parameter is specified as λ_0 under H_0, the test suggested in [Ebrahami et al., 1992] is: to reject H_0 if:

$$KL^*_{mn}(\lambda_0) = \exp(H_{mn} - \log(\lambda_0) - 1) < C, \tag{11}$$

where C is a test-threshold. It can be shown that $0 < KL^*_{mn} < 1$ and under the null hypothesis, $KL^*_{mn} \xrightarrow{P} 1$ as $n \to \infty$, $m \to \infty$, $m/n \to 0$. Therefore, the test (10) is asymptotically consistent as $n \to \infty$, $m \to \infty$, $m/n \to 0$. The statistic $KL^*_{mn}(\lambda)$ has all the properties of KL^n_{mn} described earlier. The simulated results presented in [Ebrahami et al., 1992] show that the tests (10) and (11) have the similar properties as the tests (7) and (8). That is, these tests are a very efficient decision rules provided that optimal values of m, subject to alternative distribution and the sample size n, are applied to the their statistic.

Since, in practice the alternative distribution is completely unknown, we risk choosing m that leads to a Kullback-Leibler information based test having the power that is lower than that of other known tests for normality and exponentiality. This lack of balance restricts fields of real data applications of tests based on the Kullback-Leibler information. The proposed in the next Section method can attend to this issue.

3. The Proposed Tests and Their Properties

In this section, we suggest new modifications of Kullback-Leibler information based tests for composite hypotheses of normality and exponentiality. First we consider a test of fit for normality.

In accordance with (8), we suggest the first modification of the of Kullback-Leibler information based test statistic for the composite hypothesis of normality in the form of

$$MKL^1_n = \max_{1 \le m < n/2}\left(\frac{n}{2ms\sqrt{e}}\left(\prod_{j=1}^{n}\left(X_{(j+m)} - X_{(j-m)}\right)\right)^{1/n}\right). \tag{12}$$

In addition, following the asymptotic theorems of [Vasicek, 1976], we must require $m/n \to 0$ as $m,n \to \infty$. Therefore, we propose considering m in the range $(1, n^{1-\delta})$, $0 < \delta < 1$, and hence the second modification of the test-statistic (8) has the form of

$$MKL^2_n = \max_{1 \le m < n^{1-\delta}}\left(\frac{n}{2ms\sqrt{e}}\left(\prod_{j=1}^{n}\left(X_{(j+m)} - X_{(j-m)}\right)\right)^{1/n}\right), \tag{13}$$

where $0 < \delta < 1$. Note that, we maximize the Kullback-Leibler information based test statistic KL_{mn} with respect to m.

Proposed tests for normality. Define the following decision rules. We reject the null hypothesis $H_0 : f_0(x;\mu,\sigma^2) = \dfrac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\dfrac{1}{2\sigma^2}(x-\mu)^2\right)$, μ and σ^2 are unknown, if

$$MKL^j_n < C, \tag{14}$$

where C is a test-threshold, $j = 1,2$ and MKL^j_n are the test-statistics defined in (12) and (13), respectively.

The following Proposition 1 evaluates asymptotically the operating characteristics of the test (14) with $j = 2$. (We do not prove the asymptotic consistency of the test (14) with $j = 1$, but, due to high levels of the power of this test against alternatives, which are considered in the next Section, in several situations, we can recommend the test (14) with $j = 1$ to be applied.)

Significance level of the proposed tests for normality.

Since $\sup_{\mu,\sigma} P_{H_0}\{MKL_n^j < C\} = P_{X_1,\ldots,X_n \sim N(0,1)}\{MKL_n^j < C\}$, $j = 1,2$, the type I error of the tests (14) can be calculated exactly. Set up $\delta = 0.5$ in the definition of the statistic MKL_n^2. Tables 1 and 2 present Monte Carlo roots C_α of the equations $P_{X_1,\ldots,X_n \sim N(0,1)}\{MKL_n^j < C_\alpha\} = \alpha$, for different values of α and n, $j = 1,2$, respectively. For each value of α and n the type I error results were derived from 50,000 samples of size n. Figure 1 plots these critical values C_α for different values of α and n.

Table 1. The Monte Carlo critical values for the MKL_n^1 statistic

n	Critical values C_α for the test (14), $j = 1$, for the following values of α:					
	0.01	0.025	0.05	0.10	0.15	0.2
5	0.74860	0.87878	0.98981	1.11625	1.19445	1.25167
10	1.25890	1.34138	1.41150	1.49114	1.54828	1.59814
15	1.47073	1.54501	1.61623	1.70159	1.75581	1.79678
20	1.61781	1.68925	1.75617	1.82796	1.87477	1.90914
25	1.72433	1.79548	1.85297	1.91554	1.95506	1.98436
30	1.80745	1.87430	1.92353	1.97754	2.01242	2.03886
35	1.87823	1.93242	1.97758	2.02621	2.05783	2.08178
40	1.92196	1.97615	2.01858	2.06246	2.09133	2.11327
45	1.96252	2.01206	2.05208	2.09495	2.12112	2.14154
50	2.00096	2.04405	2.08118	2.12041	2.14557	2.16476
55	2.03369	2.07363	2.10753	2.14306	2.16638	2.18401
60	2.06147	2.10004	2.13029	2.16424	2.18604	2.20251
65	2.08248	2.11974	2.14861	2.18090	2.20107	2.21667
70	2.10297	2.13653	2.16566	2.19562	2.21548	2.23038
75	2.12096	2.15301	2.17967	2.20928	2.22804	2.24229
80	2.13560	2.16835	2.19387	2.22165	2.23953	2.25357
85	2.14877	2.18096	2.20504	2.23251	2.24984	2.26313
90	2.16102	2.19035	2.21460	2.24172	2.25877	2.27187
95	2.17670	2.20459	2.22719	2.25241	2.26872	2.28096
100	2.18620	2.21482	2.23758	2.26082	2.27646	2.28861

Table 2. The Monte Carlo critical values for the MKL_n^2, $\delta = 0.5$, statistic

n	Critical values C_α for the test (14), $j = 2$, $\delta = 0.5$, for the following values of α:					
	0.01	0.025	0.05	0.10	0.15	0.2
5	0.74860	0.87878	0.98981	1.11625	1.19445	1.25167
10	1.19990	1.30599	1.38955	1.48415	1.55280	1.60152
15	1.41833	1.51801	1.59874	1.68777	1.74448	1.78860
20	1.59680	1.67865	1.74940	1.82483	1.87078	1.90562
25	1.71106	1.78204	1.84450	1.90755	1.94758	1.97816
30	1.80135	1.86647	1.91993	1.97483	2.00924	2.03607
35	1.86553	1.92600	1.97275	2.02106	2.05294	2.07601
40	1.92409	1.97408	2.01697	2.06110	2.08905	2.11067
45	1.96290	2.01055	2.05025	2.09178	2.11857	2.13851
50	2.00093	2.04353	2.07903	2.11819	2.14305	2.16230
55	2.02909	2.07264	2.10733	2.14206	2.16554	2.18264
60	2.05386	2.09308	2.12522	2.16039	2.18273	2.19939
65	2.08000	2.11724	2.14714	2.17883	2.20009	2.21527
70	2.10209	2.13755	2.16481	2.19448	2.21382	2.22879
75	2.11620	2.15097	2.17770	2.20770	2.22583	2.24028
80	2.13239	2.16438	2.19135	2.21976	2.23752	2.25122
85	2.14948	2.18020	2.20500	2.23183	2.24918	2.26242
90	2.16198	2.19147	2.21466	2.23996	2.25676	2.27029
95	2.17371	2.20269	2.22528	2.25040	2.26602	2.27827
100	2.18393	2.21198	2.23392	2.25822	2.27385	2.28579

The following Proposition 1 claims that the proposed test (14) with $j = 2$ is consistent for all $0 < \delta < 1$.

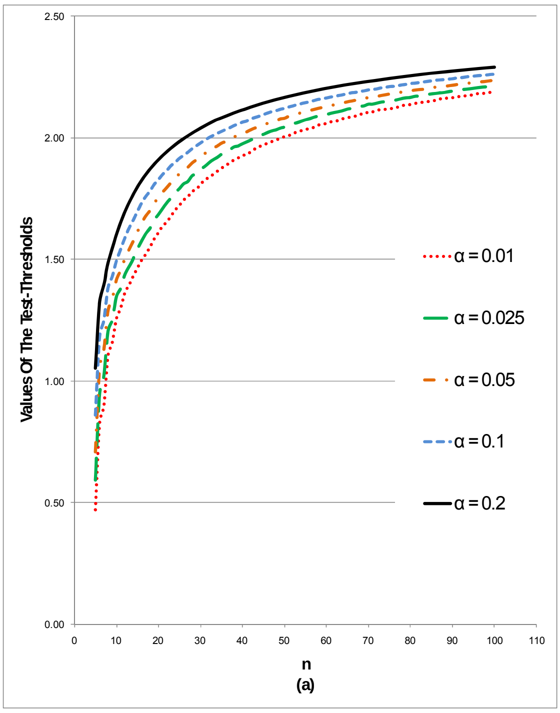
Proposition 1. For each $0 < \delta < 1$, under H_0,

$MKL_n^2 \xrightarrow{P} \sqrt{2\pi}$, as $n \to \infty$,

while, under H_1,

$MKL_n^2 \xrightarrow{P} a < \sqrt{2\pi}$, as $n \to \infty$.

The proof of the proposition 1 follows straightforwardly from the lemma 1 of [Vasicek, 1976)] The Proposition 1 shows that, with a test-threshold C_α related to the type I error $\alpha = P_{H_0}\left(MKL_n^2 < C_\alpha\right)$ in mind, $P_{H_1}\left(MKL_n^2 < C_\alpha\right) \xrightarrow[n\to\infty]{} 1$, where we define by P_{H_k} the probability measure under the hypothesis H_k , $k = 0,1$. Therefore, the suggested test (14) with $j = 2$ is the asymptotic power one (i.e., consistent) tests for all $0 < \delta < 1$.



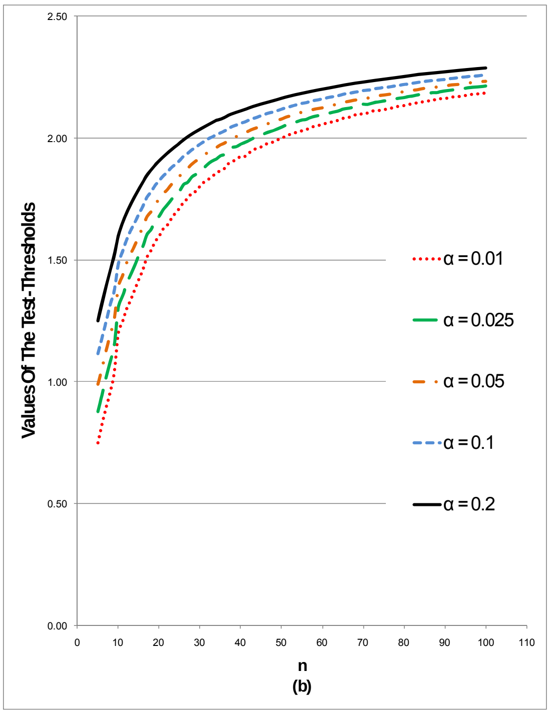


Figure 1. The curves present the values of thresholds C_α for the tests (14), corresponding to the significance levels $\alpha = 0.01, 0.025, 0.05, 0.1, 0.2$, that are plotted against the sample sizes $n = 5,10,15,...,100$. Plots (a) and (b) are related to the test-statistics MKL_n^1 and MKL_n^2 with $\delta = 0.5$, respectively

By the similar way as in (12), (13), in accordance with (10), we suggest the modifications of the of Kullback-Leibler information based test statistic for composite hypothesis of exponentiality in the form of

$$MKL_n^{*1} = \max_{1\le m<n/2}\left(\frac{n\left(\prod_{j=1}^{n}\left(X_{(j+m)} - X_{(j-m)}\right)\right)^{1/n}}{2me\overline{X}}\right), \tag{15}$$

$$MKL_n^{*2} = \max_{1\le m<n^{1-\delta}}\left(\frac{n\left(\prod_{j=1}^{n}\left(X_{(j+m)} - X_{(j-m)}\right)\right)^{1/n}}{2me\overline{X}}\right),$$

where $0 < \delta < 1$.

Proposed tests for exponentiality. Define the following decision rules. We reject the null hypothesis $H_0 : f_0(x;\lambda) = \lambda \exp(-\lambda x) I(x \ge 0)$, λ is unknown, if

$$MKL_n^{*j} < C , \tag{16}$$

where C is a test-threshold and MKL_n^{*j}, $j = 1,2$, are the test-statistics defined in (15). The following Proposition 2 evaluates asymptotically the operating characteristics of the test (16) with $j = 2$.

Significance level of the proposed tests for exponentiality.

Since $\sup\limits_{\lambda} P_{H_0}\left\{MKL^{*j}_{n} < C\right\} = P_{X_1,\ldots,X_n \sim EXP(1)}\left\{MKL^{*j}_{n} < C\right\},\ j = 1,2$,

the type I error of the tests (16) can be calculated exactly. Set up $\delta = 0.5$ in the definition of the statistic MKL^{*2}_{n}.

Tables 3 and 4 present Monte Carlo roots C_α of the equations $P_{X_1,\ldots,X_n \sim EXP(1)}\left\{MKL^{*j}_{n} < C_\alpha\right\} = \alpha$, for different values of α and n, $j = 1,2$, respectively. For each value of α and n the type I error results were derived from 50,000 samples of size n. Figure 2 plots these critical values C_α for different values of α and n.

Table 3. The Monte Carlo critical values for the MKL^{*1}_{n} statistic

n	Critical values C_α for the test (16), $j=1$, for the following values of α:				
	0.01	0.025	0.05	0.10	0.15
5	0.17655	0.22498	0.27271	0.33283	0.37572
10	0.40974	0.46185	0.50555	0.55378	0.58575
15	0.54144	0.58292	0.61791	0.65686	0.68245
20	0.61644	0.65158	0.68287	0.71539	0.73649
25	0.66636	0.70207	0.72795	0.75648	0.77466
30	0.70868	0.73688	0.76034	0.78540	0.80143
35	0.73885	0.76415	0.78496	0.80623	0.82102
40	0.76050	0.78277	0.80147	0.82277	0.83644
45	0.77870	0.80068	0.81759	0.83706	0.84947
50	0.79466	0.81335	0.82956	0.84794	0.85953
55	0.80808	0.82670	0.84207	0.85830	0.86912
60	0.81871	0.83560	0.84999	0.86546	0.87621
65	0.82917	0.84526	0.85858	0.87302	0.88285
70	0.83695	0.85206	0.86499	0.87933	0.88838
75	0.84425	0.85934	0.87174	0.88482	0.89381
80	0.85246	0.86543	0.87690	0.88952	0.89854
85	0.85841	0.87111	0.88187	0.89439	0.90297
90	0.86410	0.87673	0.88713	0.89913	0.90719
95	0.86764	0.88046	0.89105	0.90257	0.91055
100	0.87210	0.88427	0.89444	0.90622	0.91400

Table 4. The Monte Carlo critical values for the MKL^{*2}_{n}, $\delta = 0.5$, statistic

n	Critical values C_α for the test (16), $j=2$, $\delta = 0.5$, for the following values of α:				
	0.01	0.025	0.05	0.10	0.15
5	0.17655	0.22498	0.27271	0.33283	0.37572
10	0.40041	0.45116	0.49395	0.54167	0.57381
15	0.51352	0.55695	0.59391	0.63359	0.65936
20	0.60241	0.63741	0.66772	0.70119	0.72222
25	0.65593	0.68593	0.71197	0.74081	0.75877
30	0.69724	0.72594	0.74887	0.77360	0.78982
35	0.72779	0.75179	0.77226	0.79413	0.80851
40	0.75270	0.77498	0.79334	0.81357	0.82668
45	0.77105	0.79189	0.80851	0.82662	0.83858
50	0.78801	0.80621	0.82219	0.84022	0.85129
55	0.80001	0.81810	0.83346	0.84976	0.85976
60	0.81142	0.82883	0.84274	0.85773	0.86711
65	0.82438	0.83966	0.85220	0.86650	0.87546
70	0.83039	0.84610	0.85834	0.87204	0.88085
75	0.83920	0.85353	0.86528	0.87795	0.88587
80	0.84616	0.85964	0.87029	0.88211	0.88993
85	0.85262	0.86546	0.87620	0.88786	0.89552
90	0.85863	0.87097	0.88120	0.89225	0.89947
95	0.86379	0.87535	0.88470	0.89566	0.90247
100	0.86701	0.87881	0.88833	0.89857	0.90527

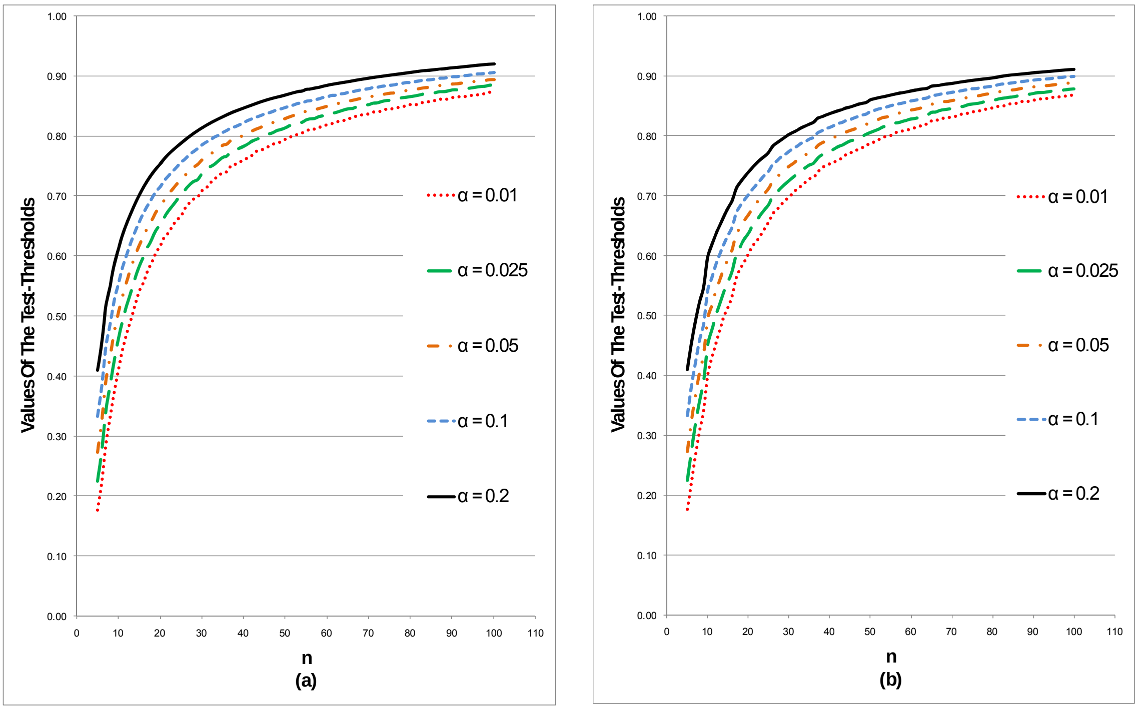


Figure 2. The curves present the values of thresholds C_α for the tests (16), corresponding to the significance levels $\alpha = 0.01, 0.025, 0.05, 0.1, 0.2$, that are plotted against the sample sizes $n = 5, 10, 15, \ldots, 100$. Plots (a) and (b) are related to the test-statistics MKL^{*1}_n and MKL^{*2}_n with $\delta = 0.5$, respectively

The following Proposition 2 claims that the proposed test (16) with $j = 2$ is consistent for all $0 < \delta < 1$.

Proposition 2. For each $0 < \delta < 1$, under H_0,

$MKL^{*2}_n \xrightarrow{P} 1$, as $n \to \infty$,

while, under H_1,

$MKL^{*2}_n \xrightarrow{P} a < 1$, as $n \to \infty$.

The proof of the proposition 2 is similar to the proof of the proposition 1 and follows from the lemma 1 of [Vasicek, 1976]. The Proposition 2 shows that, with a test-threshold C_α related to the type I error $\alpha = P_{H_0}\left(MKL^{*2}_n < C_\alpha\right)$ in mind, $P_{H_1}\left(MKL^{*2}_n < C_\alpha\right) \xrightarrow[n\to\infty]{} 1$. Therefore, the suggested test (16) with $j = 2$ is the asymptotic power one (i.e., consistent) tests for all $0 < \delta < 1$.

4. Monte Carlo Study

4.1. The Proposed Tests for Normality

To study properties of the proposed tests (14), Monte Carlo simulations were employed. For each $n = 20, 50, 100$; 50,000 samples for size n from Log-Normal, Chi-Square, T-Student, Uniform, Exponential, Gamma, Beta and Cauchy distributions were produced. The Kullback-Leibler information based test-statistic KL_{mn} for several values of *m* was calculated from each sample. The number of times H_0 was rejected by each test, for each sample size, is shown in Table 5, at the $\alpha = 0.05$ level of significance. Besides the Monte Carlo powers of the Kullback-Leibler information based tests and the proposed tests (14) with $\delta = 0.5$, Table 5 gives estimates of power of the Kolmogorov-Smirnov (KS) test to judge the proposed tests. The literature cited in the Introduction can be used to compare Table 5 with simulation results related to known tests for normality, e.g. [Vasicek, 1976], [Arizono; Ohta, 1989]. We point out that the power of the proposed tests (14) is similar or even better than that of the Kullback-Leibler information based test with optimal choice of the parameter m. Although the proposed tests tend to be superior to other tests for normality against most of the considered alternatives, in several cases we would suggest using the Kolmogorov-Smirnov or another test. For example, when a H_1-distribution is expected to be a Student $T_{(3)}$, the Kolmogorov-Smirnov test is more powerful than the Kullback-Leibler information based test including the proposed forms (14). Unfortunately, alternative distributions used to be unknown. Note that, in this article, we did not prove consistency of the test (14) with $j=1$. Table 5 displays problems with this test against the Cauchy alternative when the sample size increases. However, due to high levels of the demonstrated power of the test (14) with $j=1$ for all other considered alternatives we are afraid to reject the practical meaning of this decision rule.

Table 5. Monte Carlo power estimates of some tests for normality; *a* = 0.05

n = 20	KL_{2n}	KL_{4n}	KL_{6n}	KL_{8n}	KL_{10n}	KL_{15n}	KL_{20n}	KL_{30n}	KL_{40n}	KS	MKL_n^1	MKL_n^2
Log-Normal(0,1)	0.8990	0.9310	0.9222	0.8940	0.8200	-	-	-	-	0.7960	0.9307	0.9284
Chi-Square (3)	0.5690	0.6500	0.6270	0.5732	0.4627	-	-	-	-	0.4200	0.6495	0.6459
T-Student (3)	0.1740	0.1407	0.0969	0.0756	0.0584	-	-	-	-	0.2530	0.1271	0.1525
Uniform (0,1)	0.4780	0.4862	0.4913	0.5112	0.5152	-	-	-	-	0.1490	0.9551	0.9546
Exp (1)	0.8140	0.8642	0.8421	0.8063	0.6966	-	-	-	-	0.6020	0.8663	0.8633
Gamma (2,1)	0.4040	0.4838	0.4613	0.4154	0.3240	-	-	-	-	0.3340	0.4856	0.4803
Beta (2,1)	0.3900	0.4648	0.4593	0.4479	0.4015	-	-	-	-	0.1960	0.4664	0.4591
Cauchy (0,1)	0.7630	0.6946	0.5327	0.4150	0.3698	-	-	-	-	0.8370	0.5143	0.7118
n = 50												
Log-Normal (0,1)	0.9993	0.9999	0.9999	0.9998	0.9998	0.9990	0.9930	-	-	0.9961	0.9999	0.9999
Chi-Square (3)	0.9468	0.9804	0.9838	0.9802	0.9722	0.9263	0.8064	-	-	0.8346	0.9844	0.9845
T-Student (3)	0.3869	0.3629	0.2735	0.1800	0.1090	0.0368	0.0150	-	-	0.4702	0.0929	0.2612
Uniform (0,1)	0.8742	0.9335	0.9566	0.9684	0.9714	0.9776	0.9805	-	-	0.3206	0.9999	0.9999
Exp (1)	0.9973	0.9998	0.9998	0.9998	0.9993	0.9943	0.9708	-	-	0.9645	0.9998	0.9998
Gamma (2,1)	0.8231	0.9118	0.9253	0.9144	0.8887	0.7720	0.5982	-	-	0.7065	0.9291	0.9282
Beta (2,1)	0.8090	0.9174	0.9406	0.9449	0.9421	0.9238	0.8956	-	-	0.4789	0.9373	0.9365
Cauchy (0,1)	0.9890	0.9886	0.9794	0.9525	0.8959	0.5909	0.3342	-	-	0.9946	0.4208	0.9756
n = 100												
Log-Normal (0,1)	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9990	1.0000	1.0000	1.0000
Chi-Square (3)	0.9992	1.0000	1.0000	1.0000	1.0000	0.9999	0.9986	0.9856	0.8994	0.9921	1.0000	1.0000
T-Student (3)	0.6404	0.6777	0.6259	0.5207	0.4082	0.1668	0.0616	0.0123	0.0032	0.7198	0.0280	0.4851
Uniform (0,1)	0.9934	0.9995	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.6329	1.0000	1.0000
Exp (1)	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9929	0.9999	1.0000	1.0000
Gamma (2,1)	0.9836	0.9985	0.9996	0.9993	0.9989	0.9961	0.9863	0.9082	0.7308	0.9529	0.9989	0.9993
Beta (2,1)	0.9835	0.9992	0.9997	0.9998	0.9999	0.9998	0.9995	0.9993	0.9981	0.8313	0.9999	0.9999
Cauchy (0,1)	0.9997	0.9998	0.9998	0.9997	0.9996	0.9961	0.9645	0.6046	0.2732	0.9999	0.2601	0.9997

4.2. The Proposed Tests for Exponentiality

[Ebrahami et al., 1992] considered the Kullback-Leibler information based test for exponentiality - (10) with optimal parameters m that depend on the sample sizes n and were obtained previously for different values of n by simulations. They compared this test with two existing procedures: Van-Soest test based on the statistic $W=\sum_{i=1}^{n}\left\{F_0\left(X_{(i)};\hat{\lambda}\right)-\frac{(2i-1)}{n}\right\}^2+\frac{n}{12}$ and Finkelstein and Shafer's test based on the statistic

$$S=\sum_{i=1}^{n}\max\left\{\left|F_0\left(X_{(i)};\hat{\lambda}\right)-\frac{i}{n}\right|,\left|F_0\left(X_{(i)};\hat{\lambda}\right)-\frac{(i-1)}{n}\right|\right\},$$

where $\hat{\lambda} = 1/\overline{X}$, $F_0(x)$ is the cumulative distribution function of the observations distribution under H_0. For power comparisons, the authors considered the following alternatives:

A: a Weibull's distribution with density function $f(x;\lambda,\beta) = \beta\lambda^{\beta}x^{\beta-1}\exp\left(-(\lambda x)^{\beta}\right)$, $\beta > 0$, $\lambda > 1$, $x \geq 0$;

B: a Gamma distribution with density function $f(x;\lambda,\beta) = \dfrac{\lambda^{\beta}x^{\beta-1}\exp(-\lambda x)}{\Gamma(\beta)}$,

where $\Gamma(\beta) = \int_0^{+\infty} x^{\beta-1}\exp(-x)dx$ is the Gamma function, $\beta > 0$, $\lambda > 0$, $x \geq 0$;

C: a Log-Normal distribution with density function $f(x;\nu,\sigma^2) = \dfrac{1}{x\sigma\sqrt{2\pi}}\exp\left(-\dfrac{1}{2\sigma^2}(\log x - \nu)^2\right)$,

$-\infty < \nu < \infty$, $\sigma > 0$, $x > 0$.

For all considered distributions the parameters were chosen such that $E(X) = 1$, that is $\lambda = \Gamma(1 + 1/\beta)$ for the Weibull's distribution, $\lambda = \beta$ for the Gamma distribution and $\nu = -\sigma^2/2$ for the Log-Normal case. The presented in [Ebrahami et al., 1992] Monte Carlo power comparisons (based on 5000 samples of size n equal to 5, 10, 15 and 20) confirmed that the Kullback-Leibler information based test for exponentiality-(10) is a very efficient procedure provided that optimal values of m are found. This test tends to be superior to the Van-Soest's and Finkelstein's and Shafer's tests against all considered alternatives.

We compare the proposed tests (16) with tests considered in [Ebrahami et al., 1992]. To this end, we conduct the Monte Carlo study based on a scheme (with the alternative distributions A, B and C in mind) that is suggested by [Ebrahami et al., 1992]. In Tables 6, 7 and 8 we report a Monte Carlo comparison between the powers of the tests (16) based on the statistics MKL_n^1 and MKL_n^2 (with $\delta = 0.5$) with the Kullback-Leibler information based test-(10) for values of m, corresponding to optimal powers (KL), Van-Soest's test (W) and Finkelstein's and Shafer's test (S); at the levels of significance $\alpha = 0.01, 0.05$, for the Weibull, Gamma and Log-Normal alternatives, respectively. The estimated powers of the Kullback-Leibler information based test-(10) for values of m, corresponding to optimal powers, Van-Soest's and Finkelstein's and Shafer's tests were copied from Tables 3, 4, 5 of [Ebrahami et al., 1992].

Table 6. Monte Carlo power estimates of the W, S, KL, MKL_n^1 and MKL_n^2 ($\delta = 0.5$) tests against the Weibull's distribution

n	β	α	**W**	**S**	**KL**	MKL_n^1	MKL_n^2
5	2	0.01	0.0800	0.0795	0.1135	0.1073	0.1073
		0.05	0.2975	0.3025	0.4015	0.3076	0.3076
	3	0.01	0.2955	0.2905	0.3530	0.2829	0.2829
		0.05	0.6505	0.6660	0.7490	0.6067	0.6067
	4	0.01	0.5445	0.5415	0.5995	0.4867	0.4867
		0.05	0.8700	0.8860	0.9315	0.8205	0.8205
10	2	0.01	0.2550	0.2640	0.4250	0.3641	0.3261
		0.05	0.6135	0.6410	0.7020	0.6619	0.6223
	3	0.01	0.7650	0.8075	0.8995	0.8308	0.7979
		0.05	0.9690	0.9740	0.9855	0.9622	0.9509
	4	0.01	0.9705	0.9780	0.9930	0.9776	0.9682
		0.05	0.9995	0.9995	1.0000	0.9986	0.9982
15	2	0.01	0.4695	0.4880	0.6090	0.6163	0.5281
		0.05	0.8280	0.8420	0.8625	0.8594	0.7997
	3	0.01	0.9730	0.9800	0.9890	0.9827	0.9667
		0.05	1.0000	0.9995	0.9985	0.9993	0.9972
	4	0.01	1.0000	1.0000	1.0000	0.9998	0.9993
		0.05	1.0000	1.0000	1.0000	1.0000	1.0000
20	2	0.01	0.7585	0.7765	0.7830	0.7885	0.7373
		0.05	0.9265	0.9465	0.9290	0.9408	0.9182
	3	0.01	1.0000	1.0000	1.0000	0.9986	0.9976
		0.05	1.0000	1.0000	1.0000	0.9999	0.9999
	4	0.01	1.0000	1.0000	1.0000	1.0000	1.0000
		0.05	1.0000	1.0 000	1.0000	1.0000	1.0000

Table 7. Monte Carlo power estimates of the W, S, KL, MKL_n^1 and MKL_n^2 ($\delta = 0.5$) tests against the Gamma distribution

n	β	α	W	S	KL	MKL_n^1	MKL_n^2
5	2	0.01	0.0350	0.0330	0.0425	0.0476	0.0476
		0.05	0.1420	0.1340	0.1830	0.1650	0.1650
	3	0.01	0.0775	0.0740	0.1095	0.0855	0.0855
		0.05	0.2660	0.2625	0.3355	0.2729	0.2729
	4	0.01	0.1295	0.1175	0.1640	0.1420	0.1420
		0.05	0.3890	0.3835	0.4860	0.3813	0.3813
10	2	0.01	0.0690	0.0720	0.1355	0.1180	0.1027
		0.05	0.2500	0.2530	0.3545	0.3239	0.3001
	3	0.01	0.2025	0.2075	0.3480	0.3003	0.2664
		0.05	0.5310	0.5445	0.6370	0.6005	0.5640
	4	0.01	0.3785	0.3855	0.5765	0.5009	0.4548
		0.05	0.7615	0.7570	0.8585	0.7900	0.7590
15	2	0.01	0.1115	0.1140	0.1865	0.1983	0.1578
		0.05	0.3540	0.3580	0.4320	0.4581	0.3835
	3	0.01	0.3925	0.3940	0.5140	0.5334	0.4477
		0.05	0.7510	0.7555	0.7850	0.8074	0.7405
	4	0.01	0.6685	0.6665	0.8035	0.7847	0.7080
		0.05	0.9295	0.9350	0.9540	0.9500	0.9185
20	2	0.01	0.2200	0.2235	0.2435	0.2809	0.2377
		0.05	0.4640	0.4890	0.4850	0.5497	0.4930
	3	0.01	0.6630	0.6695	0.6900	0.7065	0.6510
		0.05	0.8870	0.8895	0.8730	0.9020	0.8696
	4	0.01	0.9185	0.9115	0.9235	0.9214	0.8886
		0.05	0.9895	0.9900	0.9850	0.9883	0.9810

Table 8. Monte Carlo power estimates of the W, S, KL, MKL_n^1 and MKL_n^2 ($\delta = 0.5$) tests against the Log-Normal distribution

n	ν	α	W	S	KL	MKL_n^1	MKL_n^2
5	-0.3	0.01	0.0270	0.0255	0.0345	0.0417	0.0417
		0.05	0.1355	0.1310	0.1670	0.1537	0.1537
	-0.2	0.01	0.0650	0.0635	0.0840	0.0745	0.0745
		0.05	0.2315	0.2260	0.3085	0.2454	0.2454
	-0.1	0.01	0.1925	0.1780	0.2425	0.2022	0.2022
		0.05	0.5430	0.5255	0.6340	0.4966	0.4966
10	-0.3	0.01	0.0555	0.0555	0.1085	0.1003	0.0899
		0.05	0.2175	0.2255	0.3080	0.2983	0.2800
	-0.2	0.01	0.1485	0.1550	0.3020	0.2543	0.2271
		0.05	0.4515	0.4560	0.6015	0.5554	0.5200
	-0.1	0.01	0.6045	0.5705	0.7840	0.7043	0.6574
		0.05	0.9225	0.9030	0.9540	0.9182	0.8997
15	-0.3	0.01	0.0795	0.0745	0.1490	0.1689	0.1431
		0.05	0.2935	0.2895	0.3675	0.4129	0.3805
	-0.2	0.01	0.3020	0.2810	0.4305	0.4534	0.3823
		0.05	0.6455	0.6240	0.7365	0.7515	0.6985
	-0.1	0.01	0.8830	0.8450	0.9285	0.9394	0.9045
		0.05	0.9890	0.9835	0.9900	0.9935	0.9884
20	-0.3	0.01	0.1695	0.1610	0.2230	0.2359	0.2201
		0.05	0.4060	0.4025	0.4780	0.4985	0.4900
	-0.2	0.01	0.5250	0.4995	0.5910	0.6205	0.5804
		0.05	0.8030	0.7845	0.8270	0.8475	0.8344
	-0.1	0.01	0.9935	0.9850	0.9910	0.9911	0.9873
		0.05	1.0000	1.0000	0.9990	0.9989	0.9995

The Monte Carlo powers of the proposed tests (16) based on the statistics MKL_n^1 and MKL_n^2 (with $\delta = 0.5$) were obtained utilizing 50,000 samples for size $n = 5, 10, 15, 20$. The critical values of the tests (16) test were calculated using Monte Carlo techniques with 50,000 replications to guarantee the levels of significance $\alpha = 0.01, 0.05$. We point out that power of the proposed tests (16) is close to that of the Kullback-Leibler information based test with optimal choice of the parameter m and almost always is better than that of the Van-Soest's, Finkelstein's and Shafer's tests for all considered alternatives. Note that we did not prove consistency of the test (16) with $j = 1$. However, by simulated study presented in Tables 6, 7, 8, this test has a high power for all considered alternatives and usually tends to be superior to other tests for exponentiality. Therefore we suggest to take into account the practical meaning of this test, but, due to consistency and high power for small sample sizes, we recommend applying the test (16) with $j = 2$ and $\delta = 0.5$.

Finally, by virtue of the simplicity for practical applications of the proposed tests (14) and (16) and high power of these tests, we conclude that in many situations the tests (14) and (16) may be a preferable alternative to existing tests of normality and exponentiality.

Conclusions

In this article, we have presented new modifications of Kullback-Leibler information based tests for composite hypotheses of normality and exponentiality. We have proved that the proposed tests for normality and exponentiality are consistent and convenient to practical applications. However, since in practice, sample sizes are limited and there is a lot of asymptotic power one tests, we would like to focus on the empirical reasoning evaluated by the Monte Carlo study. We conducted broad Monte Carlo analysis for judging our conclusions and the suggested tests. Simulation results confirm the preference of the proposed tests from a power perspective.

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HEURISTIC "LOOK-AHEAD" COST-RELIABILITY MODELS

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A hierarchical technical system functioning under random disturbances is considered. Certain elements can be refined by undertaking technical improvements. The list of the latter is pregiven as well. Assume that by means of simulation modelling (SM) it is possible to evaluate the increment of the system's reliability by implementing any set of technical improvements. The developed models centre on determining an *optimal sub-set* of technical improvements in order to either to maximize the system's reliability subject to a restricted budget assigned for the improvements' implementation (direct cost-reliability model), or, to minimize the system's budget subject to a reliability value restricted from below (dual model). Unlike existing cost-reliability models with simplified technical improvements, the developed models cover an important case of compound technical improvements with a stepwise consecutive structure. Several new heuristic algorithms, based on the combination of cost-sensitivity and "look-ahead" forecasting, are developed in order to solve both the direct and dual optimisation problems. Extensive experimentation has been undertaken to assess relative efficiency of different proposed algorithms. A numerical example is presented.

Keywords: *cost-sensitivity, direct and dual cost-reliability problems, compound technical improvement, cost-sensitivity with a "look-ahead" analysis, generalized algorithm based on switching procedures, algorithm's cost-reliability sensitivity*

1. Introduction

In the recent four decades extensive research has been undertaken in order to develop general search strategies designated as "look-ahead" approaches. The idea is to undertake decision-making by guessing the next operation to perform (usually by means of a heuristic). Note that "look-ahead" strategies are usually applied not to a row of values, but to a problem to be optimised. Thus, the term "the next operation" signifies the step, i.e., the decision-making, which brings us closer to the problem's goal to be optimised. If the decision-making does not result in progress towards the goal, then the step can be retracted and another decision-making can be tried.

N. Davies, G. Logemann and D. Loveland [4], R. Dechter [5], D. Haralick and B. Faltings [7], J. Zhang and H. Zhang [8], have developed a variety of algorithms in order to improve the objectives of problems related to trees defining the search space, as well as to branch- and bounds constraints. D. Golenko-Ginzburg and A. Gonik [6] used "look-ahead" techniques to improve the general job-shop schedule.

In our current research we consider another area, namely, to optimise one of the basic system's parameters, e.g. the system's reliability, by means of constrained technical improvements subject to the restricted budget assigned for the parameter's amendment.

We will consider henceforth the system's reliability although other basic parameters may be taken into account as well. The outlined below "look-ahead" techniques are combined with the cost-sensitivity approach [1]. This results in effective heuristic algorithms.

2. The System's Description

A complicated technical device functioning under random disturbances is considered. The device's reliability, i.e., its probability to avoid critical failures within a sufficiently long period of time, has to be extremely high since critical failures present a definite threat to people's safety, to the environment, etc., and may result in an accident or a major hazardous condition. Thus, increasing the device's reliability is considered to be an important problem of Safety Engineering, on assumption that the existing reliability value proves to be insufficient.

There exist N technical improvements (TI) to increase the device's reliability. For each k-th TI, $1 \le k \le N$, investing ΔC_k cost expenditures results in increasing the device's reliability by ΔR_k. Those parameters are obtained by means of simulation model SM and do not depend on the number of technical improvements which have already been implemented. Thus, the result of a routine k-th technical improvement does not depend on other $\{TI\}$.

Two cost-reliability problems have been formulated and solved in [1–3]:

The direct problem. Determine the optimal set of TI_{ξ_q}, $1 \le q \le Q \le N$, $\xi_q \le N$, in order to maximize the device's reliability subject to the restricted amount of costs ΔC to undertake the corresponding TI.

The dual problem. Determine the optimal set of technical improvements TI_{ξ_q}, $1 \le q \le Q \le N$, $\xi_q \le N$, which requires the minimal amount of costs to undertake the TI in order to increase the device's reliability by not less than ΔR. However, the above formulated problems outlined in previous publications [1–3] are not generalized. This is because all technical improvements TI_k, $1 \le k \le N$, are not considered to be compound. In real practice, however, technical improvements have a stepwise structure, i.e., each TI_k consists of consecutively realized steps $TI_{k1}, TI_{k2}, \dots, TI_{kn_k}$. Here each n_k denotes the number of steps for TI_k .Assume that for each routine TI_k undertaking step TI_{km} can be realized on condition that all $(m-1)$ previous steps $TI_{k\xi}$, $1 \le \xi \le m-1$, have been accomplished before. Thus, each step TI_{km} can be realized if and only if the previous step $TI_{k,m-1}$ has been finished. Assume, further, that each technical improvement TI_k can be interrupted after undertaking a routine step TI_{km}, $1 \le m \le n_k$, i.e., only part of the steps may be implemented. Each step TI_{km} is characterized by two cost-reliability parameters:

- cost ΔC_{km} to undertake TI_{km} ;
- additional reliability value ΔR_{km} which increases the total device reliability level R by implementing step TI_{km} . Value ΔR_{km} can be calculated by means of SM.

The problem under consideration is as follows:

Given a restricted cost value C, single out a set of optimal steps $\{TI_{km}\}$, $1 \le k \le N$, $m \le \xi_k$, $0 \le \xi_k \le n_k$, in order to maximize the total additional reliability $\sum_{k=1}^{N}\sum_{m=0}^{\xi_k} \Delta R_{km}$ subject to the restricted total cost investments $\sum_{k=1}^{N}\sum_{m=0}^{\xi_k} \Delta C_{km} \le C$.

Note that TI_{k0} denotes that the k-th technical improvement has not been implemented at all, i.e., not a single step of the TI_k sequence has been operated as yet.

The problem can be solved by means of integer programming involving a rather complicated algorithm. Essentially simpler quasi-optimal results may be obtained by using heuristic approaches, namely, by implementing sensitivity analysis in the corresponding optimisation algorithm, as outlined in [1]. On the basis of cost-sensitivity analysis several heuristic quasi-optimal algorithms have been developed. Note that the general idea of cost-sensitivity is based on analysing the ratio $\gamma_{km} = \dfrac{\Delta R_{km}}{\Delta C_{km}}$.

The developed algorithms are an essential extension of the direct and dual cost-sensitivity models outlined in [1–3].

3. Notation

Let us introduce the following terms:

$\{TI_k\}$ – the system's technical improvements cell;

TI_k – the k-th technical improvement to increase the system's reliability, $1 \le k \le N$;

N – the number of possible technical improvements;

R^* – the minimal acceptable system's reliability value to avoid hazardous failures (pregiven);

C – the restricted budget to undertake technical improvements (pregiven);

R_0 – system's reliability value prior to undertaking amendments (pregiven);

SM – simulation model to estimate the system's reliability.

TI_{km} – the m-th improvement step entering a compound TI_k, $1 \le m \le n_k$;

n_k – the number of improvement steps to implement the k-th stepwise technical improvement;

ΔC_{km} – the cost expenditures to implement TI_{km};

ΔR_{km} – increase of the system's reliability level due to implementing TI_{km};

γ_{km} – cost-reliability value of improvement step TI_{km}.

R_i – accumulated reliability value at a routine iteration i;

C_i – available total budget value at a routine iteration i;

$N_i \le N$ – number of remaining technical improvements at a routine iteration i;

$\{TI_k\}_i$ – "truncated", i.e., remaining technical improvements cell at a routine iteration i, before singling out the next quadruple for the solution area; $\{TI_k\}_i$ differs from $\{TI_k\}$ by excluding TI_{km} which have been previously singled out on prior iterations from the solution area;

$\{TI^I_{km}\}$ – technical improvements TI_{km} which have to be determined for the solution area (see *Section 5*) by implementing *Algorithm I*;

$\{TI^{II}_{km}\}$ – technical improvements TI_{km} which have to be singled out by implementing *Algorithm II* (see *Section 5*);

$A_{ki} \subset A_k$ – the k-th remaining area A_k of the cell $\{TI_k\}_i$ at the routine iteration i. Note that in some cases A_{ki} may be empty. All $\{A_{ki}\}$ are enumerated in an arbitrary order which remains unchanged until the cost-reliability *Algorithm* terminates. Each array A_{ki} comprises $4 \cdot n_{ki}$ quadruples;

n_{ki} – number of technical improvements steps in array A_{ki} at the i-th iteration;

$\delta C = \dfrac{C}{\sum_{k=1}^{n}\sum_{m=1}^{n_k} \Delta C_{km}}$ – the budget supplement rate in order to carry out all $\{TI_{km}\}$;

R_A – the total increase of the system's reliability by implementing the cost-reliability *Algorithm*;

C_A – the total budget expenses by implementing the cost-reliability *Algorithm*;

n_{kA} – number of improvement steps of the k-th TI which have been actually singled out in the course of implementing the algorithm;

$\eta_A = \dfrac{R_A}{C_A}$ – cost-reliability sensitivity of the algorithm.

4. The Problem's Formulation

The *direct cost-reliability problem* is as follows:
Given values R_0, C, $\{\Delta C_{km}\}$, $\{\Delta R_{km}\}$, $1 \le k \le N$, $1 \le m \le n_k$, determine values ξ_k, $0 \le \xi_k \le n_k$, in order to maximize the system's reliability level

$$\underset{\{\xi_k\}}{Max} \left\{ R_0 + \sum_{k=1}^{N} \sum_{m=0}^{\xi_k} \Delta R_{km} \right\} \tag{1}$$

subject to

$$\sum_{k=1}^{N} \sum_{m=0}^{\xi_k} \Delta C_{km} \le C \,, \tag{2}$$

$$\Delta C_{k0} = 0, \tag{3}$$

$$\Delta R_{k0} = 0. \tag{4}$$

The *dual cost-reliability problem* is as follows:
Given values R_0, R^*, $\{\Delta C_{km}\}$, $\{\Delta R_{km}\}$, $1 \le k \le N$, $1 \le m \le n_k$, determine values ξ_k, $0 \le \xi_k \le n_k$, in order to minimize the cost expenditures

$$\underset{\{\xi_k\}}{Min} \left\{ \sum_{k=1}^{N} \sum_{m=0}^{\xi_k} \Delta C_{km} \right\} \tag{5}$$

subject to (3–4) and

$$R_0 + \sum_{k=1}^{N} \sum_{m=0}^{\xi_k} \Delta R_{km} \ge R^*. \tag{6}$$

Thus, solving the direct problem delivers the maximal reliability increase subject to restricted budget, while the dual problem boil down to minimizing the budget for undertaking technical improvements subject to system's reliability level restricted from below.

5. Simplified Heuristic Algorithms for Optimising the Direct Cost-reliability Problem

As outlined above, *Problem* (1–4) may be solved by means of cost-sensitivity analysis. The general idea is as follows. Call henceforth the *cost-reliability of a technical improvement* the ratio $\gamma = \Delta R / \Delta C$. It can be well-recognized that if TI_{k_1} has a higher cost-reliability than TI_{k_2}, investing one and the same cost expenditure results in a higher increase of the reliability parameter in case of implementing the TI_{k_1} than TI_{k_2}. This consideration is used below, in the case of compound technical improvements.

Note that for any technical improvement with a stepwise structure the corresponding quasi-optimal steps to be singled out have to be implemented consecutively, one after another. Thus, to solve the problem of singling out the next step by analysing a set of several unfinished TI_k, $1 \le k \le N$, one has:

- to examine for all $\{TI_k\}$ their first non-realized steps $\{TI_{k1}\}$, $1 \le k \le N$;
- later on to calculate their corresponding values $\{\gamma_{k1}\}$, and
- to choose step $TI_{\eta 1}$ *which delivers the maximal value* to γ.

The chosen step has to be supplied with expenditure costs while the remaining budget C has to be updated. Afterwards the chosen step $TI_{\eta 1}$, being cancelled in TI_η, enters the accumulated quasi-optimal set (the algorithm's solution), while all non-implemented improvement steps in TI_η are shifted to the left in order for the second step to obtain the first position. The solution process proceeds (by examining the remaining TI_k by means of γ) until either the budget volume C decreases and cannot be assigned to any starting step of a non-accomplished TI_k, or all steps in $\{TI_k\}$ are supplied with expenditure costs in case $\sum_{k=1}^{N} \sum_{m=1}^{n_k} \Delta C_{km} \le C$ (a trivial solution).

The enlarged step-by-step procedure of the algorithm is as follows:

Step 1. (Subsidiary). Enumerate N technical improvements $\{TI\}$ in an arbitrary order. Form N corresponding arrays A_k, $1 \le k \le N$, each array A_k comprising $4 \cdot n_k$ consecutive values, i.e., n_k quadruples. Each m-th quadruple in the k-th array comprises values k, m, ΔR_{km} and ΔC_{km}, correspondingly, i.e., all parameters characterizing step improvement TI_{km}.

Step 2. Calculate N values

$$\gamma_k = \frac{\Delta R_{km_k}}{\Delta C_{km_k}}, \ 1 \le k \le N, \tag{7}$$

where m_k denotes the first quadruple of the k -th array. At the beginning of the algorithm's functioning all $m_k = 1$.

Step 3. Determine

$$\gamma_\omega = \underset{k}{Max}\, \gamma_k . \tag{8}$$

If more than one ω holds for (8), take one of them with the maximal $\Delta C_{\omega m_\omega}$.

Step 4. Send quadruple (ω, m_ω, $\Delta R_{\omega m_\omega}$, $\Delta C_{\omega m_\omega}$) to the solution array, which comprises a set of quadruples.

Step 5. Update $C - \Delta C_{\omega m_\omega} \rightarrow C$, $n_\omega - 1 \rightarrow n_\omega$, $N-1 \rightarrow N$.

Step 6. Cancel quadruple (ω, m_ω, $\Delta R_{\omega m_\omega}$, $\Delta C_{\omega m_\omega}$) and shift to the left by four values all the remaining quadruples of A_ω .

Step 7. Check whether array A_ω appears to be empty or not. If empty, go to the next step. Other-wise apply *Step 9*.

Step 8. Modify *Step 2* in order to prevent implementing value ω in calculating (1).

Step 9. Check whether relation $C \ge \underset{k}{Min}\, \Delta C_{km_k}$ holds or not. If yes, go to *Step 2*. Note that for an empty array A_ω examining the possibility of choosing the next quasi-optimal quadruple is not carried out (see *Step 8*). In case $C < \underset{k}{Min}\, \Delta C_{km_k}$ apply the next step.

Step 10. The solution process terminates. All the quadruples entering the solution array form the quasi-optimal improvement steps $\{k, m, \Delta R_{km}, \Delta C_{km}\}$.

It can be well-recognized that the quasi-optimal reliability level after implementing the above algorithm (call it henceforth *Algorithm I*) may be calculated as

$$R = R_0 + \sum_{k=1}^{N} \sum_{m=0}^{\xi_k} \Delta R_{km} . \tag{9}$$

Algorithm I has a simple modification which we will designate henceforth *Algorithm II*. The latter resembles *Algorithm I*, besides one essential detail. When calculating values γ_k (see *Step 2*) we use the following modification

$$\gamma_k = \frac{\sum_{\alpha=m_k}^{n_k} \Delta R_{k\alpha}}{\sum_{\alpha=m_k}^{n_k} \Delta C_{k\alpha}}, \ 1 \le k \le N . \tag{10}$$

Relation (10) means that cost-reliability is calculated not for each local improvement step, but for the entire technical improvement TI_k *as a whole*. It can be well-recognized that the numerator in (10) is equal to the sum of local reliability steps ΔR_{km} for all *remaining improvement steps* TI_{km}, while the de-numerator stands for the sum of the corresponding cost investment steps ΔC_{km} . Thus, using γ_k calculated by (10) prevents missing high cost-sensitivity steps at the end of A_k in case of steps with smaller cost-sensitivity at the beginning of array A_k .

The structure of *Algorithm II* is similar to the outlined above step-by-step procedure of *Algorithm I*, besides substituting (7) for (10). All other steps remain unchanged.

6. Generalized Cost-Optimisation Algorithm Based on Switching Procedures

It can be well-recognized that, unlike *Algorithm I*, *Algorithm II* is based on "look-ahead" cost-sensitivity and, thus, provides better results in cases when the total budget C covers a majority of improvement steps for most TI . However, in case when the ratio

$$\delta C = \frac{C}{\sum_{k=1}^{n}\sum_{m=1}^{n_k} \Delta C_{km}} << 1 \tag{11}$$

is very small, the pregiven budget C can usually cover only the first improvement steps. This, in turn, means that there is actually no need in "look-ahead" sensitivity analysis. In order to develop a more generalized algorithm which comprises cases of any δC values we present *Algorithm III* which implements both *Algorithms I* and *II* simultaneously on the basis of a switching procedure.

The general idea of *Algorithm III* is as follows: at each consecutive iteration i to single out the next TI_{km}, i.e., the forthcoming iteration to be undertaken, both *Algorithms I* and *II* are implemented independently for the remaining TI cell. For both algorithms the accumulated sum of reliability values $\sum_k \left[\sum_m R\left\{TI^{I}_{km}\right\} \right]$ and $\sum_k \left[\sum_m R\left\{TI^{II}_{km}\right\} \right]$ is calculated and later on compared with each other. The algorithm which results in higher accumulated reliability, is chosen to provide the next technical improvement step TI_{km}, and the corresponding quadruple is send to the solution area. After up-dating the remaining technical improvements cell $\left\{TI_{km}\right\}_i$ the competing procedure among *Algorithms I* and *II* repeats anew, until the budget value C would be exhausted. Being more complicated than both local *Algorithms I* and *II*, *Algorithm III* provides better results.

The enlarged step-by-step procedure of *Algorithm III* is as follows:

Step 1. Coincides with the corresponding steps in *Algorithms I* and *II*.

Step 2. After each routine i-th iteration check whether unification $\bigcup A_{ki}$ appears to be empty or not. If empty, go to *Step 12*. Otherwise apply the next step.

Step 3. Check whether relation $C \geq \underset{k}{Min}\left\{\Delta C_{km_{ki}}\right\}$ holds, where m_{ki} denotes the first quadruple of the k-th array. If yes, go to the next step. Otherwise apply *Step 12*.

Step 4. Determine (for the purpose of forecasting) by means of *Algorithm I* all iterations $\left\{TI^{I}_{km}\right\}$ until the algorithm terminates.

Step 5. Calculate values

$$R_i + \sum_k \sum_m \left\{\Delta R^{I}_{km}\right\} = R^{I}, \tag{12}$$

$$C_i - \sum_k \sum_m \left\{\Delta C^{I}_{km}\right\} = C^{I}, \tag{13}$$

until the budget value C^{I} ceases to cover future technical improvements. Here $\Delta R^{I}_{km} = \Delta R\left\{TI^{I}_{km}\right\}$, $\Delta C^{I}_{km} = \Delta C\left\{TI^{I}_{km}\right\}$.

Step 6. Determine by means of *Algorithm II* all iterations $\left\{TI^{II}_{km}\right\}$ in order to "look-ahead" the fitness of the algorithm.

Step 7. Similarly to *Step 5*, calculate values

$$R_i + \sum_k \sum_m \Delta R\left\{TI^{II}_{km}\right\} = R^{II}, \tag{14}$$

$$C_i - \sum_k \sum_m \Delta C\left\{TI^{II}_{km}\right\} = C^{II}. \tag{15}$$

Step 8. Compare values R^{I} and R^{II}. If $R^{I} > R^{II}$ go to *Step 10*. In case $R^{I} < R^{II}$ apply the next step. In case $R^{I} = R^{II}$ compare values C^{I} and C^{II}. If $C^{I} > C^{II}$ go to *Step 10*. Otherwise apply the next step.

Step 9. Send quadruple $\left(\omega, m_\omega, \Delta R_{\omega m_w}, \Delta C_{\omega m_w}\right)$ which has been determined in the course of i-th iteration by implementing *Algorithm II*, to the solution area. Thus, the i-th routine iteration of *Algorithm III* is accomplished. Go to *Step 11*.

Step 10. The step is similar to *Step 9*, with the exception of substituting *Algorithm II* by *Algorithm I*.

Step 11. Update the information in arrays $\{A_{ki}\}$ by diminishing value N_i by one, shifting all quadruples in Array $A_{\omega i}$ by 4 to the left, updating values

$$R_i + \Delta R_{\omega m_\omega} \Rightarrow R_{i+1}, \tag{16}$$

$$C_i - \Delta C_{\omega m_\omega} \Rightarrow C_{i+1}, \tag{17}$$

$$i+1 \Rightarrow i\ . \tag{18}$$

Go to *Step 2*.

Step 12. The work of *Algorithm III* terminates.

Note that if implementing two different algorithms results in an equal increase of the system's total reliability R, the comparative efficiency of both algorithms can be assessed by comparing the algorithms' cost-reliability sensitivity to be calculated as

$$\eta_A = \frac{\sum_{k=1}^{N}\sum_{m=1}^{n_{kA}} \Delta R_{km}}{\sum_{k=1}^{N}\sum_{m=1}^{n_{kA}} \Delta C_{km}}\ . \tag{19}$$

Corollary. For any technical improvements cell $\{TI_{km}\}$ with fixed total budget C relation

$$R^{***} \geq \max\left(R^{*},\ R^{**}\right), \tag{20}$$

where R^{*}, R^{**}, R^{***} denote the accumulated reliability values obtained by means of *Algorithms I*, *II*, and *III*, correspondingly, holds.

Proof. Assume that for a certain $\{TI_{km}\}$ relation (20) is not true. This may originate from two cases:

<u>Case 1</u>. $R^{***} < R^{*}$.

<u>Case 2</u>. $R^{***} < R^{**}$.

Examine *Case 1* first. Denote by $n\left(R^{*}, R^{***}\right) \geq 0$ the number of first coinciding iterations $\{TI_{km}\}$ when implementing *Algorithms I* and *III* independently. Since $R^{***} \neq R^{*}$, there has to be a decision point after $n\left(R^{*}, R^{***}\right)$ first iterations when the concurrence ceases to hold. Since in the course of implementing *Algorithm III* we choose at each decision point the iteration which results in the forecasted path with the maximal accumulated reliability, value R^{*}, being equal to $\sum_k \sum_m TI_{km}^{I}$, has to be less than value $\sum_k \sum_m TI_{km}^{III}$ of that forecasted path. From the other hand, in the course of implementing *Algorithm III* at each routine decision point the forecasted value R^{***} cannot diminish but only increase. Thus, at the end of implementing *Algorithm III* the actual value R^{***} has to exceed R^{*} in contradiction with the definition of *Case 1*. For *Case 2*, the logical analysis is similar to that outlined above. Thus, for both cases our assumption is false, and relation (20) holds.

7. The Dual Problem's Solution

The step-by-step procedure of the heuristic algorithm to solve problem (3–6) [call it henceforth *Algorithm IV*] is as follows:

Steps 1–2 of *Algorithm IV* fully coincide with *Steps 1–2* of *Algorithms I* or *II* (see *Section 5*).

Step 3 resembles the corresponding step of *Algorithms I, II* with one exception: choosing the maximal $\Delta C_{\omega m_\omega}$ in case of several ω satisfying (8) has to be substituted for the "*minimal*" $\Delta C_{\omega m_\omega}$.

This is done deliberately since minimizing the expenditure costs in (5) centres on preferring the minimal cost steps in the course of implementing *Algorithm IV*.

Step 4 coincides with the corresponding step in *Algorithms I* or *II*.

Step 5 has to be formulated as follows:

Update the accumulated reliability

$$R + \Delta R_{\omega m_\omega} \to R \text{ and} \tag{21}$$

accumulate the cost expenditures

$$C + \Delta C_{\omega m_\omega} \to C. \tag{22}$$

At the beginning of the *Algorithm IV* value C has to be set equal to zero, while

Steps 6–8 coincide with *Algorithms A* , *B*.

Step 9 has to be formulated as follows:

Check relation

$$R + R \geq R^*. \tag{23}$$

If (23) does not hold, go to *Step 2*. Otherwise apply the next step.

Step 10 coincides with the corresponding step in *Algorithms I* or *II*.

It can be well-recognized that the minimized cost expenditures after implementing *Algorithm IV* may be calculated as

$$Min\ C = \sum_{k=1}^{N}\sum_{m=0}^{\xi_k} \Delta C_{km}. \tag{24}$$

Note that when implementing *Algorithm IV* on the basis of the direct *Algorithm III* (see *Section 6*), the principal structure of the dual algorithm remains similar to the structure outlined above. Each improvement step of the solution array has to be determined by the look-ahead switching procedure.

8. Numerical Example

Consider a complicated technical device functioning under random disturbances. Eight technical improvements to increase the system's reliability have been considered. The corresponding 8 arrays A_k , $1 \leq k \leq 8$, [representing values $\{k, m, \Delta R_{km}, \Delta C_{km}\}$] are as follows:

A_1	:	$(1, 1, 3, 5)$	;	$(1, 2, 8, 6)$	;					
A_2	:	$(2, 1, 15, 12)$	;	$(2, 2, 4, 6)$	;	$(2, 3, 3, 5)$	;			
A_3	:	$(3, 1, 10, 13)$	;							
A_4	:	$(4, 1, 4, 5)$	;	$(4, 2, 8, 10)$	;					
A_5	:	$(5, 1, 3, 4)$	;	$(5, 2, 8, 10)$	;	$(5, 3, 2, 4)$	;			
A_6	:	$(6, 1, 4, 9)$	;	$(6, 2, 10, 15)$	;	$(6, 3, 7, 11)$	;	$(6, 4, 8, 12)$	;	
A_7	:	$(7, 1, 4, 7)$	;	$(7, 2, 5, 8)$	;	$(7, 3, 2, 3)$	;			
A_8	:	$(8, 1, 4, 6)$	;	$(8, 2, 5, 9)$	;	$(8, 3, 2, 4)$	;			

where values ΔR_{km} and ΔC_{km} are given in conditional terms, namely, the reliability unit equals 10^{-7} of the reliability value, while the cost unit equals $1,000.

The pregiven restricted budget to undertake technical improvements is $C = 125$.
The problem is to single out all possible non-contradictory improvement steps to maximize the additional device reliability subject to the restricted budget. The term "non-contradictory" means that for each array A_k all the singled out quasi-optimal steps entering that array have to be strictly consecutive (see *Sections 2*, *5*). The solution of the direct cost-optimisation problem by implementing *Algorithm I* (see *Section 5*) is carried out as represented by the below iterative computational process:

Iteration 1. The maximal cost-reliability value γ_k relates to array A_2 ($\gamma = 1.25$). Thus, step TI_{21} is singled out and enters the quasi-optimal solution's set, while the corresponding first quadruple ($2, 1, 15, 12$) is excluded from array A_2 . The remaining steps TI_{22} and TI_{23} are shifted to the left. The updated values C and R are as follows:

$C - \Delta C_{21} = 125 - 12 = 113 \rightarrow C$,

$R + \Delta R_{21} = 0 + 15 = 15 \rightarrow R$.

Other iterations are illustrated in Table 1.

Table 1. Computation process by implementing Algorithm I

Iteration No	TI_{km}	$C - \Delta C_{km} \rightarrow C$	$R + \Delta R_{km} \rightarrow R$
1	$\{TI_{21}\}$	$125 - 12 = 113$	$0 + 15 = 15$
2	$\{TI_{41}\}$	$113 - 5 = 108$	$15 + 4 = 19$
3	$\{TI_{42}\}$	$108 - 10 = 98$	$19 + 8 = 27$
4	$\{TI_{31}\}$	$98 - 13 = 85$	$27 + 10 = 37$
5	$\{TI_{51}\}$	$85 - 4 = 81$	$37 + 3 = 40$
6	$\{TI_{52}\}$	$81 - 10 = 71$	$40 + 8 = 48$
7	$\{TI_{23}\}$	$71 - 6 = 65$	$48 + 4 = 52$
8	$\{TI_{81}\}$	$65 - 6 = 59$	$52 + 4 = 56$
9	$\{TI_{11}\}$	$59 - 5 = 54$	$56 + 3 = 59$
10	$\{TI_{12}\}$	$54 - 6 = 48$	$59 + 8 = 67$
11	$\{TI_{23}\}$	$48 - 5 = 43$	$67 + 3 = 70$
12	$\{TI_{71}\}$	$43 - 7 = 36$	$70 + 4 = 74$
13	$\{TI_{72}\}$	$36 - 8 = 28$	$74 + 5 = 79$
14	$\{TI_{73}\}$	$28 - 3 = 25$	$79 + 2 = 81$
15	$\{TI_{82}\}$	$25 - 9 = 16$	$81 + 5 = 86$
16	$\{TI_{53}\}$	$16 - 4 = 12$	$86 + 2 = 88$
17	$\{TI_{83}\}$	$12 - 4 = 8$	$88 + 2 = 90$

Thus, 17 quasi-optimal technical step improvements TI_{11}, TI_{12}, TI_{21}, TI_{22}, TI_{23}, TI_{31}, TI_{41}, TI_{42}, TI_{51}, TI_{52}, TI_{53}, TI_{71}, TI_{72}, TI_{73}, TI_{81}, TI_{82}, TI_{83}, are singled out. However, the remaining budget $C = 8$ is not enough to be implemented in TI_6 (four sub-steps TI_{61} **-** TI_{64}). The maximal additional reliability value R is, thus, equal 90 by investing budget $C = 125 - 8 = 117$.

It can be well-recognized that implementing *Algorithm II* proves to be more effective and delivers better results. The corresponding computation process is illustrated in Table 2.

Table 2. Computation process by implementing Algorithm II

Iteration No	TI_{km}	$C - \Delta C_{km} \rightarrow C$	$R + \Delta R_{km} \rightarrow R$
1	$\{TI_{11}\}$	$125 - 5 = 120$	$0 + 3 = 3$
2	$\{TI_{12}\}$	$120 - 6 = 114$	$3 + 8 = 11$
3	$\{TI_{21}\}$	$114 - 12 = 102$	$11 + 15 = 26$
4	$\{TI_{41}\}$	$102 - 5 = 97$	$26 + 4 = 30$
5	$\{TI_{42}\}$	$97 - 10 = 87$	$30 + 8 = 38$
6	$\{TI_{31}\}$	$87 - 13 = 74$	$38 + 10 = 48$
7	$\{TI_{51}\}$	$74 - 4 = 70$	$48 + 3 = 51$
8	$\{TI_{52}\}$	$70 - 10 = 60$	$51 + 8 = 59$
9	$\{TI_{22}\}$	$60 - 6 = 54$	$59 + 4 = 63$
10	$\{TI_{61}\}$	$54 - 9 = 45$	$63 + 4 = 67$
11	$\{TI_{62}\}$	$45 - 15 = 30$	$67 + 10 = 77$
12	$\{TI_{63}\}$	$30 - 11 = 19$	$77 + 7 = 84$
13	$\{TI_{64}\}$	$19 - 12 = 7$	$84 + 8 = 92$
14	$\{TI_{71}\}$	$7 - 7 = 0$	$92 + 4 = 96$

Thus, only 14 iterations are required to single out 14 step improvements TI_{11}, TI_{12}, TI_{21}, TI_{22}, TI_{31}, TI_{41}, TI_{42}, TI_{51}, TI_{52}, TI_{61}, TI_{62}, TI_{63}, TI_{64}, TI_{71}. The quasi-optimal solution results in this case in additional reliability increase of $R = 96$ achieved by investing the entire total budget of $C = 125$.

A conclusion can be drawn that for the example under consideration *Algorithm II* which "looks ahead" in the course of the heuristic look-over, proves to be more effective ($R = 96$ versus $R = 90$) than *Algorithm I*.

In conclusion we present the results of implementing *Algorithm III* for the example. As proven above, *Algorithm III* cannot provide results worse than those of *Algorithm II*. The corresponding computational process is illustrated in Table 3:

Table 3. Computation process by implementing Algorithm III

Iteration No	TI_{km}	$C - \Delta C_{km} \rightarrow C$	$R + \Delta R_{km} \rightarrow R$
1	$\{TI_{21}\}$	$125 - 12 = 113$	$0 + 15 = 15$
2	$\{TI_{11}\}$	$113 - 5 = 108$	$15 + 3 = 18$
3	$\{TI_{12}\}$	$108 - 6 = 102$	$18 + 8 = 26$
4	$\{TI_{41}\}$	$102 - 5 = 97$	$26 + 4 = 30$
5	$\{TI_{42}\}$	$97 - 10 = 87$	$30 + 8 = 38$
6	$\{TI_{31}\}$	$87 - 13 = 74$	$38 + 10 = 48$
7	$\{TI_{51}\}$	$74 - 4 = 70$	$48 + 3 = 51$
8	$\{TI_{52}\}$	$70 - 10 = 60$	$51 + 8 = 59$
9	$\{TI_{81}\}$	$60 - 6 = 54$	$59 + 4 = 63$
10	$\{TI_{22}\}$	$54 - 6 = 48$	$63 + 4 = 67$
11	$\{TI_{61}\}$	$48 - 9 = 39$	$67 + 4 = 71$
12	$\{TI_{62}\}$	$39 - 15 = 24$	$71 + 10 = 81$
13	$\{TI_{63}\}$	$24 - 11 = 13$	$81 + 7 = 88$
14	$\{TI_{64}\}$	$13 - 12 = 1$	$88 + 8 = 96$

Algorithm III yields in reliability value $R = 96$ which is indeed not less than that obtained by using *Algorithm II*. However, when implementing *Algorithm III* we did not invest the entire total budget: value $\sum_k \sum_m \Delta C_{km}$ is equal 124. This means that *Algorithm III* results in a higher cost-reliability sensitivity than *Algorithm II*, namely $\frac{96}{124} = 0.774$ versus $\frac{96}{125} = 0.768$. Thus, the considered example fully supports the *Corollary* outlined in *Section 6*.

9. Experimentation

In order to check the fitness of the algorithms' cost-reliability sensitivity, extensive experimentation has been undertaken. A technical improvements cell has been simulated as follows:

a) the number N of technical improvements has been simulated as a whole number uniformly distributed within the lower and upper bounds $a_N = 5$, $b_N = 25$;

b) for each k-th technical improvement, $1 \le k \le N$, the number of consecutive improvement steps n_k has been simulated as a whole number uniformly distributed within the lower and upper bounds $a_k = 3$, $b_k = 15$;

c) each cost value ΔC_{km}, $1 \le k \le N$, $1 \le m \le n_k$, has been simulated as a whole number uniformly distributed within the distribution area $a_{cmk} = 200$, $b_{cmk} = 400$;

d) four different distribution bounds have been considered for local reliability values ΔR_{km}, $1 \le k \le N$, $1 \le m \le n_k$, namely: ΔR_{km} is a whole number uniformly distributed within lower and upper bounds:
   1. $a_{Rkm} = 20$, $b_{Rkm} = 40$;
   2. $a_{Rkm} = 40$, $b_{Rkm} = 80$;
   3. $a_{Rkm} = 60$, $b_{Rkm} = 120$;
   4. $a_{Rkm} = 80$, $b_{Rkm} = 160$.

Similarly to *Section 8*, all values ΔC_{km} and ΔR_{km} are expressed in conditional terms.

All distribution bounds do not depend on values N, n_k, k and m.

It can be well-recognized that each distribution results in four different cost-sensitivity values $\gamma_{km} = \frac{\Delta R_{km}}{\Delta C_{km}}$

1) $\gamma_1 = 0.1$; 2) $\gamma_2 = 0.2$; 3) $\gamma_3 = 0.3$; 4) $\gamma_4 = 0.4$.

e) for each simulated combination of N, n_k, ΔC_{km} and γ_{km} 9 different levels of budget supplement to carry out all technical improvements $\{TI_{km}\}$ have been examined by means of (11)

1) $\delta = 0.1$; 2) $\delta = 0.2$; 3) $\delta = 0.3$; 4) $\delta = 0.4$; 5) $\delta = 0.5$; 6) $\delta = 0.6$; 7) $\delta = 0.7$; 8) $\delta = 0.8$; 9) $\delta = 0.9$.

For $4 \times 9 = 36$ combinations of δ and γ 10,000 initial data models have been simulated, and later on algorithms' cost-reliability sensitivity rates η_A have been calculated by using each of the algorithms under comparison. Then, average sensitivity rates $\overline{\eta}_A$ have been calculated for each algorithm and each combination of δ and γ_{km}. The results are presented in the below Table 4.

Table 4. Comparison of algorithms' cost-reliability sensitivity for various δ and γ_{km}

$\overline{\eta}_{A\ III}$	$\overline{\eta}_{A\ II}$	$\overline{\eta}_{A\ I}$	δ	γ_{km}
0.1236	0.1163	0.1225	0.10	0.1
0.1169	0.1133	0.1140	0.20	0.1
0.1132	0.1109	0.1097	0.30	0.1
0.1105	0.1089	0.1069	0.40	0.1
0.1083	0.1072	0.1048	0.50	0.1
0.1064	0.1057	0.1031	0.60	0.1
0.1046	0.1042	0.1017	0.70	0.1
0.1029	0.1026	0.1005	0.80	0.1
0.1010	0.1010	0.0994	0.90	0.1
0.2491	0.2345	0.2469	0.10	0.2
0.2356	0.2284	0.2299	0.20	0.2
0.2282	0.2236	0.2213	0.30	0.2
0.2228	0.2197	0.2157	0.40	0.2
0.2184	0.2163	0.2114	0.50	0.2
0.2146	0.2131	0.2080	0.60	0.2
0.2110	0.2101	0.2052	0.70	0.2
0.2075	0.2070	0.2027	0.80	0.2
0.2038	0.2037	0.2005	0.90	0.2
0.3747	0.3527	0.3718	0.10	0.3
0.3543	0.3436	0.3458	0.20	0.3
0.3432	0.3363	0.3327	0.30	0.3
0.3351	0.3305	0.3242	0.40	0.3
0.3285	0.3253	0.3180	0.50	0.3
0.3227	0.3206	0.3129	0.60	0.3
0.3174	0.3160	0.3087	0.70	0.3
0.3121	0.3114	0.3050	0.80	0.3
0.3066	0.3063	0.3016	0.90	0.3
0.5004	0.4713	0.4964	0.10	0.4
0.4730	0.4586	0.4619	0.20	0.4
0.4584	0.4492	0.4446	0.30	0.4
0.4475	0.4414	0.4331	0.40	0.4
0.4388	0.4345	0.4246	0.50	0.4
0.4311	0.4282	0.4180	0.60	0.4
0.4240	0.4221	0.4123	0.70	0.4
0.4170	0.4160	0.4074	0.80	0.4
0.4096	0.4093	0.4030	0.90	0.4

The following conclusions can be drawn from Table 4:

1. *Algorithm III* proves to be more effective than both other cost-reliability algorithms under comparison. For any combination of δ and γ_{km} its average cost-reliability sensitivity rate is higher then by implementing *Algorithms I* and *II*.
2. If the total budget C does not cover the $\{TI\}$ cell requirement by more than 20% ($\delta \leq 0.2$) *Algorithm I* proves to be more effective than *Algorithm II* since there is practically no need in "look-ahead" sensitivity analysis. For $\delta > 0.2$ "look-ahead" techniques start to be useful, and *Algorithm II* becomes more effective than *Algorithm I*.
3. For a pair of different values of γ_{km} the ratio of corresponding algorithms' average cost-reliability rates is close to be equal the ratio of values γ_{km} themselves. This is true for any value δ and for all the algorithms under comparison.

Conclusions

The following conclusions can be drawn from the study:

1. The basic advantage of the suggested algorithms is that it makes possible to implement technical improvements of compound nature, when those improvements have a stepwise consecutive structure.
2. By comparing the several newly developed cost-reliability algorithms, it can be well-recognized that algorithms based on the "look-ahead" cost-sensitivity look-over, are more efficient than an algorithm which does not comprise forecasting techniques.
3. It has been proven theoretically that *Algorithm III* provides for the direct cost-reliability Problem's solution results being at least not worse than by implementing other algorithms. As to the algorithm's cost-reliability sensitivity rate, extensive simulation shows that *Algorithm III* provides always better results than *Algorithms I* and *II*.
4. All the suggested algorithms can be easily programmed on PC and are simple in usage. They can be applied both to the direct and the dual cost-reliability models.
5. Besides cost-reliability problems, the results obtained may be used for optimising any system's parameter by means of constrained technical improvements subject to restricted budget assigned for the parameter's amendment.

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STATISTICAL IDENTIFICATION OF AN OBSERVABLE PROCESS

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In this paper, for identifying an observable process with one of several simulation models, a uniformly most powerful invariant (UMPI) test is developed from the generalized maximum likelihood ratio (GMLR). This test can be considered as a result of a new approach to solving the Behrens-Fisher problem when covariance matrices of multivariate normal populations (compared with respect to their means) are different and unknown. The test is based on invariant statistics whose distribution, under the null hypothesis, does not depend on the unknown (nuisance) parameters.

Keywords: *observable process, simulation model, UMPI test, identification*

1. Introduction

Computational modelling has become an important tool for building and testing theories in Cognitive Science during the last years. The area of its applications includes, in particular, business process simulation, resource management, knowledge management systems, operations research, economics, optimisation, stochastic models, logic programming, operation and production management, supply chain management, work flow management, total quality management, logistics, risk analysis, scheduling, forecasting, cost benefit analysis, economic revitalization, financial models, accounting, policy issues, regulatory impact analysis, etc.

One of the most important steps in the development of a simulation model is recognition of the simulation model, which is an accurate representation of the process being studied. This procedure consists of two basic stages:

(i) establishing the form of an adequate simulation model for the process under study and then

(ii) estimating precisely the values of its parameters.

In developing strategies for the design of experiments for parameter estimation, it is customarily assumed that the correct form of the model is known. However, experimenters often do not have just one model known to be correct but have instead m > 1 rival models to consider as possible explanations of the process being investigated.

It is natural for model users to devise rules so as to identify an observable process with one of several distinct models, collected for simulation, which accurately represents the process, especially when decisions involving expensive resources are made on the basis of the results of the model.

Substantiation that a computerized model within its domain of applicability possesses a satisfactory range of accuracy consistent with the intended application of the model is usually referred to as model validation and is the definition used in this paper. It is generally preferable to use some form of objective analysis to perform model validation. A common form of objective analysis for validating simulation models is statistical hypothesis testing [1] which will be discussed in this paper.

2. Validation of a Simulation Model

Suppose that we desire to validate a k^{th} multivariate stationary response simulation model of an observable process, which has p response variables. Let $x_{ij}(k)$ and y_{ij} be the i-th observation of the $j^{\text{-th}}$ response variable of the k^{th} model and the process under study, respectively. It is assumed that all observation vectors, $\mathbf{x}_i(k) = (x_{i1}(k), ..., x_{ip}(k))'$, $\mathbf{y}_i = (y_{i1}, ..., y_{ip})'$, $i = 1(1)n$, are independent of each other, where n is a number of paired observations. Let $\mathbf{z}_i(k) = \mathbf{x}_i(k) - \mathbf{y}_i$, $i = 1(1)n$, be paired comparisons leading to a series of vector differences. Thus, for testing the validity of a simulation model of a real, observable process, it can be obtained and used a sample of n independent observation vectors $\mathbf{Z}(k) = (\mathbf{z}_1(k), ..., \mathbf{z}_n(k))$. Each sample $\mathbf{Z}(k)$, $k\in\{1, \dots, m\}$, is declared to be realization of a specific stochastic process with unknown parameters.

In this paper, for testing the validity of the k^{th} simulation model of a real, observable process, we propose a statistical approach that is based on the generalized maximum likelihood ratio. In using statistical hypothesis testing to test the validity of a simulation model under a given experimental frame and for an acceptable range of accuracy consistent with the intended application of the model, we have the following hypotheses:

$H_0(k)$: *the k^{th} model is valid for the acceptable range of accuracy under a given experimental frame*;

$H_1(k)$: *the k^{th} model is invalid for the acceptable range of accuracy under a given experimental frame.* (1)

There are two possibilities for making a wrong decision in statistical hypothesis testing. The first one, type I error, is accepting the alternative hypothesis $H_1(k)$ when the null hypothesis $H_0(k)$ is actually true, and the second one, type II error, is accepting the null hypothesis when the alternative hypothesis is actually true. In model validation, the first type of wrong decision corresponds to rejecting the validity of the model when it is actually valid, and the second type of wrong decision corresponds to accepting the validity of the model when it is actually invalid. The probability of making the first type of wrong decision will be called model builder's risk ($\alpha(k)$) and the probability of making the second type of wrong decision will be called model user's risk ($\beta(k)$). Thus, for fixed n, the problem is to construct a test, which consists of testing the null hypothesis

$$H_0(k)\text{: } \mathbf{z}_i(k) \sim \mathrm{N}_p(\mathbf{0},\mathbf{Q}(k)), \quad \forall i = 1(1)n, \tag{2}$$

where $\mathbf{Q}(k)$ is a positive definite covariance matrix, versus the alternative

$$H_1(k)\text{: } \mathbf{z}_i(k) \sim \mathrm{N}_p(\mathbf{a}(k),\mathbf{Q}(k)), \quad \forall i = 1(1)n, \tag{3}$$

where $\mathbf{a}(k) = (a_1(k), ..., a_p(k))' \neq (0, ... ,0)'$ is a mean vector. The parameters $\mathbf{Q}(k)$ and $\mathbf{a}(k)$ are unknown.

It will be noted that the result of Theorem 1 given below can be used to obtain test for the hypothesis of the form H_0: $\mathbf{z}_i(k)$ follows $\mathrm{N}_p(\mathbf{a}(k),\mathbf{Q}(k))$ versus H_a: $\mathbf{z}_i(k)$ does not follow $\mathrm{N}_p(\mathbf{a}(k),\mathbf{Q}(k))$, $\forall I = 1(1)n$. The general strategy is to apply the probability integral transforms of $w_r(k)$, $\forall k = p + 2(1)n$, to obtain a set of i.i.d. U(0,1) random variables under H_0 [2]. Under H_a this set of random variables will, in general, not be i.i.d. U(0,1). Any statistics, which measures a distance from uniformity in the transformed sample (say, a Kolmogorov-Smirnov statistics), can be used as a test statistics.

Theorem 1. **(***Characterization of the multivariate normality***).** Let $\mathbf{z}_i(k)$, $i = 1(1)n$, be n independent p-multivariate random variables ($n \geq p + 2$) with common mean $\mathbf{a}(k)$ and covariance matrix (positive definite) $\mathbf{Q}(k)$. Let $w_r(k)$, $r = p + 2, \dots, n$, be defined by

$$w_r(k) = \frac{r-(p+1)}{p}\frac{r-1}{r} \times \left(\mathbf{z}_r(k) - \overline{\mathbf{z}}_{r-1}(k)\right)' \mathbf{S}_{r-1}^{-1}(k)\left(\mathbf{z}_r(k) - \overline{\mathbf{z}}_{r-1}(k)\right) =$$
$$= \frac{r-(p+1)}{p}\left(\frac{|\mathbf{S}_r(k)|}{|\mathbf{S}_{r-1}(k)|} - 1\right), \quad r = p+2, ..., n, \tag{4}$$

where

$$\overline{\mathbf{z}}_{r-1}(k) = \sum_{i=1}^{r-1}\mathbf{z}_i(k)/(r-1), \tag{5}$$

$$\mathbf{S}_{r-1}(k) = \sum_{i=1}^{r-1}(\mathbf{z}_i(k) - \overline{\mathbf{z}}_{r-1}(k))(\mathbf{z}_i(k) - \overline{\mathbf{z}}_{r-1}(k))', \tag{6}$$

then the $\mathbf{z}_i(k)$ ($i = 1, \dots, n$) are $\mathrm{N}_p(\mathbf{a}(k),\mathbf{Q}(k))$ if and only if $w_{p+2}(k), \dots, w_n(k)$ are independently distributed according to the central F distribution with p and $1, 2, \dots, n - (p + 1)$ degrees of freedom, respectively.

Proof. The proof is similar to that of the characterization theorems [3–4] and so it is omitted here. □

3. GMLR Statistics

In order to distinguish the two hypotheses ($H_0(k)$ and $H_1(k)$), a generalized maximum likelihood ratio (GMLR) statistics is used. The GMLR principle is best described by a likelihood ratio defined on a sample space Z with a parameter set Θ, where the probability density function of the sample data is maximized over all unknown parameters, separately for each of the two hypotheses. The maximizing parameter values are, by definition, the maximum likelihood estimators of these parameters; hence the maximized probability functions are obtained by replacing the unknown parameters by their maximum likelihood estimators. Under $H_0(k)$, the ratio of these maxima is a $\mathbf{Q}(k)$-free statistics. This is shown in the following.

Let the complete parameter space for $\theta(k) = (\mathbf{a}(k),\mathbf{Q}(k))$ be $\mathbf{\Theta} = \{(\mathbf{a}(k),\mathbf{Q}(k))\text{: } \mathbf{a}(k)\in\mathbf{R}^p,\ \mathbf{Q}(k)\in \mathrm{Q}_p\}$, where Q_p is a set of positive definite covariance matrices, and let the restricted parameter space for $\theta(k)$, specified by the $H_0(k)$ hypothesis, be $\mathbf{\Theta}_0 = \{(\mathbf{a}(k),\mathbf{Q}(k))\text{: } \mathbf{a}(k) = \mathbf{0},\ \mathbf{Q}(k)\in \mathrm{Q}_p\}$. Then one possible statistics for testing $H_0(k)$: $\theta(k)\in\mathbf{\Theta}_0$ versus $H_1(k)$: $\theta(k)\in\mathbf{\Theta}_1$, where $\mathbf{\Theta}_1 = \mathbf{\Theta} - \mathbf{\Theta}_0$, is given by the generalized maximum likelihood ratio

$$\mathrm{LR} = \frac{\max_{\theta(k)\in\mathbf{\Theta}_1} \mathrm{L}_{H_1(k)}(\mathbf{Z}(k);\theta(k))}{\max_{\theta(k)\in\mathbf{\Theta}_0} \mathrm{L}_{H_0(k)}(\mathbf{Z}(k);\theta(k))}. \tag{7}$$

Under $H_0(k)$, the joint likelihood for $\mathbf{Z}(k)$ is given by

$$\mathrm{L}_{H_0(k)}(\mathbf{Z}(k);\theta(k)) = (2\pi)^{-np/2}\left|\mathbf{Q}(k)\right|^{-n/2} \times \exp\left(-\sum_{i=1}^{n}\mathbf{z}_i'(k)[\mathbf{Q}(k)]^{-1}\mathbf{z}_i(k)/2\right). \tag{8}$$

Under $H_1(k)$, the joint likelihood for $\mathbf{Z}(k)$ is given by

$$\mathrm{L}_{H_1(k)}(\mathbf{Z}(k);\theta(k)) = (2\pi)^{-np/2}\left|\mathbf{Q}(k)\right|^{-n/2} \times \exp\left(-\sum_{i=1}^{n}(\mathbf{z}_i(k)-\mathbf{a}(k))'[\mathbf{Q}(k)]^{-1}(\mathbf{z}_i(k)-\mathbf{a}(k))/2\right). \tag{9}$$

It can be shown that

$$\max_{\theta(k)\in\mathbf{\Theta}_0} \mathrm{L}_{H_0(k)}(\mathbf{Z}(k);\theta(k)) = (2\pi)^{-np/2}\left|\hat{\mathbf{Q}}_0(k)\right|^{-n/2}\exp(-np/2) \tag{10}$$

and

$$\max_{\theta(k)\in\mathbf{\Theta}_1} \mathrm{L}_{H_1(k)}(\mathbf{Z}(k);\theta(k)) = (2\pi)^{-np/2}\left|\hat{\mathbf{Q}}_1(k)\right|^{-n/2}\exp(-np/2), \tag{11}$$

where

$$\hat{\mathbf{Q}}_0(k) = \mathbf{Z}(k)\mathbf{Z}'(k)/n, \tag{12}$$

$$\hat{\mathbf{Q}}_1(k) = (\mathbf{Z}(k) - \hat{\mathbf{a}}(k)\mathbf{u}')(\mathbf{Z}(k) - \hat{\mathbf{a}}(k)\mathbf{u}')'/n, \tag{13}$$

and $\hat{\mathbf{a}}(k) = \mathbf{Z}(k)\mathbf{u}/\mathbf{u}'\mathbf{u}$ are the well-known maximum likelihood estimators of the unknown parameters $\mathbf{Q}(k)$ and $\mathbf{a}(k)$ under the hypotheses $H_0(k)$ and $H_1(k)$, respectively, $\mathbf{u} = (1, ..., 1)'$ is the n-dimensional column vector of units. A substitution of (10) and (11) into (7) yields

$$\mathrm{LR} = \left|\hat{\mathbf{Q}}_0(k)\right|^{n/2}\left|\hat{\mathbf{Q}}_1(k)\right|^{-n/2}. \tag{14}$$

Taking the $(n/2)$th root, this likelihood ratio is evidently equivalent to

$$\mathrm{LR}_{\bullet} = \left|\hat{\mathbf{Q}}_0(k)\right|\left|\hat{\mathbf{Q}}_1(k)\right|^{-1} = \left|\mathbf{Z}(k)\mathbf{Z}'(k)\right| / \left|\mathbf{Z}(k)\mathbf{Z}'(k) - (\mathbf{Z}(k)\mathbf{u})(\mathbf{Z}(k)\mathbf{u})'/\mathbf{u}'\mathbf{u}\right|. \tag{15}$$

Now the likelihood ratio in (15) can be considerably simplified by factoring out the determinant of the $p \times p$ matrix $\mathbf{Z}(k)\mathbf{Z}'(k)$ in the denominator to obtain this ratio in the form

$$\mathrm{LR}_{\bullet}=\frac{\left|\mathbf{Z}(k)\mathbf{Z}'(k)\right|}{\left[\left|\mathbf{Z}(k)\mathbf{Z}'(k)\right|\left(1-\dfrac{(\mathbf{Z}(k)\mathbf{u})'[\mathbf{Z}(k)\mathbf{Z}'(k)]^{-1}(\mathbf{Z}(k)\mathbf{u})}{\mathbf{u}'\mathbf{u}}\right)\right]}=1\Big/\Big(1-(\mathbf{Z}(k)\mathbf{u})'[\mathbf{Z}(k)\mathbf{Z}'(k)]^{-1}(\mathbf{Z}(k)\mathbf{u})/n\Big). \tag{16}$$

This equation follows from a well-known determinant identity. Clearly (16) is equivalent finally to the statistics

$$v_n(k)=\left(\frac{n-p}{p}\right)(\mathrm{LR}_{\bullet}-1)\;=\left(\frac{n-p}{p}\right)n\hat{\mathbf{a}}'(k)[\mathbf{T}(k)]^{-1}\hat{\mathbf{a}}(k), \tag{17}$$

where $\mathbf{T}(k)=n\hat{\mathbf{Q}}_1(k)$. It is known that $(\hat{\mathbf{a}}(k),\mathbf{T}(k))$ is a complete sufficient statistics for the parameter $\theta(k)$ = ($\mathbf{a}(k)$,$\mathbf{Q}(k)$). Thus, the problem has been reduced to consideration of the sufficient statistics $(\hat{\mathbf{a}}(k),\mathbf{T}(k))$. It can be shown that under H_0, $v_n(k)$ is a $\mathbf{Q}(k)$-free statistics which has the property that its distribution does not depend on the actual covariance matrix $\mathbf{Q}(k)$. This is given by the following theorem.

Theorem 2. **(***PDF of the statistics $v_n(k)$***).** Under $H_1(k)$, the statistics $v_n(k)$ is subject to a noncentral F-distribution with p and $n-p$ degrees of freedom, the probability density function of which is

$$f_{H_1(k)}(v_n(k);n,q)=\left[\mathrm{B}\left(\frac{p}{2},\frac{n-p}{2}\right)\right]^{-1}\left[\frac{p}{n-p}\right]^{\frac{p}{2}}\times\; v_n(k)^{\frac{p}{2}-1}\left[1+\frac{p}{n-p}v_n(k)\right]^{-\frac{n}{2}}$$

$$\times\,\mathrm{e}^{-nq/2}{}_1F_1\left(\frac{n}{2};\frac{p}{2};\frac{nq(k)}{2}\left[1+\frac{n-p}{pv_n(k)}\right]^{-1}\right),\; 0<v_n(k)<\infty. \tag{18}$$

where ${}_1F_1(b;c;x)$ is the confluent hypergeometric function, $q(k)=\mathbf{a}'(k)[\mathbf{Q}(k)]^{-1}\mathbf{a}(k)$ is a non-centrality parameter. Under H_0(k), when $q(k)$ = 0, (18) reduces to a standard F-distribution with p and $n–p$ degrees of freedom,

$$f_{H_0(k)}(v_n(k);n)=\left[\mathrm{B}\left(\frac{p}{2},\frac{n-p}{2}\right)\right]^{-1}\times\left[\frac{p}{n-p}\right]^{\frac{p}{2}}\; v_n(k)^{\frac{p}{2}-1}\left[1+\frac{p}{n-p}v_n(k)\right]^{-\frac{n}{2}},\;0<v_n(k)<\infty. \tag{19}$$

Proof. The proof follows by applying Theorem 1 [5–7] and being straightforward is omitted. □

4. GMLR Test

The GMLR test of $H_0(k)$ versus $H_1(k)$, based on $v_n(k)$, is given by

$$v_n(k)\begin{cases}\geq h(k), & \text{then } H_1(k),\\[2ex] <h(k), & \text{then } H_0(k),\end{cases} \tag{20}$$

and can be written in the form

$$\vartheta(v_n(k))=\begin{cases}1, & \text{if } v_n(k)\geq h(k)\quad (H_1(k)),\\[2ex] 0, & \text{if } v_n(k)<h(k)\quad (H_0(k)),\end{cases} \tag{21}$$

where $h(k)>0$ is a threshold of the test which is uniquely determined for a prescribed level of significance $\alpha(k)$ so that

$$\sup_{\theta(k)\in\Theta_0}\mathrm{E}_{\theta}\{\vartheta(v_n(k))\}=\alpha(k). \tag{22}$$

When the parameter $\theta(k)$ = ($\mathbf{a}(k)$, $\mathbf{Q}(k)$) is unknown, it is well known that no the uniformly most powerful (UMP) test exists for testing $H_0(k)$ versus $H_1(k)$ [8]. However, it can be shown that the test (20) is UMPI for a natural group of transformations on the space of observations. Here the following theorem holds.

Theorem 3. (*UMPI test*). For testing the hypothesis $H_0(k)$: $q(k) = 0$ versus the alternative $H_1(k)$: $q(k) > 0$, the test given by (20) is UMPI.

Proof. The proof is similar to that of Nechval [8] and so it is omitted here. □

5. Robustness Property

In what follows, as one more optimality of the $v_n(k)$-test, a robustness property can be studied in the following set-up. Let $\mathbf{Z}(k) = (\mathbf{z}_1(k), ..., \mathbf{z}_n(k))'$ be an $n \times p$ random matrix with a PDF φ, let C_{np} be the class of PDF's on $\mathbf{R}^{np}$ with respect to Lebesque measure $d\mathbf{Z}(k)$, and let H be the set of nonincreasing convex functions from $[0,\infty)$ into $[0,\infty)$. We assume $n \geq p + 1$. For $\mathbf{a}(k) \in \mathbf{R}^p$ and $\mathbf{Q}(k) \in \mathrm{Q}_p$, define a class of PDF's on $\mathbf{R}^{np}$ as follows:

$$\mathrm{C}_{np}(\mathbf{a}(k), \mathbf{Q}(k)) = \left\{ \begin{array}{l} f \in \mathrm{C}_{np} : f(\mathbf{Z}(k); \mathbf{a}(k), \mathbf{Q}(k)) = \left|\mathbf{Q}(k)\right|^{-n/2} \\ \times \eta\left(\sum_{i=1}^{n} (\mathbf{z}_i(k) - \mathbf{a}(k))'[\mathbf{Q}(k)]^{-1}(\mathbf{z}_i(k) - \mathbf{a}(k)) \right), \\ \eta \in \mathrm{H} \end{array} \right\}. \tag{23}$$

In this model, it can be considered the following testing problem:

$$H_0(k): \varphi \in \mathrm{C}_{np}(\mathbf{0}, \mathbf{Q}(k)), \mathbf{Q}(k) \in \mathrm{Q}_p \tag{24}$$

versus

$$H_1(k): \varphi \in \mathrm{C}_{np}(\mathbf{a}(k), \mathbf{Q}(k)), \mathbf{a}(k) \neq \mathbf{0}, \mathbf{Q}(k) \in \mathrm{Q}_p, \tag{25}$$

and shown that $v_n(k)$-test is UMPI. Clearly if ($\mathbf{z}_1(k)$, ..., $\mathbf{z}_n(k)$) is a random sample of $\mathbf{z}_i(k) \sim \mathrm{N}_p(\mathbf{a}(k),\mathbf{Q}(k))$, $i = 1(1)n$, or $\mathbf{Z}(k) \sim \mathrm{N}_{np}(\mathbf{ua}'(k),\mathbf{I}_n \otimes \mathbf{Q}(k))$, where $\mathbf{u} = (1, ..., 1)' \in \mathbf{R}^n$, the PDF φ of $\mathbf{Z}(k)$ belongs to $\mathrm{C}_{np}(\mathbf{a}(k),\mathbf{Q}(k))$. Further if $f(\mathbf{Z}(k);\mathbf{a}(k),\mathbf{Q}(k))$ belongs to $\mathrm{C}_{np}(\mathbf{a}(k),\mathbf{Q}(k))$, then

$$g_*(\mathbf{Z}(k); \mathbf{a}(k), \mathbf{Q}(k)) = \int_0^\infty f(\mathbf{Z}(k); \mathbf{a}(k), r\mathbf{Q}(k))dG_*(r), \tag{26}$$

also belongs to $\mathrm{C}_{np}(\mathbf{a}(k),\mathbf{Q}(k))$ where G_* is a distribution function on $(0,\infty)$, and so $\mathrm{C}_{np}(\mathbf{a}(k),\mathbf{Q}(k))$ contains the (np-dimensional) multivariate t-distribution, the multivariate Cauchy distribution, the contaminated normal distribution, etc. Here the following theorem holds.

Theorem 4. (*Robustness property*). For the problem (24)–(25), $v_n(k)$-test is UMPI and the null distribution of $v_n(k)$ is F-distribution with p and $n-p$ degrees of freedom.

Proof. The proof is similar to that of Nechval [8] and so it is omitted here. □

In other words, for any $\mathbf{Q}(k) \in \mathrm{Q}_p$ and any $\varphi \in \mathrm{C}_{np}(\mathbf{0},\mathbf{Q}(k))$, the null distribution of $v_n(k)$ is exactly the same as that when $\mathbf{Z}(k) \sim \mathrm{N}(\mathbf{0},\mathbf{I}_n \otimes \mathbf{Q}(k))$, that is, the distribution of $v_n(k)$ under $H_0(k)$ is the F-distribution with p and $n-p$ degrees of freedom. In this sense, the $v_n(k)$-test is robust against departures from normality.

6. Risk Minimization

For fixed n, in terms of the above probability density functions in (18) and (19), the probability of making the first type of wrong decision (model builder's risk ($\alpha(k)$)) is found by

$$\alpha(k)[h(k); n] = \int_{h(k)}^{\infty} f_{H_0(k)}(v_n(k); n)dv_n(k) \tag{27}$$

and the probability of making the second type of wrong decision (model user's risk ($\beta(k)$)) by

$$\beta(k)[h(k); n, q(k)] = \int_0^{h(k)} f_{H_1(k)}(v_n(k); n, q(k))dv_n(k). \tag{28}$$

This implies that the model is a perfect representation of the process with respect to its mean behaviour. Any value of $\mathbf{a}(k)$ will result in a value for $q(k)$ that is greater than zero. As the value of $\mathbf{a}(k)$ increases, the value of $q(k)$ will also increase. Hence, the non-centrality parameter $q(k)$ is the validity measure for the above test (20). Let us assume that for the purpose for which the simulation model is intended, the acceptable range of accuracy (or the amount of agreement between the model and the process) can be stated as $0 \le q(k) \le q^{\bullet}(k)$, where $q^{\bullet}(k)$ is the largest permissible value. In the statistical validation of simulation models, for pre-assigned $n = n^{\bullet}$ ($n^{\bullet} > p$) determined by a data collection budget, if we let $w_{\alpha(k)}$ and $w_{\beta(k)}$ be the unit weight (cost) of the model builder's risk ($\alpha(k)$) and the model user's risk ($\beta(k)$), then the optimal threshold of test, $h^*(k)$, can be found by solving the following optimisation problem:

Minimize:

$$R[h(k);n^{\bullet},q^{\bullet}(k)] = w_{\alpha(k)}\alpha(k)[h(k);n^{\bullet}] + w_{\beta(k)}\beta(k)[h(k);n^{\bullet},q^{\bullet}(k)]\,. \tag{29}$$

Subject to:

$$h(k) \in (0,1), \tag{30}$$

where $R[h(k);n^{\bullet},q^{\bullet}(k)]$ is a risk representing the weighted sum of the model builder's risk and the model user's risk. It can be shown that $h^*(k)$ satisfies the equation

$$w_{\alpha(k)} f_{H_0(k)}(h^*(k);n^{\bullet}) = w_{\beta(k)} f_{H_1(k)}(h^*(k);n^{\bullet},q^{\bullet}(k)). \tag{31}$$

In the statistical validation of simulation models, the model user's risk is more important that the model builder's risk, so that $w_{\alpha(k)} \le w_{\beta(k)}$.

For instance, let us assume that $p = 10$, $n^{\bullet} = 40$, $q^{\bullet}(k) = 0.5$, and $w_{\alpha(k)} = w_{\beta(k)} = 1$. It follows from (31) that the optimal threshold $h^*(k)$ is equal to 0.365.

If the sample size of observations, n, is not bounded above, then the optimal value n^* of n can be defined as

$$n^* = \inf n : \left(\begin{array}{l} \alpha(k)[h^*(k);n] + \beta(k)[h^*(k);n,q^{\bullet}(k)] \le r^{\bullet}(k), \\ \\ h^*(k) = \arg \min\limits_{h(k)\in(0,1)} R[h(k);n,q^{\bullet}(k)] \end{array} \right), \tag{32}$$

where $r^{\bullet}(k)$ is a preassigned value of the sum of the kth model builder's risk and the kth model user's risk.

7. Process Identification

Let us assume that there is available a sample of measurements of size n from each simulation model. The elements of a sample from the kth model are realizations of p-dimensional random variables $\mathbf{x}_i(k)$, $i = 1(1)n$, for each $k \in \{1, \ldots, m\}$. We are investigating an observable process on the basis of the corresponding sample of size n of p-dimensional measurements $\mathbf{y}_i = (y_{i1}, \ldots, y_{ip})'$, $i = 1(1)n$. We postulate that this process can be identified with one of the m simulation models but we do not know with which one. The problem is to identify the observable process with one of the m specified simulation models. When there is the possibility that the observable process cannot be identified with one of the m specified simulation models, it is desirable to recognize this case.

Let $\mathbf{y}_i$ and $\mathbf{x}_i(k)$ be the ith observation of the process and kth model variable, $k \in \{1, \ldots, m\}$, respectively. It is assumed that all observation vectors, $\mathbf{y}_i = (y_{i1}, \ldots, y_{ip})'$, $\mathbf{x}_i(k) = (x_{i1}(k), \ldots, x_{ip}(k))'$, $I = 1(1)n$, are independent of each other, where n is a number of paired observations. Let $\mathbf{z}_i(k) = \mathbf{x}_i(k) - \mathbf{y}_i$, $i = 1(1)n$, be paired comparisons leading to a series of vector differences. Thus, for identifying the observable process with one of the m specified simulation models, it can be obtained and used samples of n independent observation vectors $\mathbf{Z}(k) = (\mathbf{z}_1(k), \ldots, \mathbf{z}_n(k))$, $k = 1(1)m$. It is assumed that under $H_0(k)$, $\mathbf{z}_i(k) \sim \mathrm{N}_p(\mathbf{0},\mathbf{Q}(k))$, $\forall i = 1(1)n$, where $\mathbf{Q}(k)$ is a positive definite covariance matrix. Under $H_1(k)$, $\mathbf{z}_i(k) \sim \mathrm{N}_p(\mathbf{a}(k),\mathbf{Q}(k))$, $\forall i = 1(1)n$, where $\mathbf{a}(k) = (a_1(k), \ldots, a_p(k))' \ne (0, \ldots ,0)'$ is a mean vector. The parameters $\mathbf{a}(k)$ and $\mathbf{Q}(k)$, $\forall k = 1(1)m$, are unknown. For fixed n, the problem is to identify the observable process with one of the m specified simulation models. If the observable process cannot be identified with one of the m specified simulation models, it is desirable to recognize this case.

The test of $H_0(k)$ versus $H_1(k)$, based on the GMLR statistics $v_n(k)$, is given by (20). Thus, if $v_n(k) \geq h(k)$ then the kth simulation model is eliminated from further consideration.

If $(m-1)$ simulation models are so eliminated, then the remaining model (say, kth) is the one with which the observable process may be identified.

If all simulation models are eliminated from further consideration, we decide that the observable process cannot be identified with any model of the m specified simulation models.

If the set of simulation models not yet eliminated has more than one element, then we declare that the observable process may be identified with simulation model k^* if

$$k^* = \arg\max_{k\in D} (h(k) - v_n(k)), \tag{33}$$

where D is the set of simulation models not yet eliminated by the above test.

8. Applications

8.1. Application 1

This subsection discusses an application of the above test to the following problem. An airline company operates more than one route. It has available more than one type of airplanes. Each type has its relevant capacity and costs of operation. The demand on each route is known only in the form of the sample data, and the question asked is: which aircraft should be allocated to which route in order to minimize the total cost (performance index) of operation? This latter involves two kinds of costs: the costs connected with running and servicing an airplane, and the costs incurred whenever a passenger is denied transportation because of lack of seating capacity. (This latter cost is "opportunity" cost.) We define and illustrate the use of the loss function, the cost structure of which is piecewise linear. Within the context of this performance index, we assume that a distribution function of the passenger demand on each route is known as certain component of a given set of predictive models. Thus, we develop our discussion of the allocation problem in the presence of completely specified set of predictive demand models. We formulate this problem in a probabilistic setting.

Let $A_1, ..., A_g$ be the set of airplanes which company utilize to satisfy the passenger demand for transportation en routes 1, ..., h. It is assumed that the company operates h routes which are of different lengths, and consequently, different profitabilities. Let $f_{ij}^{(k)}(s)$ represent the predictive probability density function of the passenger demand S for transportation en route j, $j\in\{1, ..., h\}$, at the i-th stage ($i\in\{1, \ldots, n\}$) for the kth predictive model ($k\in\{1, \ldots, m\}$). It is required to minimize the expected total cost of operation (the performance index)

$$J_i(\mathbf{U}_i) = \sum_{j=1}^{h}\left[\sum_{r=1}^{g} w_{rij}u_{rij} + c_j\int_{Q_{ij}}^{\infty}(s - Q_{ij})f_{ij}^{(k)}(s)ds\right] \tag{34}$$

subject to

$$\sum_{j=1}^{h} u_{rij} \leq a_{ri}, \quad r = 1, \ldots, g, \tag{35}$$

where

$$Q_{ij} = \sum_{r=1}^{g} u_{rij}q_{rj}, \quad j = 1, \ldots, h, \tag{36}$$

$\mathbf{U}_i = \{u_{rij}\}$ is the $g \times h$ matrix, u_{rij} is the number of units of airplane A_r allocated to the jth route at the ith stage, w_{rij} is the operation costs of airplane A_r for the jth route at the ith stage, c_j is the price of a one-way ticket for air travel en jth route, q_{rj} is the limited seating capacity of airplane A_r for the jth route, a_{ri} is available the number of units of airplane A_r at the ith stage.

Let us assume that $\mathbf{U}_i^* = \{u_{rij}^*\}$ is the optimal solution of the above-stated programming problem. Since an information about the passenger demand is not known precisely, this result provides only approximate solution to a real airline system. To depict the real, observable airline system more accurately, the test proposed in this paper, might be employed to validate the results derived from the analytical model (34)–(36). In this case

$$Z_{ij}(k) = X_{ij}(k) - Y_{ij}, \quad j = 1(1)h, \quad \forall i \in \{1, \ldots, n\}, \tag{37}$$

where

$$X_{ij}(k) = c_j \left[\int_0^{Q_{ij}^*} s f_{ij}^{(k)}(s) ds + Q_{ij}^* \int_{Q_{ij}^*}^{\infty} f_{ij}^{(k)}(s) ds \right], \tag{38}$$

is the expected gain (ensured by the service of a passenger demand on the jth route at the ith stage) derived from the analytical model (34)–(36),

$$Q_{ij}^* = \sum_{r=1}^{g} u_{rij}^* q_{rj}, \quad j = 1, \ldots, h, \tag{39}$$

Y_{ij} is the real gain ensured by the service of a passenger demand on the jth route at the ith stage (an observation of the airline system response variable).

Thus, the methodology proposed in this paper allows one to determine whether the analytical model (34)–(36) is appropriate for minimizing the total cost of airline operation.

8.2. Application 2

Failures of any high speed rotating components (jet engine rotors, centrifuges, high speed fans, etc.) can be very dangerous to surrounding equipment and personnel (see Fig. 1), and must always be avoided.

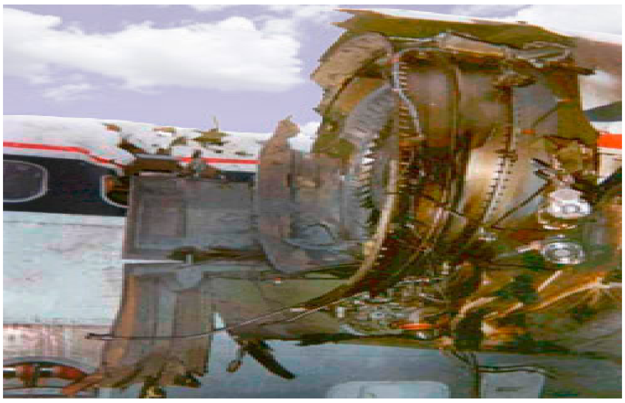

Figure 1. Jet engine fan section failure

Jet engine disks operate under high centrifugal and thermal stresses. These stresses cause microscopic damage as a result of each flight cycle as the engine starts from the cold state, accelerates to maximum speed for take-off, remains at speed for cruise, then spools down after landing and taxi. The cumulative effect of this damage over time creates a crack at a location where high stress and a minor defect combine to create a failure initiation point. As each flight operation occurs, the crack is enlarged by an incremental distance. If allowed to continue to a critical dimension, the crack would eventually cause the burst of the disk and lead to catastrophic failure (burst) of the engine. Engine burst in flight is rarely survivable.

Non-destructive testing methods like ultrasonic testing, X-ray, etc., are generally useful for the purpose. These methods are costly and time consuming for long components, e.g., railway tracks, long pipelines, etc. Vibration-based methods can offer advantages in such cases [9]. This is because measurement of vibration parameters like natural frequencies is easy. Further, this type of data can be easily collected from a single point of the component. This factor lends some advantages for components, which are not fully accessible. This also helps to do away with the collection of experimental data from a number of data points on a component, which is involved in a prediction based on, for example, mode shapes.

Suppose that we desire to compare a target vibration signal and a k^{th} reference vibration signal, which have p response variables. Let $x_{ij}(k)$ and y_{ij} be the ith observation of the j^{th} response variable of the k^{th} reference signal and the target signal, respectively. It is assumed that all observation vectors, $\mathbf{x}_i(k) = (x_{i1}(k), \ldots, x_{ip}(k))'$, $\mathbf{y}_i = (y_{i1}, \ldots, y_{ip})'$, $i = 1(1)n$, are independent of each other, where n is a number of paired observations. Let $\mathbf{z}_i(k) = \mathbf{x}_i(k) - \mathbf{y}_i$, $i = 1(1)n$, be paired comparisons leading to a series of vector differences. Thus, in order to compare the above signals, and return the likelihood whether the two signals are similar or not, it can be obtained and used a sample of n independent observation vectors $\mathbf{Z}(k) = (\mathbf{z}_1(k), ..., \mathbf{z}_n(k))$. Each sample $\mathbf{Z}(k)$, $k \in \{1, \ldots, m\}$, is declared to be realization of a specific stochastic process with unknown parameters. It is assumed here that $\mathbf{z}_i(k)$, $i = 1(1)n$, are independent p-multivariate normal random variables ($n \geq p + 2$) with common mean $\mathbf{a}(k)$ and covariance matrix (positive definite) $\mathbf{Q}(k)$.

A goodness-of-fit testing for the multivariate normality is based on Theorem 1.

For testing that the two signals (target signal and reference signal) are similar, there can be used the statistical approach described above.

Conclusions

Validation is a central aspect to the responsible application of models to scientific and managerial problems. The importance of validation to those who construct and use models is well recognized. However, there is little consensus on what is the best way to proceed. This is at least in part due to the variety of models, model applications, and potential tests. The options are manifold, but the guidelines are few.

The main idea of this paper is to find a validation test statistics whose distribution, under the null hypothesis, does not depend on unknown (nuisance) parameters. This allows one to eliminate the unknown parameters from the problem.

The authors hope that this work will stimulate further investigation using the approach on specific applications to see whether the obtained results with it are feasible for realistic applications.

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RELIABILITY ANALYSIS OF AN N-UNIT PARALLEL STANDBY SYSTEM UNDER IMPERFECT SWITCHING USING COPULA

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This paper studies a problem consisting of two n-unit standby redundant system designated N and S with repair facility incorporating the concept of imperfect switching under human failure and all repairs except one follow general repair time distribution. Both the systems suffer from two types of failures namely constant failure and human failure. All failures of the system follow exponential time distribution. The failure rate of units and repair of the switch are statistically independent. In contrast to earlier models which mainly considered only single transition between two adjacent states here authors have taken an important aspect of repairs, which is consistent with actual failures of switching by assuming two different types of repair between adjacent states. Using supplementary variable technique, Laplace technique and Gumbel-Hougaard family of copula the transition state probabilities, reliability and MTSF of the system is evaluated. Inversions have also been carried out so as to obtain time dependent transition probabilities, which determine availability of the system at any time.

Keywords: *copula, Gumbel-Hougaard family of copula, reliability and MTSF*

1. Introduction

In reliability analysis of redundant repairable systems, several authors carried out analysis of system with three modes and different repair policies assuming that after repair the system work like a fresh one. Tuteja *et al.* [1] have analysed a two single-unit model with three modes and different repair policies of repairmen who appear and disappear randomly. Gupta *et al.* [2] have evaluated the reliability and MTTF of a two duplex-unit standby system with two types of repair. Melchiori [3] described different families of Archimedean copula and discusses which family of Archimedean copula is suitable one? Singh [4] used Melchiori work in stochastic dependence modelling in environmental hydrology. Bedford [5] describes a method of assessing minimally informative copulas using observable quantities, which gives expert guidance on the range allowed for coherence. Models [1, 2] considered single transition between consequent stages. Here authors propose to take double transition between two consequent stages and evaluate the reliability characteristics of a complex system.

In this paper we consider two *n*-unit standbys redundant system designated N and S with repair facility. The system is assumed to have imperfect switching. It is also assumed that the system can fail due to human error. The failure rates of units and repair of the switch are statistically independent. In contrast to earlier models presented by authors [1, 2] here authors have taken an important aspect of repairs which is consistent with actual failures of switching by assuming two different types of repair namely exponential and constant between adjacent states S_2 and S_3.

By employing Gumbel-Hougaard family of copula, supplementary variable technique and Laplace transformation technique, various transition state probabilities, reliability and mean time to system failure (M.T.S.F) of the system have been evaluated which is helpful to system designers as well as operational managers.

Inversions have also been carried out so as to obtain time dependent probabilities, which determine availability of the system at any time.

2. Assumptions and Notations

2.1. Assumptions

(1) Initially the system is in good state.
(2) The system consists of two subsystems namely N and S. At time t = 0, all the units of N starts operating while the subsystem S is kept in standby.
(3) Switching over devices for standby system is not perfect.
(4) All failures for the system follow exponential time distribution.
(5) Both the sub-systems suffer from two types of failure namely constant failure and human failure.
(6) The whole system can fail from normal state directly due to human failure.
(7) After repair system works like a new one and never damages anything.
(8) Transition from S_2 to S_3 follows two different distributions.
(9) Joint probability distribution of repair rate from S_2 to S_3 follows Gumbel-Hougaard family of copula.

STATES	SUB SYSTEMS		SYSTEM STATE
	N	S	
S_0	O	S	O
S_1	F	S	F
S_2	F	Q	F
S_3	F	O	O
S_4	F	F	F
S_5	H	H	F

O: operable; S: standby; F: failed; Q: switching device under repair; H: human error.

2.2. Notations

The following notations are associated with this model:

a : Probability of successful operation of switching over device.

$\overline{a}$: Probability of failure of switching over device.

$\lambda_{h_1}/\lambda_{h_2}$: Constant failure rate due to human error from state S_0 to S_5 and S_3 to S_5.

λ_i/λ_j : Constant failure rate of normal standby subsystem.

$R/K = e^x$: Constant and exponential repair rates of switching over device.

$\eta_i(x)\Delta/\eta_j(y)\Delta/\eta_H(z)\Delta$: First order probability that the system is repaired in time interval (x, x + Δ)/(y, y + Δ)/(z, z + Δ).

$\beta(x)$: Switching rate from state S_1 to S_3.

$P_{o,s}(t)$: The probability at time t, the system is in the operable state S_0.

$P_{F_1,S}(x,t)\Delta$: The probability that at time t, the system is in the state S_2 and the elapsed repair time lies in the interval (x, x + Δ).

$P_{F,S}(x,t)\Delta$: The probability that at time t, the system is in state S_1 due to failure of subsystem N and elapsed repair time lie in the interval (x, x + Δ).

$P_{F,O}(x,t)\Delta$: The probability that at time t, the system is in the state S_3 and elapsed repair time lies in the interval (x, x + Δ).

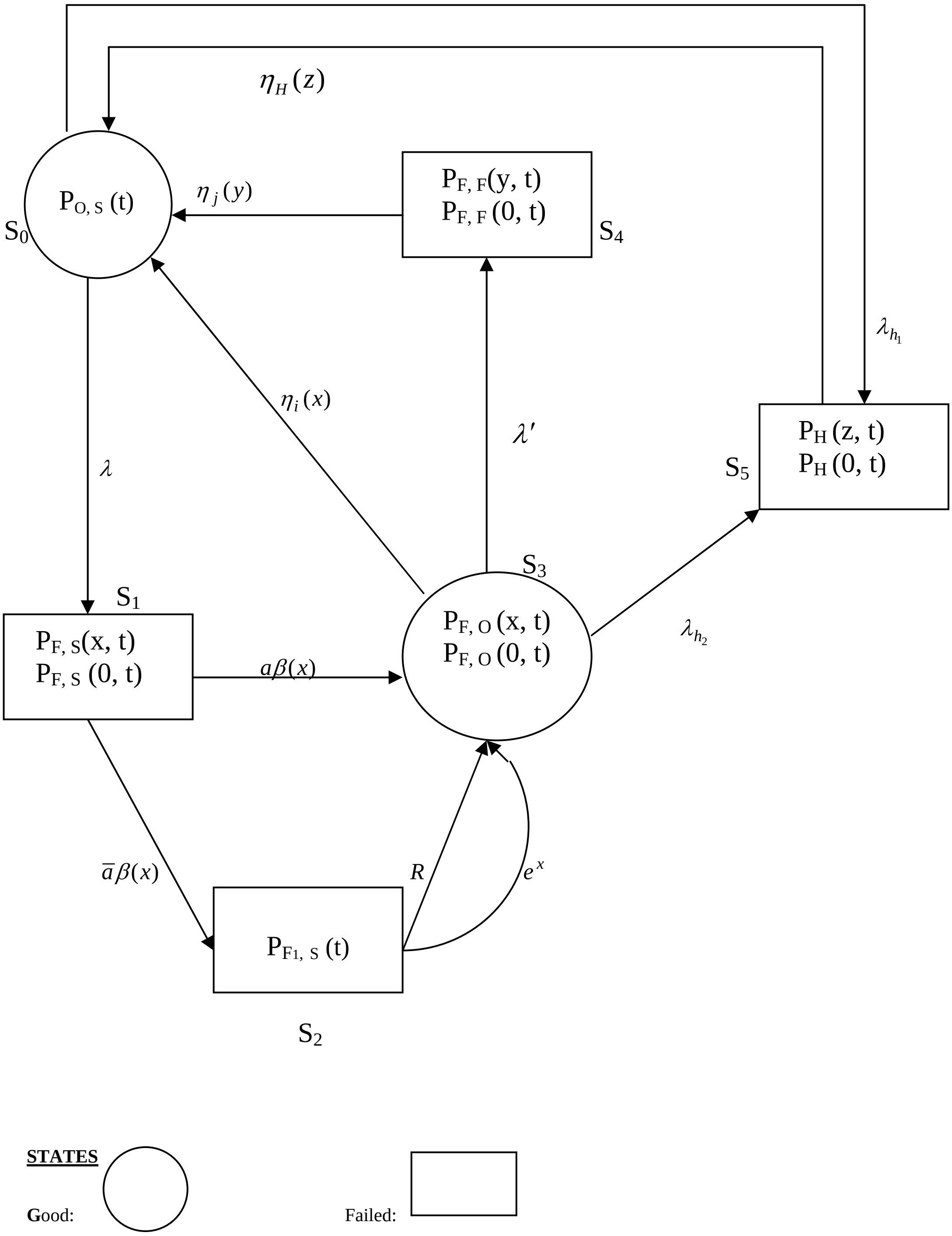

Figure 1. Transition State Diagram

$P_{F,F}(y,t)\Delta$: The probability that at time t, the system is in the state S_4 due to the failure of both the subsystems an elapsed repair time lies in the interval (y, y + Δ).

$P_H(z,t)\Delta$: The probability that at time t, the system is in failed state S_5 due to human error and elapsed repair time lies in the interval (z, z + Δ).

$F_X(x)/F$: Marginal distribution of random variables, where $F_X(x) = e^x$ and $F = R$.

Also $\sum_{i=1}^{N}\lambda_i = \lambda$, $\sum_{j=1}^{N}\lambda_j = \lambda'$. Here, we let $F_X(x) = e^x$ and $F = R$ (constant). Then the joint probability according to Gumbel-Hougaard family is

$$\exp[x^\theta + (\ln R)^\theta]^{1/\theta} \ . \tag{1}$$

3. Formulation of Mathematical Model

By probability considerations and continuity arguments, the following difference-differential equations governing the behaviour of the system may seem to hold good:

$$\left[\frac{\partial}{\partial t} + \lambda + \lambda_{h_1}\right]P_{O,S}(t) = \sum_{i=1}^{N}\int_0^\infty P_{F,O}(x,t)\eta_i(x)dx + \int_0^\infty P_H(z,t)\eta_H(z)dz + \sum_{J=1}^{N}\int_0^\infty P_{F,F}(y,t)\eta_j(y)dy \tag{2}$$

$$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \beta(x)\right]P_{F,S}(x,t) = 0 \tag{3}$$

$$\left[\frac{\partial}{\partial t} + \exp\left\{x^\theta + \left(\ln R\right)^\theta\right\}^{1/\theta}\right]P_{F_1,S}(t) = \overline{a}\int_0^\infty \beta(x)P_{F,S}(x,t)dx \tag{4}$$

$$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \lambda' + \lambda_{h_2} + \eta_i(x)\right]P_{F,O}(x,t) = 0 \tag{5}$$

$$\left[\frac{\partial}{\partial y} + \frac{\partial}{\partial t} + \eta_j(y)\right]P_{F,F}(y,t) = 0 \tag{6}$$

$$\left[\frac{\partial}{\partial z} + \frac{\partial}{\partial t} + \eta_H(z)\right]P_H(z,t) = 0 \tag{7}$$

Boundary conditions

$$P_{F,S}(0,t) = \lambda P_{O,S}(t) \tag{8}$$

$$P_{F,O}(0,t) = \exp[x^\theta + (\ln R)^\theta]^{1/\theta} P_{F_1,S}(t) + a\int_0^\infty \beta(x)P_{F,S}(x,t)dx \tag{9}$$

$$P_{F,F}(0,t) = \lambda' P_{F,O}(t) \tag{10}$$

$$P_H(0,t) = \lambda_{h_1}P_{O,S}(t) + \lambda_{h_2}P_{F,O}(t) \tag{11}$$

Initials conditions are

$$P_{O,S}(0) = 1 \tag{12}$$

and other state probabilities are zero at t = 0.

4. Solution of the Model

Taking Laplace transformation of equations (2) through (11) and using equation (12), we obtain

$$\left[s + \lambda + \lambda_{h_1}\right]\overline{P}_{O,S}(t) = 1 + \sum_{i=1}^{N}\int_0^\infty \overline{P}_{F,O}(x,s)\eta_i(x)dx + \int_0^\infty \overline{P}_H(z,s)\eta_H(z)dz + \sum_{j=1}^{N}\int_0^\infty \overline{P}_{F,F}(y,s)\eta_j(y)dy \tag{13}$$

$$\left[\frac{\partial}{\partial x}+s+\beta(x)\right]\overline{P}_{F,S}(x,s)=0 \tag{14}$$

$$\left[s+\exp\left\{x^{\theta}+(\ln R)^{\theta}\right\}^{1/\theta}\right]\overline{P}_{F_1,S}(t)=\overline{a}\int_0^{\infty}\beta(x)\overline{P}_{F,S}(x,s)dx \tag{15}$$

$$\left[\frac{\partial}{\partial x}+s+\lambda'+\lambda_{h_2}+\eta_i(x)\right]\overline{P}_{F,O}(x,s)=0 \tag{16}$$

$$\left[\frac{\partial}{\partial y}+s+\eta_j(y)\right]\overline{P}_{F,F}(y,s)=0 \tag{17}$$

$$\left[\frac{\partial}{\partial z}+s+\eta_H(z)\right]\overline{P}_H(z,s)=0 \tag{18}$$

$$\overline{P}_{F,S}(0,s)=\lambda\overline{P}_{O,S}(s) \tag{19}$$

$$\overline{P}_{F,O}(0,s)=\exp[x^{\theta}+(\ln R)^{\theta}]^{1/\theta}\,\overline{P}_{F_1,S}(s)+a\int_0^{\infty}\beta(x)\overline{P}_{F,S}(x,s)dx \tag{20}$$

$$\overline{P}_{F,F}(0,s)=\lambda'\overline{P}_{F,O}(s) \tag{21}$$

$$\overline{P}_H(0,s)=\lambda_{h_1}\overline{P}_{O,S}(s)+\lambda_{h_2}\overline{P}_{F,O}(s) \tag{22}$$

Now on integrating equations (13) through (18) and using equation (19) through (22), one may obtain the following:

$$\overline{P}_{O,S}(s)=\frac{1}{\gamma(s)}\,, \tag{23}$$

$$\overline{P}_{F,S}(s)=\frac{\lambda}{\gamma(s)}\cdot D_{\beta}(s)\,, \tag{24}$$

$$\overline{P}_{F_1,S}(s)=\frac{\overline{a}\lambda}{\gamma(s)}\cdot\frac{\overline{S}_{\beta}(s)}{s+\exp[x^{\theta}+(\ln R)^{\theta}]^{1/\theta}}\,, \tag{25}$$

$$\overline{P}_{F,O}(s)=\frac{\lambda}{\gamma(s)}\cdot\left[a+\frac{\overline{a}\exp\{x^{\theta}+(\ln R)^{\theta}\}^{1/\theta}}{s+\exp\{x^{\theta}+(\ln R)^{\theta}\}^{1/\theta}}\right]\cdot\overline{S}_{\beta}(s)\cdot D_{\eta_i}(s+\lambda'+\lambda_{h_2})\,, \tag{26}$$

$$\overline{P}_{F,F}(s)=\frac{\lambda\lambda'}{\gamma(s)}\cdot\left[a+\frac{\overline{a}\exp\{x^{\theta}+(\ln R)^{\theta}\}^{1/\theta}}{s+\exp\{x^{\theta}+(\ln R)^{\theta}\}^{1/\theta}}\right]\cdot\overline{S}_{\beta}(s)\cdot D_{\eta_i}(s+\lambda'+\lambda_{h_2})\cdot D_{\eta_j}(s) \tag{27}$$

$$\overline{P}_H(s)=\frac{1}{\gamma(s)}\cdot\left[\lambda_{h_1}+\lambda\lambda_{h_2}\left\{a+\frac{\overline{a}\exp\{x^{\theta}+(\ln R)^{\theta}\}^{1/\theta}}{s+\exp\{x^{\theta}+(\ln R)^{\theta}\}^{1/\theta}}\right\}\cdot\overline{S}_{\beta}(s)\cdot D_{\eta_i}(s+\lambda'+\lambda_{h_2})\right]\cdot D_{\eta_H}(s)\,, \tag{28}$$

where

$$\gamma(s)=s+\lambda+\lambda_{h_1}-\sum_{i=1}^{N}\lambda\left[a+\frac{\overline{a}\exp\{x^{\theta}+(\ln R)^{\theta}\}^{1/\theta}}{s+\exp\{x^{\theta}+(\ln R)^{\theta}\}^{1/\theta}}\right]\cdot\overline{S}_{\beta}(s)\cdot S_{\eta_i}(s+\lambda'+\lambda_{h_2})$$

$$-\sum_{j=1}^{N}\lambda\lambda'\left[a+\frac{\overline{a}\exp\{x^{\theta}+(\ln R)^{\theta}\}^{1/\theta}}{s+\exp\{x^{\theta}+(\ln R)^{\theta}\}^{1/\theta}}\right]\cdot\overline{S}_{\beta}(s)\cdot D_{\eta_i}(s+\lambda'+\lambda_{h_2})\cdot S_{\eta_j}(s)$$

$$-\left[\lambda_{h_1}+\lambda\lambda_{h_2}\left\{a+\frac{\overline{a}\exp\{x^{\theta}+(\ln R)^{\theta}\}^{1/\theta}}{s+\exp\{x^{\theta}+(\ln R)^{\theta}\}^{1/\theta}}\right\}\cdot\overline{S}_{\beta}(s)\cdot D_{\eta_i}(s+\lambda'+\lambda_{h_2})\right]\cdot S_{\eta_H}(s) \tag{29}$$

It is worth noticing that

$$\overline{P}_{O,S}(s)+\sum_{i=1}^{N}\overline{P}_{F,O}(s)+\overline{P}_{F,S}(s)+\overline{P}_{F_1,S}(s)+\sum_{j=1}^{N}\overline{P}_{F,F}(s)+\overline{P}_{H}(s)=\frac{1}{s}. \tag{30}$$

5. Ergodic Behaviour of the System

Using Abel's lemma; viz. $\lim_{s\to 0} s\overline{F}(s)=\lim_{t\to\infty} A(t)=F(say)$, provided the limit on R.H.S exists, in equation (23) through (28), the time independent probabilities are obtained as follows:

$$P_{O,S}=\frac{1}{\gamma'(0)} \tag{31}$$

$$P_{F,S}=\frac{\lambda}{\gamma'(0)}M_{\beta} \tag{32}$$

$$P_{F_1,S}=\frac{\overline{a}\lambda}{\gamma'(0)}\cdot\frac{1}{\exp[x^{\theta}+(\ln R)^{\theta}]^{1/\theta}} \tag{33}$$

$$P_{F,O}=\frac{\lambda}{\gamma'(0)}\cdot D_{\eta_i}(\lambda'+\lambda_{h_2}) \tag{34}$$

$$P_{F,F}=\frac{\lambda\lambda'}{\gamma'(0)}\cdot D_{\eta_i}(\lambda'+\lambda_{h_2})\cdot M_{\eta_j} \tag{35}$$

$$P_{H}=\frac{1}{\gamma'(0)}\cdot\left[\lambda_{h_1}+\lambda\lambda_{h_2}D_{\eta_i}(\lambda'+\lambda_{h_2})\right]M_{\eta_H}, \tag{36}$$

where $\gamma'(0)=\left[\frac{d}{ds}\gamma(s)\right]$ at s = 0 and $M_K=-\overline{S}_K'(0)$ for all K.

6. Particular Cases

6.1. Constant Repair Rates

When all repairs follows exponential time distribution. Setting $\overline{S}_{\eta_H}(s)=\frac{\eta_K}{s+\eta_K}$ where (k = i, j, H) in equation (31) through (36), we get

$$\overline{P}_{O,S}(s)=\frac{1}{E(s)}, \tag{37}$$

$$\overline{P}_{F,S}(s)=\frac{\lambda}{E(s)}\cdot\frac{1}{(s+\beta)}, \tag{38}$$

$$\overline{P}_{F_1,S}(s)=\frac{\overline{a}\lambda\beta\left[s+\exp\left\{x^{\theta}+(\ln R)^{\theta}\right\}^{1/\theta}\right]^{-1}}{E(s)\cdot[s+\beta]}, \tag{39}$$

$$\overline{P}_{F,O}(s)=\frac{\lambda}{E(s)}\cdot\left[a+\frac{\overline{a}\exp\left\{x^{\theta}+(\ln R)^{\theta}\right\}^{1/\theta}}{s+\exp\left\{x^{\theta}+(\ln R)^{\theta}\right\}^{1/\theta}}\right]\cdot\frac{\beta}{(s+\beta)}\cdot\frac{1}{(s+\lambda'+\lambda_{h_2}+\eta_i)}, \tag{40}$$

$$\overline{P}_{F,F}(s)=\frac{\lambda\lambda'}{E(s)}\cdot\left[a+\frac{\overline{a}\exp\left\{x^{\theta}+(\ln R)^{\theta}\right\}^{1/\theta}}{s+\exp\left\{x^{\theta}+(\ln R)^{\theta}\right\}^{1/\theta}}\right]\cdot\frac{\beta}{(s+\beta)}\cdot\frac{1}{(s+\lambda'+\lambda_{h_2}+\eta_i)(s+\eta_j)}, \tag{41}$$

$$\overline{P}_{H}(s)=\frac{1}{E(s)}\cdot\left[\lambda_{h_1}+\lambda\lambda_{h_2}\left(a+\frac{\overline{a}\exp\left\{x^{\theta}+(\ln R)^{\theta}\right\}^{1/\theta}}{s+\exp\left\{x^{\theta}+(\ln R)^{\theta}\right\}^{1/\theta}}\right)\cdot\frac{\beta}{(s+\beta)\cdot(s+\lambda'+\lambda_{h_2}+\eta_i)}\right]\times$$

$$\times\frac{1}{s+\eta_H}, \tag{42}$$

where

$$E(s)=s+\lambda+\lambda_{h_1}-\sum_{i=1}^{N}\lambda\left[a+\frac{\overline{a}\exp\left\{x^{\theta}+(\ln R)^{\theta}\right\}^{1/\theta}}{s+\exp\left\{x^{\theta}+(\ln R)^{\theta}\right\}^{1/\theta}}\right]\cdot\frac{\beta\eta_i}{(s+\beta)}\cdot\frac{1}{(s+\lambda'+\lambda_{h_2}+\eta_i)}$$

$$-\sum_{J=1}^{N}\lambda\lambda'\left[a+\frac{\overline{a}\exp\left\{x^{\theta}+(\ln R)^{\theta}\right\}^{1/\theta}}{s+\exp\left\{x^{\theta}+(\ln R)^{\theta}\right\}^{1/\theta}}\right]\cdot\frac{\beta\eta_j}{(s+\beta)}\cdot\frac{1}{(s+\lambda'+\lambda_{h_2}+\eta_i)(s+\eta_j)}$$

$$-\left[\lambda_{h_1}+\lambda\lambda_{h_2}\left(a+\frac{\overline{a}\exp\left\{x^{\theta}+(\ln R)^{\theta}\right\}^{1/\theta}}{s+\exp\left\{x^{\theta}+(\ln R)^{\theta}\right\}^{1/\theta}}\right)\cdot\frac{\beta}{(s+\beta)\cdot(s+\lambda'+\lambda_{h_2}+\eta_i)}\right]\cdot\frac{\eta_H}{s+\eta_H} \tag{43}$$

6.2. Perfect Switch over Device

When the switching device is perfect, the results are obtained by putting a = 1 in foregoing analysis.

6.3. Non-Repairable System

If the system is non-repairable then taking probabilities independent of x and joint repair rate zero, we obtain

$$\overline{P}_{up}(s)=\frac{1}{s+\lambda+\lambda_{h_1}}\cdot\left[1+\frac{\lambda a\beta}{(s+\beta)(s+\lambda'+\lambda_{h_2})}\right]$$

$$\therefore P_{up}(t)=e^{-(\lambda+\lambda_{h_1})t}\left[1+\frac{\lambda a\beta}{(\beta-\lambda-\lambda_{h_1})(\lambda'+\lambda_{h_2}-\lambda-\lambda_{h_1})}\right]$$

$$+\lambda a\beta\left[\frac{e^{-\beta t}}{(\lambda+\lambda_{h_1}-\beta)(\lambda'+\lambda_{h_2}-\beta)}+\frac{e^{-(\lambda'+\lambda_{h_2})t}}{(\beta-\lambda'-\lambda_{h_2})(\lambda+\lambda_{h_1}-\lambda'-\lambda_{h_2})}\right] \tag{44}$$

also $\overline{R}(s)=\dfrac{1}{(s+\lambda+\lambda_{h_1})}$

$$\therefore R(t)=e^{-(\lambda+\lambda_{h_1})t} \tag{45}$$

and

$$MTSF=\lim_{s\to 0}\overline{R}(s)=\frac{1}{(\lambda+\lambda_{h_1})} \tag{46}$$

7. Numerical Computation

7.1. Availability Analysis

Setting the numerical values say:

$\lambda = 0.01,\ \ \lambda' = 0.02,\ \ \lambda_{h_1} = 0.015,\ \ \lambda_{h_2} = 0.025,\ \ a = 1,\ \ \beta = 0.4,\ \ t = 0,1,2,...$ in equation (44), we can compute the Table 1 for $P_{up}(t) = 1.533e^{-0.025t} + 0.03e^{-0.4t} - 0.5633e^{-0.045t}$

7.2. Reliability Analysis

Setting $\lambda = 0.01,\ \ \lambda_{h_1} = 0.015,\ \ t = 0,1,2,...$ in equation (45), we compute the Table 2 for $R(t) = e^{-0.025t}$.

Table 1. Availability of the system at different time

t	$P_{up}(t)$
0	1
1	0.97703
2	0.95716
3	0.93937
4	0.92294
5	0.90738
6	0.89242
7	0.87785
8	0.86357
9	0.84948
10	0.8355

Table 2. Reliability of the system at different time

t	R(t)
0	1
1	0.97531
2	0.95122
3	0.92774
4	0.90484
5	0.88249
6	0.8627
7	0.83945
8	0.81873
9	0.79852
10	0.7788

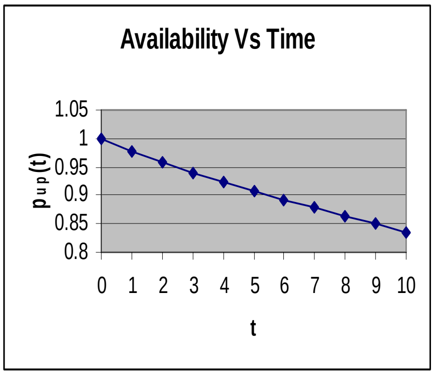

Figure 2

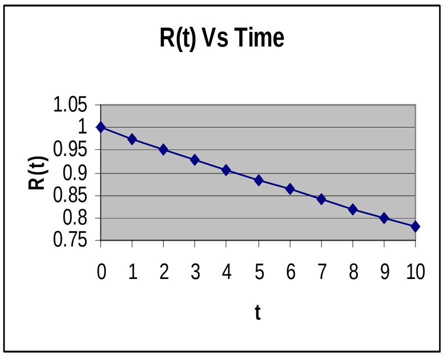

Figure 3

Conclusions

The graphs (Fig. 2 and Fig. 3) corresponding to Table 1 and Table 2 reveal that availability as well as reliability of the system decreases with time. It is observed that during the starting stages the decrement is faster in comparison to latter stages. As time passes the availability and the reliability of the system looks to be moving towards stability. It is also found that incorporation of the Gumbel-Hougaard family copula helps in improving the reliability of the system.

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MATHEMATICAL STUDY OF EVOLUTION OF RUSSIAN LANGUAGE

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Zipf's laws are used to study characteristic properties of the Russian language and its evolution during XIX and XX centuries. It is shown that Zipf's constant characterizes stylistic properties of works creation of different authors and at the same time allows to observe peculiarities of creative work for one author in different periods. It is obtained that structural parameters of The Russian language did not show a pronounced tendency to change during XIX and XX centuries.

Keywords: *Zipf's laws, the Russian language*

1. Introduction

Mathematical linguistics (ML) [1] is developed as a separate field of the artificial intellect. In this field mathematical models are used for description of the structure of natural languages. Each language is considered as a mechanism that determines the human verbal activity leading to creation of the so-called "right" texts [2]. Besides that ML includes the theory of formal grammars that investigates the whole totality of "right" texts and creates the "formal grammar" for creation of individual texts of corresponding structure. ML is closely linked to computer and information technologies.

The important step in ML development was an application of Markov's chains [3]. These methods allowed:

- To understand deep correlations between letters and words in a wide range of contexts.
- To perform an analysis of many samples of prose and verses and to clarify their internal structure.
- To find the origin of unknown ancient literature works.
- To find authorship for some literature works.

The next example of quantitative approaches within linguistics is an application of laws by Zipf [4–6]. Zipf's laws allow solving many problems of linguistics. It was revealed the existence of universal characteristics of all human texts independently on the language groups.

New fundamental results have been obtained in such fields as origin of languages, peculiarities of different language groups, deciphering of ancient texts, theory of translation, and information searching. Nowadays mathematical approaches in these fields of linguistics led to a new level of understanding of central questions of linguistics. Below the main ideas of Zipf's laws are described.

1.1. The First Zipf's Law

The first law of Zipf gives a definition of frequency, ω for the given word if this word is repeated in the text ω times. The frequencies ω_i for all words must be found ($i = 1, 2 \ldots n$, where n is the total number of different words in the text). For some text one can get a series:

$$\omega_1 > \omega_2 > \omega_3 \ldots > \omega_n . \qquad (1)$$

For frequency ω_k the parameter k is named as a rank of the frequency ω_k. According to the first Zipf's law:

$$\frac{k \cdot \omega_k}{n} = Const \quad . \tag{2}$$

The result means that within each group of languages this constant is the same for any text. For example, within English texts this constant is 0.1 and for Russian texts it is 0.06–0.07.

1.2. The Second Zipf's Law

In the first Zipf's law, different words in the text that can have the same frequency ω were not accounted for. In real conditions we meet the situation when m words in the text have the same frequency ω_k. It was shown that there is a universal link between the parameters of m and ω_k. This link is the second Zipf's law and can be expressed by formula:

$$\ln m = \chi \ln \omega_k \quad . \tag{3}$$

This linear dependence is true for all language groups! The differences are observed only in the values of the coefficient χ (See Figure 1). One can see that significant words in the text can be selected using this dependence.

It is reasonable to note that Zipf's laws are true not only for language sciences. For example, a number of cities depend on the number of their population in accordance with Zipf's laws. Ratings of Internet sites also satisfy these laws. Now scientists come to conclusion that Zipf's laws reflect the human origin of the object. Some very old manuscripts were investigated using Zipf's laws. It was even unclear if these materials are texts in principal. The investigations with application of Zipf's laws answered this question. Computer programs allow on the basis of Zipf's laws to differentiate between women's and men's texts and to solve psychological problems.

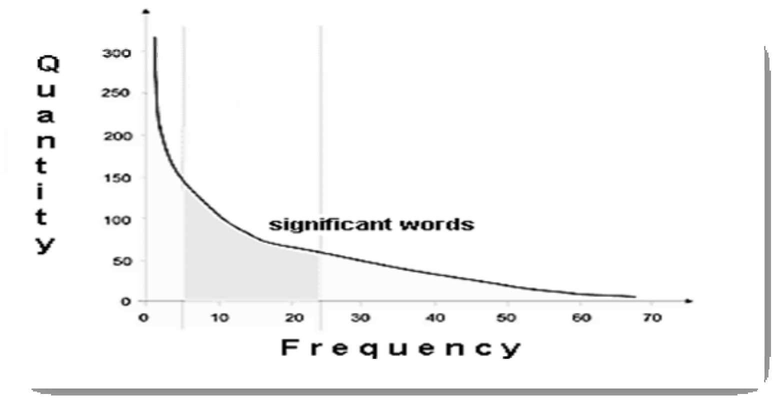

Figure 1. The dependence between parameters m and ω_k

2. Results and Discussion

2.1. Description of Computer Program

Using C# language a computer program was developed for calculation of Zipf's constant in the first Zipf's law. The algorithm of the program consists of the following steps:

- Selection of different words in the text.
- Determination of the quantity (and frequency) for each word.
- Arrangement of all different words according to requirement (1).
- Calculation of Zipf's constant according to formula (2) for each rank of words.
- Calculation of the average Zipf's constant for the given text.

On this stage the developed program is applicable only for analysis of Russian texts and demands an additional improvement. It is necessary to increase the accuracy of Zipf's constant determination accounting suffixes and other peculiarities of words with the same meaning.

2.2. Results of the Russian Language Study

This work is aimed to apply Zipf's laws for revealing the differences between the structures of different Russian texts that belong to the same author or to different well known authors. The next task was to follow the possible changes of Zipf's constant for the Russian language during XIX and XX centuries. We have chosen the authors whose works creation reflects typical characteristics of the Russian language of their time: A. S. Pushkin, A. P. Chehov, A. I. Kuprin, Gr. Rasputin and B. Akunin. The results are demonstrated in Figure 2.

Figure 2. The changes of Zipf's constant (W) during 1800–2000 year

One can see that no strongly pronounced tendency during the studied period was revealed in time changes of Zipf's constant. This constant fluctuates relatively the mean value of Zipf's constant for the Russian language (0. 006) with deviations ≤ 0.002. All these data are obtained for prosaic texts. It is interesting to note that for poetic texts (A. S. Pushkin, Evgeniy Onegin) this deviation is 0. 004. The last result can reflect the general property of poetic texts to express thoughts in the expressed form.

The meaning of Zipf's constant allows comparing stylistic features of authors. Really, the epic style of Rasputin leads to the increasing value of Zipf's constant. At the same time we see the characteristic expressed style of Chehov and Kuprin. We observe changes of Zipf's constant for the same author comparing his small works (small stories) and large novels. The distinct differences of the value of Zipf's constant are seen between prosaic and poetic works of the same author. An increase of statistics and an improvement of computer program will amplify the predictive possibilities of our approach.

Conclusions

The obtained results showed that Zipf's laws can be used for mathematical analysis of structural changes in the literature works during the arbitrary historical period. It is found that differences between Zipf's constants for the works of different authors are linked to their stylistic properties. The further analysis will give the detailed information about structural changes of the Russian language from the beginning of language formation.

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SIMULATION OF VIDEO OBSERVATION SYSTEMS

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The problems of observing system's simulation (media dots in a wide view) are considered. Classification of system of video observation, base function of such systems and processes of analysing media dates are described. The approach to the analysing of video observation systems is proposed. Some simulation models based on stochastic Petri nets and results are discussed.

Keywords: *video observation, Petri nets, simulation model, media dots, program agents*

1. Introduction

Now in all spheres of life systems of video observation are actively applied. They allow conducting round-the-clock supervision over object, supervising access, operating technical devices remotely, fixing emergencies and informing secure structures, so providing safety and safety of people and material assets. The main source of the information in such systems is video stream, but not only this one. The various media-information sources can be generalized, using media-dots concept – an elements, which are capable to accept, reproduce and process media data. Much plenty of resources (memory and computing capacities) is required for processing and storing of media data. One of problem is how to find optimum for using resources (processors, memory) so that there is the least number of refusals at processing the information. One of variants of the decision of this problem is the maximal processing of video information in the point of its reception.

2. Media Dots and Video Observing Systems

Let's consider various media data sources.

1. Systems of video-supervision, which can be advanced up to monitoring systems of city or area (Fig. 1). The example of the project of the monitoring system of area of city is described, for example, in [8].
2. The systems of "clever" buildings too can be a rather volumetric media-data source) are considered [2, 11]. The monitoring system and managements of a "clever" building can contain some media-data sources intended for supervision over a condition of rooms and technical subsystems, and also management blocks (Fig. 2).
3. The project of a mobile system of supervision (Fig. 3).
4. The systems of security supervision, in which increasing popularity win IP-cameras [6, 4]. The modern variants of systems of video-supervision and security signal system include algorithms of recognition of the people, cars and some situations (sharp acceleration of movement, fall of the man etc.) [7].
5. The monitoring system of warehouses and rooms, with adjustable zones of the control. One of popular tasks in these systems is the systems of the person's recognition [3].
6. Systems of supervision and control used in educational institutions, for example, at schools [12], where the cameras are used for the control of study rooms and sports platforms.
7. Project of a control system educational content of higher-school (Fig. 4).

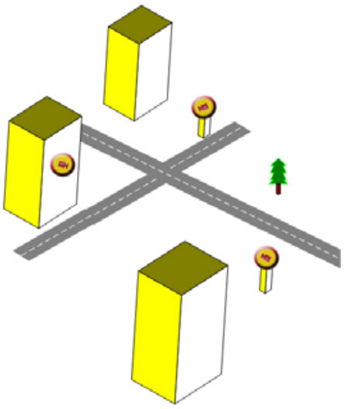

Figure 1. Town control system

Figure 2. Media- dots in a smart house

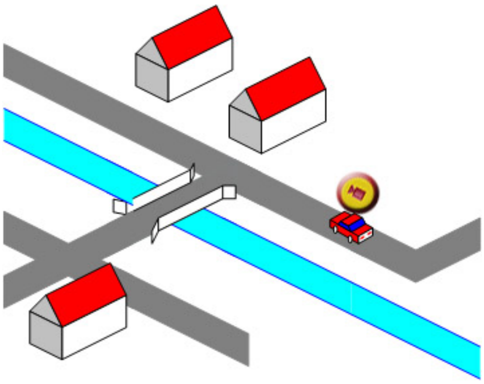

Figure 3. Mobile M-dots

Figure 4. Graphical user interface for control media dots in higher school

The various media-information sources can be generalized, using media-dots concept [10]. Let's name media or multi-media dot (M-dot) an element, which is capable to accept and/or reproduce and/or to process media data.

Some of the bright examples of media dots are video observing systems (VOS). They are carried out functions: protection of object, systems “clever house”, monitoring systems of access, monitoring systems behind movement. Video observing system often used in transport, banks, industrial and strategic objects, in a safe city systems, for supervision through the Internet.

Now base functions of VOS are – monitoring, record, motion detection, a digital compression, the audio-control, the control of a condition of gauges and management of external devices, the notification on e-mail, to phone or SMS, the remote video-control and video observation over the Internet by means of a usual browser, protection of access to system. In the future quantity of function will increase. Modern systems will be more intellectual. The important function of newest systems is pattern recognition in more comprehensive sense (recognition of faces, recognition of conditions and so on). In a system where some media dots are working together all these functions are implemented simultaneously and asynchronously.

3. Media Data Processes

The standardization and the unification of systems of media-information processing will require the decision of many tasks, beginning from the user interface and finishing researches and development of the programs of intellectual media data processing.

One of the problems is the huge plenty of data stream from the media-dots. Frequently are used only thousand shares of percent of received media data from the dot of view. Therefore, a development of means which could separate useful media-information from all huge flow is urgent. One of variants of the decision of this problem is the maximal processing of video information in the point of its reception.

The special program-agent at receipt of video signal processes it according to algorithm (allocates the important elements, depending on functions of system). The program carries out a compression of the sorted stream. And then sends on a server. At the presence of such huge plenty of data stream this way of preliminary video information processing allows sending a minimal data stream on a server. For this reason using processor and memory resources at the server are decreasing.

We will consider processes which are possible at a receipt and processing of data arrived from media dot. An approximate list is resulted below.

A – Detecting
A1 – Motion
A2 – Sound
A3 – Light
A4 – Temperature
A5 – Mass
A6 – Volume
B – Recognition by using some frames (1–10 frames)
B1 – Image recognition (appear/disappear of object)
B2 – Faces recognition
C – Pattern recognition
C1 – Pattern recognition by using 1–2 frames (with a purpose to select an algorithm of further actions)
C2 – Recognition
H – Pattern recognition (situations) expecting execution of hundreds and thousands frames
D – Record
D1 – One frame
D2 – Some frames
D3 – Sound
D4 – Video stream (thousands and millions frames)
E – Data processing (image/sound processing filters, video/sound compressing/decompressing, rendering)

For getting the result it is necessary to execute a few processes. It connects to a chain of media date processes.

Many combinations of processes are possible. But, as a rule, chain of processes started from detecting some signal. We can consider the following chains of processes.

A1 - > B1 - > C1 - > B2
A2 - > D2 - > C2
A5 - > C1 or C2
A3 - > D1 - > B1 - > B2
And so on.

These chains of processes are implemented simultaneously and asynchronously. A control and implementation of process's chains is carried out by program agents.

The aims of investigation are to find optimal distribution of agent's parts in media dots space and to find algorithms of agents. The article considers the first goal – distribution of the parts of agents and how it influences on the demands to server capacity.

To modelling asynchronously processes often Petri nets (PN) are used [8]. In our case we use extension of classic PN – a stochastic PN (SPN). Transitions in SPN can be executed immediately or after random time interval. The program HPSim [1] is used for the simulation.

4. Models of Processes Media Data Implementation

Let's consider models of program agent's job at the following assumptions.

All processes except A1 are executed on the server.

Part of processes (A1 and B1) is executed directly in a point of data reception.

Let's estimate requirements to computing capacity of servers in the first and second cases at parallel execution of several chains of processes.

Model 1. Type (a) – one chain of processes (Fig. 5).

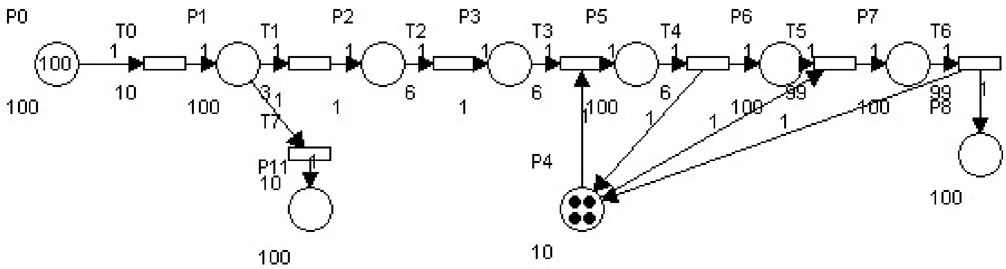

Figure 5. Chain of processes (a) type. One chain of processes

The elements of model (Fig. 5) means as follows:
P0 – Input video signal
T0 – Receipt of a signal (exponent's distribution law)
T1 – Motion detection (A1)
T2 – Recognition appear/disappear of object (B1)
T3, T4 – Pattern recognition by using 1–2 frames (C1). T3 – start of process, T4 – end
T5, T6 – Face recognition (B2) start and end
P4 – Server's resources
P11 – Raw applications

Model 2 of two chains of processes (a) type.

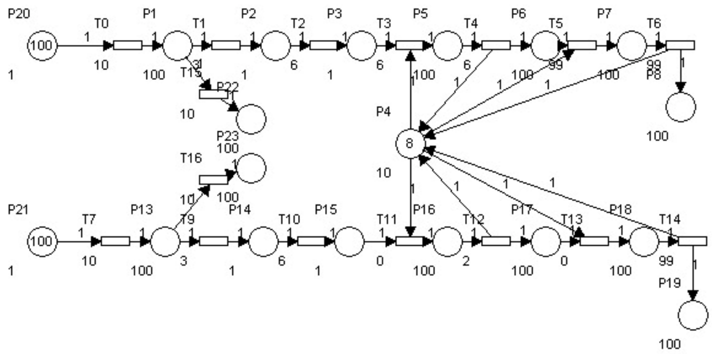

Figure 6. Chains of processes (a). Two media dots. All of the processes do with server resources (with exception the A1)

Model 3 of two chains of processes (a) type. The processes A1 and B1 do with media dots resources.

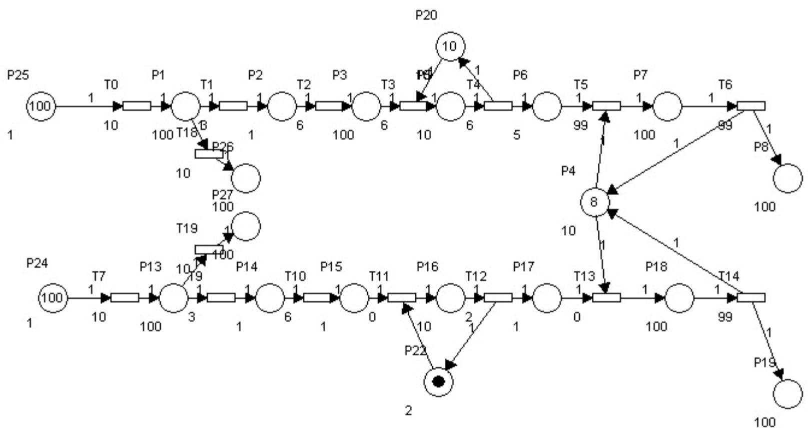

Figure 7. Chain of processes (a). Two media dots. The processes A1 and B1 do with media dots resources

5. Results

Start of models has shown, that server's resources increasing at the big intensity of the entrance dataflow, practically does not render influence for the period of processing and quantity(amount) of the lost applications (about 3000 units of time and 80%, accordingly). During a lot of time, increase in buffer memory of the server or media dot allows to reduce considerably number of the lost applications (Fig. 8).

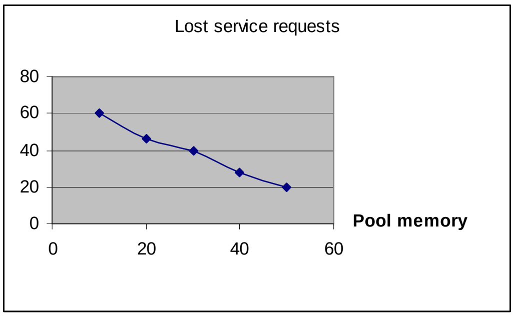

Figure 8. Lost service requests

6. Tasks of the Future Investigations

The purposes of the nearest researches – the analysis and modelling of other process's chains (b), (c), (d) and their combinations at various requirements to computing media dot's and servers capacity.

The aim of future investigation is development and research of the agents existing in media-dots space with a different degree of intelligence and various functional filling.

Such agents could solve problems of concrete object's search (static or moving), problems of certain attributes object's search, recognition of situations. One of the ways is use of self-learning agents and agents of a live organisms simulating behaviour [5]. We suggest that for a rating of various agents behaviour's strategy also is possible to use imitating models. On the other hand, we could use Markov's circuits for modelling chains of the processes described above. It would be interesting to compare these two approaches. The other interesting problem of future investigation is the analysis of reliability of video observation systems.

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MDA PIM-TO-PIM TRANSFORMATION USING EXTENDED AUTOMATA

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This paper proposes a generic Platform-Independent-Model (PIM) for realization of OMG Model Driven Architecture (MDA) standard. Our model defines precise execution semantic for UML state diagrams with OCL constraints. We describe a decomposition technique for state diagrams that separates platform-independent and platform-specific parts and facilitates further transformations to various implementation platforms.

Our method is based on the notion of Parallel Hierarchical Statechart Automaton (PHSA) introduced in this paper. This kind of extended automata can be a common tool for transforming UML diagrams to a PIM suitable for further transformations to PSM automata and to compilable code. The architecture for a general UML-to-PHSA PIM-to-PIM transformation for UML state diagrams allows portable execution on various specific implementation platforms and interoperability between PSMs based on common PHSA-based PIM.

Keywords*: MDA, Model transformation, extended automata, UML, OCL, XMI.m*

1. Introduction

MDA (Model Driven Architecture) [1] is a standard proposed by OMG (Object Management Group) in order to separate application logic from the technology of implementation platform (J2EE, .NET, EJB, CORBA, Web-based platforms etc.). The UML (Unified Modelling Language) [2] is a set of several diagrammatic notations addressing various aspects of system development. Each kind of diagrams comes with its own distinct syntax and semantics, which can be used to convey different aspects of the modelled system (e.g., Statecharts and Activity Diagrams, Sequence Diagrams and Collaboration Diagrams).

In the process of software development MDA proposes to develop a model completely independent of technology, known as PIM (Platform Independent Model), and for each implementation platform to develop a model specific to the destination platform, called PSM (Platform Specific Model). Further, it is possible to generate from PSM compilable source code (see Fig. 1) [4]. Thus, in the MDA approach the transformations are the heart of the software development. Many kinds of transformation tools were developed in order to implement the MDA approach. Wide overviews and taxonomy of existing transformation techniques is be found in [5, 6].

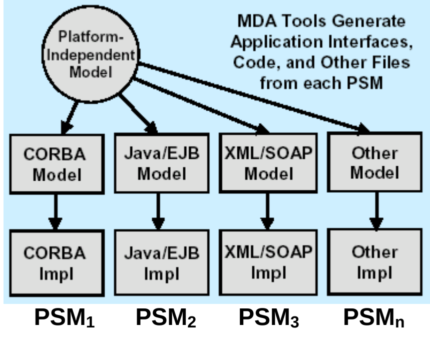

Figure 1. MDA portability through PIM-PSM (from [3])

According to the types of models being transformed, there exist seven kinds of transformations corresponding to combinations shown in Fig. 2. Besides, there exists a possibility of transformations from PIM directly to code, called "automatic transformations" in [1].

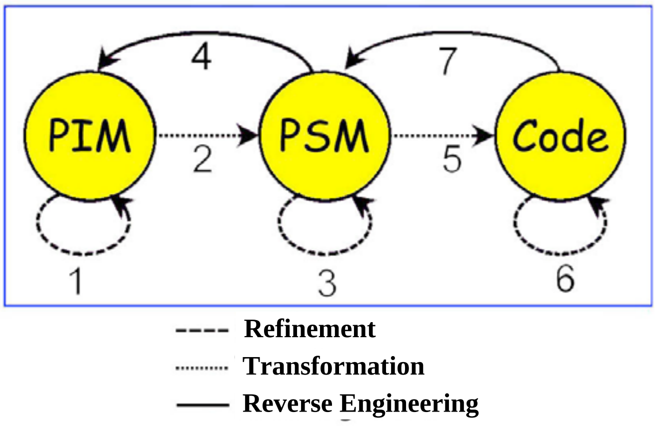

Figure 2. Kinds of MDA Transformations (from [7])

The purpose of PIM-to-PIM transformations is to add or subtract some information about models, or to re-organize this information and to represent it in a different form. This is the kind of transformation presented in this paper.

2. Proposed Approach to Modelling Application Behaviour in PIM

Our goal is transforming a UML state diagram to a form which will have a precise execution semantic and that will facilitate further transformations to target platforms in a generic way. UML state diagrams, based on the notion of Harel statecharts [8], capture behaviour of the modelled object at PIM level (further we will use terms "state diagram" and "statechart" interchangeably). We transform a statechart into another PIM, which separates between platform-independent and platform-dependent aspects of the modelled behaviour. In fact, we decompose the system functionality described by a statechart to four components, each one implementing a particular aspect of the system behaviour. The four components are: Moore automaton [9], Memory, Calculator and Input-output system (Fig. 3). We use the term SSA (Statechart Sequential Automaton) for the composition of the four components. The word "sequential" means that such a model does not handle composite states of UML statecharts, which can be either hierarchical (containing a single inner region) or concurrent (containing multiple regions). To include hierarchical states, we represent each region as an SSA and, in order to compose the SSA's in a whole system adequate to the source statechart, introduce some additional ("dummy") states and transitions. This extended model is called HSA (Hierarchical Statechart Automaton). Concurrent states are handled by further extending of the model, resulting in PHSA (Parallel Hierarchical Statechart Automaton). These automata will be explained in section 3.

2.1. The SSA: Statechart Sequential Automaton

The SSA consists of four components (see Fig. 3). The first is a reactive component $\mathcal{A}$ which is essentially a Moore automaton, whose states correspond to those of the UML statechart and the role of input alphabet play combinations of an incoming event and a guard, that is, a Boolean expression involving system variables. The output symbols are actions performed by the automaton when it enters a destination state. (Note that if we had chosen to work with transition actions instead of entry actions, we would get a Mealy [10] automaton instead of Moore. Since there is a well-known equivalence between Mealy and Moore machine, our preference for a Moore machine does not affect the generality of class of systems that is possible to represent by our model.)

The second component is a stateless transformational scheme $\mathcal{C}$, which centralizes computation of conditions and is responsible for executing local methods. The third is a memory register $\mathcal{M}$ for storing the system variables. The fourth component, I/O, performs input-output routines.

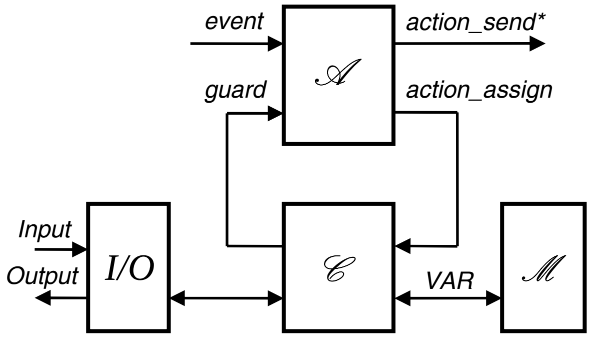

Figure 3. The SSA structural scheme

This decomposition helps us to separate the platform-independent part of the modelled system (the Moore automaton) from other components, which are to some extent platform-dependant: implementation of the memory and the conditional scheme depends on data types supported in the target platform, and the I/O system is completely platform-dependant.

2.2. The HSA: Extending the SSA Model to Include Hierarchy ("Or" Composite States)

The HSA (Hierarchical Statechart Automaton) is an SSA extension that treats hierarchical composite states of a UML statechart, and PHSA goes forth to deal with parallel execution of the sub-automata.

An application defined by UML statechart may contain composite states. Such states are themselves statecharts, so the application can be represented by a hierarchical structure. In general, a composite state may contain several inner statecharts (or "regions"), which in this case must be executed concurrently, but at this step of the modelling we suppose that each composite state contains exactly one statechart, that is, we deal at this step only with hierarchy, but not with concurrency (parallelism). Here, we describe a formal model that we call Hierarchical Statechart Automaton (HSA).

Let us consider a hierarchy of K+1 automata made of a main automaton A_0 and sub-automata A_1, …, A_K. The only restriction we put on the structure of the system is that it has to be a tree, that is, each sub-automaton belongs only to one parent.

The hierarchical behaviour of a system means that each transition is on its definite level, so each transition changes the state of *one* definite sub-automaton only, and access to other sub-automata allowed only through their initial states. Therefore, each transition takes the form:

$$state_{km} \xrightarrow{event_i \,/\, cond_j} state_{kn}\,,\; k = 0,1,\ldots,K\,, \qquad (1)$$

where $state_{km}$, $state_{kn} \in STATES_k$ (the set of the states of the automaton A_k).

We assume that the statecharts $CHART_0$, $CHART_1$, … , $CHART_K$ of the main automaton A_0 and its sub-automata A_1, …, A_K are given. Some of the vertices of $CHART_0$ (i.e. states of A_0) are composite states of A_0 and are interpreted as sub-automata from the list A_1, …, A_K ; the same may occur for some of the states of any one of sub-automata. Our restriction means that all the connections in the system are arranged in a tree.

For each statechart $CHART_k$ there is a subset H_k of its vertices that present composite states of A_k that are to be interpreted as sub-automata from the same list above A_1 ,…, A_K, i.e. we mean that there is given the set of functions f_k:

$$\begin{aligned} &f_k :\; H_k \to \{1, \ldots, K\} \\ &state_{km} \mapsto f_k(m)\,. \end{aligned} \qquad (2)$$

Each function f_k maps the indices of the composite state in A_k into the indices of their corresponding automata on the next (lower) level of hierarchy. In the simplest case, we suggest that no automaton in the list may correspond to more then one composite state of any other automata, i.e. our net of automata form a tree structure.

Now, in order to represent hierarchical automaton (HSA) as composition of components which are SSA automata, we add for each composite state $state_{km}$ four new elements:

1) entry action $dummyAction_{km}$ in $state_{km}$ to start execution of the sub-automaton $A_{f_k(m)}$,
2) "dummy" state $dummyState_{km}$,
3) transition from the state $state_{km}$ to the $dummyState_{km}$. It is triggered by
4) event $dummy\ Event_{km}$ that each sub-automaton $A_{f_k(m)}$ produces when it reaches its final state.

The purpose of the $dummyAction_{km}$ is to move down to the lower level of hierarchy, while the rest three two elements cause the automaton to return to the previous level.

Introducing the dummy states is simply made by extending the original set of UML state diagram states S_k with:

$$\widetilde{S}_k = S_k \cup (\bigcup_{i \in H_k} \{DummyState_{ki}\}) . \quad (3)$$

That is, for every automaton A_k which contains a set H_k of composite states, the set S_k has to be extended with only one dummy state for each composite state.

Appropriately, we will need to add new entities in the XML document representing the table of events in the serialized form of our automaton.

$$\widetilde{E}_k = E_k \cup (\bigcup_{i \in H_k} \{DummyEvent_{ki}\}) . \quad (4)$$

The event $DummyEvent_{ki}$ must be triggered when the sub-automaton $A_{f(k,i)}$ reaches its final state. This requires adding an entry action of "send event" kind in the final state of every sub-automaton in all the hierarchy. In this way we can reduce each composite state in UML statechart to a composition of SSA blocks.

2.3. The PHSA: Including Parallelism ("AND" States)

The Parallel Hierarchical Statechart Automaton (PHSA) goes forth relative to HSA, allowing a composite state to have more than one child sub-automata of HSA form. These represent concurrent processes, or threads. The sub-automata start executing concurrently on entering the composite state, and the main automaton resumes its execution on completion of all the sub-automata.

For each such composite state $state_{km}$ we define in the parent automaton a Boolean vector Par_{km} with number of components equal to number of parallel sub-automata. Initially, $Par_{km} = \mathbf{0}$. Whenever an i-th sub-automaton within is finished, on entry to its final state it will perform $dummyAction_{km}(i)$, which will update the i-th component of Par_{km} to 1, and check if all the components are equal to 1. Meeting this condition means that all parallel branches are finished. This leads us to the situation similar to that treated in the previous section. As earlier, the $dummyEvent_{km}$ should be sent to the parent automaton, triggering transition to $dummyState_{km}$, which will substitute $state_{km}$ for events and transitions that were originated from $state_{km}$ before the decomposition.

3. Integrating PHSA in the MDA Transformation Process

Transformations in MDA are developed according to pattern illustrated in Fig. 4 (adopted from the OMG MDA Guide [1]). This framework permits to transform any source model Model 1 that conforms to Metamodel A into its appropriate destination model Model 2 conforming to Metamodel B.

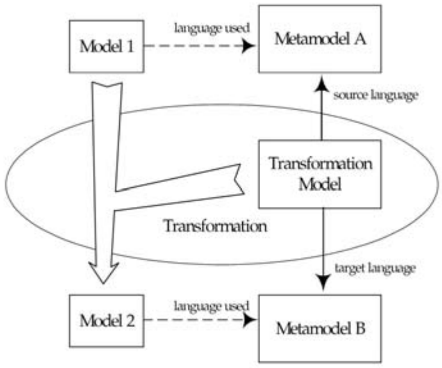

Figure 4. Metamodel Mapping Transformation (Fig. 5–1 from [1])

In our approach, the role of the source model for transformation tool, shown as "Model 1", play UML state diagrams. The language used in which these state diagrams are written is a subset of the UML, specifically the package StateMachine and related packages. The role of "Metamodel A" plays the subset of UML metamodel describing the StateMachine package (see Figure 15.2 in [2]). The destination "Model 2" is PIM based on PHSA metamodel, which stands for "Metamodel B".

Both the source model and the destination model are written in format XMI, which is the textual representation of the UML diagrams, including constraints in OCL [11]. The transformation itself is written in a language which conforms to the Transformation Model.

From the above explanation follows that the first step in realization of our approach should be definition of a platform-independent metamodel of the PHSA. Therefore the first transformation in the process of model implementation will be of kind PIM-to-PIM, i.e. from PIM given in UML state diagram to PIM in terms of PHSA model.

4. The Platform-Independent Metamodel of PHSA

The PIM metamodel of PHSA must express the four main PHSA components and the interactions between them. The class diagram of the metamodel is depicted in Figure 5. Below we explain the metamodel classes and interactions between them.

4.1 Representation of PHSA Components and Interactions

The Automaton class and related classes in the Moore Automaton box implement the set of Moore automata with the parallel and hierarchical structure described in Section 2. An Automaton object contains a collection of states. There must be at least two states: exactly one initial pseudo-state and at least one final state. For each state are defined a number of transitions (possibly zero in a final state). A state knows both its outgoing and incoming transitions. A transition may be guarded, that is, be subject to a condition represented by a Guard object. On entering a state, a number of entry actions (scripts) may be performed; this is the collection of EntryAction objects associated with that state. The aggregation between the State and Automaton classes describes the relation between a composite state and its sub-automata, allowing for hierarchy and concurrency.

The Memory class stores a collection of variables. The Variable class has attributes for variable name, type and run-time value. Events are considered as variables, since each event corresponds to a Boolean flag which indicates whether this event is present. Raising an event is performed by just setting its flag on. This approach permits to model signal events both in synchronous (real-time) and asynchronous systems. Certainly, a concrete mechanism of sending asynchronous or synchronous (fugitive or persistent) events must be rendered in mappings of PIM to PSM's for a specific destination platform.

The Calculator and related classes in Calculation Scheme serve for evaluation of guard expressions and parsing and performing of action scripts. A script is a sequence of statements handled by Statement class. Statements contain expressions that are parsed and evaluated using Expression class, which also is responsible for evaluating guards, since they are Boolean expressions. As a result of expressions parsing may be identified function calls or input or output commands. A function call is resolved by referencing a function stored in FunctionLibrary and passing required arguments. The concrete form of functions stored the in FunctionLibrary – whether they will be interfaces for calling methods from the runtime environment, manually coded sources of application-specific functions or just black-boxes for later implementation – may vary with mappings to different platforms or according to concrete application. In the case that the parser identifies input or output operation, is created an IOAction object, which is passed by the Calculator to the IOSystem for execution. After function calls and input operations, if any, are resolved, is created an object of class VarAction. Since statements of this type may include only commands of assignments to system variables or raising events (which in our model is actually the same thing, as described above in the context of Memory), it can be directly executed.

The purpose of IOSystem is execution of simple I/O instructions like input a unique variable or output one value. The IOOperation class translates these instructions in a destination language. The IOLibrary class plays the role of dictionary of this translation. The ActionIO class verifies the validity of the I/O statement before its execution. If expressions appear in the IO instruction, they are calculated by the FunctionLibrary class and the value is sent to the IOSystem component.

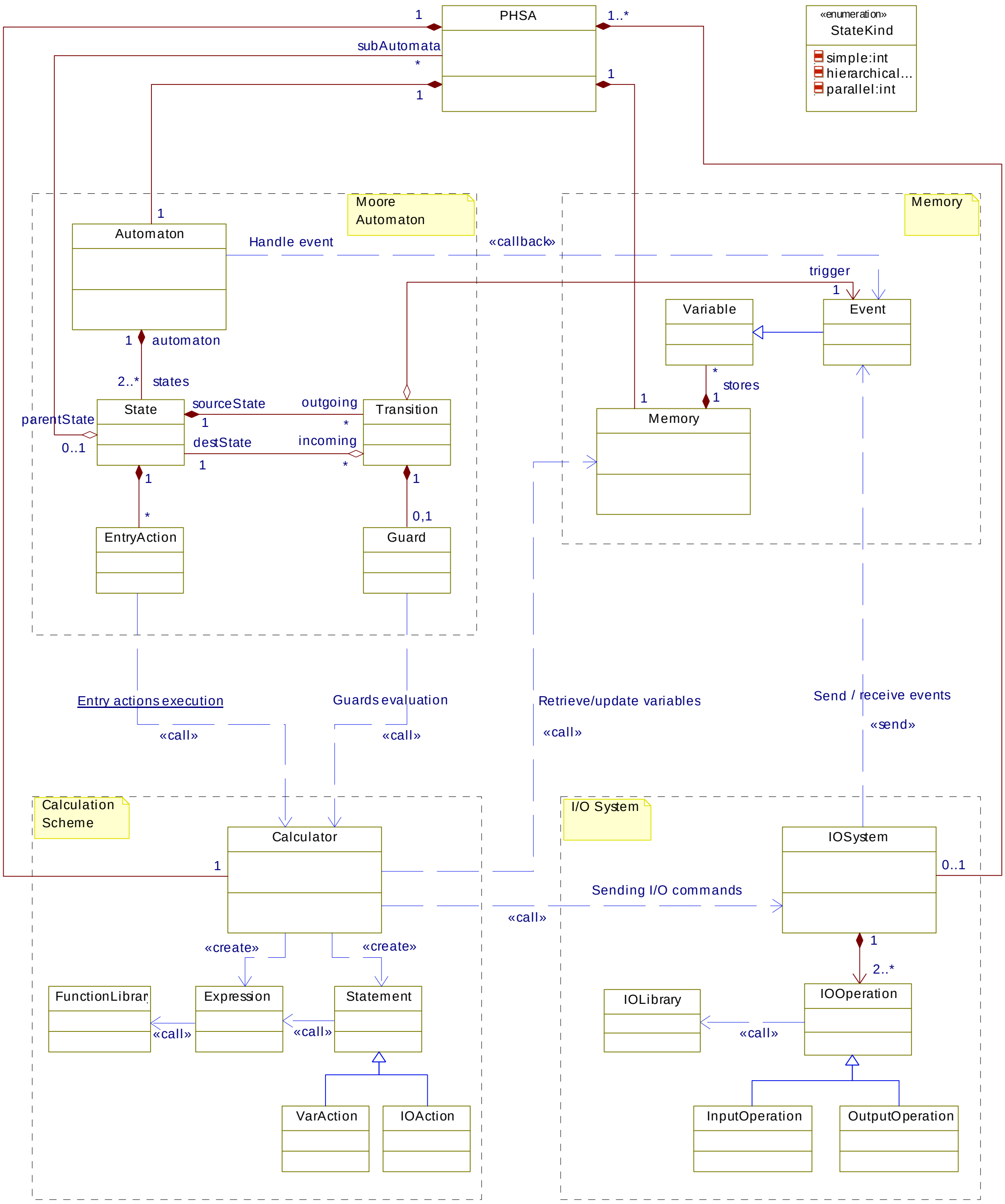

Figure 5. The PHSA Metamodel

4.2. Handling Hierarchy

When the source statechart contains composite states (hierarchical or concurrent), we must define an hierarchical structure on a set of SSA entities, i.e. a parent-child relation between an SSA state and its sub-automata of SSA kind, representing inner regions of the state machine. If only hierarchical composite states are present, we have an HSA case, so each state contains at most one SSA sub-automaton. On the other hand, each SSA automaton, excluding the main one, belongs to exactly one parent state. The Fig. 6 depicts a fragment of the PHSA metamodel from Fig. 5, focusing on the hierarchy handling in the restricted case of HSA.

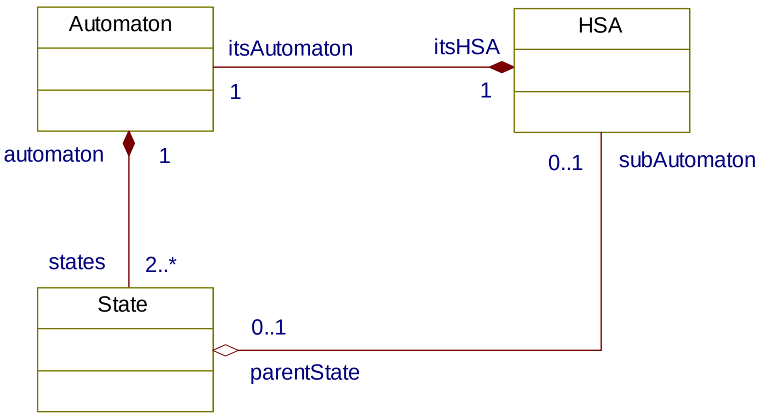

Figure 6. Metamodel of hierarchy handling in HSA

4.3. Handling Concurrency

In case of presence of concurrent composite states in the source statechart, the only difference in hierarchical structure of SSA entities is that a parent state may contain multiple SSA sub-automata, so we have an extended automaton conforming to the PHSA model. The PHSA metamodel fragment that focuses on concurrency handling is shown in Fig. 7.

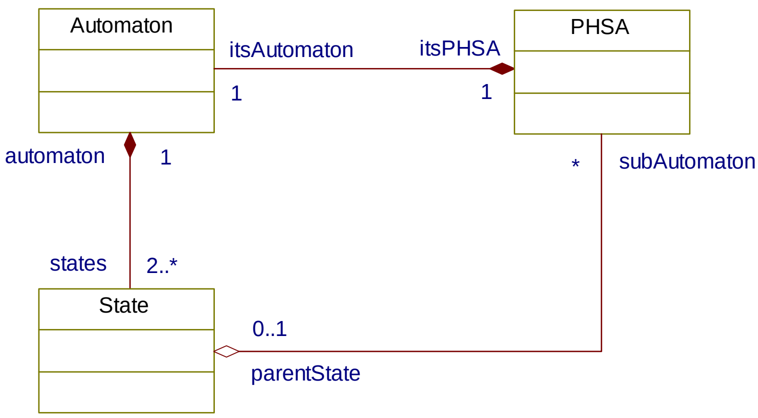

Figure 7. Metamodel of concurrency handling in PHSA

5. PIM-to-PIM Mapping from UML Statechart to PHSA

We have two PIM-level metamodels: the UML state machine and the PHSA. In order to write the transformation definition, we have to list all the elements of the source metamodel and to define mapping rules that describe how to transform each element to corresponding elements in the destination metamodel.

Currently, we have multiple options for a choice of a transformation language. We have examined and experimented with XSLT[12], ATL [13] and Rhapsody Rules Composer [14]. Here we intend to present our transformation in a concise and explanative form, independent of implementation. Below we give a short explanation of each rule and then a high-level pseudocode.

5.1. Transformation Rules

The mapping rules that constitute the transformation definition are summarized in the following Table 1.

Table 1. Mapping rules.

Rule Name	Type of Input Element in UML Metamodel	Type of Output Element in PHSA Metamodel
SM2PHSA	StateMachine	PHSA
Reg2Auto	Region	Automaton
Pseudo2Init	Pseudostate	State
Final2Final	FinalState	State
Simple2Simple	State	State
State2HierState	State	State
State2ParState	State	State
Trans2Trans	Transition	Transition
Signal2Event	Signal	Event
Guard2Guard	Guard	Guard
EntryAct2EntryAct	OpaqueBehavior	EntryAction
Attr2Var	Class	Variable
CreateFunc	LiteralString	FunctionLibrary
CreateIO	LiteralString	IOLibrary

Rule 1: SM2PHSA
Extended automata of PHSA kind roughly correspond to the metaclass Region in the package BehaviourStatemachines of the UML 2.1 metamodel. The main PHSA corresponds to the outermost region of the state machine.
This rule finds the StateMachine element in the source model (this element appears as ownedBehaviour sub-element of the modelled class). Then the rule translates its outer Region element named ROOT, which wraps all the states of state machine, into the PHSA element. The Automaton element is created by the next rule Reg2Auto (Rule 2). All other PHSA components – Memory, Calculator and IOSystem – are created at this stage and populated during execution of rules Signal2Event (Rule 9), Guard2Guard (Rule 10) and EntryAct2EntryAct (Rule 11).

Rule 2: Reg2Auto
This rule translates each region of the state machine into an Automaton element of the PHSA. First, it translates the states by invoking rules Pseudo2Init (Rule 3), Final2Final (Rule 4), Simple2Simple (Rule 5), State2HierState (Rule 6) and State2ParState (Rule 7). Then it translates the transitions by invoking rule Trans2Trans (Rule 8).

Rule 3: Pseudo2Init
The unique initial state of each region is represented by element of kind Pseudostate which attribute name is ROOT. Its translation is a simple PHSA state with name which is a concatenation of the PHSA name and the fixed string "S0".

Rule 4: Final2Final
A final state of each region is represented by element of type FinalState. Its translation is a simple PHSA state with name which is a concatenation of the PHSA name and the fixed string "End".

Rule 5: Simple2Simple
This rule applies to elements of type State that have no inner regions. Its translation is a State element in Automaton part of the PHSA. After the output element is created, relevant attributes are copied from the input state. If an entry action is present, it is translated by calling the rule EntryAct2EntryAct (Rule 11).

Rule 6: State2HierState
This rule applies to elements of type State having exactly one sub-element of type Region. First is called the rule Simple2Simple (Rule 5), which creates the output State element in the PHSA, copies its attributes and handles an entry action. Then the inner region is translated to the sub-automaton of PHSA kind belonging to this state. At last, are added dummy elements necessary for decomposition of hierarchical composite state.

Rule 7: State2ParState
This rule applies to State elements that have more than one Region sub-elements. As earlier, first of all the common actions of State element translation are performed by the rule Simple2Simple (Rule 5). Then each inner region is translated to a parallel sub-automaton, and dummy elements are added according to our decomposition technique for parallel composite states.

Rule 8: Trans2Trans
This rule translates statechart Transition elements to PHSA transitions. The Transition elements of the source statechart belong to Region's and are referenced from the containing region as transition. The source and target states are referenced in the Transition element as source and target, and states also contain references to outgoing and incoming transitions as multiple association ends. The present rule must translate all this structure of reciprocal references. To make it possible, the rule must be executed after applying the rule Reg2Auto (Rule 2), which creates all the states of the region. Then the present rule creates transitions and fills appropriate references in Region, Transition's and State's. If the current Transition contains a sub-element of kind Trigger, the rule Signal2Event (Rule 9) is called, and if there is present a Guard sub-element, it must be processed by Guard2Guard (Rule 10).

Rule 9: Signal2Event
This rule is invoked from Trans2Trans (Rule 8) when the translated transition is caused by a trigger. The trigger sub-element in the current Transition references a SignalEvent element, which references an element of type Signal. This Signal element has a unique name, and is translated to a PHSA element of type Event, which is stored in the PHSA Memory component.

Rule 10: Guard2Guard
This rule, as the previous one, is invoked from Trans2Trans (Rule 8) when the translated transition is guarded. The body of the guard expression is found via the path Transition → ownedRule → specification → value (A → B means that A contains B). The ownedRule element is of type uml:Constraint. The value is stored in a Guard element in PHSA Automaton component. Further, the PHSA Calculator component will parse the string for its evaluation.

Rule 11: EntryAct2EntryAct
This rule may be invoked from rules Simple2Simple (Rule 5), State2HierState (Rule 6) or State2ParState (Rule 7) in order to translate entry actions. The body of the entry action script is found via the path State → entry → body. The script is translated to an EntryAction element in PHSA Automaton component, to be parsed later by the Calculator.

Rule 12: Attr2Var
This rule translates the attributes of the modelled class into elements of type Variable in PHSA Memory. These attributes are found in the element of type uml:Class which represents the modeled class and contains the state diagram being translated as ownedBehaviour sub-element.

Rule 13: CreateFunc
This rule is invoked either from rule Guard2Guard (Rule 10) or rule EntryAct2EntryAct (Rule 11). When an expression in guard or action script contains a function call, e.g. cos(x) or pow(x, y), the function calling format (the signature) is added to the FunctionLibrary. This collection of function calls is used later in transformations to PSM's.

Rule 14: CreateIO
This rule is similar to the previous and is invoked in the same context. When a call to input or output operation is identified in the process of parsing, the operation format is stored as InputOperation or OutputOperation object in the IOLibrary component of PHSA IOSystem.

5.1. The Transformation Algorithm

The algorithm for applying the transformation rules is based on basic depth-first-search graph algorithm. It traverses the PIM given in XMI file that represents the state diagram, possibly containing hierarchical and parallel states. The main rule SM2PHSA calls the procedure Reg2Auto which is called recursively from the rules State2HierState and State2ParState, and it calls all the other rules. Following is the pseudocode of the algorithm.

```
SM2PHSA (sm: StateMachine, phsa: Phsa)
begin
  phsa.copyAttributes(sm)
  reg ← sm.getRootRegion()
  Reg2Auto (reg, phsa.getAutomaton())
end

Reg2Auto (reg: Region, auto: Automaton)
begin
  auto.copyAttributes(reg)
  foreach subvertex s in reg
  begin
    phsaState ← auto.currentAutomaton.makeNewState()
// translate the states
    if s.getAttr("xmi:type") = "uml:Pseudostate" and s.getAttr("name") = "ROOT" then
      Pseudo2Init(s, phsaState)
    else if s.getAttr("xmi:type") = "uml:FinalState" then
      Final2Final(s, phsaState)
    else if s.getAttr("xmi:type") = "uml:State" then
      begin
        regions ← s.countRegions()
        if regions = 0 then
          Simple2Simple(s, phsaState)
        else if regions = 1 then
          State2HierState(s, phsaState)
        else
          State2ParState((s, phsaState)
// translate the state entry action, if present
        entry ← s.getChild("entry")
        if entry <> nil then
           EntryAct2EntryAct(entry, newState)
      end
// translate the region transitions (also signals and guards)
    foreach Transition tr in reg.transitions
      Trans2Trans(tr, auto.currentAutomaton.makeNewTransition())
  end  // foreach subvertex s in reg
end  // Reg2Auto

State2ParState(umlState: State, phsaState: State)
begin
  foreach Region reg in umlState.getInnerRegions()
    begin
      newAuto ← phsaState.addSubAutomaton()
      Reg2Auto(reg, newAuto)
    end
  HandleParallelism(phsaState)
end
```

```
State2HierState(umlState: State, phsaState: State)
begin
   newAuto ← phsaState.addSubAutomaton()
   Reg2Auto(umlState.getInnerRegion(), newAuto)
   HandleHierarchy(phsaState)
end

Trans2Trans(umlTransition: Transition, phsaTransition: Transition)
begin
   phsaTransition.copyAttributes(umlTransition)
   phsaTransition.source.addOutgoingTransition(phsaTransition)
   phsaTransition.target.addIncomingTransition(phsaTransition)
   if umlTransition.getAttr("guard") <> nil then
   begin
     Guard guard ←
              umlTransition.getChild("ownedRule").getChild("specification").getAttr("value")
     Guard2Guard(guard, phsaTransition.addGuard())
   end
   if umlTransition.getChild("trigger") <> nil then
   begin
     Trigger trigger ← umlTransition.getChild("trigger")
     SignalEvent sigEv ← reg.getClassNode().FindById(trigger.getAttr("event"))
     Signal signal ← reg.getClassNode().FindById(sigEv.getAttr("signal"))
     phsaEvent ← phsa.addEvent()
     Signal2Event(signal,  phsaEvent)
     phsaTransition.addEvent(phsaEv)
   end
end

EntryAct2EntryAct(entry: OpaqueBehavior, state: PhsaState)
begin
  state.addEntryAction(entry)
  CreateFunc(entry.getBody(), state.getPhsa().getFunctionLibrary())
  CreateIO(entry.getBody(), state.getPhsa().getIOLibrary())
end

Guard2Guard(umlGuard: Guard, phsaGuard: Guard)
begin
  phsaGuard.copyAttributes(phsaGuard)
  CreateFunc(phsaGuard.getBody(), state.getPhsa().getFunctionLibrary())
  CreateIO(phsaGuard.getBody(), state.getPhsa().getIOLibrary())
end
```

6. Conclusions and Related Works

The theoretical and technical approaches that are developed in this paper facilitate MDA process using UML state diagrams as an input and executable automata as output. On every step of our technique, the process is efficient in sense that we preserve the states of a source statechart and the only added states are minimal, so the model is transparent for an engineer and developer-friendly. Though the conflict analysis is beyond the scope of this paper, we can point out that our model does not add any new conflicts compared to the source state diagram. We show an algorithm of transformation a state diagram to a Parallel Hierarchical Statechart Automaton.

Our work is closely related to the notion of Parallel Automata [15]. We elaborated this concept to approach it to UML statecharts and to make it more suitable to serve a basis for MDA transformations.

Currently we are developing transformations of the PHSA-based PIM to PSM's [16] and to compilable source code for different target platforms.

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A SIMULATION STUDY OF ON-LINE AUTOMATIC SENSING AND DETECTING DAMAGED DETERIORATING INVENTORY WITH RANDOM LIFETIME AND DEMAND

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Managing perishable items listed in an inventory becomes nowadays a quite important and challenging task. Although each perishable item is usually assigned a corresponding expiration-date label, unexpected events like human errors, dynamic temperature conditions, improper maintenance, etc., can drastically deteriorate products quality and properties and cause significant losses both to supplier and customer. Analytic inventory models for deteriorating items with stochastic demand and various environmental characteristics have been developed. Another efficient solution includes automatic sensing and detecting of spoiled items. Most sophisticated labels that combine both chemical and electronic functionality with radio frequency identification (RFID) technology could remotely warn users and automated systems whenever stability problems have occurred. Yet, technological solutions involving automatic labelling devices require additional scientific investigation and scrutiny by means of the simulation theory, which mainly has not been attempted so far. Our research presents a simulation study of providing a single product type of perishable inventory with general AD (automatic device), which on-line senses and detects spoiled items. Experimental design is outlined and numerical results presented and analyzed.

Keywords: *perishable inventory, automatic sensing, FQI, TTI, RFID, random lifetime, common expiration date*

1. Introduction

Generally, a manufacturer assigns each perishable item in inventory a label that denotes its *expiration-date* E, after which the item is formally not proper for original usage [2, 4, 7]. We consider a single product type in which all items entering the inventory system are assigned a common expiration date. Items that are left in the system after the expiration date are considered obsoletes, therefore are drawn out from the system with disposal costs. Within the scope of supply chain of producing and distributing products, unexpected events can have major influence on the quality status of products presented for sale [2, 6]. Examples of such events include human errors made in production, dynamic temperature conditions at which products are being stored, damages that happen while delivery is carried on, bad maintenance, unknown factors, etc. Unexpected events can also improve expected quality level of products. For example, better storage conditions compared to standard ones, better maintenance, rapid delivery, etc.

In some cases, where unexpected events cause few items to be damaged before expiring, usually neither management nor customer is aware of. A consequence is that customers might purchase damaged products. In such a case the customer who bought a product with quality deterioration might suffer damage, sometimes a severe one such as health problems or safety risks. The organization, as a result, looses reputation and often will have to compensate the customer with a new product. Sometimes the compensation process that is carried by the organization involves extremely larger costs than the primary cost of the product itself. An example can be compensation as a result of lawsuit carried by a customer. On the other hand, products that unexpected events further increased their expected quality might be left unsold because of approaching expiration date, which is usually the only indicator for quality status of a specific item. This situation again causes management to leave some good items unsold with substation losses.

Since usually information about expiration date is completely provided to customers [5, 12], a possible reaction of customers is sorting products that are placed on the shelf by a decreasing interval of time until the expiration date. In order to avoid such situations we assume in this work that all items arriving to the inventory system possess a common expiration date. Moreover, non-obsolete items that are left un-sold at the replenishment-date will simultaneously be sold to another supplier for a lower price. These actions will prevent customers from sorting products as mentioned above.

Analytic inventory models for deteriorating items with stochastic demand and various environmental characteristics were also developed by [1, 12, 18–19]. Since a considerable work has been done on deteriorating inventory system, the details of which can be found in [4, 10, 14]. In [2] is used input-output analysis, Laplace transforms and MRP approach, the problem of stabilizing cold logistic chain can be transferred to LP parametric problem, which enables to obtain actions against perturbations that can appear on the different stages of a cold chain. Inventory models for fixed lifetime perishable products have been studied by [6, 11, 13]. The analysis of random lifetime perishable inventory system is considerably more difficult as compared to its lifetime counterpart [4]. In [9] is analysed a model with an *Erlang* lifetime distribution, a renewal demand process and a zero lead time. In [5] is studied a continuous review (s, S) inventory system with Poisson demand where commodities are assumed to be damaged due to decay and disaster. They considered the lifetime of an item and times between the disasters to be exponential. In [15] is investigated a model with a *Poisson* demands process, a constant lifetime and random lead times. He assumed that the aging of the fresh stock begins only after all remaining old units are depleted.

Another possible reaction of customers to having full information is not to purchase the product at all. The advent of automated identification (Auto-ID) technology has enfeebled electronic labelling and wireless identification of objects, which facilitates real-time product visibility and accurate tracking at all levels of the product life cycle [8]. RFID technology facilitates electronic product codes that provide unique identification for each product. The need to organize and make decisions based on the data provided by the RFID tags is prominent [17]. In the work of [16] benefits that RFID technology can provide to supply chain processes are described. One among many advantages of RFID tags is better management of perishable items. Additionally automatic location of the oldest stock is on-line tracked. A specific type of sensing device is food quality indicator (FQI) in the form of a label that can be affixed inside any clear food packaging or outside a breathable or gas-permeable food packaging and read by anyone to determine the quality of the packaged food.

Automatic sensing and detecting of spoiled items is required to efficiently support supply chain. For example, stored food in refrigerator at $10\,C^0$ might deteriorate from two to 12 times faster than if stored at $0C^0$, depending on the mechanism of deterioration [3]. As storage conditions are stochastic and unpredictable, setting expiration date is actually an inaccurate way to inform a customer about the quality or safety of the product. Cases of food poisoning continue to rise as well as illness. In resent years within the scope of inventory management, a technology called *TTI* (Time Temperature Indicator) was developed [7]. This technology enables to on-line determine the actual quality situation of some product types compared to their date of expiration. In situations where the product is unexpectedly damaged, the *TTI* instrument should alert management. The technology reduces some of the situations of selling damaged products.

Inventory management of time-sensitive materials is very critical due to their shelf lives [17]. Incorporating perishable items modelling together with automatic device for sensing and detecting spoiled items is rarely addressed in literature up to date. In [17] a comparative analysis between several fixed inventory policies and a dynamic, forecast-integrated inventory model that includes RFID data in a simulation environment. Results show that the integrated model can adapt to the system dynamics more efficiently then current fixed practice. The results demonstrates the overall benefits and effectiveness of RFID technologies in providing low-cost manufacturing solutions, reduced inventory levels, and lower overall waste. Inventory model for fixed lifetime products and deterministic demand was developed by [7]. They developed a model for inventory management of refrigerated food items. Their model assumed the existence of a technology that allows dynamic and exact monitoring of expiration data. They addressed deterministic demand and LSFO (Least Shelf-life First Out) rule for scheduled sells. Statistical results in that research concluded that increasing item cost by 5% because of the cost of TTI tag (representing the technology that deterministically predicts actual lifetime) will decrease substantially the preference of using TTI tags (from 98% to 78%).

TTI and similar devices use chemical reactions to exhibit an irreversible change of colour in response to the combined effects of time and temperature. They are applied also as labels. They offer customers better protection and contribute environmentally to reduce the amount of food waste. It should be an interest of retailers to gain these advantages. Most sophisticated labels that combine both chemical and electronic functionality with radio frequency identification (RFID) technology could remotely warn users and automated systems whenever stability problems have occurred. The labelling system includes circuitry that measures and calculates, and indicators that signal to pay attention to the time has come for discounted sale, and later, that the time has come for disposal rather than sale. Optionally, the circuitry may act as an "over-temperature alarm" system, to measure, calculate and indicate when a one-time temperature violation has occurred that is of such a magnitude that the item is immediately considered compromised or spoiled [8].

In this paper we analyse by a simulation study an incorporation of single product type of perishable inventory with general *AD* (automatic device), which on-line senses and detects spoiled items. The modelling frame is introduced in *Section 2*. Experimental design and results are introduced in *Section 3*, while *Section 4* concludes the paper.

2. Modelling Frame

In this section we develop the modelling frame for describing a single perishable product type of expiring inventory. All items possess identical age with complete information about expiration date, while the operating system operates in two optional ways. The first way uses *AD*, while the other does not.

2.1. Modelling Frame

The behaviour of the inventory system is randomly generated by three sources of uncertainty; *Demand process*, which schedules the exact timing of consumer request for an item. It is assumed that the expected rate of demand d does not change with time; *Spoilage process* which schedules the time of an event causing an item in inventory to be damaged. To model the spoilage process each item entering the inventory system will be damaged randomly within expected time of $1/\mu_D$. The rate of on-line expiring items generally increases with the number of fresh products in inventory. This rate generally depends also on time t since products that stay longer on the shelf poses higher probability of being damaged. *Selection process* is the third source of uncertainty, in which consumers choose randomly and uniformly a single item from total inventory. When *AD* is applied the selection can be choosing only fine product. In case that *AD* is not applied customers might select damaged products. Generally, this selection process causes consumers to purchase some of the damaged items, while the rest of the damaged items remain in the system.

Since a comparison to a policy that does not use *AD* will be carried in this work, we measure the total inventory level I_t^d that includes also damaged ones (sub index d for "with damaged"). The accumulated damaged items since last replenishment D_t until time t and by P_t to denote accumulated damaged items selected by consumers until time t. The events that construct process P_t are scheduled always at a sub-set of demand time-points referring process I_t^d. In cases where *AD* is applied we define transient inventory level of un-damaged items, I_t where the initial number of items is denoted by I_0. From the above notation it follows $I_t = \max\{0, I_t^d - D_t\}$, and $P_t \le D_t$, $\forall t$.

We assume a periodic review policy where inventory is replenished to level I_0 in a periodic cycle of T units of time. Items are sold to the market, each in price p. Each item is purchased in cost c. Holding an item in storage for a unit of time costs h, while ordering cost is denoted by K and is independent of quantity. In situations of shortage the system is empty and do not supply the product. The inventory system in these situations losses good will of $\hat{\pi}$ for each unit of time regardless of demand.

Since the product is not backlogged, backlog quantity is not considered. Therefore, by definition processes I_t^d and I_t are always non-negative.

In this paper we model current and future use of sensing and detecting technology devices by assuming on-line automatic sensing and detection (*AD*) of spoiled products when the device is operated. Expiration date that is set is unchanged regardless of environmental conditions. Its use is to inform management the last date after which the product is disposed from inventory. Information on expiration date is also open to all customers, while an identical and time-independent utility from each individual item gained by customers is assumed. This last assumption results in a constant expected demand with time approaching expiation-date, while agrees with the ability of technology (*AD*) to prevent damaged products from being sold.

2.2. Profit Performances

The analysis is separated into two different cases as each one of them includes some different components. The first relates to scheduling an arrival of a shipment before the expiring-date, i.e., $T < E$. The other case relates to the opposite situation, i.e., $T \geq E$.

(a) $T < E$.

Here, we assume that items that are left in the system at time T are simultaneously sold in price $\hat{p}$, where obviously $\hat{p} < p$. The motivation for purchasing the product may result from earlier agreement if it refers to the original supplier. In the case that the supplier who purchases the product in time T is not the original one, other benefits of purchasing product with age T are motivating the buyer. One possibility of purchasing the product can be the comparably long interval of time that is left until expiring in a specific application. Another possibility is the attraction of the offered lower price $\hat{p}$. Here we assume that all items are eventually sold at time T even if some are considered damaged (happens in the case that damaged items are not be detected on-line). All items are simultaneously sold in order to have only single-aged items, i.e., preventing customers from sorting items according to their age, and as a result preventing the need for price discrimination.

(a.1) With AD

When applying *AD* we assume that damaged items are detected and drawn immediately from the system as a specific spoilage event occurs (its specification depends on the technology of the detecting device). Modelling the costs of initially purchasing *AD* is carried here by an increase of product direct cost by c^{AD}. The expected profit function $f_a^{AD}(I_0, T)$ measured for a unit of time is of the following form:

$$f_a^{AD}(I_0,T) = \frac{1}{T} E\left(\begin{array}{l} p\left[I_0 - I_T^d(\omega)\right] + \hat{p} I_T(\omega) + \\ - \left[\begin{array}{l} K + \left(c + c^{AD}\right) I_0 + h \int_0^T I_t(\omega) dt + \\ + \hat{\pi} \int_0^T Sign\left(I_t(\omega)\right) dt + c^{out} D_T(\omega) \end{array} \right] \end{array} \right), \qquad (1)$$

where function $Sign(x)$ is defined as follows $Sign(x) = \begin{cases} 0 & x > 0 \\ 1 & x \leq 0 \end{cases}$, ω denotes a possible scenario of time points of demand and of detecting damaged items, $I_t^d(\omega)$ denotes transient inventory level without considering the unexpected spoilage effect i.e., $I_t(\omega) = \max\{0, I_t^d(\omega) - D_t(\omega)\}$, c^{out} is the cost of disposal from the system a damaged item that is detected before T. Note that when $I_t(\omega)$ becomes 0,

$I_t^d(\omega)$ must remain in its current level since demand cannot be supplied even if required in such a scenario ω.

(a.2) Without AD

As items might be damaged by unexpected events, while customers and management are not aware of, damaged items are purchased. Customers will be aware of the damage caused by their own purchase only later. We model compensation penalty for each damaged item that was actually purchased by η, where $\eta \geq c$. The expected profit function $f_a(I_0, T)$ measured for a unit of time takes the following form:

$$f_a(I_0,T) = \frac{1}{T} E \left(\begin{array}{l} p\left[I_0 - I_T^d(\omega)\right] + \hat{p} I_T^d(\omega) + \\ - \left[K + cI_0 + h\int_0^T I_t^d(\omega)dt + \hat{\pi}\int_0^T Sign\left(I_t^d(\omega)\right)dt + \eta D_T(\omega) \right] \end{array} \right). \tag{2}$$

Note that since *AD* is not applied here some of the damaged items are sold. Some of these damaged items, $P_T(\omega)$ are sold in an ordinary purchase process to consumers, while the rest, $D_T(\omega) - P_T(\omega)$ are sold to an outer supplier. Similarly, the outer supplier will be aware of the damaged items only later, i.e., after purchase is made. We assume for simplicity that penalty cost η for each delivered damaged item is the same for an inner customer and for an outer one (outer supplier).

(b) $T \geq E$.

Here items, at expiration date E are simultaneously drawn out from inventory, each with salvage cost c^{out}. Therefore, at interval of time $t \in [E, T]$ the system is empty, i.e., in shortage situation.

(b.1) With AD

The expected profit function $f_b^{AD}(I_0, T)$ measured for a unit of time is of the following form:

$$f_b^{AD}(I_0,T) = \frac{1}{T} E \left(\begin{array}{l} p\left[I_0 - I_E^d(\omega)\right] + \\ - \left[\begin{array}{l} K + \left(c + c^{AD}\right)I_0 + h\int_0^E I_t(\omega)dt + \\ \hat{\pi}\int_0^E Sign\left(I_t(\omega)\right)dt + \hat{\pi}\left(T - E\right) + c^{out}\left(D_E(\omega) + I_E(\omega)\right) \end{array} \right] \end{array} \right). \tag{3}$$

Note as before, that when $I_t(\omega)$ becomes 0, $I_t^d(\omega)$ must remain in its current level since demand cannot be supplied even if required in such a scenario ω.

(b.2) Without AD

Here some of the damaged items $P_E(\omega)$ are purchased by the consumers, while the rest of them $D_E(\omega) - P_E(\omega)$ are drawn out from the system at expiration date E. The expected profit function $f_b(I_0, T)$ measured for a unit of time is of the following form:

$$f_b(I_0,T) = \frac{1}{T} E \left(\begin{array}{l} p\left[I_0 - I_E^d(\omega)\right] + \\ - \left[\begin{array}{l} K + cI_0 + h\int_0^E I_t^d(\omega)dt + \hat{\pi}\int_0^E Sign(I_t^d(\omega))dt + \hat{\pi}\left(T - E\right) + \\ + \eta P_E(\omega) + c^{out} I_E^d(\omega) \end{array} \right] \end{array} \right). \tag{4}$$

3. Experimental Study

Computerized experiments that are carried in this research mainly intend to achieve statistically validity for applying *AD* when compared to the obvious alternative of not using the technology. Since other powerful environmental control factors influence the performance of inventory system, such as the ordering policy parameters, we will first set their preferred values before making any comparison. Specifically, we are mainly interested in understanding the performance change when applying the *AD* technology. The simulation model was written in C++ programming.

3.1. Experimental Design

In order to include within our simulation analysis large number of possibly influencing factors on system performance, each simulated factor was assigned only with two levels; lower and upper. The experimental design includes a full factorial type 2^k. Each experiment is characterized by setting a specific level of all factors. The considered 11 factors in experiments are:

(1) Applying *AD* or not.
(2) Length of planning horizon.
(3) Ordering quantity.
(4) Price of selling items that are left in the system at time T.
(5) Cost of disposal of damaged item that is detected before T.
(6) Cost for applying *AD*.
(7) Cost of backlog in a unit of time.
(8) Penalty cost for selling a damaged item.
(9) Variability of time between demand events.
(10) Variability of time until spoilage events.
(11) Expected time until item spoilage.

Simulation parameters, environmental parameters, control parameters and probability distributions are listed in *Tables 1–4* below. Lower (among the two alternatives) value of order quantity was estimated by EOQ model with setting demand rate parameter that includes two components. The first component is the expected demand (left hand side value of demand) and the other is the expected spoilage rate. This policy minimizes the possibility of having a priory shortage or increased inventory situations. Nevertheless these parameters will allow situations of shortage. Upper value of order quantity was estimated similarly with referring to the right hand side value of demand rate.

Table 1. Parameters

Parameter	E	c	K	h	μ	p
Value	4	50	500	10	10	150

Table 2. Environmental parameters

Parameter	**Description**	**Value**
$\hat{p}$	Price of selling an item that is left in the system at time T	0, 25
c^{out}	Cost of disposal a damaged item detected before T	0, 5
c^{AD}	Additional item costs for applying *AD* technology	5, 15
$\hat{\pi}$	Cost of backlog of a unit of time	50, 200
$1/\mu_D$	Expected time until a item is damaged	4, 5
σ_D	Variability parameter for the time until spoilage	2, 4
σ_d	Variability parameter for the time between demand events	0.01, 0.05
η	Penalty cost for selling a damaged item	200, 400

Table 3. Control parameters

Parameter	Description	Value
AD	Using *AD*	No, Yes
T	Planning horizon	3, 5
I_0	Ordering quantity	36, 45

Table 4. Statistical distributions in the simulation

Variable description	Statistical distribution
Time between demand events	*Uniform* $\left[1/\mu - \sigma_d , \frac{1}{\mu} + \sigma_d \right]$ hours, where $\sigma_d = \frac{0.1}{\mu}$ for the lower level of demand variance and $\sigma_d = \frac{0.5}{\mu}$ for the higher level of demand variance (the value of μ is found in *Table 1*).
Time until spoilage event	*Uniform* $\left[1/\mu_D - \sigma_D , \frac{1}{\mu_D} + \sigma_D \right]$ hours, where $\sigma_D = 2$ for the lower level of variance and $\sigma_E = 4$ for the higher level of variance (the value of μ_D is found in *Table 2*).

As there are 11 factors, our design includes a 2^{11} full factorial experiment. In order to obtain better confidence of results each experiment was carried with 100 replications, each with different seed.

3.2. Summarizing Results

Previous analysis of variance (ANOVA) results with 0.99 confidence level show, as expected, that the three control factors in *Table 3* substantially influence the expected profit. In *Table 5* below a comparison between profit measures with\without applying *AD* are introduced. Results show that expected profit is higher when applying *AD* then the alternative. Large variance of profit results because the other two effecting control factors of ordering policy which were not determined in a specific level.

Table 5. Profit with\without AD

***AD* = 0**	***AD* = 1**	
44.07	335.76	Avg.
406.02	148.45	Sdv.

Results introduced in *Table 6* below show the same comparison after setting control parameters of ordering policy by their preferred profit values, i.e., $T = 5$ and $I_0 = 36$ (among all 4 possibilities).

Table 6. Profit with\without AD after setting preferred policy parameters $T = 5$ and $I_0 = 36$

***AD* = 0**	***AD* = 1**	
202.88	380.91	Avg.
224.41	111.87	Sdv.

Table 6 above shows that generally applying *AD* significantly improves the expected profit relative to the alternative of not applying it. Another result is that variation of the profit is substantially reduced.

The rest of results are summarized in *Tables 7–14*, while only conclusions with at least of 0.99 confidence level are stated. Some of the influencing major factors and influencing interactions between factors regarding the *AD* factor are introduced. Among the 11 parameters introduced in *Tables 2* and *3* most of them are effective. Among the effective factors all policy control parameters AD, T, I_0 are also included.

The parameters T and I_0 determine the feedback ordering policy. As a direct conclusion we will set the levels in these 2 control parameters before further statistical analysis is carried on. Specifically the levels at which best results were obtained are the upper level of planning horizon and lower level of ordering quantity, i.e., $T = 5$ and $I_0 = 36$ respectively. Considering that updating of control parameters $T = 5$ and $I_0 = 36$ respectively, some statistical results regarding *AD* are introduced in *Tables 7–14* below.

Table 7 below compares also the influence of *AD* cost on profits. It follows that increasing c^{AD} to the upper level (in this parameter settings it is 30% of item cost) reduces expected profits by 17.3% when using *AD*. Nevertheless, the expected profits remain higher when *AD* is used. When *AD* is not used there is no relevance of the values of c^{AD}.

Table 7. Profit for different levels of c^{AD}

***AD* = 0**	***AD* = 1**		**Parameter C^{AD}**
204.26	415.80	Avg.	Low
224.44	105.15	Sdv.	
204.26	343.80	Avg.	High
224.44	105.15	Sdv.	

Table 8 below compares the influence of shortage costs on the profit. From the results we conclude that increasing $\hat{\pi}$ to the upper level reduces expected profits by 13.3% when *AD* is applied. Similar relative influence results are obtained when *AD* is not applied.

Table 8. Profit for different levels of shortage parameter $\hat{\pi}$

***AD* = 0**	***AD* = 1**		**Parameter $\hat{\pi}$**
225.30	406.78	Avg.	Low
223.89	103.30	Sdv.	
183.33	352.74	Avg.	High
223.76	111.90	Sdv.	

Parameter η represents cost penalty when a customer purchases a damaged item. From *Table 9* below it follows that this environmental parameter significantly decreases profits when *AD* is not applied. *Table 10* below shows similar conclusions regarding the variance of the expiry date.

Table 9. Profit for different levels of shortage parameter η

***AD* = 0**	***AD* = 1**		**Parameter η**
287.58	379.61	Avg.	Low
131.93	111.68	Sdv.	
123.43	379.61	Avg.	High
260.60	111.68	Sdv.	

Table 10. Profit for different levels of expiry date variance

***AD* = 0**	***AD* = 1**		**Variance of expiry date**
334.16	451.50	Avg.	Low
129.38	71.24	Sdv.	
76.56	312.12	Avg.	High
218.92	98.18	Sdv.	

Results that are presented in *Table 11* below show as expected, that longer expected expiry date increases expected profits. This conclusion results from the reduced number of accumulated damaged items in a cycle and higher revenue gained from the outer supplier when *AD* is applied. In the case that *AD* is not applied, lower presence of damaged items increases profits.

Table 11. Profit measures for different levels of excepted expiry date

AD = 0	*AD* = 1		Expected expiry date
106.60	330.23	Avg.	Low
223.29	109.26	Sdv.	
306.62	433.38	Avg.	High
161.14	87.36	Sdv.	

A more detailed analysis of the results includes comparison between similar environmental characteristics as presented in *Tables 12–15* below. The results in *Table 12* follow previous ones. In an outer environmental conditions where expected expiry date is in the lower level (i.e., short), and the variance of the expiration date is large the difference between applying *AD* or not is the largest. On the opposite, in environmental conditions where the expected expiry date is in the upper level and the variance of expiry date is in its lower level the difference between applying *AD* or not is the smallest.

Table 12. Profit for different levels of expectation and variance of expiry date

AD = 0	*AD* = 1		Expected expiry date	Variance of expiry date
242.93	406.27	Avg.	4	SMALL
124.16	65.14	Sdv.		
421.76	495.41	Avg.	5	
49.02	47.53	Sdv.		
-37.96	258.26	Avg.	4	LARGE
219.05	86.15	Sdv.		
192.74	374.62	Avg.	5	
146.52	73.72	Sdv.		

As expected, in *Table 13* below the environmental conditions of larger variance and shorter expectation of expiry date deteriorate expected profits. Moreover, in situations where η increases that result of further deterioration of profits accelerates.

Table 13. Profit for different levels of expectation and variance of expiry date and η

AD = 0	*AD* = 1		η	Expected expiry date	Variance of expiry date
315.07	404.38	Avg.	200	4	SMALL
68.23	69.02	Sdv.			
432.46	496.34	Avg.	200	5	
34.15	46.38	Sdv.			
178.56	404.38	Avg.	400	4	
129.07	69.02	Sdv.			
413.21	496.34	Avg.	400	5	
54.89	46.38	Sdv.			
119.83	254.15	Avg.	200	4	LARGE
94.31	89.12	Sdv.			
283.52	370.41	Avg.	200	5	
74.56	77.28	Sdv.			
-212.16	254.15	Avg.	400	4	
183.76	89.12	Sdv.			
115.15	370.41	Avg.	400	5	
146.46	77.28	Sdv.			

A detailed analysis that is showed below in *Table 14* strengthens the previous results. From the results we conclude that the best environmental conditions are smaller variance of expiry, longer expectation of expiry, and smaller value of shortage parameter $\hat{\pi}$, while *AD* is applied. The expected profit is reduced by 53% if the variance of the expiry date and the value of shortage parameter $\hat{\pi}$ are in their higher level, while expectation of expiry in its lower value. If *AD* is not applied the difference between expected profits in extreme environmental conditions is more the 145%. From the results we conclude that the worst environmental conditions are larger variance of expiry, shorter expectation of expiry, and larger value of shortage parameter $\hat{\pi}$, while *AD* is not applied. In those specific environmental conditions, the expected profit is at its lowest value while still displays very large variance.

Table 14. Profit for different levels of expectation and variance of expiry date, η and $\hat{\pi}$.

***AD* = 0**	***AD* = 1**		$\hat{\pi}$	η	**Expected expiry date**	**Variance of expiry date**
338.47	431.89	Avg.	50	200	4	SMALL
64.52	64.15	Sdv.				
454.81	518.41	Avg.	50	200	5	
24.41	43.30	Sdv.				
204.72	431.89	Avg.	50	400	4	
126.88	64.15	Sdv.				
436.81	518.41	Avg.	50	400	5	
47.40	43.30	Sdv.				
296.44	380.26	Avg.	200	200	4	
62.91	67.14	Sdv.				
412.69	474.82	Avg.	200	200	5	
23.69	44.24	Sdv.				
162.69	380.26	Avg.	200	400	4	
125.32	67.14	Sdv.				
394.69	474.82	Avg.	200	400	5	
46.76	44.24	Sdv.				
147.80	288.54	Avg.	50	200	4	LARGE
94.70	78.25	Sdv.				
304.70	397.64	Avg.	50	200	5	
65.74	71.72	Sdv.				
-177.20	288.54	Avg.	50	400	4	
189.07	78.25	Sdv.				
136.45	397.64	Avg.	50	400	5	
130.92	71.72	Sdv.				
105.67	223.42	Avg.	200	200	4	
94.68	83.34	Sdv.				
262.51	342.74	Avg.	200	200	5	
65.47	76.10	Sdv.				
-219.33	223.42	Avg.	200	400	4	
189.08	83.34	Sdv.				
94.26	342.74	Avg.	200	400	5	
130.71	76.10	Sdv.				

4. Discussion

4.1. Validity

Results presented in *Tables 5–14* show that applying *AD* significantly improves system's profit with at least 0.99 level of confidence. The set of data on which the simulated experiments are based on reflects an ordinary one among all possible ones. Most of the parameters data is identical to both alternatives, one that uses *AD* and the other. The main differences in data between the alternatives regards the *AD* parameters such as c^{AD} and η. The lower value of 200$ for η considers loss of product revenue of 150$ (parameter p) and additional minor loss of reputation of 50$, while the upper value includes sever penalty cost of 400$. Stated results in *Tables 13* and *14* above are expected to strengthen as the value of η increases. The lower value of c^{AD} is 5$, representing 10% of the product cost, while the upper value of c^{AD} is 15$, representing 30% of product cost (parameter c). In both levels when applying *AD*, profits are significantly higher than the alternative. When the value of c^{AD} is further increased, deterioration of profits is expected**.** From further experiments only when c^{AD} is set to 33$ applying *AD* is not preferred to the alternative (for the smaller level of η). Such a value of c^{AD} is comparable to about 65% of product cost in our simulation experiments. It means that unless the cost of applying *AD* for a unit is relatively large, the expected profits are higher then the alternative.

4.2. Conclusions

The results introduced in *Tables 5–14* demonstrate that applying *AD* result generally better-profits performance when compared to the alternative, i.e., not applying *AD*. This conclusion seems also correct when different environmental conditions are considered. While basing our main conclusion of applying *AD* preference we set control parameters regarding ordering policy in there approximated preferred values of $T = 5$ and $I_0 = 36$. Another result is that variation of the profit is substantially reduced when preferred policy is applied. If we had set these control parameters of inventory policy in their non-preferred values within the scope of the experimental design data, i.e. $T = 4$ and $I_0 = 45$, the expected profits decreases and variability increases both significantly. Also, in such non-preferred conditions expected profits poses larger gap between the alternatives as introduced in *Table 15* below.

Table 15. Profit with/without *AD* for control parameters $T = 4$ and $I_0 = 45$

***AD* = 0**	***AD* = 1**	
-254.28	163.81	Avg.
518.96	105.93	Sdv.

The above results, which are strengthened also by a pre-experimental analysis (ANOVA), it is suggested to find first the optimal ordering policy as profit accelerator. A direct conclusion that results from all experiments relates profit variability. The conclusion is that when applying *AD* profit variability is significantly reduced. Different environmental conditions influence that variability magnitude differently only when *AD* is not applied. More specifically, increased variance of time until expiration date, or increased penalty parameter η, or decreased expected expiration date, increases significantly the profit variability. A direct practical conclusion results from *Table 11*. In a strategy of not applying *AD* expected profits are increased when suppliers offer products possessing long expected expiration. The conclusion is expected since the probability of finding a damaged product within planning horizon reduces. Opposing to the results obtained in [7] preference of applying *AD* remains significantly strong even after razing costs of applying *AD* up to 30% of item cost.

4.3. Summary

A basic assumption that is considered in the current research is the jointly expiration date for all items. This policy implies that in case of a product having naturally very long expiration duration, rarely inventory replenishments are necessary, while inventory costs increase. In such a case this policy of setting jointly expiration date might not be preferred. Another assumption made in this research is that expected demand does not change according to time. In actual situations, especially where expiration date is known customers might prefer newer products rather older ones. This assumption follows the applicability of *AD*, which reduces the risk of purchasing a damaged item that is in inventory.

This research enables managers to consider application of using sensing and detecting technology despite its costs. It also encourages technology producers to develop the suitable and economic devices, which can eventually be accepted by distributes. Since 11 factors are considered, higher order interactions might influence profits. In this study only main factors and pair-wise interactions were considered. Further research can take into account several directions. Some of these directions include analysing the case of multiple expiration dates, generalizing the conclusions for a multiple-product type system, and developing different strategies of presenting expiration information given on-line to customers.

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THE INFLUENCE OF BUSINESS RESULTS ON EXECUTIVE SALARIES IN ISRAEL

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The purpose of this paper is to examine the correlation between executive salary levels and business results for companies in Israel. The examination of this issue included a survey of professional literature and relevant legislature, a study of various executive compensation methods in use in Israel and abroad, and the performance of an empirical analysis of the research hypothesis by use of classic regression.

The analysis of the results shows a low correlation between changes in executive salaries and changes in business effectiveness. But given that, there are steps that can be taken which may improve this correlation such as: unified and proper disclosure of salaries, linkage between executive salaries and business results under the supervision of the Board of Directors, and increased taxation on higher salaries.

Keywords: *executive salary, business results, executive compensation methods, company performance, stock options, stock allocations, bonus distribution*

1. Introduction

The relation between executive salaries and company performance is an issue which has attracted significant public attention. The various media in Israel and in the Western World in general, provide extensive coverage on this issue, due both to the economic aspects relating to compensation and to the natural curiosity of the general public concerning executive salaries.

2. Purpose of Research and Hypothesis

The purpose of this study is to examine the relationship between executive salaries and business results for companies traded on the Tel Aviv Stock Exchange (TASE).

The research hypothesis is that the yearly rate of increase in executive salaries is only partially dependent on the business performance as expressed by the yearly increase in sales and net profits.

3. Significance of Study

There are several significant aspects concerning the study of the relationship between executive salary levels and company performance. One of these aspects deals with evaluating the assumption commonly held by the Israeli public that existing executive compensation is economically unjustified. It can be shown that this is a based assumption if we examine salary and profit data for public companies in 1994. An examination of the salary data reveals a surprising result, even for cynics. The median wage for executives in 234 companies was $200,000 for that year, while the median profit for those same companies was $310,000. In addition, the median salary for the five top executives in those companies reached $560,000 – almost twice the median profit of all shareholders.

An additional significant aspect for examining the relationship between executive salaries and company performance relates to the problem of executives whose actions are not necessarily for the benefit of the shareholders. When control of the company is dispersed among different shareholders, the influence over the executives is lessened.

4. Examining the Subject

The subject was examined by empirical research utilizing reports on salary expenses of public bodies (since 1994) and data on public companies traded on the TASE. Research data was analysed by performing a linear regression. Relevant professional literature was surveyed, the existing legislative basis concerning proper disclosure of executive salaries was examined and common executive compensation methods in Israel and abroad were studied.

5. Survey of Professional Literature

To examine this subject, relevant professional literature was surveyed. From the literature concerning the relationship between executive salaries and company performance it was found that in most studies, executive compensation is only partially dependent on business results for corporations [Hauser; Solomon, 1995]. In the study by Bar Yoseph and Talmor [Bar; Talmor, 1997] it was found that the change in real terms of executive salaries among 1991–1994 was almost always positive, even though the change in profits and cash flows were negative. In addition, in Bar Yoseph and Talmor's study [Bar; Talmor, 1998] it was found that there was no relationship between executive compensation and company performance.

However, in Hauser and Gizbar's research [Hauser; Gizbar, 1992] it was found that economic success as measured by net profit, sales, and excess returns or changes in these, are only of secondary importance in determining salary levels. These economic success indices explained salary levels but not changes in real terms of executive salaries over time, except for the changes in associated benefits.

6. The Existing Legislative Basis for Proper Disclosure of Executive Salaries and Its Significance

In addition to the survey of the literature, the existing legislative basis for proper disclosure of executive salaries was studied, since this may influence the readiness of corporations to conceal executive salaries even if companies don't succeed.

By current Israel law, companies covered under the Securities Law are obligated to specify in their periodic reports salaries and benefits paid to company officers. The disclosure of executive salaries is of utmost importance from the following aspects:

1. It assists in creating competition in the market for executives which on average decreases the executive salaries, thus serving the public interest [Brockway, 1984].
2. Disclosure enables public scrutiny which reduces the ability of executives to set their own salaries to their heart's desire (with the consent of the members of the board of directors), and provides an incentive to pay executives according to their abilities and their contribution to the company [Chaikin, 1992].
3. Proper disclosure of executive salaries is important to us as citizens and taxpayers, whose interest is that companies stay in business and not go bankrupt. That is because those companies provide employment and supply needed resources to the national economy [Globerson, 1995].

7. Common Executive Compensation Methods in Israel and Abroad

Executive compensation methods are more different than those used for employees. The difference results from the nature of the position and the executive's contribution to the financial strength of the company on the market.

The following is a description of the common types of executive compensation in the Israeli economy and abroad:

1. Stock options (the right to purchase stocks in the future at a certain price) – this form of compensation is prevalent in the hi-tech sector. It is designed to tie the executive to the fate of the company and thus serves both the employer and the employee.
2. Stock allocations – this method links the executives directly to the company profits and turns them into small shareholders, causing them to act in the interest of the owners and thus raising the stock price.
3. Bonus distribution – this compensation is based mainly on indices, such as: increase in yearly profit per share, yearly increase in company value.

It should be pointed out that existing academic circles severely criticize the commonplace practice of setting bonuses based on short term accounting indices (such as yearly increase in net profits or profit per share), this since these indices are easily manipulated [Lev, 1985].

8. Research Method (Model)

This section describes the empirical method of testing the research hypothesis as presented above. The sample chosen – out of all the companies traded on the TASE between 1997 and 2000, where full data was available, a sample (layer sample) of 20 companies was taken from the three following sectors: food, hotels and electrical/electronic ($\alpha = 0.05$).

Research method – the "classic regression" method was used for testing the research hypothesis and analysing the data. A regression line is the best possible straight line for predicting one variable based on another variable. If all data points fall on a straight line, we can predict one variable based on another variable.

In this case, the regression line will be a straight line connecting all data points. However, the linear connection isn't always perfect; therefore the regression line is a straight line where the average of the squares of the deviations of the prediction from the actual value is minimal. This is called the least squares approach.

The significance level used to test the research hypothesis will be 5% ($\alpha = 0.05$) as is usually acceptable. The following are the statistical indices to be used in testing the research hypothesis:

R Square – indicates the goodness of fit of regression line to real data points (coefficient of determination) – this index is equal to the relation between the explained variance of Y (the explained variable) and the total variance of Y. The index can have values between 0 and 1. An index closer to 1 indicates a lower unexplained variance of Y and therefore better explanation.

Significance F – indicates the significance of the regression. If this index is lower than 0.05, the regression is significant.

9. Definitions

W – Executive Wages
Wi – Wages of Top five Corporate Executives
S – Total Corporate Sales
A – Total Corporate Assets
G – Total Corporate Profits.

To test the hypothesis, the following four regressions are run:

1. The rate of change in average yearly wages of the executive as a function of the rate of change in the yearly sales levels and total corporate assets

W = f(S + A),

where W is an dependent variable but A and G are independent variables. These variables are defined as follows:

W – Executive wages, A – Total Corporate Assets, S – Total Corporate Sales

2. The rate of change in average yearly wages of the executive as a function of the rate of change in the net yearly profits and total corporate assets

W = f(G + A),

where W is an dependent variable but A and G are independent variables. These variables are defined as follows:

W – Executive wages, A – Total Corporate Assets, G – Total Corporate Profits

3. The rate of change in average yearly wages of the top five executives as a function of the rate of change in the net corporate yearly sales levels and total corporate assets

$$\sum_{i=1}^{5} Wi = f(S + A),$$

where $\sum_{i=1}^{5} Wi$ expresses the wages of five corporate executives. This is a dependent variable whereas S and A are independent variables. These variables are defined as follows:

W – Executive wages, A – Total Corporate Assets, S – Total Corporate Sales

4. The rate of change in average yearly wages of the top five executives as a function of the rate of change in the net corporate yearly profits and total company assets

$$\sum_{i=1}^{5} Wi = f(G + A),$$

where $\sum_{i=1}^{5} Wi$ expresses the wages of five corporate executives. This is a dependent variable whereas S and A are independent variables. These variables are defined as follows:
W – Executive wages, A – Total Corporate Assets, G – Total Corporate Profits.

10. Results and Conclusions

An analysis of the results of the regression described above shows that the coefficient of determination (R Square) is very low, 3.5% to 10% (except for the regression of the rate of change of executive salaries as a function of the rate of change in yearly sales levels where the coefficient of determination was 23%). This indicates that the size of the corporation (total balance) and the rate of change in the levels of sales and net profits are not principal variables relating to the rate of change in the level of executive salaries.

All the regressions where found to be insignificant (Significance F > 0.05), except the regression used to test the relationship between the rate of change in executives' yearly salaries as a function of the rate of change in the yearly sales levels. In this regression the variable is explanatory; the rate of change in sales levels is significant.

To summarize, the hypothesis is supported by the data and the model, that is, the research shows a partial relationship only between yearly rate of change in executive salaries and yearly rate of change in total sales and net profit.

11. Discussion

In this section we test the correlation between the research conclusions and their significance and the professional literature discussed above.

The results show a partial relationship only between the yearly rate of change in executive salaries and the yearly rate of change in total sales and net profit. This is in contrast to the traditional economic viewpoint, where salaries are based on the economic contribution of the employee to the company. Employees with a greater contribution to the profitability of the company should receive greater compensation than those with a lesser marginal contribution.

However, in Lev's article "Salaries and Performance" [Lev, 1985], the author wonders how it is possible that certain companies which are managed poorly and suffered heavy losses until they reach bankruptcy, continue to employ the same executives and even reward them with exaggerated salaries and benefits. This question is also asked in this paper.

According to [Chet, 1998], the criticism concerning the salaries of executives is related, among others things, to the perception that they are set by the holders of the controlling interest in the companies (often serving as executives themselves), without consideration for the shareholders from the general public. This may be the simple, natural explanation for the lack of relationship between success and incentives for executives as expressed by a low R Square indicating a low correlation between company success and executive success.

12. Recommendations

The following are some steps which may improve the relationship between executive salaries and company performance:

1. Uniform and proper disclosure of salaries and benefits – clause 123 a' of the Companies Directive (Integrated Version), 1983, including a requirement to specify the salaries of the five top salary holders in the corporation.

   In spite of this requirement, this study has shown that the reporting of salaries and benefits is not uniform, and in many cases is unclear. Therefore the legislative basis in the matter should be corrected to require uniform, clear and detailed salary disclosure.

2. Creating linkage, by law, between the rate of change in business results and the rate of change in executive salaries, under the supervision of the Board of Directors. This is essential since, according to [Chet, 1995], in 1994 the salaries of the five top executives were 25% more than the companies' profits.

3. Setting a maximum wage in the public sector (for example five times the average national wage) while in the private sector this will be the maximum amount recognized for tax purposes (any amounts paid to an employee exceeding 5 times the average wage will not be recognized as an expense by the tax authorities).

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THE METHODOLOGY OF WEIGHTING COEFFICIENTS' CALCULATION FOR BUSINESS' EVALUATION

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It is recommended by International Standards of Estimation to employ different methods and approaches to business evaluation. The obtained results herewith can significantly differ from each other. The weighting coefficients are applied for derivation of final business' cost. The methodology for the calculation of these coefficients has not been developed yet. The aim of the present research is to offer a variant of such a methodology.

Keywords: *business evaluation, weighting coefficient, methodology*

The occurring problem is becoming progressively topical owing to the extending purview of business` cost evaluation results` application. Thus, according to ISE regulations "the estimation of businesses might be required in the variety of events, particularly in purchasing and liquidation of separate businesses, in estimation of shareholders` shares in proprietary and all that ([1], p. 118, paragr. 5.1). The normative acts of Latvia also order the estimation of business and its share in the variety of events. Thus, a Commercial Law of the Republic of Latvia (CL) requires written estimation of property deposits into joint capital of joint stock company (JSC) and into corpus of limited liability company (LLC) ([2], article 154). These deposits can be shares in a capital of other commercial society. Their market-value is being determinate by the estimation of business` cost method. Besides this, one of the approaches to business` estimation is mentioned in the Commercial Law with formulation of rules of compensation calculation for the succeeding party of the dead company`s contributor ([2], article 353) and of the compensation for those contributors of companies, who have voted against the incipient restructuring and who have refused to contribute in it ([2], article 353), and also, in the events, when contributor of JSC or LLC wants to sell his share and he settles its cost himself ([2], article 188 and article 238). The norms of business` estimation standards are applied in the calculation of coefficients of the restructuring companies share exchange to its full extent ([2], article 339, part 1). An example of such calculation is given in Figure 1.

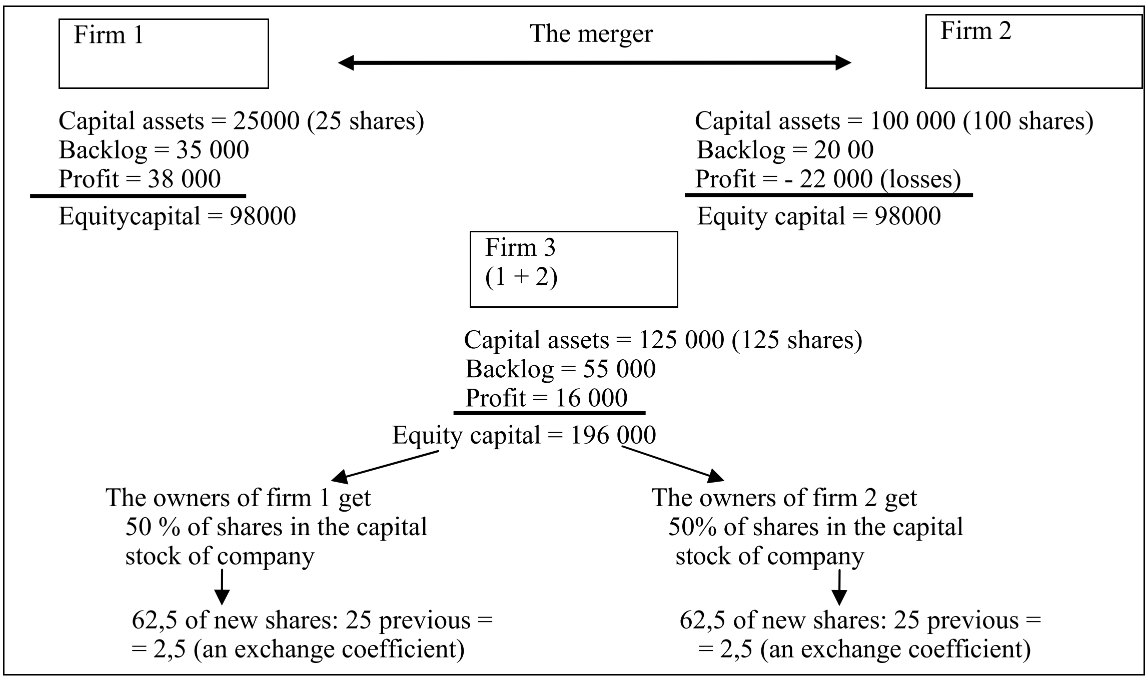

Figure1. Business estimation in the calculation of share exchange coefficient in conditions of restructuring (merger)

There are 3 approaches to the business` cost estimation that are envisaged by the standards: spending, profitability and market.

The first approach is an approach to business estimation on the bases of capital employed of the balance. The spending that is connected to asset sales and shutting down of the business must be cumulated by this type of evaluation [ISE, 2000, p. 127, paragr. 6.7]. The main idea is noted without entering into details of technology of application of this approach, it is: business cost as much as their owners could get by means of liquidation quota, if their firm had not been closed down today. That is why it is necessary to evaluate for the calculation of business` cost of commercial society the possible cost of its asset sales and to deduct the liabilities to creditors from the amount received. The remaining amount is the business cost of this society.

For instance, while evaluating the market-value of deemed LLC "X" (Table 1), it must be admitted, that it will not be possible to realize even one lat for immaterial investment and differed charges. Total negative profit is 100 000 LVL. Total asset sales can bring 200 000 LVL and this totality has to be returned to creditors. Thus, estimation of LLC "X" business within first approach "capital employed of balance" by the calculation` method of deemed liquidation quota gives us number 0 LVL.

Table 1. Date of balance of LLC "X" * (LVL)

Asset		Passive	
1. Long-term investment		1. Equity capital	
Intangible assets	50 000	Capital asset	20 000
Capital goods	100 000	Backlog	20 000
Loans to associated enterprises	50 000	Retained profit	60 000
Total of section 1:	200 000	Total section 1:	100 000
2. Current asset	50 000	2. Savings	0
Store	50 000	3. Creditors	
Provisions for liabilities and charges	50 000	Long-term	100 00
		Short-term	100 00
Total of section 2:	100 00	Total of section 3:	200 000
Balance:	300 000	Balance:	300 000

*The table has been made by the author using relative data

The second approach is a profitable approach to the business` estimation set by the standards [ISE, 2000, p. 128, paragr.6.7.2.1]. The expected benefits of estimable business are determined within this approach. There are two the most widespread methods within profitable approach: capitalization of earnings power, capitalization of earnings, dividend reinvestment and discounting of money flow.

The main idea is noted without entering into technology of application of each method. The valuator determines the amount of money that is reasonable to pay for procurators (estimable) business today in order to exact expected profit (or dividends) in a monetary form each year afterwards. The capitalization rates and discount rates record inflator money depreciation, profitability of alternative ways of investment, the risk levels and other factors.

For instance, when the capitalization rate is 20% and guaranteed annual profit is 60 000 LVL (see Table 1) the cost of LLC"X" business comes to 300 000 LVL (60 000 LVL are divided by 20% and are multiplied by 100%). That result gives a method of profit capitalization.

It will not be mentioned how much does the business cost by using the method of discount money flow, but it will be specified, that according to standards of estimation the discount rate is higher than the capitalization rate [ISE, 2000, p. 129–130, paragr. 6.7.2.2]. It means that the cost of LLC "X"`s business by using this method will be lower than 300 000 LVL.

In addition there is *a market approach* to business estimation. There are exchange share quotations of estimable companies in the basis of this approach. If neither LLC “X”`s shares nor company`s shares are quoted at stock-exchange, this approach is not valid for the purposes of this research. It has to be noted, that by the estimation of business there is a possibility of cost` considerations known as “goodwill”. By the recording of the business the number found by the methods of expected benefits can be higher substantially. The different methodologies of goodwill` considerations in the business` estimation are offered in literature. The reference to this kind of problem for the purposes of present article is not reasonable. Therefore, it is assumed that there is no positive goodwill in estimable business. As it is seen from Table 2 the calculation of business` cost differs very significantly.

Table 2. The results of estimation of LLC “X”`s business

№	Method and approach	Cost (thousand LVL)
1	An approach on the basis of capital employed (the method of relative liquidation quota)	0
2	An approach on the basis of expected profit (the method of profit capitalization)	300

* The above-mentioned has been calculated by the author with help of data from Table1 and regulations of ISE

The harmonization of estimation of the results is the main unsolved problem at the moment what is the final resulting cost of LLC “X” business: is it zero LVL or three hundred thousands LVL?

It is defined by the standards, that “the choice of appropriate approaches, methods and calculation techniques and affiance to them depend on valuator`s assertion” [IES, 2000, p. 132, paragr. 6.8.2.1]. Therefore, the standard does not give any concrete instructions concerning the possibility of using the results from only one of the methods or necessity to conduct a derivation of average quantity on the basis of obtained results. On the example of LLC “X” it means that the valuator can give an opinion that business cost nothing, but also he/she can stop at valuation 300 000 LVL or (relying on the positive goodwill) it can be higher .

It is true as well, that the standard appoints at the fact that valuator can develop the bottom-line price with consideration of particular specific gravities, it a sort of credibility coefficient to each method of exploit evaluation. “The valuator, in this case, has to adduce objectivity and explanation for the methods used, and also for gravities attached to different methods, one of which the valuator has to use in order to define an agreed quantity of value”(briefing, total – the remark of O.L.) [ISE, 2000, p. 132, paragr. 6.8.2.2].

On the basis of the mentioned above, it is clear, that the choice of gravities, i.e. credibility coefficient to each of derived estimation results, has to be justified conclusively. An assertion of valuator must have an argumentation based on objective indexes, facts, tendencies. Otherwise the dispute`s results about finding out if LLC “X”‘s business cost 300 000 LVL or 150 000 LVL, or it does not cost anything depend on declamatory abilities of parties but not on the arguments they have available.

It is necessary to choose the objective indexes that could be used for calculation of reliance (weighting) coefficients to each of the evaluation` results obtained. In addition to this, the principle of precaution asks for more careful verification of this approach, the results of which delivers much higher results of cost of the business. It is required in this case, that the probability of attending the profit (60 000 LVL) is high indeed. In connection to this it is necessary to conduct a valuation of estimable business` stability and effectiveness. In this case it might be possible to use the normative indexes of commercial societies` stability set by the government and the indexes of relative effectiveness of the actual merchant. After the comparison between stability and effectiveness indexes and normative indexes, it is possible to find the reliance coefficient to the profitable approach of business` cost evaluation and than, the weighting (reliance) coefficient at to extravagant approach of capital employed correspondingly. But the choice of normative indexes might be undertaken by buyer of business and its seller together in view of the results of agreement about their own postures on this question.

In capacity of one variant of choice of such indexes the stability coefficients of enterprises accepted in Latvia as normative standards might be used [The Rules of Cabinet of Ministers of the Republic of Latvia No 548 from 14.08.2007, paragr. 7]. The content of these standards is seen on Figure 2.

- The provision index of long-term liquidity = Equity capital + Long-term creditors – Long-term investment

  Ratio > 0

- The coefficient of provision of equity capital $= \frac{\text{Equity capital – Long-term investment}}{\text{Current asset}}$

  Ratio ≥ 0,1

- The single liquidity coefficient $= \frac{\text{Current asset}}{\text{Short-term shares}}$

  Ratio ≥ 1

- The equity capital is bigger than the amount of payables
- The equity capital is a positive figure
- The absence of loans given to associated and joint enterprises

Figure 2. Latvian normative standards of financial stability of enterprise

The normative indexes of financial stability of enterprise stated by Rules of Cabinet of Ministers are intended for use in cases, when the practicability of government`s or self-government`s refuse from the collecting of debts in taxes from merchants is estimated. The government (self-government) takes the decision to take a share in capital of debtor`s company and to sell this share later. In the nature of things the buyer hopes for this share only in case if taken into by him possession business is stable enough. That is why the tough concrete requirements are determined to the indexes in company`s balance (former tax debtor) after capitalization of debts (i.e. conversion into capital asset of firm).

The first six indexes in Table 3 are applied to these standards, which range is defined in the mentioned Rules of Cabinet of Ministers. Index number 7 is documented in Commercial Law of LR [CL, 2006, art. 161, 182]. The rest three indexes are offered by the author of this article to be used as supplementary ones. They characterize the effectiveness of company`s performance.

After estimation of stability of LLC "X" in this table on the data base of its balance for the last three years, the reliance coefficient (weight) might be calculated as to business` estimation that has been made on the basis of calculation of expected profits. It is assumed, that this analysis for the relative LLC "X" has already been conducted, and the results of this are gathered in one resulting table (see Table 3).

As it is seen from the example, only three indexes of stability and effectiveness out of ten correspond to company`s standards. It might be accepted accordingly, that the reliance (weigh) as to results of estimation with this method comes to 0,3 (30%). The reliance (weight) as to method of relative liquidation quota comes to 0,7 (70%).

In this case the summary valuation comes to 0 LVL * 0,7 + 300 000 LVL * 0,3 = 0 + 90 000 LVL = = 90 000 LVL.

Thus, the method of determination of weights (reliance coefficient) offered in this table might be used in derivation of total business` cost.

Table 3. The prediction of LLC "X"`s stability and estimation of reliance coefficient as to the valuation of business by method of capitalization of earnings – power*

№	The index and its standard value	The mark of accomplishment in LLC "X"
1	Equity capital is more than 0 (in LLC "X" it is 100 000 LVL)	+
2	Equity capital is bigger than an amount of creditor`s debts	-
3	Single liquidity coefficient is less than 1	-
4	Provision coefficient of equity is not less than 0,1	-
5	Provision coefficient of long-term liquidity is more than 0	-
6	A lack of debts that are given away to associate and joint enterprises	-
7	Equity capital is bigger than capital asset (in LLC "X" 100 000 is more than 20 000)	+
8	Equity capital grows faster than the creditor`s equity	
9	Return on assets (in LLC "X"= 60 000/300 000*100% = 20%) is bigger than inflation rate (in present time it is around 15% in Latvia) and bank deposits rates (around 100%)	+
10	Net profit ratio is bigger than average sectorial rate (it is bigger than average in region)	-
	In total the quantity of rates corresponds to normative standards	3
	Reliancy (weighting) coefficients as to estimation of business by method of profit capitalization	0,3 (30%)

*The table has been projected by the author with help of data from Table 1 and Rules of Cabinet of Ministers of LR N548 from 14.08.2007.

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CUMULATIVE INDEX

COMPUTER MODELLING and NEW TECHNOLOGIES, volume 12, No. 1, 2008

(Abstracts)

A. Baublys. Statistical Probability Models for Transport Means Fleet, *Computer Modelling and New Technologies*, vol. 12, No 1, 2008, pp. 7–13.

Some algorithms to forecast repair time of damaged vehicle, time losses for lack of drivers and various accidental hindrances-during goods handling are proposed using methods of mathematical statistics and probability theory.

Statistical information on transport means fleet is renewed and replenished in the course of time. With the growth of information amounts the costs of its storage increase as well. Therefore the relevant algorithms for obtaining required statistical assessments with the least statistical information are presented in the article. It is deduced that in the modelling of transport networks and freights as well as the flows of transport means in them, it is analytically proper to describe random factors by the non-parametric assessment.

Keywords: *transport means, repair time, statistical probability algorithms*

G. Gurevich, A. Davidson. Standardized Forms of Kullback-Leibler Information Based Statistics for Normality and Exponentiality, *Computer Modelling and New Technologies*, vol. 12, No 1, 2008, pp. 14–25.

Our aim in this article is to present clear forms of well-known Kullback-Leibler information based tests. We focus on tests for normality and exponentiality, common tasks in applied studies. The proposed approach standardized the Kullback-Leibler information based tests. We show that the proposed tests are asymptotically consistent and convenient to practical applications. We conduct broad Monte Carlo analysis for judging our conclusions and the suggested tests. Thus, we can infer that the proposed tests for normality and exponentiality are powerful procedures that can be applied to real data studies.

Keywords: *empirical distribution, entropy, goodness-of-fit tests, normality tests, exponentiality tests, Kullback-Leibler information*

D. Golenko-Ginzburg , Z. Laslo, D. Greenberg , A. Ben-Yair. Heuristic "Look-Ahead" Cost-Reliability Models, *Computer Modelling and New Technologies*, vol. 12, No 1, 2008, pp. 26–37.

A hierarchical technical system functioning under random disturbances is considered. Certain elements can be refined by undertaking technical improvements. The list of the latter is pregiven as well. Assume that by means of simulation modelling (SM) it is possible to evaluate the increment of the system's reliability by implementing any set of technical improvements. The developed models centre on determining an *optimal sub-set* of technical improvements in order to either to maximize the system's reliability subject to a restricted budget assigned for the improvements' implementation (direct cost-reliability model), or, to minimize the system's budget subject to a reliability value restricted from below (dual model). Unlike existing cost-reliability models with simplified technical improvements, the developed models cover an important case of compound technical improvements with a stepwise consecutive structure. Several new heuristic algorithms, based on the combination of cost-sensitivity and "look-ahead" forecasting, are developed in order to solve both the direct and dual optimisation problems. Extensive experimentation has been undertaken to assess relative efficiency of different proposed algorithms. A numerical example is presented.

Keywords: *cost-sensitivity, direct and dual cost-reliability problems, compound technical improvement, cost-sensitivity with a "look-ahead" analysis, generalized algorithm based on switching procedures, algorithm's cost-reliability sensitivity*

In this paper, for identifying an observable process with one of several simulation models, a uniformly most powerful invariant (UMPI) test is developed from the generalized maximum likelihood ratio (GMLR). This test can be considered as a result of a new approach to solving the Behrens-Fisher problem when covariance matrices of multivariate normal populations (compared with respect to their means) are different and unknown. The test is based on invariant statistics whose distribution, under the null hypothesis, does not depend on the unknown (nuisance) parameters.

Keywords: *observable process, simulation model, UMPI test, identification*

This paper studies a problem consisting of two n-unit standby redundant system designated N and S with repair facility incorporating the concept of imperfect switching under human failure and all repairs except one follow general repair time distribution. Both the systems suffer from two types of failures namely constant failure and human failure. All failures of the system follow exponential time distribution. The failure rate of units and repair of the switch are statistically independent. In contrast to earlier models which mainly considered only single transition between two adjacent states here authors have taken an important aspect of repairs, which is consistent with actual failures of switching by assuming two different types of repair between adjacent states. Using supplementary variable technique, Laplace technique and Gumbel-Hougaard family of copula the transition state probabilities, reliability and MTSF of the system is evaluated. Inversions have also been carried out so as to obtain time dependent transition probabilities, which determine availability of the system at any time.

Keywords: *copula, Gumbel-Hougaard family of copula, reliability and MTSF*

Zipf's laws are used to study characteristic properties of the Russian language and its evolution during XIX and XX centuries. It is shown that Zipf's constant characterizes stylistic properties of works creation of different authors and at the same time allows to observe peculiarities of creative work for one author in different periods. It is obtained that structural parameters of the Russian language did not show a pronounced tendency to change during XIX and XX centuries.

Keywords: *Zipf's laws, the Russian language*

The problems of observing system's simulation (media dots in a wide view) are considered. Classification of system of video observation, base function of such systems and processes of analysing media dates are described. The approach to the analysing of video observation systems is proposed. Some simulation models based on stochastic Petri nets and results are discussed.

Keywords: *video observation, Petri nets, simulation model, media dots, program agents*

This paper proposes a generic Platform-Independent-Model (PIM) for realization of OMG Model Driven Architecture (MDA) standard. Our model defines precise execution semantic for UML state diagrams with OCL constraints. We describe a decomposition technique for state diagrams that separates platform-independent and platform-specific parts and facilitates further transformations to various implementation platforms.

Our method is based on the notion of Parallel Hierarchical Statechart Automaton (PHSA) introduced in this paper. This kind of extended automata can be a common tool for transforming UML diagrams to a PIM suitable for further transformations to PSM automata and to compilable code. The architecture for a general UML-to-PHSA PIM-to-PIM transformation for UML state diagrams allows portable execution on various specific implementation platforms and interoperability between PSMs based on common PHSA-based PIM.

Keywords: *MDA, model transformation, extended automata, UML, OCL, XMI.m*

A. Herbon, D. Golenko-Ginzburg, A. Ben-Yair. A Simulation Study of on-Line Automatic Sensing and Detecting Damaged Deteriorating Inventory with Random Lifetime and Demand, *Computer Modelling and New Technologies,* vol. 12, No 1, 2008, pp. 77–88.

Managing perishable items listed in an inventory becomes nowadays a quite important and challenging task. Although each perishable item is usually assigned a corresponding expiration-date label, unexpected events like human errors, dynamic temperature conditions, improper maintenance, etc., can drastically deteriorate products quality and properties and cause significant losses both to supplier and customer. Analytic inventory models for deteriorating items with stochastic demand and various environmental characteristics have been developed. Another efficient solution includes automatic sensing and detecting of spoiled items. Most sophisticated labels that combine both chemical and electronic functionality with radio frequency identification (RFID) technology could remotely warn users and automated systems whenever stability problems have occurred. Yet, technological solutions involving automatic labelling devices require additional scientific investigation and scrutiny by means of the simulation theory, which mainly has not been attempted so far. Our research presents a simulation study of providing a single product type of perishable inventory with general AD (automatic device), which on-line senses and detects spoiled items. Experimental design is outlined and numerical results presented and analyzed.

Keywords: *perishable inventory, automatic sensing, FQI, TTI, RFID, random lifetime, common expiration date*

O. Barkai. The Influence of Business Results on Executive Salaries in Israel, *Computer Modelling and New Technologies,* vol. 12, No 1, 2008, pp. 89–93.

The purpose of this paper is to examine the correlation between executive salary levels and business results for companies in Israel. The examination of this issue included a survey of professional literature and relevant legislature, a study of various executive compensation methods in use in Israel and abroad, and the performance of an empirical analysis of the research hypothesis by use of classic regression.

The analysis of the results shows a low correlation between changes in executive salaries and changes in business effectiveness. But given that, there are steps that can be taken which may improve this correlation such as: unified and proper disclosure of salaries, linkage between executive salaries and business results under the supervision of the Board of Directors, and increased taxation on higher salaries.

Keywords: *executive salary, business results, executive compensation methods, company performance, stock options, stock allocations, bonus distribution*

O. Lukashina. The Methodology of Weighting Coefficients' Calculation for Business' Evaluation, *Computer Modelling and New Technologies*, vol. 12, No 1, 2008, pp. 94–98.

It is recommended by International Standards of Estimation to employ different methods and approaches to business evaluation. The obtained results herewith can significantly differ from each other. The weighting coefficients are applied for derivation of final business' cost. The methodology for the calculation of these coefficients has not been developed yet. The aim of the present research is to offer a variant of such a methodology.

Keywords: *business evaluation, weighting coefficient, methodology*

COMPUTER MODELLING and NEW TECHNOLOGIES, 12.sējums, Nr. 1, 2008 (Anotācijas)

A. Baublys. Statistiskās varbūtības modeļi transporta līdzekļu parkam, *Computer Modelling and New Technologies,* 12.sēj., Nr.1, 2008, 7.–13. lpp.

Rakstā autors parāda dažus algoritmus, lietojot matemātiskās statistikas un varbūtības teorijas metodes, kas palīdz noteikt transporta līdzekļu tehniskās pārbaudes laiku, laika zudumu gadījumos, kad trūkst transporta līdzekļu vadītāji, kā arī dažādus negadījumus kravas pieņemšanas laikā.

Laika gaitā transporta līdzekļu parka statistiskā informācija tiek atjaunota un paplašināta. Līdz ar informācijas daudzuma pieaugumu tā uzglabāšanas izmaksas arī palielinās. Tādēļ atbilstoši algoritmi, lai apgūtu nepieciešamo statistisko izvērtējumu ar vismazāko pieejamo statistisko informāciju, tiek piedāvāti dotajā rakstā.

Atslēgvārdi: *transporta līdzekļi, tehniskās pārbaudes laiks, statistiskās varbūtības algoritmi*

G. Gurevich, A. Davidson. Kulbaka-Leiblera standartizētās datu statistikas formas bāzētas uz informāciju normalitātei un eksponenciālismam, *Computer Modelling and New Technologies,* 12.sēj., Nr.1, 2008, 14.–25. lpp.

Šī raksta autoru mērķis ir parādīt plaši pazīstamo Kulbaka-Leiblera testu, kas pamatoti uz informāciju, skaidrās formas. Autori akcentē uzmanību uz testiem, kas paredzēti normalitātei un eksponenciālismam, kopējiem uzdevumiem lietišķajās studijās. Piedāvātā pieeja unificē Kulbaka-Leiblera testus, bāzētus uz informāciju. Autori parāda, ka piedāvātie testi ir asimptotiski konsekventi un ērti praktiskai pielietošanai. Autori izmanto plašu Monte Karlo analīzi, lai spriestu par veiktajiem secinājumiem un piedāvātajiem testiem. Tādējādi mēs varam secināt, ka piedāvātie testi normalitātei un eksponenciālismam ir spēcīgas procedūras, kas var būt pielietotas reālajās datu studijās.

Atslēgvārdi: *empīriskā sadale, entropija, pilnīga derīguma testi, normalitātes testi, eksponenciālisma testi, Kulbaka-Leiblera informācija*

D. Golenko-Ginzburgs, Z. Laslo, D. Grinbergs, A. Ben-Jears. Heiristiskie „paredzamie" izmaksu-drošuma modeļi, *Computer Modelling and New Technologies,* 12.sēj., Nr.1, 2008, 26.–37. lpp.

Rakstā tiek izskatīta hierarhiskā tehniskā sistēma, kas funkcionē nejaušībās. Noteikti elementi var tikt izcelti, veicot tehniskus uzlabojumus. Šo uzlabojumu saraksts tiek pievienots. Pieņemam, ka ar imitācijas modelēšanas palīdzību ir iespējams novērtēt sistēmas drošuma pieaugumu, ieviešot jebkuru tehnisko uzlabojumu virkni. Attīstītie modeļi centrējas uz noteiktas tehnisko uzlabojumu optimālas zem-virknes, lai vai nu maksimizētu sistēmas drošuma priekšmetu ierobežotā budžetā, kas asignēts uzlabojumu ieviešanai (tiešais izmaksu-drošuma modelis), vai arī lai minimizētu sistēmas budžeta priekšmetu līdz drošuma vērtības ierobežošanai no apakšas (duālais modelis). Pretēji eksistējošiem izmaksu drošuma modeļiem ar vienkāršotiem tehniskiem uzlabojumiem, attīstītie modeļi sedz sarežģītu tehnisko uzlabojumu ar pakāpenisku sekojošu struktūru svarīgu daļu. Rakstā tiek piedāvāti arī skaitliskie piemēri.

Atslēgvārdi: *izmaksu-jutīgums, tiešās un duālās izmaksu drošuma problēmas, sarežģīti tehniskie uzlabojumi, izmaksu jutīgums ar „paredzamības" analīzi, algoritma izmaksu-drošuma jutīgums*

N. A. Nechval, K. N. Nechval. Vērā ņemama procesa statistiskā identifikācija, *Computer Modelling and New Technologies,* 12.sēj., Nr.1, 2008, 38.–46. lpp.

Autori šajā rakstā, lai identificētu vērā ņemamu procesu ar vienu no vairākiem simulācijas modeļiem, izstrādā pēc vispārinātas maksimuma varbūtības proporcijas vienmērīgi visspēcīgāko invarianta testu. Šo testu var uzskatīt par jaunas pieejas rezultātu, lai risinātu Behrena-Fišera problēmu, t.i., kovariācijas multivariāciju normālās populācijas (salīdzinot attiecībā uz to līdzekļiem) formas ir dažādas un nezināmas. Tests ir pamatots uz statistisko invariantu, kura sadale saskaņā ar nenozīmīgu hipotēzi nav atkarīga no nezināmiem (apgrūtinājums) parametriem.

Atslēgvārdi: vērā ņemams process, simulācijas modelis, identifikācija

A. Kumars, S. B. Singhs. N-vienības paralēlās rezerves sistēmas drošuma analīze pie nepilnīgas pārslēgšanās, izmantojot kopulu, *Computer Modelling and New Technologies,* 12.sēj., Nr.1, 2008, 47.–55. lpp.

Autori rakstā izskata problēmu, kas sastāv no divu n-vienību rezerves sistēmas, nosakot N un S ar remonta iekārtu, iekļaujot nepilnīgas pārslēgšanās konceptu pie cilvēciskas kļūdīšanās, un visi remonti ar viena izņēmumu seko vispārējai atjaunošanas laika sadalei. Abas sistēmas cieš no divu veidu kļūdām, un konkrēti, konstantās kļūdas un cilvēciskās kļūdas. Visas sistēmas kļūdas seko eksponenciālā laika sadalei. Vienību kļūdas norma un slēdža remonts ir statistiski neatkarīgi. Pretēji agrākiem modeļiem, kuri galvenokārt paredzēja tikai vienu pāreju starp diviem blakusesošiem stāvokļiem, šeit autori ir pieņēmuši svarīgu labošanas aspektu, kas ir saskanīgs ar faktiskām pārslēgšanas kļūdām, pieņemot divus atšķirīgus atjaunošanas veidus starp blakusesošiem stāvokļiem. Lietojot papildu mainīgā lieluma tehnisko paņēmienu, Laplasa paņēmienu un Gumbela-Hugārda kopulu saimi pārejas stāvokļa varbūtības, drošums un sistēmas *MTSF* tiek izvērtēti. Bez tam arī inversijas tika izstrādātas, tā lai iegūtu laika atkarīgās pārejas varbūtības, kuras nosaka sistēmas pieejamību jebkurā laikā.

Atslēgvārdi: *kopula, Gumbela-Hugārda kopulu saime, drošums un MTSF*

A. Kivs, D. Gončarenko, J. Sedovs, L. Bodnars, N. Jaremčuks. Krievu valodas evolūcijas matemātiskā izpēte, *Computer Modelling and New Technologies,* 12.sēj., Nr.1, 2008, 56.–59. lpp.

Krievu valodas raksturīgās īpašības un tās evolūcija XIX un XX gadsimtā tiek pētīta, izmantojot Zipfa likumus. Tiek parādīts, ka Zipfa konstante raksturo dažādu autoru darbu radīšanas stilistiskās īpašības un vienlaicīgi pieļauj novērot viena autora radošā darba īpatnības dažādiem periodiem. Tiek panākts, ka krievu valodas strukturālie parametri neuzrāda izteikto tendenci mainīties XIX un XX gadsimtā.

Atslēgvārdi: *Zipfa likums, krievu valoda*

E. Gluhovska, R. I. Muhamedjevs. Video novērošanas sistēmu modelēšana, *Computer Modelling and New Technologies,* 12.sēj., Nr.1, 2008, 60.–64. lpp.

Rakstā tiek izskatīti jautājumi par video novērošanas sistēmu modelēšanu. Video novērošanas sistēmu klasifikācija, šādu sistēmu pamatfunkcijas un vides datu procesu analīze ir veikta dotajā pētījumā. Rakstā tiek piedāvāta video novērošanas sistēmu analīzes pieeja. Bez tam tiek arī iztirzāti daži simulācijas modeļi, pamatojoties uz stohastiskajiem Petri tīkliem un tiek apspriesti to rezultāti.

Atslēgvārdi: *video novērošana, Petri tīkli, simulācijas modelis, vides punkti, programmu aģenti*

D. Dajans, R. Kaplinskis, A. Vīzens, S. Blohs, H. Mendelbaums. Uz modeli virzītās arhitektūras *(MDA)* platformas-neatkarīgā-modeļa pret platformas-neatkarīgo-modeli *(PIM-to-PIM)* transformācija, lietojot paplašināto automātu, *Computer Modelling and New Technologies,* 12.sēj., Nr.1, 2008, 65.–76. lpp.

Autori šajā rakstā piedāvā vispārēju Platformas-Neatkarīgo-Modeli (*PIM*), lai realizētu pēc *OMG* Modeļa virzītās arhitektūras (*MDA*) standarta. Mūsu modelis definē precīzu izpildes semantiku *UML* stāvokļa diagrammām ar *OCL* ierobežojumiem. Mēs aprakstām dekompozīcijas tehniku stāvokļa diagrammām, kas atdala platformas-neatkarīgās no platformas-specifiskām daļām un veicina tālākās transformācijas dažādām ieviešanas platformām.

Mūsu metode ir pamatota uz priekšstatu par Paralēliem Hierarhiskiem Stāvokļa-Diagrammas Automātiem (*PHSA*), kas tiek izskatīti šajā rakstā.

Atslēgvārdi: *MDA*, modeļa transformācija, paplašinātais automāts, *UML*, *OCL, XMI.m*

A. Herbons, D. Golenko-Ginzburgs, A Ben-Jiers. Automātiskās zondēšanas izpētes modelēšana tiešsaistē un sabojāta inventāra ar nejaušu dzīves ilgumu un pieprasījumu atklāšana, *Computer Modelling and New Technologies,* 12.sēj., Nr.1, 2008, 77.–88. lpp.

Sabojāto vienību vadīšana, kas atrodas inventarizācijas sarakstā, mūsdienās kļūst par ļoti svarīgu un izaicinājuma pilnu nodarbi. Lai gan katrai sabojātai vienībai ir parasti pievienota klāt atbilstoša zīme ar izlietojuma datumu, neparedzēti gadījumi tādi kā cilvēciskās kļūdas, dinamiski temperatūras apstākļi, nepareizs pielietojums utt. var drastiski sabojāt produktu kvalitāti un to īpašības un izraisīt nozīmīgus zaudējumus gan kā piegādātājam, tā arī pircējam.

Piedāvātais pētījums sniedz ieskatu, modelējot vienkāršu produkta veidu no bojājošamies inventāra ar vispārēju *AD* (automātiskā ierīce), kura tiešsaistē zondē un novērš sabojātās vienības. Eksperimentāls dizains ir īpaši izcelts un skaitliskie rezultāti tiek piedāvāti un analizēti.

Atslēgvārdi: bojājošamies inventārs, automātiskā zondēšana, *FQI, TTI, RFID,* nejaušs dzīves ilgums; kopējais izlietošanas datums

O. Barkajs. Lietišķo rezultātu ietekme uz izpildes algām Izraēlā, *Computer Modelling and New Technologies,* 12.sēj., Nr.1, 2008, 89.–93. lpp.

Šī raksta nolūks ir izpētīt savstarpējo korelāciju starp izpildes algu un biznesa rezultātiem firmās Izraēlā. Pētījumā tiek analizēta kā profesionālās literatūras un atbilstošās likumdošanas, tā arī dažādu izpildes kompensācijas metožu pielietošanas izpēte un pētījuma hipotēžu, lietojot klasisko regresiju, empīriskā analīze.

Rezultātu analīze parāda zemu korelāciju starp izmaiņām izpildes algās un biznesa efektivitātē.

Atslēgvārdi: izpildes alga, biznesa rezultāti, izpildes kompensācijas metodes, akciju opcijas, bonusu sadale

O. Lukašina. Svēršanas koeficientu aprēķināšanas metodoloģija biznesa vērtējumam, *Computer Modelling and New Technologies,* 12.sēj., Nr.1, 2008, 94.–98. lpp.

Starptautiskie vērtēšanas standarti rekomendē izmantot dažādas metodes un pieejas biznesa novērtējumam. Šajā pētījumā iegūtie rezultāti var lielā mērā atšķirties viens no otra. Tiek pielietoti svēršanas koeficienti, lai noteiktu galīgās biznesa izmaksas. Minēto koeficientu aprēķināšanas metodoloģija vēl joprojām nav izstrādāta. Šī pētījuma mērķis ir piedāvāt vienu variantu no iespējamās metodoloģijas.

Atslēgvārdi: *biznesa vērtēšana, svēršanas koeficients, metodoloģija*

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