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Editors' Remarks

Sir Isaac Newton

Under a spreading apple tree,
The village genius stands;
His mind conceives of wondrous things,
He writes them with his hands;
His fame goes forth to all the world--
He's known in many lands.

.....

He talked of falling bodies
and his famous Laws of Motion,
And of colors seen in bubbles
and the tides upon the ocean.
And his crowning jewel, "Principia,"
created *great* commotion.

Yes, Newton's brilliant mind, it was
a trunk with many twigs-
His mind branched out in every way
(right through his powdered wigs).
His greatest contribution, though,
was cookies made from figs.

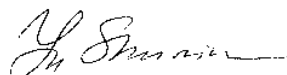
Sir Isaac Newton, David Arns, 1977

This 12th volume No.3 is purposed to various questions of **applied statistics** and **analytical management**, to problems, which are really for this day actual. In particular, we present actual papers from Israel, Lithuania and Latvia.


Our journal policy is directed on the fundamental and applied sciences researches, which is the basement of a full-scale modelling in practice.

This edition is the continuation of our publishing activities. We hope our journal will be interesting for research community, and we are open for collaboration both in research and publishing.

EDITORS



Yu.N. Shunin



I.V. Kabashkin



BAYES-FIDUCIAL APPROACH FOR MATHEMATICAL STATISTICS PROBLEM SOLUTION

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Review of using of Bayes-Fiducial (BF) approach for solution of some important statistical problem is given. BF decision is always a function of sufficient statistics (even in case when sufficient statistics coincides with the sample itself (for example, for Weibull's distribution)). By contrast with maximum likelihood method BF decision is based on the use of specific loss function. By contrast with Bayes decision it does not need a priori distribution of unknown parameters.

For the distributions with location and/or scale parameter the solutions of the following problems are considered: point estimates of location and scale parameters, p-bound (prediction limit) calculation and specified life nomination with specific loss function. Numerical examples are given.

Keywords: parameter estimation, testing statistical hypotheses, prediction limit

1. Introduction

In the name of Bayes-Fiducial (BF) approach the word "Bayes" means that the unknown distribution parameter is considered as random variable (just as in Bayes approach); the word Fiducial means that instead of a priori or a posterior distribution the fiducial distribution is used. Let us remind the main ideas of BF approach, which initially (some version of this approach was initially called quasi-Bayesian) has been offered in 1973 (see [1]). Then it was developed in [2, 3]. In [4] application of this approach to the problem of unbiased estimation when the sufficient statistics coincides with the sample itself was considered.

Let us remind the main idea of this approach for solution of the problem of looking for optimal statistical decision when some loss function is defined. Let's assume that the unknown parameter, θ , of some cumulative distribution function (cdf) of a random vector (rv) $X = (X_1, \dots, X_n)$, $F(x, \theta)$, is itself a random variable (rv), $\tilde{\theta}$, the probability density function (pdf) of which is fiducial pdf $f_{\tilde{\theta}}(s; x)$. And suppose also that the consequence of taking decision d when the cdf of rv X is $F(x, \theta)$ is a loss, which can be expressed as a nonnegative real number $L(\theta, d)$. Let a decision $d = \delta(x)$, that is, a function whose domain is the set of values of X and whose range is the set of possible decisions. An optimum procedure that minimizes so called Bayes-Fiducial risk of δ

$$R(\theta, \delta) = \int L(s, \delta(x)) f_{\tilde{\theta}}(s; x) ds \quad (1)$$

is called a BF solution of the given decision problem.

Let us remind also a definition of fiducial distribution. It is easy to see that sometimes cdf of random variable X , $F_X(x, \theta)$, with one dimensional parameter θ is such that at the fixed x the function

$$F_{\tilde{\theta}}(\theta; x) = 1 - F_X(x, \theta)$$

is a cdf of r.v. $\tilde{\theta}$. This distribution is called the fiducial distribution by Fisher and it has been used for construction of interval for unknown parameter similar to confidential interval. Easily we can use this definition of fiducial distribution and for the case when we have some sample of size $n > 1$ if there is one dimensional sufficient statistics. For example, if r.v. Y_i has normal distribution $N(\theta, 1)$, $i = 1, \dots, n$,

$$X = \sum_{i=1}^n Y_i / n, \text{ then}$$

$$F_X(x, \theta) = \Phi((x - \theta) / \sqrt{1/n})$$

and

$$F_{\tilde{\theta}}(s, x) = 1 - \Phi((x - s) / \sqrt{1/n})$$

is some cdf of r.v. $\tilde{\theta}$.

In more complex case when sufficient statistics coincides with the sample itself (for example if cdf is the smallest extreme value distribution) but unknown parameters are location and scale parameters are considered in [2, 3].

Let

$$F_{x_i}(x, \theta) = F_x\left(\frac{x - \theta_0}{\theta_1}\right), i = 1, \dots, n, \quad (2)$$

where $F_x(\cdot)$, are known c.d.f. of X , θ_0, θ_1 – are unknown location and scale parameters. If both parameters θ_0 and θ_1 are unknown then fiducial pdf of $(\tilde{\theta}_0, \tilde{\theta}_1)$ is defined by equation

$$f_{\tilde{\theta}_0, \tilde{\theta}_1 | x}(s_0, s_1) = h \frac{1}{s_1^{n+1}} \prod_{i=1}^n f\left(\frac{x_i - s_0}{s_1}\right). \quad (3)$$

If θ_1 is known then without loss of generality we can set $\theta_1 = 1$ then fiducial pdf of $\tilde{\theta}_0$ is defined by equation

$$f_{\tilde{\theta}_0 | x}(s) = h_0 \prod_{i=1}^n f(x_i - s), \quad (4)$$

if θ_0 is known then without loss of generality we can set $\theta_0 = 0$ then fiducial pdf of $\tilde{\theta}_1$ is defined by equation

$$f_{\tilde{\theta}_1 | x}(s) = h_1 \frac{1}{s^{n+1}} \prod_{i=1}^n f(x_i / s), \quad (5)$$

where h, h_0, h_1 are just normalization factors, $x = (x_1, \dots, x_n)$.

In this paper we consider application of BF method to parameter estimation, p-bound calculation and specified life nomination.

2. Parameter Estimation

Usually for parameter estimation the loss function is defined by equation

$$L(\theta, \hat{\theta}) = (\theta - \hat{\theta})^2. \quad (6)$$

In this case BF risk is equal to

$$R(\theta, \hat{\theta}) = \int (s - \hat{\theta})^2 f_{\tilde{\theta}}(s; x) ds.$$

For the case when there is only one unknown parameter θ_0 this equation takes the following form

$$R(\theta_0, \hat{\theta}_0) = \int (s - \hat{\theta}_0)^2 f_{\tilde{\theta}_0}(s; x) ds, \quad (7)$$

where $f_{\tilde{\theta}_0}(s; x)$ is defined by equation (4).

It is easy to see that minimum of BF risk in this case is reached at

$$\hat{\theta}_0 = \int_{-\infty}^{\infty} s f_{\tilde{\theta}_0}(s; x) ds. \quad (8)$$

Using equation (4) we get finally

$$\hat{\theta}_0 = \int_{-\infty}^{\infty} s \prod_{i=1}^n f(x_i - s) ds / \int_{-\infty}^{\infty} \prod_{i=1}^n f(x_i - s) ds. \quad (9)$$

This estimate coincides with the Pitmen's estimate of location parameter, which provides the minimum of risk in the class of corrected estimate (see Section 3).

Pitmen's estimate of the scale parameter we get if we use the following definition of loss function

$$L(\theta, \hat{\theta}) = (\theta - \hat{\theta})^2 / \theta^2. \quad (10)$$

Using the BF approach we get

$$\hat{\theta}_1 = \int_0^\infty s^{-(n+2)} \left(\prod_{i=1}^n f(x_i / s) \right) ds / \int_0^\infty s^{-(n+3)} \left(\prod_{i=1}^n f(x_i / s) \right) ds.$$

3. P-bound for Random Variables

3.1. Definition of P-Bound for RV

To make possible the common approach for solution of the both problem SL nomination and IP development we need to remind the p -set function definition [4]. It is a special statistical decision of P-bound for random variable is a specific case of a p -set function, definition of which was introduced by the author [4]. In this paper we limit our self by only application of BF approach for calculation of p-bound.

P-bound is defined in the following way.

Definition. Let Z is a random variable and X is random vector of n dimension and we suppose that it is the known class $\{P_\theta, \theta \in \Omega\}$ to which the probability distribution of the random vector $W = (Z, X)$ is assumed to belong. Of the parameter θ , which labels the distribution, it is assumed known only that it lies in a certain set Ω , the parameter space.

a) Function $\tau(x)$ is called a p-bound for r.v. Z if

$$\sup_{\theta \in \Theta} P_\theta \{Z < \tau(X)\} = p. \quad (11)$$

b) Function $\tau(x)$ is called a parameter-free (p.f.) p-bound for r.v. Z if

$$P_\theta \{Z < \tau(X)\} = p \text{ for all parameters } \theta \in \Omega. \quad (12)$$

c) P-bound for r.v. Z is called a right-hand binary (r.h.b. p-bound), if for each possible observation x of r.v. X , function $\tau(x)$ assigns only one of two decisions:

$$\tau(x) = -\infty \text{ if } x \in S; \tau(x) = \tau^*, \text{ if } x \in S^*, \quad (13)$$

where τ^* is some number, S^* and S are two complementary regions of the sample space.

So we see that the definition of p-bound can be considered as some generalization of the definition of prediction limit. But it is some statistical decision function, which covers both prediction limit and, in some way, testing statistical hypotheses.

We can say also that p.f. p-bound $\tau(x)$ is a p-quantile estimate of cdf $F_Z(x)$ and, as function of p , it is an estimate of inverse cumulative distribution function $F_Z^{-1}(p)$, but very specific estimate: expectation value $E(F_Z(\tau(X))) = p$.

It is easy to get $\tau(x)$ for distribution with location and scale parameters. As the main application of the problem under question we'll consider a problem of SL nomination for some fatigue-prone airframe structure. We suppose to have observations of fatigue lives of some identical units of this structure as a result of full-scale fatigue tests. Usually for fatigue life data processing both a lognormal and Weibull's distributions are used. If we use logarithm scale (if we use $X = \ln(T)$ instead of T) then both these distributions become distributions with location and scale parameters. So we can say, that r.v. X has the following structure: $X = \theta_0 + \theta_1 \overset{0}{X}$, where θ_0, θ_1 are unknown parameters, r.v. $\overset{0}{X}$ has either standard normal c.d.f. $F_{\overset{0}{X}}(x) = \Phi(x)$ or standardized the smallest extreme value (sev) c.d.f. $F_{\overset{0}{X}}(x) = 1 - \exp(-\exp(x))$ for lognormal or Weibull's distributions of T correspondingly. For this case for the specified life nomination problem the following theorem can be used (we give it without proof).

Theorem 1. Let

$$F_{X_i}(x, \theta) = F_x \left(\frac{x - \theta_0}{\theta_1} \right), i = 1, \dots, n, \quad F_Z(x, \theta) = F_z \left(\frac{x - \theta_0}{\theta_1} \right), \quad (14)$$

where $F_x(\cdot)$, $F_z(\cdot)$ are known c.d.f. of $\overset{\circ}{X}$, $\overset{\circ}{Z}$, θ_0, θ_1 – are unknown location and scale parameters. And let the random variables, estimations of θ_0, θ_1 , as function of $X = (X_1, X_2, \dots, X_n)$ be described by the similar structural formulas:

$$\hat{\theta}_0 = \theta_0 + \theta_1 \overset{\circ}{\theta}_0, \quad \hat{\theta}_1 = \theta_1 \overset{\circ}{\theta}_1, \quad (15)$$

where $\overset{\circ}{\theta}_0, \overset{\circ}{\theta}_1$ – are random variables, corresponding to the estimates of θ_0, θ_1 using a sample of the same size n but when $\theta_0 = 0$, $\theta_1 = 1$. We refer to this type of estimates as “correct” estimates.

Then p.f. and r.h.b. p-bounds are described accordingly by formulae

$$\tau_1(x) = \hat{\tau}_1, \quad \tau_2(x) = \begin{cases} -\infty, & \hat{\tau}_2 \leq \tau^* \\ \tau^*, & \hat{\tau}_2 > \tau^* \end{cases} \quad (16)$$

where $\hat{\tau}_i = \hat{\theta}_0 + t_i \hat{\theta}_1$, $i = 1, 2$,

t_1 is p -quantile of r.v. $V_Z = (Z - \theta_0) / \theta_1$, t_2 is the root of equation: $\xi(t) = p$,

$$\xi(t) = \sup_c F_z(c) (1 - F_z(c)) = \sup_c F_z(c) F_{V_c}(t),$$

$$\overset{\circ}{\tau}(t) = \hat{\theta}_0 + \hat{\theta}_1 t, \quad V_C = (c - \hat{\theta}_0) / \hat{\theta}_1.$$

3.2. Optimality Criterion for P.F. P-bound Used for Aircraft Specified Life Nomination

Now we turn to a discussion of some preference orderings of decision procedures: choice of function $\tau(x)$. In framework of theorem 1 it is really the choice of estimates $\hat{\theta}_0$, $\hat{\theta}_1$ and loss function. Let $X = (X_1, X_2, \dots, X_n)$, where X_i , $i = 1, \dots, n$, are fatigue lives of aircraft in (full-scale) laboratory test, $Z = \min(Y_1, Y_2, \dots, Y_m)$, where Y_j , $j = 1, \dots, m$, are fatigue lives of aircraft in operation, $F_{X_i}(t) = F_{Y_j}(t)$, $i = 1, \dots, n$, $j = 1, \dots, m$; p - allowed probability of failure in operation of at least one aircraft.

In application to the problem of required SL confirmation, when τ^* is required SL, we are interested in increasing of probability that $\tau(x) = \tau^*$. It is something similar to increasing of power of some test in testing some statistical hypothesis.

In application to the problem of some SL nomination we should get the maximum of expectation value of $\tau(X)$ provided that reliability requirements are met, it is if $\tau(X)$ is a p -bound for Z . To study the optimality of $\tau(x)$ we can use the Jensen's inequality. This inequality says that the function of complete sufficient statistics, which is unbiased estimation of its own mathematical expectation, provides the minimal risk if the correspondent loss-function is convex. Consider the simplest case, when θ_1 is a known parameter. Let $\theta_i = \theta_0 + t\theta_1$ is some quantile. Random variable $\hat{\theta}_i = \tau(x) = \hat{\theta}_0 + t\theta_1$ is unbiased estimate of its own expectation (which in general *-case is not equal to θ_i). In problem under question the function $F_z(\tau)$ can be considered as the loss-function. Then the expectation $E_x \{F_z(\hat{\theta}_i)\} = P(Z < \tau(X))$ is the risk function. For normal and the smallest extreme value (sev) distributions of Y_j $j = 1, \dots, m$, $F_z(\tau)$ is convex (and increasing one) if its value is small enough and we have minimum of $E_x \{F_z(\hat{\theta}_i)\} = P(Z < \tau(X)) = p$ at the fixed expectation value of $\hat{\theta}_i = \tau(X)$, if $\tau(x)$ is a function of sufficient statistic. And, on the contrary, if $\tau(x)$ is a function of sufficient statistic and $P(Z < \tau(X)) = p$ then we have maximum of expectation value of $\tau(X)$ if p is small enough and probability $P(\tau(X) < c)$ is high enough for such c , that $F_z(z)$ is

convex if $z < c$. For example, for normal distribution $\Phi(z)$ is convex if $z < 0$. The generalization of the Jensen's inequality for the case of multivariate sufficient statistic can be found in [7].

For the case when sufficient statistics coincides with the sample itself (for example, Weibull's or the smallest extreme value (sev) distribution usually for prediction interval the Monte Carlo (MC) method is used [10]. Here we show that for the problem of p.f. p-bound, $\tau(x)$, calculation analytic solution can be found using Bayes-fiducial (BF) approach.

3.3. Bayes-Fiducial Approach for P.F. P-bound Calculation

Let the problem is to estimate p-quantile $\tau_p(\theta)$ for cdf $F_Z((x-\theta_0)/\theta_1)$ and loss function $L(\tau_\theta(\theta), \tau_x(x)) = (F_Z((\tau_p - \theta_0)/\theta_1) - F_Z((\tau_x(x) - \theta_0)/\theta_1))^2 = (p - F_Z((\tau_x(x) - \theta_0)/\theta_1))^2$ when we have sample $x = (x_1, x_2, \dots, x_n)$ from cdf $F_X((x - \theta_0)/\theta_1)$.

Let us denote by $\tau_x(x, p)$ the solution of BF equation, corresponding to the considered loss function

$$E_{\tilde{\theta}}\{F_Z((\tau_x(x, p) - \tilde{\theta}_0)/\tilde{\theta}_1)\} = p, \quad (17)$$

where $\tilde{\theta} = (\tilde{\theta}_0, \tilde{\theta}_1)$, r.v. $\tilde{\theta}_0, \tilde{\theta}_1$ have fiducial distribution. Here $E_x(f(X))$ is expected value of $f(X)$ in accordance with cdf of X .

We can simplify solution of Eq. 8. Instead of vector $x = (x_1, \dots, x_n)$ without loss of information we can consider vector $\varpi = (\hat{\theta}_0, \hat{\theta}_1, w_1, \dots, w_{n-2})$, where $\hat{\theta}_0, \hat{\theta}_1$ are correct parameter estimates (see (15)), $w_i = (x_i - \hat{\theta}_0)/\hat{\theta}_1$, $i = 1, \dots, n - 2$. Then conditional fiducial distribution (at the fixed invariant (w_1, \dots, w_{n-2})) of random variables $\tilde{\theta}_0, \tilde{\theta}_1$ can be defined in the following form [3]

$$f_{\tilde{\theta}_0, \tilde{\theta}_1 | w_1, \dots, w_n}(s_0, s_1) = h \frac{\hat{\theta}_1^{n-1}}{s_1^{n+1}} \prod_{i=1}^n f\left(\frac{\hat{\theta}_0 + \hat{\theta}_1 w_i - s_0}{s_1}\right) ds_0 ds_1,$$

where h is just normalization factor. (Note: $w_{n-1}, w_n, w_i = (x_i - \hat{\theta}_0)/\hat{\theta}_1$, are functions of vector ϖ).

If in (17) we use new notations:

$$U_0 = (\hat{\theta}_0 - s_0)/\hat{\theta}_1, \quad U_1 = \hat{\theta}_1/s_1, \quad \tau(x, p) = (\tau(x, p) - \hat{\theta}_0)/\hat{\theta}_1,$$

then instead of (17) we get equation

$$E_{w_1, \dots, w_n} E_{U_0 | U_1 | w_1, \dots, w_n} \left(F((\tau(x, p) - U_0)/U_1) \right) = p, \quad (18)$$

where random variables U_0, U_1 has conditional pdf

$$f_{U_0, U_1 | w_1, \dots, w_n}(u_0, u_1) = h_w u_0^{n-2} \prod_{i=1}^n f(u_0 + w_i u_1), \quad (19)$$

where h_w is just normalization factor which depends only on invariant vector $w = (w_1, \dots, w_{n-2})$.

If $\tau(x, p)$ is solution of the equation

$$E_{U_0 | U_1 | w_1, \dots, w_n} \left(F((\tau(x, p) - U_0)/U_1) \right) = p, \quad (20)$$

then

$$\tau_x(x, p) = \hat{\theta}_0 + \tau(x, p) \hat{\theta}_1 \quad (21)$$

is solution of Eq. (20) and Eq.(17) because equation (20) takes place for every vector $w = (w_1, \dots, w_{n-2})$, cdf of which does not depend on $\theta = (\theta_0, \theta_1)$. So if (20) is true, consequently (17) is true also.

It is very important that $\tau(x, p)$ in (20) does not depend on the value of $\theta = (\theta_0, \theta_1)$ and for solution of this equation we can set $\theta_0 = 0, \theta_1 = 1$. If $\hat{\theta}_0, \hat{\theta}_1$ have the structures defined by (15), then probability $P(Z < \tau(X, p))$ does not depend on $\theta = (\theta_0, \theta_1)$ and we can find p_1 for which

$$P(Z < \tau(X, p_1)) = p.$$

So $\tau_x(x, p_1)$ is p-bound for random variable Z.

As it is easy to see (see [3; p. 84]) the pdf (19) is conditional pdf of $\hat{\theta}_0, \hat{\theta}_1$ at the fixed $w = (w_1, \dots, w_{n-2})$ for the case when $\theta_0 = 0, \theta_1 = 1$. This means that the values of p_1 and p coincide.

It is very important also that result does not depend on the choice of the type of correct statistics $\hat{\theta}_0, \hat{\theta}_1$ (see eq. 23.a and 23.b), because vector $x = (x_1, \dots, x_n)$ and vector $\varpi = (\hat{\theta}_0, \hat{\theta}_1, w_1, \dots, w_{n-2})$ have one-one mapping at any choice of correct statistics.

3.3.1. Example 1. P-bound for Lognormal Distribution

Let r.v. T have a lognormal distribution and $t = (t_1, t_2, t_3) = (45\ 952, 54\ 143, 65\ 440)$ is the sample from the same distribution. Then r.v. $X = \log(T)$ has a normal distribution $N(\theta_0, \theta_1^2)$ and $x = (x_1, x_2, x_3) = (10.735\ 10.899\ 11.089)$ is the sample from this distribution. The problem is to calculate the p.f. p-bound for independent r.v. $Z = \min(Y_1, \dots, Y_m)$, where r.v. $Y_i, i = 1, \dots, m$, has the normal distribution $N(\theta_0, \theta_1^2)$ also. We consider here just the case, when $m = 1$, because for this case there is a general analytical solution (see, for example, [3; p. 172])

$$\tau(x) = \hat{\theta}_0 + \hat{\theta}_1 t_{n-1, p} (1 + 1/n)^{1/2}, \quad (22)$$

where

$$\hat{\theta}_0 = \bar{x}, \quad \hat{\theta}_1 = (\sum (x_i - \bar{x})^2 / (n-1))^{1/2}$$

are estimates of expected value and standard deviation, $t_{k, q}$ is q-quantile from Student's distribution with k degree of freedom. So we can make comparison of this solution with the solution which we get by using new approach.

For considered data, using equation (22) for $p = 0.01$ we calculate $t_{st} = \exp(\tau(x)) = 13\ 162$, which is the value of p-bound for r.v. T on the base of observations (t_1, t_2, t_3) .

Now let us consider the new approach. For normal distribution the conditional pdf has the following form

$$f_{U_0, U_1 | w_1, \dots, w_n}(u_0, u_1) = h_w u_0^{n-2} \prod_{i=1}^n \varphi(u_0 + w_i u_1),$$

where $\varphi(x) = \exp(-x^2/2)/(2\pi)^{1/2}$. After transformation the equation (11) has the following form

$$1 - a(\tau, \bar{z}, D_z) / \Gamma((n-1)/2) = p,$$

where

$$a(\tau, \bar{z}, D_z) = \int_0^\infty u^{(n-3)/2} \exp(-u) \Phi\left((2u/D_z(n+1))^{1/2}(\bar{z} - \tau)\right) du,$$

$$\bar{z} = \sum_{i=1}^n z_i / n, \quad D_z = \sum_{i=1}^n (z_i - \bar{z})^2 / n,$$

$\Gamma(\cdot)$ is gamma function, $\Phi(\cdot)$ is cdf of standard normal distribution.

Consider two types of statistics $\hat{\theta}_0, \hat{\theta}_1$, which for the considered data have the following values:

$$a) \hat{\theta}_0 = \bar{x} = 10.908, \quad \hat{\theta}_1 = (\sum (x_i - \bar{x})^2 / (n-1))^{1/2} = 0.177, \quad (23.a)$$

$$\text{b) } \hat{\theta}_0 = x_{1,n} = 10.735, \hat{\theta}_1 = x_{n,n} - x_{1,n} = 0.354, \quad (23.b)$$

where $x_{i,n}$ is i -th order statistics of vector $x = (x_1, \dots, x_n)$.

In case a) we have $\tau^0 = -7.889$, in case b) we have $\tau^0 = -3.560$.

Corresponding values of p-bound for r.v. T on the base of observations (t_1, t_2, t_3) are as follows:

$$t_a = \exp(\tau(x)) = 13\,523, t_b = \exp(\tau(x)) = 13\,050.$$

It seems that the difference between t_a, t_b and $t_{St} = 13\,162$ is produced only by the problem to get the required calculation accuracy.

3.3.2. Example 2. P-bound for SEV and WEIBULL'S distribution

Let we have the same sample $t = (t_1, t_2, t_3) = (45\,952, 54\,143, 65\,440)$ or $x = (x_1, x_2, x_3) = (10.735, 10.899, 11.089)$ but r.v. T has a Weibull's distribution and, correspondingly $X = \log(T)$ has distribution of the smallest extreme value (sev) with cdf $F_x(x) = 1 - \exp(-\exp((x - \theta_0)/\theta_1))$. In this case the equation (20) has the following form

$$1 - a(\tau, \bar{z}, D_z) / b(\bar{z}, D_z) = p,$$

where

$$a(\tau, \bar{z}, D_z) = \int_0^\infty u^{(n-2)} \left(\exp(-u \sum_{i=1}^n z_i) / (\sum_{i=1}^n \exp(uz_i) + m \exp(u\tau))^n \right) du,$$

$$b(\bar{z}, D_z) = \int_0^\infty u^{(n-2)} \left(\exp(-u \sum_{i=1}^n z_i) / (\sum_{i=1}^n \exp(uz_i))^n \right) du,$$

$$\bar{z} = \sum_{i=1}^n z_i / n, D_z = \sum_{i=1}^n (z_i - \bar{z})^2 / n.$$

For $m = 1, p = 0.01$, using statistics (23.a) we get $\tau^0 = -11.929$, using statistics (23.b) we get $\tau^0 = -5.424$. Corresponding values of p-bound for r.v. T on the base of observations (t_1, t_2, t_3) are as follows:

$$t_a = \exp(\tau(x)) = 6\,616, t_b = \exp(\tau(x)) = 6\,752.$$

For $m = 500, p = 0.2$ using statistics (23.a) we get $\tau^0 = -12.889$, using statistics (23.b) we have $\tau^0 = -5.970$. Corresponding values of p-bound for r.v. T on the base of observations (t_1, t_2, t_3) are:

$$t_a = \exp(\tau(x)) = 5\,584, t_b = \exp(\tau(x)) = 5\,568.$$

Again, it seems that the difference between t_a and t_b is produced only by the problem to get required calculation accuracy.

Considered data really were considered in several papers and for $m = 500, p = 0.2$ Lowless (1973) obtained prediction limit of 5623, Mee and Kushary (1994) – 5225. The Mann and Saunders (1969) result was only 766. For these calculations the Monte Carlo method was used [8].

4. Using Bayes-Fiducial Method for SL Nomination with Optimality Criterion of Economics

Let the income of aircraft successful service during time t is equal to t but in case of failure the loss is equal to some negative value $-b$, where b is some large positive value. Then the income of one aircraft service, r.v. U , is defined by formula

$$U = \begin{cases} t_{SL}, & \text{if } T > t_{SL}, \\ T - b, & \text{if } T \leq t_{SL}, \end{cases}$$

where T is random fatigue life, t_{SL} is some SL.

Expectation value of U

$$u(t_{SL}, \theta, b) = \int_0^{t_{SL}} (t - b) dF_T(t, \theta) + t_{SL} (1 - F_T(t, \theta)),$$

where $F_T(t, \theta)$ is c.d.f. of T .

In general case maximum of $u(t_{SL}, \theta, b)$ is reached at t_{SL}^* , which is the root of the equation:

$$bf_T'(t)/(1 - F_T(t, \theta)) = 1.$$

For normal distribution of $X = \ln T$ it can be written in the following way

$$\theta_0 = t_{SL}^* - \theta_1 \lambda^{-1}(t_{SL}^* \theta_1 / b),$$

where $\lambda(z) = \varphi(z)/(1 - \Phi(z))$ is failure rate function for standard normal distribution, $\lambda^{-1}(\cdot)$ is inverse function. This equation allows very easy to get θ_0 as function of t_{SL}^* at the fixed θ_1 and then to find the inverse function:

$$t_{SL}^* = S^*(\theta_0, \theta_1, b).$$

For $b = 346\,000$, $\theta_1 = 0.346$ and $\theta_0 = 9.948$ we have: $t_{SL}^* = 7936$ (flights). It is of interest to note that this value corresponds to the failure probability equal to 0.0026. This can be interpreted in the following way. The failure of 2.6 aircraft (in flight) from 1000 aircraft can be considered as equivalent to the loss of 346000 hours of service time or loss of $346000/7936 = 43.6$ aircraft (on the ground) of this types (the value $t_{SL}^* = 7936$ can be considered as the price of one aircraft of this type). Or in other words, failure of one aircraft (in flight) is equivalent to loss of $43.6/2.6$ (approximately 16) aircraft of the same type (on the ground).

But we do not know the parameters of c.d.f. of T and should estimate them using fatigue test data. Usually maximum likelihood estimate is considered as most appropriate. We show here that for considered problem the offered by a Bayes-fiducial approach is much more appropriate.

In accordance with Bayes's approach the parameter θ_0 is r.v. For the case of airframe it can be interpreted in the following way. Design stress analysis of an airframe should meet some standard requirements (FAR, ...). These requirements in fact define only some mean value of θ_0 but of course, in every case there are some "occasional mistakes" and we have some specific (random) value of θ_0 for every aircraft type. And then there is a scatter of r.v. X (specific random fatigue life of some specific aircraft) at this random θ_0 . The parameter θ_1 is function of technology level, and if one is not changed, then the parameter θ_1 is not changed as well. So we suppose that θ_1 is known constant but θ_0 is random variable, $\tilde{\theta}_0$. Let $\pi(\theta_0)$ is a priori distribution density for $\tilde{\theta}_0$. Then c.d.f. of r.v. X will be

$$\tilde{F}_X(x) = \int_{-\infty}^{\infty} F_X((x - \theta_0) / \theta_1) \pi(\theta_0) d\theta_0.$$

It is well known, that if θ_1 is constant but r.v. $\tilde{\theta}_0$ has normal distribution with known both mean τ_0 and standard deviation τ_1 , then distribution of X will be again normal with mean value τ_0 and standard deviation $((\tau_1)^2 + (\theta_1)^2)^{1/2}$. In this case t_{SL} again will be defined by equation (1), but parameter θ_1 should be replaced by $\theta_{1r} = ((\tau_1)^2 + (\theta_1)^2)^{1/2}$.

In fact we do not know a priori distribution of $\tilde{\theta}_0$. For this case FB approach is offered. Instead of posterior distribution of $\tilde{\theta}_0$ we offer to use fiducial distribution. In considered case fiducial distribution of $\tilde{\theta}_0$ again is normal with mean \bar{x} and standard deviation $\theta_1/n^{1/2}$. Then for the purpose of calculation t_{SL} we again can use the same equation (1), but θ_0 , θ_1 should be replaced by $\hat{\theta}_0 = \bar{x}$ and $\theta_1 (1 + 1/n)^{1/2}$ correspondingly. So using sample $x = (x_1, \dots, x_n)$, result of full-scale fatigue test, in case of ML approach the nominated SL is equal to $S^*(\bar{x}, \theta_1, b)$, but for BF approach $t_{SL}(x) = S^*(\bar{x}, \theta_1 (1 + 1/n)^{1/2}, b)$. By the use of Monte Carlo method for $\theta_0 = 9.948$, $\theta_1 = 0.346$, $b = 346,000$ we have got that the expectation value of r.v. U_x is equal to 2310, 4122, 5571, 6904 for BF approach but it is equal to -8624, 809, 4422, 6935

for ML approach for the same sample sizes $n = 1, 2, 4, 100$. We see that for small n the expectation value of r.v. U_x is much more for BF than for ML approach.

Conclusions

BF approach has the following advantages:

1. As in a case of using a maximum likelihood (ML) estimates BF solution is always a function of sufficient statistics, but in contrast to ML the BF solution takes into account the specific loss function.
2. By contrast with Bayes's decision it does not need a priori distribution of unknown parameters.

It is given approximate analytical solution of the problem to get the maximum of expected value of SL is given of economics optimality criterion, it is shown also that for the considered type of loss function the BF approach is more preferable than the direct use of ML estimates. Numerical examples are given.

For the distributions with location or scale parameters it is shown that the BF point estimates of these parameters are equal to the corresponding Pitmen's estimates. BF approach can be used also for solution of the problem to get unbiased estimate as function of sufficient statistics when the sufficient statistics coincides with the sample itself.

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ESTIMATION OF RISK INSURANCE IN PRACTICE OF WORK OF ACTUARIALS

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The problem of estimating various probabilistic characteristics of risk in activity of actuaries of the insurance company is considered in the article. The model of collective risk is discussed more detailed. As well as in models of individual risk, in models of collective risk ruin is defined by total payments S of the insurance company of kind $S = Y_1 + Y_2 + Y_3 + \dots + Y_v$, and the probability of ruin of the company are defined as $P(Y_1 + Y_2 + Y_3 + \dots + Y_v > U)$, where U – size of actives of the company. Exact analytical estimation of this probability for any distributions of random variables v and Y_v are far not always possible to receive. In the article is offered and numerically realized the algorithm of an estimation of probability of ruin of the insurance company under conditions of model of collective risk at use practically any distribution of size of damage and trivial discrete (non-negative integer) distribution of the number of insurance cases.

Keywords: *statistics, insurance, estimation, collective risk model*

1. Introduction

The estimation of financial risk of the insurance company begins with a choice and construction of reasonable mathematical models. In models of collective risk which here are discussed, the basic characteristic of a portfolio of contracts of insurance is not the number of concluded contracts N , but total number v of insurance cases for the analysed period. It is clear that v is a random variable, and random variables of damage owing to insurance cases $Y_1, Y_2, Y_3 \dots Y_v$ believe independent and equally distributed. As well as in models of individual risk, in models of collective risk ruin is defined by total payments S of the insurance company of kind $S = Y_1 + Y_2 + Y_3 + \dots + Y_v$. The probability of ruin of the company is defined as

$$P(Y_1 + Y_2 + Y_3 + \dots + Y_v > U), \quad (1)$$

where U – size of actives of the company.

2. Main Results

Knowledge of distribution of number of insurance cases and sizes of individual damage Y_v allows establishing many characteristics of risk in activity of the insurance company. However it is possible to make it far not always and often depends on distributions which participate in model. Let's consider, being based on [1], for example, a case when distribution of size of individual damage of the insurance company has normal distribution with known parameters μ, σ^2 , those $Y \sim N(\mu, \sigma^2)$. Thus discrete not negative distribution of number of insurance cases is set by the following table:

Table 1. Distribution of number of insurance cases

N	$P(N = n)$
0	0.5
1	0.2
2	0.2
3	0.1

Value $S_0 = 100$, which characterizes actives of the insurance company, is known as well. The problem of definition of probability $P(S > S_0)$ is put, i.e. probabilities when total damage in all insurance cases will exceed actives of the company, as will lead to ruin at last. The decision of this problem can be made on the basis of the formula of full probability of a kind

$$P(S > S_0) = \sum_{i=0}^n P(N = i) \cdot P(X^{*i} > S_0), \quad (2)$$

where S – total damage, S_0 – a preset value describing actives of the company, X^{*i} – distribution of the sum of individual damage at number composed i .

Coming back to a case of normal distribution $N(\mu, \sigma^2)$, it is possible to find simply enough distribution r.v. $X^{*i} = X_1 + X_2 + \dots + X_i$, which has distribution $N(i \cdot \mu, i \cdot \sigma^2)$.

At concrete values $\mu = 120$, $\sigma^2 = 9$, $S_0 = 100$ the formula (2) turns to the formula of a kind

$$\begin{aligned} P(S > 100) &= \sum_{i=0}^3 P(N = i) \cdot P(X^{*i} > 100) = \\ &= \sum_{i=0}^3 P(N = i) \cdot P(X^{*i} > 100) = 0.5 \cdot 0 + 0.2 \cdot P(X > 100) + 0.2 \cdot P(X^{*2} > 100) + 0.1 \cdot P(X^{*3} > 100). \end{aligned}$$

Clearly that r.v. X has distribution $N(120, 9)$, and $X^{*2} = X_1 + X_2 \sim N(240, 18)$,

$$X^{*3} = X_1 + X_2 + X_3 \sim N(360, 27).$$

Therefore

$$P(S > 100) = 0.2 \cdot P\left(Z > \frac{100 - 120}{3}\right) + 0.2 \cdot P\left(Z > \frac{100 - 240}{\sqrt{18}}\right) + 0.1 \cdot P\left(Z > \frac{100 - 360}{\sqrt{27}}\right).$$

Generalization of the considered approach is the finding of probability of excess of total damage S of set constant S_0 in conditions of any distribution of size of damage. Thus our problem consists of the following. Demanded distribution k random variables is offered to be found in the formula (2) numerically, being based on the formula of convolution

$$F_{X^{*k}}^k(x) = \int_0^x F_{X^{*(k-1)}}^{(k-1)}(x-y) f_X(y) dy, \quad (3)$$

where $F_{X^k}^k(x)$ – distribution of the sum k random variables, each of which has density of distribution of damage $f_X(y)$.

For better understanding we shall consider the following example. Let distribution of damage of the insurance company, have a concrete, for example, exponential distribution $f(x) = \lambda \cdot e^{-\lambda x}$ with known parameter λ . Discrete distribution of the number of the insurance cases, set by Table 2 is known as well.

Table 2. Distribution of number of insurance cases for the exponential distribution of damage

n	$P(N = n)$
0	0.45
1	0.25
2	0.10
3	0.08
4	0.12

As well as earlier, it is necessary to define probability of that total losses S in model of collective risk will exceed some preset value C , which characterizes actives of the company. For the decision of a problem under conditions of a random number of insurance cases it is possible to take advantage, as well as in case of normal distribution, representation of a kind

$$\begin{aligned} P(S > C) &= \sum_{i=0}^n P(N = i) P(X_*^i > C) = \\ &= 0.25 \cdot P(X_*^1 > C) + 0.1 \cdot P(X_*^2 > C) + 0.08 \cdot P(X_*^3 > C) + 0.12 \cdot P(X_*^4 > C), \end{aligned} \quad (4)$$

where X_*^i means distribution of the sum of i composed.

Distribution X_i^* is defined by the sum i exponential random variables that gives density of Erlang distribution [3]

$$F(x) = \begin{cases} 0, & x < 0 \\ 1 - \sum_{i=0}^{l-1} \frac{(\lambda * x)^i}{i!} \cdot e^{-\lambda x}, & x \geq 0. \end{cases}$$

Realizing analytical expression (4) with use of the formula for $F(x)$, we can receive required probability of ruin of the company, for example, at $C = 10$ probability $P(S > C) = 0.048$.

Consecutive application recurrent formulas (3) in MatchCad environment for the above described example, is presented in the following program listing.

```

probn(c) :=
  f(x) ← λ · exp(-λ · x)
  F1(x) ← 1 - exp(-λ · x)
  F2(x) ← ∫0x F1(x - y) · f(y) dy
  F3(x) ← ∫0x F2(x - y) · f(y) dy
  F4(x) ← ∫0x F3(x - y) · f(y) dy
  pr ← (1 - F1(c)) · p1 + (1 - F2(c)) · p2 + (1 - F3(c)) · p3 + (1 - F4(c)) · p4
  probn(10) = 0.048
  
```

Figure 1. Realization of the formula (4) in MatchCad environment

Correctness of the decision is provided with concurrence of results both in analytical, and in case of numerical realization that is presented on the schedule for diapason of the values $1 < c < 20$

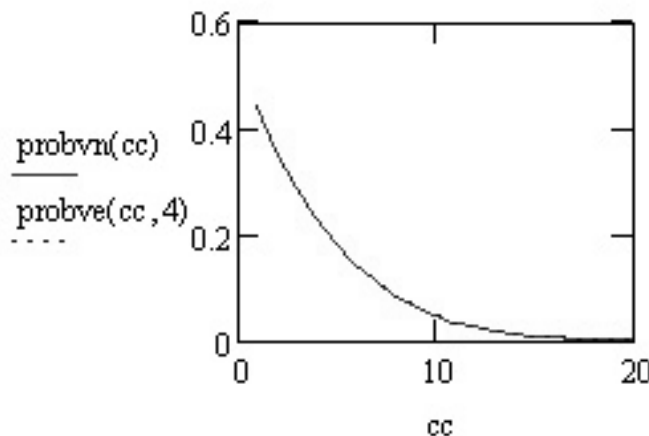


Figure 2. Concurrence of the analytical and numerical decision

The major consequence of the aforesaid is the opportunity to duplicate the considered approach with use of function of convolution for any densities which describe distribution of individual damage. At all there is no necessity to achieve analytical representation of results for the numerical approach becomes equivalent.

We shall consider one more example, where number of insurance cases has discrete distribution which is set by the following Table 3:

Table 3. Distribution of number of insurance cases

n	$p(N = n)$
0	0.6
1	0.2
2	0.2

The density of damage is described by density of distribution of a kind

$$f(x) = \begin{cases} x \cdot e^{-x}, & x \geq 0 \\ 0, & x < 0 \end{cases}.$$

It is necessary to find $P(S > 10)$, that is probability of that total damage S will exceed preset value $S_0 = 10$.

To find required probability, we shall take advantage of the formula considered earlier:

$$p(S > 10) = \sum_{i=0}^2 p(N = i) \cdot p(X^{*i} > 10),$$

where X^{*i} means distribution of the sum i composed.

$$\text{So } P(S > 10) = 0.2 \cdot (1 - F(10)) + 0.2 \cdot (1 - F_2(10)).$$

To calculate required probability, it is necessary for us to find the function of distribution, and also the function of distribution of the sum of two random values. By integration we shall define an analytical sight of functions of distribution:

$$F(x) = \int_0^x x \cdot e^{-x} dx = -x \cdot e^{-x} - e^{-x} + 1.$$

The function of distribution for the sum of two values will be defined by expression of a sight

$$F_2(x) = \int_0^x \int_0^x F(x-y) \cdot y \cdot e^{-y} dy = 1 - e^{-x} - x \cdot e^{-x} - \frac{x^2 e^{-x}}{2} - \frac{x^3 e^{-x}}{6}. \quad (5)$$

Having substituted in the formula (5) found functions of distributions, we can define required probability:

$$p(S > 10) = 0.2 \cdot (1 - F(10)) + 0.2 \cdot (1 - F_2(10)) = 0.00217. \quad (6)$$

The same problem is easily solved numerically with the use of recurrent formula (3). Not making comments in details techniques of realization of the formula (3) in MatchCad, we shall mention, that the result will be identical to the previous result (6).

Other important and modern approach of application of the model of collective risk is the model based on formula Panjer [2]. Recommendations and some ideas on introduction of this algorithm in practice of work of insurance firm are briefly discussed here. In particular, we shall result to more exact the formulation of the statement Panjer. Let total payments S of the insurance company are defined by expression of a sight

$$S = Y_1 + Y_2 + Y_3 + \dots + Y_v, \quad (7)$$

where $Y_1, Y_2, Y_3, \dots, Y_v$ – random values of losses owing to insurance cases are independent and equally distributed. Let also a random variable v satisfy to property of recursive of a kind

$$P(v = n) = (a + b/n)P(v = n - 1). \quad (8)$$

Then for model of collective risk of kind $S = Y_1 + Y_2 + Y_3 + \dots + Y_v$, where v has the specified property (8), $Y_i, i = 1, 2, \dots, v$, positive random variables and $f_S(0) = f_v(0)$, it is possible to find distribution r.v. S and thus the formula takes place

$$f_S(r) = \sum_{j=1}^r (a + bj/r) f_Y(j) f_S(r-j), \quad r = 1, 2, \dots \tag{9}$$

where

$f_S(r)$ – density (or probability) random variable S ; $f_Y(y)$ – density (or probability) a random variable of damage Y .

Let's consider application of this formula on a concrete example, thus realization of formula (9) we shall make as program in package MatchCad.

Let's preliminary notice also, that, if a random variable v has, for example, Poisson distribution, property of recursive (9) is easily carried out. Really, let

$$P(v = n) = \frac{\lambda^n e^{-\lambda}}{n!} = (0 + \lambda/n) P(v = n-1), \quad \text{i.e. } a = 0, b = \lambda. \tag{10}$$

Further we shall consider applicability and numerical realization of algorithm on a concrete example [2].

Example 1. The size of damage S has compound Poisson distribution i.e.

$S = Y_1 + Y_2 + Y_3 + \dots + Y_v$, thus value v has Poisson distribution with parameter $\lambda = 12$, and value Y accepts values $\{1, 5, 10\}$ with corresponding probabilities $\{0.2, 0.3, 0.5\}$.

In this case the formula (9) turns to the formula of a kind

$$P(S = r) = \sum_{j=1}^{\min(r,10)} (\lambda * j/r) P_Y(j) P(S = r-j) = \frac{12}{r} \sum_{j=1}^{\min(r,10)} j P_Y(j) P(S = r-j)$$

Below the program in package MatchCad, realizing the specified formula, i.e. distribution r.v. S is presented. S is easily calculated at any r

```

PS_0 := e^-3      fx_1 := 0.2      fx_5 := 0.3      fx_10 := 0.5

prupr (r) :=
  PS_0 ← e^-12
  for j ∈ 1..min(r, 10)
    12 · ∑_{k=1}^j fx_k · k · PS_{j-k}
    PS_j ← —————
              (j)
  sum1 ← PS_r if r ≤ 10
  for j ∈ 11..r if (r > 10)
    12 · ∑_{k=1}^{min(r, 10)} fx_k · k · PS_{j-k}
    PS_j ← —————
              (j)
  sum1 ← PS_r
  
```

Figure 3. Realization of formula Panjer in MatchCad environment for an example 1

Conclusions

In the conclusion we shall notice, that we offer and numerically realize algorithm of an estimation of probability of ruin of the insurance company under conditions of model of collective risk at use practically any distribution of size of damage and trivial discrete distribution of number of insurance cases. The Panjer formula is lead also to numerical realization. The discussed algorithms with its numerical realization can be recommended for introduction in practice of the insurance companies work.

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SELECTION OF CONSTRUCTION PROJECT MANAGERS BY APPLYING COPRAS-G METHOD

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There are a number of attributes and associated sub-factors influencing the matching of managers to construction projects. Attributes and sub-attributes were identified based on a thorough review of the related literature and interviews of management personnel involved in the selection of project managers. Project managers' characteristics are considered to be less important for effective project management. The model is based on multi-attribute evaluation of project managers. The evaluation embraces the identified attributes influencing the process of construction project manager selection. This paper considers the application of grey relations methodology for defining the utility of alternatives, and a multiple criteria method of COMplex PROportional ASsessment of alternatives with Grey relations (COPRAS-G) is offered. In this model, the parameters of the alternatives are determined by the grey relational grade and expressed in intervals. A case study presents the selection of construction project manager. The results obtained show that this method may be used as an effective decision aid in multi-attribute selection.

Keywords: COPRAS, grey relations, manager, selection

1. Introduction

In recent years, the number of construction projects has been growing rapidly. Therefore, it is very important to find the right project managers for such projects. It has become a major task in project implementation. Different projects require different skills and capabilities on the part of the project manager. All stakeholders, consultants, and contractors are looking for a few good project managers available. They are indeed hard to find and even a searching firm is hardly capable of finding the suitable staff even though the target candidate (a good project manager) can practically write his own pay. This paper presents the analysis of matching managers to construction projects.

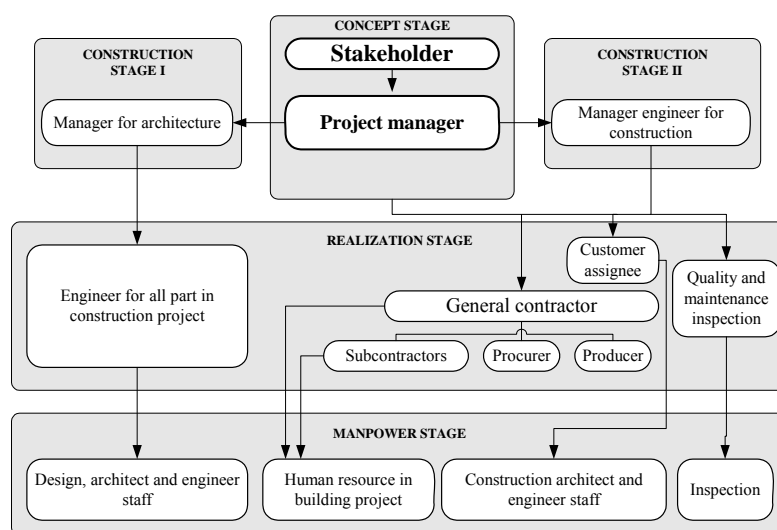


Figure 1. The influence of project manager on construction process

2. The Analysis of Factors Influencing Manager Selection for Construction Projects

The role of construction project manager is very important in the process of construction. The construction process is risky and its success largely depends on the choice of the right project manager. Project manager influence on construction process is shown on Figure 1.

Project management is a complicated task. Every building project differs in place, size, time, cost, etc. The factors influencing project manager selection mentioned in the review of the related literature are presented in Table 1. Collins takes a holistic view of the project manager candidates, which also provides for the addition of any selection criteria deemed relevant to a specific project. The results are scored and in the case of a close score between candidates, the candidates' availability could help swing the decision.

Table 1. Factors taken from the review of the related literature which are relevant to project manager selection [1-12]

No.	FACTORS	RELATED LITERATURE SOURCE											
		[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]
1.	Education level	•			•		•			•			•
2.	Age				•					•			•
3.	Racial stock				•								•
4.	Insufficient time spent in family								•				•
5.	Gender											•	•
6.	Personal skills		•		•	•	•	•	•	•	•	•	•
6.1.	Mobilizing											•	•
6.2.	Verbal communications		•		•		•	•	•	•	•	•	•
6.3.	Coping with situation											•	
6.4.	Delegating authority											•	
6.5.	Political sensitivity											•	•
6.6.	Conflict resolution diplomacy		•						•	•			
6.7.	High self-esteem											•	•
6.8.	Enthusiasm									•	•	•	•
7.	Dependability	•											
8.	Experience (in similar projects)	•		•	•	•		•	•			•	•
9.	Self views			•		•		•	•				
10.	Self relevant goals			•		•		•	•				•
11.	Paperwork								•				
12.	Job stress								•	•			•
13.	Pay									•			•
14.	Problem specification, selection, analysis of alternatives						•	•	•			•	
15.	Conceptual and organizational skills											•	•
15.1.	Planning											•	•
15.2.	Organizing											•	•
15.3.	Strong goal orientation											•	
16.	Project management skills		•	•		•	•	•	•	•	•	•	•
16.1.	Leadership of team		•						•				
16.2.	Developing resource plans		•				•			•			
16.3.	Knowledge of project implementation process		•	•		•	•	•	•			•	
17.	Business skills (markets)		•				•	•					•
17.1.	Strategic thinker		•				•						
17.2.	Ability to carry out the requirement of the customer		•				•						
17.3.	Business case development		•				•						
17.4.	Internal investments							•					
17.5.	Venture capital							•					
18.	Technical skills		•			•	•	•				•	
18.1.	Engineering background		•			•	•					•	
19.	Appropriate computer tools developed						•					•	•
20.	Control						•		•				•
21.	Quality	•					•		•				

While this has merit, it must be noted that using the criteria in the table could result in selecting a project manager for the wrong reasons. Collins states: “The process focuses on the premise that a successful project manager must master two primary skill sets: the project manager's technical skills and leadership skills. Technical skills mentioned by Collins include the following items: integration management, scope management, time management, cost management, quality management, risk management and procurement management. Leadership skills include items like individual influence, integrity, strategic leadership, teamwork and

collaboration, communication and tenacity. He concludes: "As these [project managers] continue to improve, they can become more proficient in the role and assume a broader spectrum of complex projects [2]. Lorda and Brown define self-concept as a broad amalgam of knowledge, experience, self views and possible selves, self relevant goals that individuals see as self relevant or self descriptive [3].

3. Research Methodology

3.1. A Method of Multiple Criteria Complex Proportional Assessment

In order to evaluate the overall efficiency of a project, it is necessary to identify selection attributes, to assess information, relating to these attributes, and to develop methods for evaluating the attributes to meet the participants' needs. Decision analysis is concerned with the situation in which a decision-maker has to choose among several alternatives by considering a particular set of attributes. The COPRAS method presented in the paper uses a stepwise ranking and evaluating procedure of the alternatives in terms of significance and utility degree. This method has been applied to the solution of various problems in construction [13–15].

3.2. A Method of Multiple Criteria Complex Proportional Assessment with Values Expressed in Intervals

In 1982, Deng developed the Grey system theory [16]. In 1988, Deng [17] presented grey decision-making systems. According to Deng [18], the Grey relational analysis has some advantages: it involves simple calculations and requires a smaller number of samples; a typical distribution of samples is not needed; the quantified outcomes from the Grey relational grade do not result in contradictory conclusions to qualitative analysis; the Grey relational grade model is a transfer functional model that is effective in dealing with discrete data.

The idea of COPRAS-G method is based on the real conditions of decision-making and applications of the Grey systems theory. In 2008, Zavadskas *et al.* developed the COPRAS-G method [19]. COPRAS-G method was applied to the selection of the effective walls for a dwelling house [19].

The procedure of using the COPRAS-G method consists in the steps shown on Figure 2.

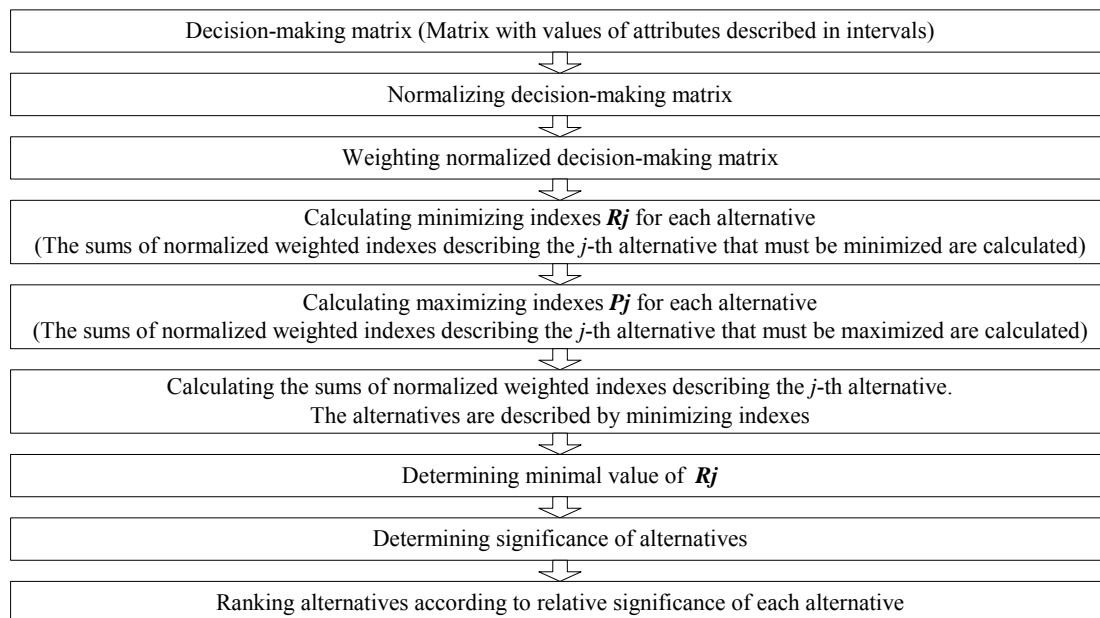


Figure 2. Ranking of alternatives by applying COPRAS-G method

The procedure of using the COPRAS-G method with attribute values expressed in intervals consists in the following steps:

1. Selecting the set of the most important attributes, describing the alternatives;
2. Constructing the decision-making matrix X :

$$X = \begin{bmatrix} [w_{11}; b_{11}] & [w_{12}; b_{12}] & \dots & [w_{1m}; b_{1m}] \\ [w_{21}; b_{21}] & [w_{22}; b_{22}] & \dots & [w_{2m}; b_{2m}] \\ \vdots & \vdots & \dots & \vdots \\ [w_{n1}; b_{n1}] & [w_{n2}; b_{n2}] & \dots & [w_{nm}; b_{nm}] \end{bmatrix}; \quad j = \overline{1, n}; \quad i = \overline{1, m}, \quad (1)$$

where w_{ij} is the smallest value, the lower limit, b_{ij} is the biggest value, the upper limit.

3. Determining weights of the attributes q_j .

4. Normalizing the decision-making matrix \overline{X} . The normalized values of this matrix [20; 21] are calculated as follows:

$$\begin{aligned} \overline{w}_{ij} &= \frac{w_{ij}}{\frac{1}{2} \left(\sum_{j=1}^n w_{ij} + \sum_{j=1}^n b_{ij} \right)} = \frac{2w_{ij}}{\sum_{j=1}^n w_{ij} + \sum_{j=1}^n b_{ij}}; \\ \overline{b}_{ij} &= \frac{b_{ij}}{\frac{1}{2} \left(\sum_{j=1}^n w_{ij} + \sum_{j=1}^n b_{ij} \right)} = \frac{2b_{ij}}{\sum_{j=1}^n (w_{ij} + b_{ij})}; \end{aligned} \quad (2)$$

$i = \overline{1, n}$ and $j = \overline{1, m}$;

In formula (2), w_{ij} is the lower value of the j attribute in the i alternative of the solution; b_{ij} is the upper value of the j attribute in the i alternative of the solution; m is the number of attributes; n is the number of the alternatives compared.

Then, the decision-making matrix is normalized:

$$\overline{X} = \begin{bmatrix} [\overline{w}_{11}; \overline{b}_{11}] & [\overline{w}_{12}; \overline{b}_{12}] & \dots & [\overline{w}_{1m}; \overline{b}_{1m}] \\ [\overline{w}_{21}; \overline{b}_{21}] & [\overline{w}_{22}; \overline{b}_{22}] & \dots & [\overline{w}_{2m}; \overline{b}_{2m}] \\ \vdots & \vdots & \dots & \vdots \\ [\overline{w}_{n1}; \overline{b}_{n1}] & [\overline{w}_{n2}; \overline{b}_{n2}] & \dots & [\overline{w}_{nm}; \overline{b}_{nm}] \end{bmatrix}. \quad (3)$$

5. Calculating the weighted normalized decision matrix \hat{X} . The weighted normalized values \hat{x}_{ij} are calculated as follows:

$$\begin{aligned} \hat{w}_{ij} &= \overline{w}_{ij} \cdot q_j; \\ \hat{b}_{ij} &= \overline{b}_{ij} \cdot q_j. \end{aligned} \quad (4)$$

In formula (4), q_j is the significance (weight) of the j -th attribute.

Then, the decision-making matrix is normalized:

$$\hat{X} = \begin{bmatrix} [\hat{w}_{11}; \hat{b}_{11}] & [\hat{w}_{12}; \hat{b}_{12}] & \dots & [\hat{w}_{1m}; \hat{b}_{1m}] \\ [\hat{w}_{21}; \hat{b}_{21}] & [\hat{w}_{22}; \hat{b}_{22}] & \dots & [\hat{w}_{2m}; \hat{b}_{2m}] \\ \vdots & \vdots & \dots & \vdots \\ [\hat{w}_{n1}; \hat{b}_{n1}] & [\hat{w}_{n2}; \hat{b}_{n2}] & \dots & [\hat{w}_{nm}; \hat{b}_{nm}] \end{bmatrix}. \quad (5)$$

6. Calculating the sums P_j of the attribute values, whose larger values are more preferable (optimisation direction is maximization), for each alternative (each row of the decision-making matrix):

$$P_j = \frac{1}{2} \sum_{i=1}^k (\hat{w}_{ij} + \hat{b}_{ij}). \quad (6)$$

In formula (6), k is the number of attributes which must be maximized (it is assumed that, in the columns of decision-making matrix, the attributes with optimisation direction maximum are placed first and only then the attributes with optimisation direction minimum are inserted).

7. Calculating the sums R_j of attribute values, whose smaller values are more preferable (optimisation direction is minimization), for each alternative (each row of the decision-making matrix):

$$R_j = \frac{1}{2} \sum_{i=k+1}^m (\hat{w}_{ij} + \hat{b}_{ij}); \quad i = \overline{k, m}. \quad (7)$$

In formula (7), $(m - k)$ is the number of attributes, which must be minimized.

8. Determining the minimal value of R_j :

$$R_{\min} = \min_j R_j; \quad j = \overline{1, m}. \quad (8)$$

9. Calculating the relative weight of each alternative Q_j :

$$Q_j = P_j + \frac{R_{\min} \sum_{j=1}^n R_j}{R_j \sum_{j=1}^n \frac{R_{\min}}{R_j}}. \quad (9)$$

Formula (9) can be written as follows:

$$Q_j = P_j + \frac{\sum_{j=1}^n R_j}{R_j \sum_{j=1}^n \frac{1}{R_j}}. \quad (9^*)$$

10. Determining the optimality criterion K :

$$K = \max_j Q_j; \quad j = \overline{1, n}. \quad (10)$$

11. Determining the priority of the project. The greater is the significance (relative weight of alternative) Q_j , the higher is the priority (rank) of the project. The relative significance Q_j of project j indicates the satisfaction degree of the needs of the project participants. In the case of Q_{\max} , the satisfaction degree is the highest compared to the relative significance of other projects.

12. Calculating the utility degree of each alternative. The degree of project utility is determined by comparing the analysed projects with the best project. The values of the utility degree range from 0% to 100% between the worst and the best alternatives. The utility degree N_j of each alternative j is calculated as follows:

$$N_j = \frac{Q_j}{Q_{\max}} 100\%, \quad (11)$$

where Q_j and Q_{\max} are the significances of projects obtained from Eq. (9*).

The decision approach proposed in this section allows the evaluation of the direct and proportional dependence of the significance and utility degree of the alternatives on a system of attributes, weights and attributes values.

4. Case Study: The Selection of Managers for Construction Projects Applying COPRAS-G Method

Based on the review of literature, six key indicators were identified for using in project manager selection. In order to determine the significance of the above indicators, a survey of five experts was conducted. The results obtained are presented in Table 2. The initial decision-making values and intermediary calculation data and the weighted normalized values of the attributes describing the compared alternatives, are also presented in Table 2.

All the attributes are scored. Optimisation directions of the selected attributes are as follows:

- $x_1, x_2, x_3, x_4, x_5 \xrightarrow{\text{optimization direction}} \max ;$
- $x_6 \xrightarrow{\text{optimization direction}} \min .$

The stakeholders of construction projects having the experience in construction project administration of five and more years has been chosen as respondents. The determination of quantitative values of attributes is based on the data provided by similar project managers.

Table 2. Initial decision-making matrix with the attribute values described in intervals and weighted normalized values of the attributes describing the compared alternatives

Alternative No	Personal skills (Score)		Project management skills (Score)		Business skills (Score)		Technical skills (Score)		Quality skills (Score)		Time of decision-making (Score)	
Optimization direction	<i>max</i>		<i>max</i>		<i>max</i>		<i>max</i>		<i>max</i>		<i>min</i>	
Attribute weight – q_j	0.25		0.15		0.12		0.20		0.13		0.15	
	x_1		x_2		x_3		x_4		x_5		x_6	
	w_1	b_1	w_2	b_2	w_3	b_3	w_4	b_4	w_5	b_5	w_6	b_6
Project manager 1	50	60	40	55	10	20	50	70	50	45	30	40
Project manager 2	70	80	60	70	40	45	60	75	70	80	70	60
Project manager 3	60	70	55	70	30	40	70	80	55	65	40	50
	Normalized weighted values of the attributes describing the compared alternatives – matrix \hat{X}											
	\hat{w}_1	\hat{b}_1	\hat{w}_2	\hat{b}_2	\hat{w}_3	\hat{b}_3	\hat{w}_4	\hat{b}_4	\hat{w}_5	\hat{b}_5	\hat{w}_6	\hat{b}_6
Project manager 1	0.064	0.077	0.034	0.047	0.013	0.026	0.049	0.069	0.036	0.032	0.031	0.041
Project manager 2	0.090	0.103	0.051	0.060	0.052	0.058	0.059	0.074	0.050	0.057	0.071	0.062
Project manager 3	0.077	0.090	0.047	0.060	0.039	0.052	0.069	0.079	0.039	0.046	0.041	0.052

According to the calculation results, project manager 2 is the best. Project manager 2 is also the best in terms of the utility degree that equals 100 %. Project manager 3 with the utility degree of 87.66 % is ranked the second. Project manager 1 with the utility degree of 66.93 % is ranked the third. The vector of optimality criterion values $N_j = [65.93; 100; 87.66]$. According to the N_j , the ranks obtained in the procedure of project manager selection are as follows: Project manager 2 \succ Project manager 3 \succ Project manager 1.

Conclusions

- In actual multi-attribute modelling of multi-alternative assessment problems, attribute values referring to the future can be expressed in intervals.
- COPRAS-G is a newly developed method for assessment of alternatives by multiple-attribute values expressed in intervals.
- This approach is intended to support decision-making and to increase the efficiency of the resolution process.
- The method COPRAS-G may be applied to the solution of a wide range of problems associated with the selection of construction project managers by using discrete multi-attribute assessment technique.

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RISK LEVEL EVALUATION OF CONSTRUCTION INVESTMENTS PROJECTS

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Management of investment risk is a usual practice of any investment project or business. The estimation of risk must be carried out at various project stages. In practice it is impossible to avoid not exhaustive and inaccurate information, therefore, unfavourable risky situations occur, the consequences of which can be very damaging to the project or business. Due to close cooperation of the participants of the project the risk occurring in one stage of the project can transfer to other stages and one type of risk can change into another one. This means that chain reaction is characteristic to the risk and it decreases efficiency and safety of any project or business. Various types of risk can be caused by different factors. Classification of risk is determined by efficiency of risk management organisation. Risk classification is understood as risk allotment according to certain features into concrete groups for reaching the set objectives. Conceptually reasoned risk classification allows defining the role of each risk in the total system of all risks. This paper presents methods of multi-attribute comparative analysis (CLARA and SAW methods) of variants of investment classified risks in construction. A practical case of illustrating the methods work is presented.

Keywords: investment, project, risk, multi-attribute analysis

1. Introduction

Decision-making problems in construction management often involve a complex process of decision-making in which multiple requirements and conditions have to be taken into consideration simultaneously. Thus quantitative and qualitative assessments are often required to deal with uncertainty, subjective and imprecise data [1]. Organizational and technological complexity of construction projects generates enormous risks. Investment risk managing theory allows planning investment problems [2, 3]. Managing the risk of investments means presence of an effective control for all procedures in any phase of the project, when varying factors are taking place, which influence the realization of the project. In most cases, any investment project possesses several parameters of efficiency. Conditions of investor works continuously change assessment. For this reason rules of investment projects quality at this moment can be based only on the investor's leadership politics. The principle of quality valuation is based on the intuition and experience of the decision maker [4, 5, 6].

In practical use task of getting expert knowledge in many cases can be formulated like task of classification, because expert intelligence is to sort objects (alternatives, state of object) through classes of decision. Elements formatting some whole to be classified may have different origin. It can be different physical objects, cases of choice or condition of some object [7, 8, 9, etc.]. In recent years, multi-criteria evaluation methods have been widely used in solving both theoretical and practical problems. Actually, these methods are universal. They allow us quantitatively evaluate any complicated object described by a set of criteria. Another advantage of these methods is their ability to combine both maximizing and minimizing attributes expressed in various dimensions into one integrated criterion. The maximizing attributes imply that, if their values are growing, the situation is getting better, while for minimizing attributes this means a worsening situation [1].

A role of a risk valuation during decision-making becomes particularly essential [4, 5, 7, 10, 11, 12]. Different methods of multi-purpose choice of effective resource-saving investment are applied to select alternative, from the certain set of possible variants. For the majority of the problems solutions (LINMAP, TOPSIS, SAW, ZAPROS (ЗАПРОС), ORKCLASS (ОРКЛАСС), PARK (ПАРК), CIKL (ЦИКЛ), etc. [3, 4, 12, 13] the qualitative or quantitative information is used. However, in praxis there are problems for which description the ordinal (serial) information or the information of both characters is necessary at the same time. Practical problems of the building investment project are solved at presence or absence of data on the importance of efficiency parameters.

Various methods for such problems solution are known [1, 2, 11, 12, 14, 15, 16, 17]. Multi-attribute decision-making methods have different characteristics [9, 10, 18]. In this article, we present CLARA and SAW (Ustinovichius *et al.* 2006a; Ustinovichius *et al.* 2008b) methods to multi-attribute comparative analysis of investments risk alternatives in construction in the given work. The offered methods have been successfully applied for the assessment of building projects .

2. Verbal Decision Methods for Classification of Alternatives

Describing the method of prescription of the object to a certain class of decision is complicated because of inverbality of the strategy expert uses. Anyway, these inverbal skills are effectively and promptly used, when expert solves task of classification in his sphere of knowledge. Classification is a very important aspect in decision-making [3, 4, 6, 7, 12]. One of the tasks preparing base for classification is setting of numerous criteria (attributes), which are able to describe any object. Scale of all criteria is formed by setting finite set of possible values [3]. If in certain task scale of values of one or more criteria is infinite, it can be modified to finite by cutting it to finite set of intervals. Finally, on the base of expert knowledge must be organised classification of definite intervals and its components i.e. must be formulated rules according which any object can be prescribed to one of the predefined classes. Classified projects are described by assessing various efficiency criteria that could be both qualitatively and quantitatively expressed.

The problem may be formally represented in the following way:

1. G is the property satisfying the target criterion of the problem.
2. $K = \{K_1, K_2, \dots, K_Q\}$ is a set of evaluating criteria of an object.
3. $S_q = \{k_1^q, \dots, k_{w_q}^q\}$ for $q = 1, \dots, Q$ is a set of estimates based on the criterion K_q , w_q is the number of graduation marks on the scale of the criterion K_q ; the scales are arranged in the order of distinctness of the property G .

4. $Y = S_1 \times \dots \times S_Q$ is the state space of the objects to be classified. Every object is described by a number of estimates based on the criteria K_1, \dots, K_Q . In this way, a set of alternatives $\{y_1, y_2, \dots, y_L\}$ is defined, where

$$L = |Y| = \prod_{q=1}^Q w_q \text{ is cardinality of a set } Y, \text{ (the number of alternatives).}$$

5. $C = \{C_1, C_2, \dots, C_M\}$ is a set of classes to be obtained by breaking down the set Y^a , which should be arranged in the ascending order of distinctness of the property G (in the class C_{n+1} this property is more distinct, while in the class C_n it is less distinct).

6. $Y^a \subseteq Y$ is a set of *admissible* real objects.

Since the estimates based on each criterion are ordered, then, the scale showing the order of classes S_q can be compared with the numerical scale $B^q = \{1, 2, \dots, w_q\}$, where $b_i^q < b_j^q$, if b_i^q is less preferable for a decision maker (DM), then b_j^q . The information of the DM preferences determines the relationships of rigorous preference (or dominance) P^0 in the set Y : $P^0 = \{(y_i, y_j) \in Y \times Y \mid \forall q \in K \ b_i^q \geq b_j^q \wedge \exists q^0 : b_i^{q^0} > b_j^{q^0}\}$ implying that the alternative $x \in Y$ is dominant over the alternative $y \in Y$.

On the other hand, it is known that the classes of solution are ordered for the DM. It means that any alternative belonging to the class $n+1$ is more preferable for the DM than any alternative of the class n . This is shown by the following binary preference relationship in the set Y :

$$P^1 = \{(y_i, y_j) \in Y \times Y \mid y_i \in Y_k, y_j \in Y_l, k > l\}.$$

It can be assumed that none of the vector estimates in the set Y , dominating over the given one, should be referred to a less preferable class. This statement is known as the ‘‘hypothesis of distinctness’’. It can be formally expressed as follows:

$$(y_i, y_j) \in P^0 \Rightarrow (y_j, y_i) \notin P^1. \tag{1}$$

Definition. Breaking down a set of vector estimates Y into the M ordered classes is consistent if the condition (1) is *satisfied* for any $y_i, y_j \in Y$.

Based on the preferences of decision maker, it is *required* to construct a consistent representation of $F: Y^a \rightarrow \{Y_l\}, l = 1, 2, \dots, M$, such that $Y^a = \bigcup_{l=1}^M Y_l; Y_l \cap Y_k = 0$, where $k \neq l, Y_l$ is a set of the vector estimates from Y , assigned to the class C_l .

There are some most frequently used verbal ordinal classification methods. Let us consider several most commonly used methods in more detail.

ORCLASS [3, 12]. This method (Ordinal CLASSification) allows us to build a consistent classification, to check the information and to obtain general decision rules. The method relies on the notion of the most informative alternative, allowing a great number of other alternatives to be implicitly assigned to various classes. ORCLASS takes into account possibilities and limitations of the human information processing system.

Method assessment: The main disadvantage of the method is low effectiveness due to the great number of questions to DM needed for building a comprehensive classification.

DIFCLASS [3, 12]. This method is the first to use dynamic construction of chains covering Y space for selecting questions to DM. However, the area of DIFCLASS application is restricted to tasks with binary criteria scales and two decision classes.

CYCLE [7, 12]. CYCLE (Chain Interactive Classification) algorithm overcomes DIFCLASS restrictions, generalizing the idea of dynamic chain construction to the area of ordinal classification task with arbitrary criteria scales and any number of decision classes. The chain here means an ordered sequence of vectors $\langle x_1, \dots, x_d \rangle$, where $(x_{i+1}, x_i) \in P$ and vectors x_{i+1} and x_i differ in one of the components.

Method assessment: As comparisons demonstrate, the idea of dynamic chain construction allows us to get an algorithm close to optimal by a minimum number of questions to DM necessary to build a complete classification. The application of ordinal classification demonstrates that problem formalization as well as introduction of classes and criteria structuring allows solution of classification problems by highly effective methods.

CLARA [3, 4, 6]. This method (CLAssification of Real Alternatives) is based on ORCLASS, but is designed to classify a given subset rather than a complete set of alternatives (Y space). Another common application of CLARA is classification of full set with large number of exclusions, i.e. alternatives with impossible combinations of estimations. In both cases CLARA demonstrates high effectiveness.

Nowadays, computer software can assist many management techniques like sensitivity analysis and improve the efficiency of the analysing process. Computer simulation packages are thought to be more realistic than the theoretical calculations. The method/program CLARA can be successfully applied to classification of investment projects when the decision classes and the criteria used are thoroughly revised.

3. Simple Additive Weighing (Saw) Method

Stage 1. Decision-making matrix's forming.

$$P = \begin{matrix} & x_1 & x_2 & \dots & x_n \\ \begin{matrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{matrix} & \left[\begin{matrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{matrix} \right] \end{matrix}, \quad (2)$$

where: m – number of alternatives; n – number of attributes.

$i = 1, \dots, m; j = 1, \dots, n$.

We find the best values of each parameter according to the formula (3)

$$\begin{aligned}
 x_j^* &= \min_j x_{ij}, \text{ if preferable is minimum of } j^{\text{th}} \text{ attribute,} \\
 x_j^* &= \max_j x_{ij}, \text{ if preferable is maximum of } j^{\text{th}} \text{ attribute.}
 \end{aligned}
 \tag{3}$$

Stage 2. Performing normalization of the decision-making matrix. The normalization values of normalized decision-making matrix \bar{P} are calculated according to the formula (4)

$$\begin{aligned}
 \bar{x}_{ij} &= \frac{x_{ij}}{\max_j x_{ij}}, \text{ if preferable value of the } j^{\text{th}} \text{ attribute is maximum,} \\
 \bar{x}_{ij} &= \frac{\min_j x_{ij}}{x_{ij}}, \text{ if preferable value of the } j^{\text{th}} \text{ attribute is minimum.}
 \end{aligned}
 \tag{4}$$

Stage 3. Defining weighted normalized matrix \hat{P} . Values of the \hat{P} matrix are calculated multiplying values of \bar{P} matrix by corresponding weights of significances of each attribute:

$$\hat{P} = \begin{bmatrix} q_1 \bar{x}_{11} & q_2 \bar{x}_{12} & \dots & q_n \bar{x}_{1n} \\ q_1 \bar{x}_{21} & q_2 \bar{x}_{22} & \dots & q_n \bar{x}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ q_1 \bar{x}_{m1} & q_2 \bar{x}_{m2} & \dots & q_n \bar{x}_{mn} \end{bmatrix}
 \tag{5}$$

Stage 4. Defining efficiency criterion for each alternative:

$$K_i = \frac{1}{n} \sum_{j=1}^n \hat{x}_{ij}, \quad i = 1, \dots, m; \quad j = 1, \dots, n.
 \tag{6}$$

Optimum variant and ranks of the alternatives are established by size K_i .

$$K^* = \left\{ a_i \left| \max_i \sum_{j=1}^n q_j \bar{x}_{ij} \right. \right\}, \quad i = 1, \dots, m; \quad j = 1, \dots, n; \quad \sum_{j=1}^n q_j = 1.
 \tag{7}$$

After parameters are defined, it is necessary to estimate its weights. The expert method of pair comparison is applied to determine of attributes Saaty for this purpose [19].

It is known, that in a basis of human perception of surrounding reality, the decomposition and synthesis present. While studying any system, the person makes its decomposition to subsystems. Having revealed attitudes between subsystems make its synthesis. Decomposition of a problem is made on the basis of the risk qualifier (presented in the form of Table 1). We make the synthesis by applying SAW method.

To determine a priority it is recommended to use an importance scale, which has been offered by Saaty [19]. The group valuation can be considered enough reliable only in the case, when opinions of interrogated experts are consentaneous. Therefore, investigating the information received from experts statistically, it is necessary to value a coordination of their opinions and to determine the information heterogeneity reasons [20].

4. Case Study: Determination of the Most Suitable Alternative for Investments

Organizational and technological complexity of construction projects generates enormous risks. Investment risk managing theory allows planning investment problems. Managing the risk of investments means presence of an effective control for all procedures in any phase of the project, when varying factors are taking place, which influence the realization of the project. In most cases, any investment project possesses several parameters of efficiency. Conditions of investor works continuously change assessment. Investment projects' quality rules can be based only on the investor's leadership politics at this moment. The quality evaluation principle based on the intuition and experience of the decision maker. We present investments problem to the construction in this article.

Every construction project is unique and each has different risk allocation, capital requirements, management teams, construction methods etc. All these factors can affect project cost, and thus it is necessary to identify and analyse the risks associated with project budget and realization.

4.1. Application of Clara Method to Multi-Attribute Comparative Analysis of Investments Risk Alternatives

After a few iteration series expert (DM) can choose final decisions – Final class decisions (Fig. 3). Detailed description of these groups is provided in the first hierarchy level. Further the classification of the possible investment project risks must be established taking into consideration all levels of their multi-purpose quality descriptions – the second hierarchy level. During that quality of the received results must be checked as well.

Risk level might be established using the composed classificatory, but a lot of criteria must be compared. It is a very difficult task for any person (for expert too), besides it takes a lot of time. Therefore, it is possible to use computer program CLARA (classification of real alternatives). This method (program) allows evaluating constructional investment project according to accurately established classes with the respectful offered criteria for risk size evaluation.

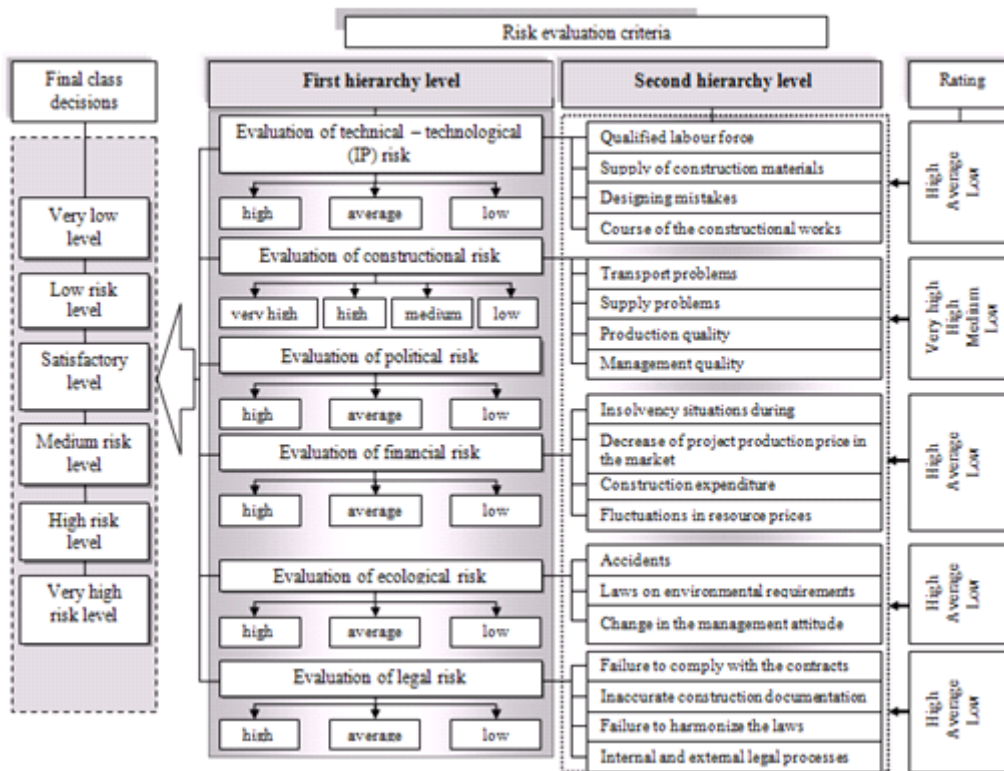


Figure 1. The classificatory of investment risks level evaluation

Classificatory establishment course. Data input into the program.

Stage 1. – For the second hierarchy level evaluation criteria are introduced:

- Criterion 1 – qualified labour force;
- Criterion 2 – supply of construction materials;
- Criterion 3 – designing mistakes;
- Criterion 4 – course of the constructional works.

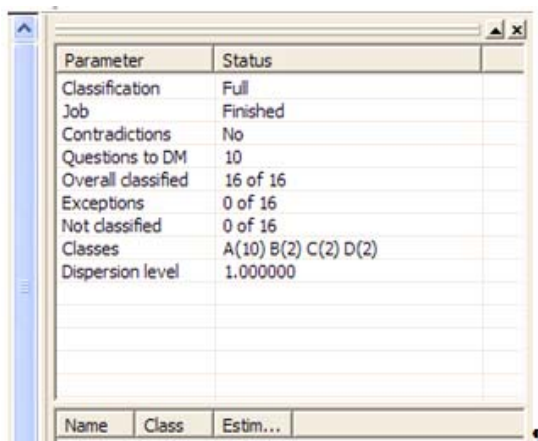
Criteria evaluation classes:

- Class A – high;
- Class B – average;
- Class C – low.

Criteria 1–4 are chosen for evaluation of technical-technological risk. While analysing two projects (2 alternatives) the expert determines the following: where the chosen labour force is qualified enough, where permanent continuous supply of materials will be ensured during the construction, what is the estimated course of works? After the project is analysed, it is determined if there are no mistakes in it. Other stages are input adequately to stage 1.

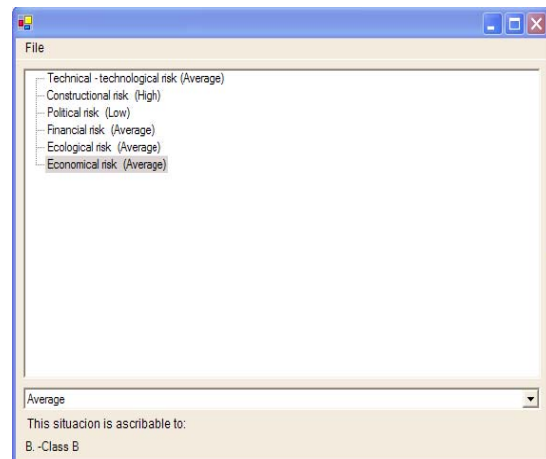
Analytical Management

Classification implementation in the program. After introducing all the criteria that will be taken into consideration while evaluation 2 investment alternatives, the last stage is performed, i.e. the criteria are compared. The comparison is made in the following way: the program selects one evaluation of each criterion and composes their combinations. The expert assigns the available evaluation combination to the respectful class. When the assigning is finished, a transfer is made to the next stage (by pushing the button "NEXT"). Another evaluation combination is provided. This is done up to a moment, until all the combinations are allotted to the respectful class. During the work the expert might make a mistake or change his opinion, therefore, contradictions might appear in his answers. In such case, the program shows a warning that contradictions have occurred and it will ask to confirm the new answer or to change it. After the work is finished, the program saves all the data, performs analysis and shows the number of the given DM questions, the number of eliminated combinations. It also shows how many of evaluates combinations have been allotted to classes A, B or C. Evaluating of all the second hierarchy level criteria have been established in an analogous way.



Parameter	Status
Classification	Full
Job	Finished
Contradictions	No
Questions to DM	10
Overall classified	16 of 16
Exceptions	0 of 16
Not classified	0 of 16
Classes	A(10) B(2) C(2) D(2)
Dispersion level	1.000000

Figure 2. The data of program



File
Technical - technological risk (Average)
Constructional risk (High)
Political risk (Low)
Financial risk (Average)
Ecological risk (Average)
Economical risk (Average)

Average
This situation is ascribable to:
B.-Class B

Figure 3. Data base (I hierarchy level)

Final solving analysis. The final analysis is performed according to the evaluations of the first hierarchy level. After the final analysis is performed, we get evaluation data, i.e. we establish risk levels (Fig. 3).

4.2. Application of Saw Method to Multi-Attribute Comparative Analysis of Investments Risk Alternatives

An example of the implementation of the proposed method is included provided below and will provide the reader with a better understanding of the proposed methodology.

The investment company engaged in investments considered five possible alternatives of investments into construction of different objects. Projects have various volumes of investments and complexity of realization:

1. Very big and very complicated object – A the first alternative;
2. Two complicated objects – A the second alternative;
3. Three objects of average complexity – A the third alternative;
4. Six objects of average complexity – A the fourth alternative;
5. Eleven simple objects – A the fifth alternative.

The aim of the investor is to assess a risk level of projects and to choose one and the most effective project. After some iterations, as final classes of solutions for a valuation investment risk problem there have been chosen (Table 1) the following:

- *The Highest category of quality:* investors all obligations performance is practically assured, the credit line is opened for the investor, and the limit of crediting is established.
- *High category of quality:* the in-depth analysis of company activity and the investment project shows high probability of the borrower (investor) performance of all contracted obligations.

- *Satisfactory category of quality*: the investor can have some difficulties with performance of contracted obligations.
- *Low category of quality*: the investor can have the certain difficulties with performance of treaty obligations.
- *Unprofitable category of quality*: the investor is not capable to make repayment of the basic duty independently.

Realization of risk classification in possible investment projects on all levels of the multi-purpose quality description. Firstly, the risk level at the second level of hierarchy is defined. The valuation of parameters occurs on a scale of risk definition (from 0 up to 9). Further an orderliness of parameters and classification of risks on top-level hierarchies takes place. The final result – by the received quantitative results the most comprehensible project is defined. As a whole the analysis of the investment risk project by the SAW method is carried out in 3 stages. According to the calculations presented in the article, the most comprehensible from possible alternative amount has been chosen the 5th variant, i.e. eleven simple objects.

Table 1. The result table of experts’ interrogation – a matrix of decision-making

			<i>q</i>	<i>a</i> ₁	<i>a</i> ₂	<i>a</i> ₃	<i>a</i> ₄	<i>a</i> ₅	
10	Technological risk	<i>x</i> ₁	Mistakes of designing	0.0411	7	6	5	3	2
11		<i>x</i> ₂	Lacks of technologies	0.0365	5	4	4	2	1
12		<i>x</i> ₃	Erroneous calculation of capacity	0.0350	5	4	3	2	2
13		<i>x</i> ₄	Mistakes of management	0.0328	7	4	5	2	1
14		<i>x</i> ₅	Shortage of the qualified labour	0.0321	7	5	5	3	2
15		<i>x</i> ₆	Failure of building materials delivery	0.0318	4	3	2	2	1
16		<i>x</i> ₇	Non-observance by contractors (subcontractors) of	0.0314	5	4	3	2	1
17		<i>x</i> ₈	Changes in prices of materials and energy carriers	0.0313	4	3	2	2	1
18		<i>x</i> ₉	Increase in charges at a wages	0.0308	5	4	3	2	2
19		<i>x</i> ₁₀	Increase in the prices of equipment	0.0306	3	2	2	1	1
	Construction								
21	<i>A- Period before termination of construction works</i>	<i>x</i> ₁₁	Delays in construction	0.0305	7	6	4	2	2
22		<i>x</i> ₁₂	Default from obligations of the supplier	0.0305	5	4	2	2	1
23		<i>x</i> ₁₃	Stop of civil work on fault of the contractor’s	0.0302	5	4	2	2	1
24		<i>x</i> ₁₄	Risk of building materials shortage	0.0299	3	2	2	1	1
25		<i>x</i> ₁₅	Availability of the contractor	0.0299	3	3	2	2	1
31	<i>B-Period after termination of construction works</i>	<i>x</i> ₁₆	Quality of production	0.0291	3	4	5	6	6
32		<i>x</i> ₁₇	Quality of management	0.0289	4	3	3	5	6
33		<i>x</i> ₁₈	Realization of production	0.0289	5	4	3	5	6
34		<i>x</i> ₁₉	Export – import	0.0288	6	5	4	6	7
35		<i>x</i> ₂₀	Losses	0.0282	4	3	2	2	1
36		<i>x</i> ₂₁	Transport	0.0281	4	3	3	2	3
37		<i>x</i> ₂₂	Deliveries	0.0277	6	5	4	5	6
38	<i>x</i> ₂₃	Incomparability of equipment	0.0277	6	5	3	2	1	
41	Financial risk	<i>x</i> ₂₄	Inconstancy of economy in the country	0.0276	4	3	3	2	2
42		<i>x</i> ₂₅	Inflation	0.0274	5	5	4	4	5
43		<i>x</i> ₂₆	Situation payment delay in what or sphere of	0.0274	4	3	2	1	1
51	Political risk	<i>x</i> ₂₇	Changes in tax system currency transactions	0.0268	4	4	2	2	2
52		<i>x</i> ₂₈	Changes on sales and the customs control	0.0268	6	5	3	2	1
53		<i>x</i> ₂₉	Changes of legislative system	0.0267	6	5	4	3	2
61	Ecological risk	<i>x</i> ₃₀	Lacks of legislative system	0.0267	6	5	4	3	2
62		<i>x</i> ₃₁	Failures	0.0263	5	4	3	4	5
63		<i>x</i> ₃₂	Change of a position of the state on changes in the	0.0249	5	4	3	2	1
71	Legal risk	<i>x</i> ₃₃	Incompatibility of laws	0.0242	6	6	5	4	3
72		<i>x</i> ₃₄	Discrepancies in the documentation	0.0234	5	4	3	2	1
			Optimisation direction for all attributes is minimum						
			<i>K</i> _{<i>i</i>}		1	2	3	4	5
					0.42	0.41	0.57	0.63	0.89
			Ranks of alternatives		4	5	3	2	1

Conclusions

In conditions of market relations, an introduction of technical innovations and acceptance of effective decisions is necessary. Some courageous, not trivial decisions increase risk, however it does not mean, that it is necessary to avoid risk. It is necessary to be able to value the degree of risk and to operate it. The general conceptual approach for managing the investment risk in construction consists of the following stages:

- a) revealing possible consequences of investment activity in a risky situation;
- b) development of measures, which are not supposing, preventing or reducing damage from influence up to the end of not considered risky factors, unforeseen circumstances;
- c) such risk consideration system realization in business, where not only negative probable results can be neutralized or compensated, but also maximum chances of the high income are used.

Investment risk in construction can be evaluated efficiently enough using CLARA method. This method allows classifying all possible constructional investment projects presented by evaluations on the predefined criteria into several accurately defined classes reflecting the project risk level. The algorithm CLARA (Classification of Real Alternatives) is based on the dichotomy of the alternatives chains, beginning with the longest chain.

Criteria of the classificatory and the evaluations are introduced into verbal decision analysis support system CLARA, which allows performing criteria combination classification rather quickly. After all the above mentioned actions have been performed, by the person who wants to evaluate the investment risk it is enough to introduce the respectful evaluations into the composed program data base and the program will provide the result – the risk level.

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KNOWLEDGE MANAGEMENT MODEL FOR CONSTRUCTION PROJECTS

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In the past there has been no structured approach to learning from construction projects once they are completed. At present the construction industry is adapting concepts of knowledge management to improve the situation. In this paper knowledge management benefits to construction industry organizations and projects are discussed.

The main purpose of this paper is to present knowledge management model for construction projects. Paper consists of three parts. In the first part the concept of knowledge management in construction is discussed. In the second part different knowledge management models presented in scientific literature are discussed and compared as well as the new model, developed by the authors, is presented. In the third part, basing on the proposed model, the architecture of Knowledge Based Decision Support System for Construction Projects Management has been created as well as Multiply Criteria Analysis Method COPRAS application for construction decisions support have been discussed.

Keywords: knowledge management, construction projects, knowledge based decision support system, multiply criteria analysis

1. Introduction

In the recent times, construction projects have turned into a more complicated, dynamic and interactive scenario. Project managers are constantly required to speed-up reflective decision-makings on time. Knowledge therefore is noted to be one of the most important resources contributing towards managerial decision-making and enhancing the competitive advantage of organizations carrying out such projects [1, 2].

The construction industry is a workplace that is dominated by heuristics. Construction companies and their personnel refer to carry out their project management tasks based upon their past experiences, rather than following a textbook approach, or established analytical approaches [3]. Indeed the costs of attracting, recruiting, and retaining talented employees are expensive [4, 5]. This is further complicated by the fact that in the coming years, the construction industry is expected to loose a large portion of its skilled and knowledgeable workforce. Conversely, there is no single strategy in place, to handle construction management problems that arise. One of the most effective and powerful tools for strengthening industrial and organizational competition is through systematic identification, in the best practice of knowledge utilization and distribution.

The significance of knowledge management in construction industry is proved and researched extensively in the scientific literature. Indeed, authors present different points of view to knowledge management as well as different knowledge management models. This article covers a wide range of issues, from basic definitions and fundamental concepts, to the role of information technology and different knowledge management models presented in literature.

The main purpose of this research – is to develop knowledge management model for construction projects and to use it for the knowledge based decision support system architecture.

2. The Concept of Knowledge Management

Knowledge has been described as information, which has been used and becomes a part of a person's knowledge-based experience and behavioural patterns [6, 7]. Individuals have different knowledge-based capacity and experience, thus leading to different problem solving approaches and decision-making. When choosing a construction project manager, knowledge and experience are significant [8]. Project managers must therefore be capable of knowing how to use, manage, and utilize such knowledge.

Before specifying the knowledge management (KM) models, the KM concept has to be defined first. KM has however been defined in different ways in scientific literature. According to Qunitas et al. [9], KM means to manage all knowledge continuously to meet various requirements in an organization. Coleman [10] defines KM as an umbrella term for a wide variety of interdependent and interlocking functions consisting of: knowledge creation; knowledge valuation and metrics; knowledge mapping and indexing; knowledge transport, storage and distribution; and knowledge sharing. Gurteen [11] comprehensively defined KM as an emerging set of organizational design and operational principles, processes, organizational structures, applications and technologies that helps knowledge workers dramatically leverage their creativity and ability to deliver business value.

According to Robinson [12], knowledge management relates to unlocking and leveraging the different types of knowledge so that it becomes available as an organisational asset. Implementing KM enables an organisation to learn from its corporate memory, share knowledge, and identifies competencies in order to become a forward thinking and learning organization.

Other authors mentioned more KM benefits to projects management. Kamara et al. [5], Love et al. [13] state that the role of effective management of knowledge is evident in producing innovation, reducing project time, improving quality and customer satisfaction. According to Siemieniuch and Sinclair [14], through knowledge management an organisation's intangible assets can be better exploited to create value, with both internal and external knowledge being leveraged to the benefit of the organisation. In projects, knowledge management can improve communications within teams, and provide more informed knowledge by sharing best practice documents, lessons learned, project management and system engineering methodologies, examples of review packages, and the rationale for strategic decisions. Kaklauskas and Kanapeckiene [15] distinguish such KM benefits as productive information use, activity improvement, intelligence enhancement, intellectual capital storage, strategic planning, flexibility acquisition, best practice gathering, success probability enhancement and productive collaboration. Authors have used the systemized approach to KM definition (see Fig. 1).

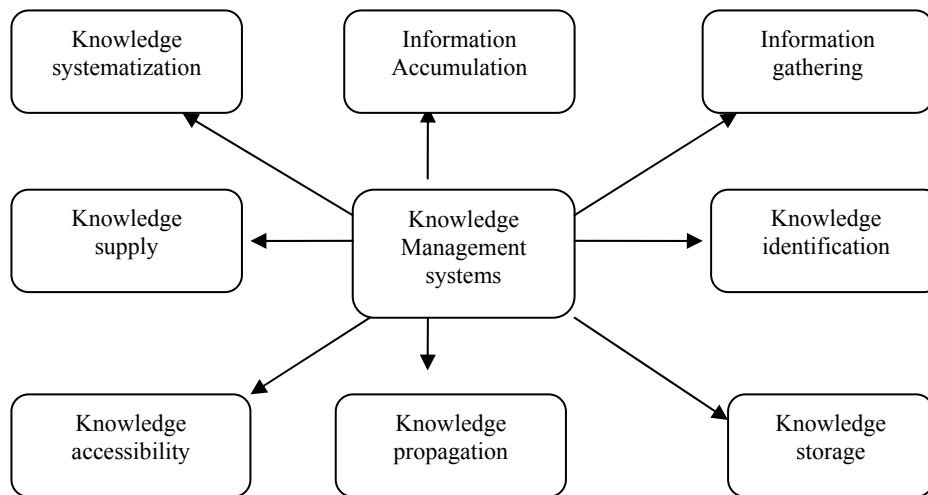


Figure 1. Functions of knowledge management systems [15]

The different definitions of KM in the literature result from the various perspectives and contexts that are specific to the authors and their research fields. Within construction, KM can be difficult to define precisely as there is not a general consensus on a single unified meaning of the concept. However, Egbu [16] explains that knowledge is an important resource for construction organisations due to its ability to provide market leverage and contributions to organisational innovations and project success. The idea of knowledge as a competitive resource within project-oriented industries is a concept shared by numerous authors: Nonaka and Takeuchi [2], Egbu [16], Egbu and Botteril [17], Oltra [18], etc.

The potential benefits of effectively utilising their knowledge has meant that an increasing number of construction companies have identified the need to implement KM initiatives. However, the difficulties associated with understanding and managing organisational knowledge has meant that organisations experience numerous problems in successfully implementing and sustaining their initiatives [16, 18]. Egbu and Botteril [17] state that due to the project-oriented nature of construction organisations, cultural

considerations are important for successful KM. They continue by stating that the short-term, task-focussed work can promote a culture, which inhibits continuous learning.

It can be concluded that though academics and industrial organisations have recognised the need for KM, there can be confusion over specific definitions of knowledge and KM within construction organisations. As a result there is danger that KM initiatives can become misguided and not serve their desired purpose. It is important for the whole organisation to understand what KM is and why it is important. The organisation should take a recognised and accepted generic definition, apply it to their specific context, and tailor it to accommodate specific business objectives. This will require support, agreement and communication from the top. To ensure an alignment with its business objectives and strategies, the organisation should consider the type of work they carry out, their culture, dynamics, politics and practices, as well as the added value that is required from the KM initiative [19].

Basing on these assumptions authors of this article are aiming to develop the generalized and easily adaptive KM model, which is presented further.

3. Development of the Knowledge Management Model for Construction Projects

Different KM models developed by different authors emphasizing various aspects can be found in scientific literature. Most of them are usually activity oriented. Four major dimensions for the process of KM activities presented by Nonaka and Takeuchi [2] and Davenport and Prusak [21] are usually adopted for the general models structure of KM in enterprises. These four dimensions are knowledge creation, knowledge diffusion, knowledge transfer and knowledge inventory.

Maqsood et al. [3] developed the integrated knowledge management, organisational learning and innovation model. This model explains the transformation of the organisation over time by illustrating organisational learning. It shows three transformation stages that are indicative of the transformation process, which is a continuous process: before transformation, transformation, ideal transformed state, e.g. existing knowledge of organization, knowledge after certain learning, knowledge in the organization after further learning.

Korsvold and Russak [21] in the proposed generic model distinguish three necessary arenas for knowledge development, being identified as “collective knowing”, relational knowledge and knowing how. Consequently, the relationship between the three conditions or the knowledge content of the arenas for knowledge creation in constituting a generic model for creating organizational innovation in the operative accomplishment of the building process as a whole is intrinsically dynamic and interdependent. This implies a continuous process of internalisation and externalisation between tacit (embedded knowledge: the common frame of reference as Web-based communicative and reflective device in the operative accomplishment of the building process) and encultured knowledge (encultured: the common frame of reference as shared collective understanding of the building process as a whole).

It should be noticed, however, that not so much of the proposed models are adapted to construction sector.

Teerajetgul and Charoenngam [22] research addressed the concerns of practicing knowledge management in construction projects by examining the relationships between knowledge factors and the knowledge creation process composed of socialization, externalisation, combination, and internalisation. A framework was employed to test these relationships and the empirical evidence supports the relationships. Findings from this study confirmed that three selected factors (IT, incentive, and individual competency) affect the overall knowledge creation process in Thai construction projects. From the research results it can be assumed that KM in construction projects is impossible without IT and human interaction.

Tserng and Li [23] presented more detail framework of knowledge management used in construction projects. Authors distinguished three construction project’s KM spheres, namely, content management, experience management and process management and six management stages:

- problem happening;
- create knowledge;
- share knowledge;
- record knowledge;
- knowledge storage;
- knowledge reused.

Also authors proposed activity based model.

The model presented by authors is developed basing on the synthesis of the above discussed models, indeed it is more concentrated on IT and construction project’s life-cycle as well as decision support (see Fig. 2).

Analytical Management

The given view to KM in construction projects is generalized by distinguishing four the most important knowledge management stages: project information and knowledge gathering, knowledge acquisition, best practice knowledge data base creation and knowledge based decision support for implementation of other projects.

Project information and knowledge gathering as well as knowledge acquisition stages are strongly connected with all construction project life-cycle activities, including: conceptual planning, design, procurement, construction, operation and maintenance. It should be noticed, that the information and knowledge must be gathered from the all different bodies and organizations participating in the project e.g. clients, designers, consultants, contractors, and inspectors because inter- and intra-discipline communication between these distinctive professionals is often problematic. The lack of integration and co-ordination between the industry's distinct professions can be perceived as a major contributory factor to poor project performance [24].

An effective knowledge strategy is required to acquire and manage both explicit and tacit knowledge.

Explicit knowledge is the type of knowledge that is readily available in the organisation in the form of books, procedures and can be appropriately archived for use when required. Indeed, tacit knowledge is embedded in organizational routines and processes and employees heads. It is a very complex type of knowledge. The challenge of knowledge management is to make it explicit through the balanced use of technology, and soft human-related factors like leadership, vision, strategy, reward systems and culture. Researches results revealed the importance of tacit knowledge in relation to organizational performance and achievement of competitive advantage and has further highlighted the relevance of tacit knowledge in the construction industry by considering its intrinsic characteristics [25].

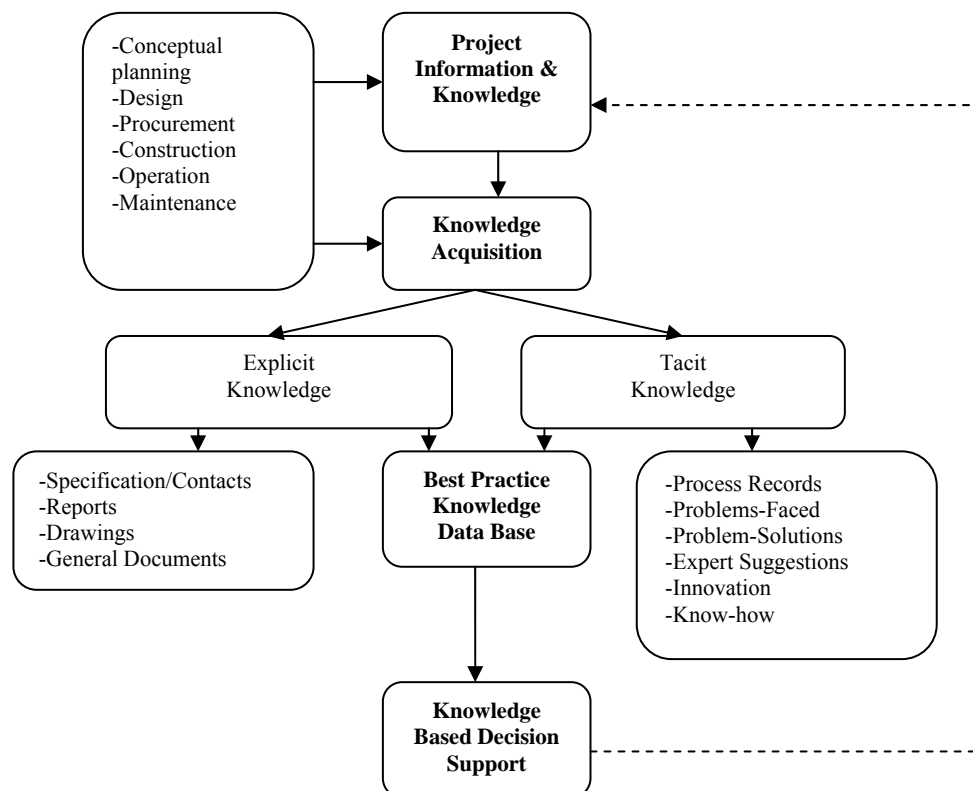


Figure 2. Knowledge management model for construction projects

Tserng and Lin [23] distinguished the main problems indicated in construction phases by acquiring and using tacit and explicit knowledge. According to authors, problems for tacit knowledge are loss of experience, loss of Know-how, problem-solution loss, loss of innovation. Problems connected with explicit information are mainly connected with information saving problems: information can be recorded incompletely or partly.

Analytical Management

The above mentioned problems can be solved by the information technology use and tacit data coding as well as other technology based measures: videos, interviews, etc.

When the knowledge is collected the next step is the best practice data base creation avoiding insignificant or less worthy information. It should be noticed that usually construction projects are not universal. Therefore the standardisation of all project life-cycle phases is needed. Furthermore the data base must be periodically updated for new information and knowledge acquisition.

When the best practice data base is created, the second step is knowledge application and reuse in order to make knowledge based decisions in construction projects. For this purpose authors propose to use computerized knowledge based decision support system, which is discussed in the next chapter.

4. Knowledge Based Decision Support System for Construction Projects Management

Basing on the above discussed knowledge management model for construction projects, the architecture of computerized Knowledge Based Decision Support System for Construction Projects (DSS-CP) can be created (see Figure 3).

DSS-CP consists of a database, database management system, model-base, model-base management system and a user interface.

The DSS-CP database management system allows users to: present information of the general physical and functional state of the building process; present information of the physical state of the building's envelope; calculate the volume of work to be carried out; rationalize the energy consumption of the building; propose the required measures to increase the quality of air and indoor environment and analyse the construction processes scenarios by taking into account the system of criteria.

A module base allows the DSS-CP's user to select the most suitable construction alternatives by comparing the measures that promote the greatest value to all interested bodies and organizations.

The following models of a model-base aim at performing the functions of: a model for developing the alternative variants of a building's enclosures, a model for determining the initial weight of the criteria (with the use of expert methods), a model for the establishment of the criteria weight, a model for the multi-variant design of a building construction alternatives, a model for multiple criteria analysis and for setting the priorities, a model for the determination of a project's utility degree and market value, a model for negotiations.

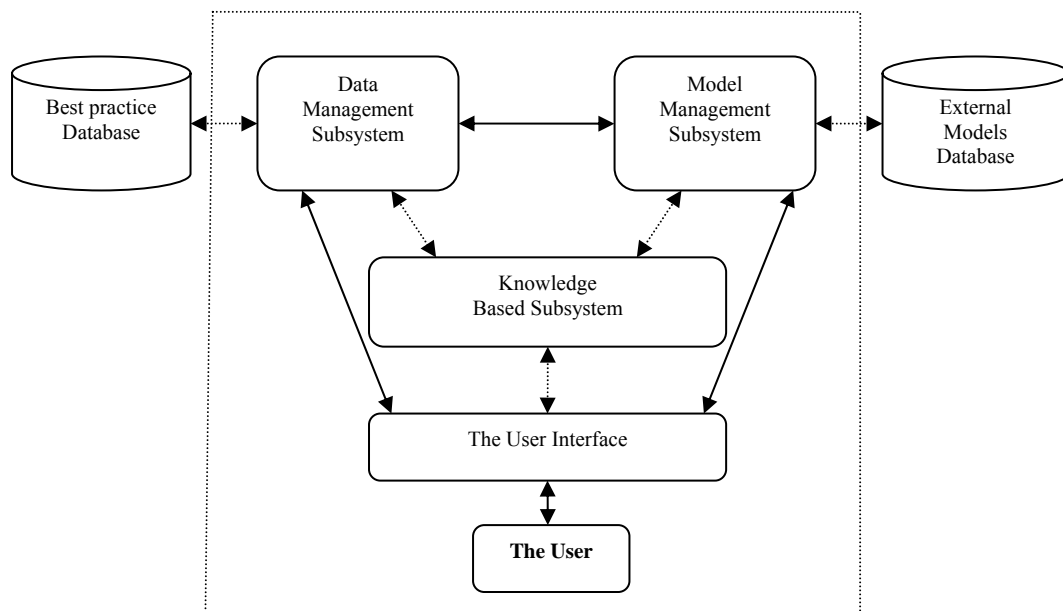


Figure 3. Architecture of Knowledge Based Decision Support System for Construction Projects (DSS-CP)

The best construction alternatives selection in the presented DSS-CP is based on the Complex Proportional Assessment method (COPRAS) [26, 27].

This method assumes direct and proportional dependence of significance and priority of investigated versions on a system of criteria adequately describing the alternatives and on values and significances

of the criteria. The system of criteria is determined and the values and initial significances of criteria are calculated by experts. All this information can be corrected by interested parties (customer, users, etc.) taking into consideration their pursued goals and existing capabilities. Hence, the assessment results of alternatives fully reflect the initial refurbishment data jointly submitted by experts and interested parties.

The determination of significance and priority of alternatives is carried out in four stages.

Stage 1: The weighted normalized decision-making matrix D is formed. The purpose of this stage is to receive dimensionless weighted values from the comparative indexes. When the dimensionless values of the indexes are known, all criteria, originally having different dimensions, can be compared. The following formula is used for this purpose:

$$d_{ij} = \frac{x_{ij} \cdot q_i}{\sum_{j=1}^n x_{ij}}, \quad i = \overline{1, m}; \quad j = \overline{1, n}, \quad (1)$$

where x_{ij} – the value of the i -th criterion in the j -th alternative of a solution; m – the number of criteria; n – the number of the alternatives compared; q_i – significance of i -th criterion.

The sum of dimensionless weighted index values d_{ij} of each criterion x_i is always equal to the significance q_i of this criterion:

$$q_i = \sum_{j=1}^n d_{ij}, \quad i = \overline{1, m}; \quad j = \overline{1, n}. \quad (2)$$

In other words, the value of significance q_i of the investigated criterion is proportionally distributed among all alternative versions a_j according to their values x_{ij} .

Stage 2: The sums of weighted normalized indexes describing the j -th version are calculated. The versions are described by minimizing indexes S_{+j} and maximizing indexes S_{-j} . The lower value of minimizing indexes is better and the greater value of maximizing indexes is better. The sums are calculated according to the formula:

$$S_{+j} = \sum_{i=1}^m d_{+ij}; \quad S_{-j} = \sum_{i=1}^m d_{-ij}, \quad i = \overline{1, m}; \quad j = \overline{1, n}. \quad (3)$$

In this case, the values S_{+j} (the greater is this value (project “pluses”), the more satisfied are the interested parties) and S_{-j} (the lower is this value (project “minuses”), the better is goal attainment by the interested parties) express the degree of goals attained by the interested parties in each alternative project. In any case the sums of “pluses” S_{+j} and “minuses” S_{-j} of all alternative alternatives are always respectively equal to all sums of significances of maximizing and minimizing criteria:

$$S_{+} = \sum_{j=1}^n S_{+j} = \sum_{i=1}^m \sum_{j=1}^n d_{+ij}, \quad (4)$$

$$S_{-} = \sum_{j=1}^n S_{-j} = \sum_{i=1}^m \sum_{j=1}^n d_{-ij}, \quad i = \overline{1, m}, \quad j = \overline{1, n}.$$

Stage 3: The significance (efficiency) of comparative versions is determined on the basis of describing positive alternatives (“pluses”) and negative alternatives (“minuses”) characteristics. Relative significance Q_j of each alternative a_j is found according to the formula:

$$Q_j = S_{+j} + \frac{S_{-\min} \cdot \sum_{j=1}^n S_{-j}}{S_{-j} \cdot \sum_{j=1}^n \frac{S_{-\min}}{S_{-j}}}, \quad j = \overline{1, n}. \quad (5)$$

Stage 4: Priority determination of alternatives. The greater is the Q_j the higher is the efficiency (priority) of the refurbishment alternative.

The analysis of the method presented makes it possible to state that it may be easily applied to evaluating the projects and selecting most efficient of them, being fully aware of a physical meaning of the process. Moreover, it allowed formulating a reduced criterion Q_j which is directly proportional to the relative effect of the compared criteria values x_{ij} and significances q_i on the end result.

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Significance Q_j of project a_j indicates satisfaction degree of demands and goals pursued by the interested parties – the greater is the Q_j the higher is the efficiency of the project.

The degree of project utility is directly associated with quantitative and conceptual information related to it. If one project is characterized by the best comfort ability, aesthetics, price indices, while the other shows better maintenance and facilities management characteristics, both having obtained the same significance values as a result of multiple criteria evaluation, this means that their utility degree is also the same. With the increase (decrease) of the significance of project analysed, its degree of utility also increases (decreases). The degree of project utility is determined by comparing the project analysed with the most efficient project. In this case, all the utility degree values related to the project analysed will be ranged from 0% to 100%. This will facilitate visual assessment of project efficiency.

The formula used for the calculation of alternative a_j utility degree N_j is given below:

$$N_j = \left(Q_j : Q_{\max} \right) \cdot 100\% . \quad (6)$$

Previously discussed DSS-CP architecture concept has been used for Knowledge Based Decision Support System for Buildings Refurbishment, developed in Vilnius Gediminas Technical University by Zavadskas and Kaklauskas (see the fragment on Fig. 4) [6, 15, 28].

 Ltd "Fasadų renovacija" | Ltd "Bairamkavas" | Ltd "Statveksas" | Ltd "Kreivėlė Vilnius" | Ltd "Imparatas" | Ltd "Atrubotas" | Ltd "Alkosta" | Ltd "Naujas Fasadai" |

</thead>
1
 Price | Lt | - | 0.6 | 390425 | 400125 | 394790 | 395275 | 458900 | 354050 | 383150 | 392850 | 41 |2
 Adhesive (glue-d) joint strength (concrete/thermal insulating board) | N/mm² | + | 0.0148 | 0.12 | 0.1 | 0.5 | 0.1 | 0.5 | 0.1 | 0.1 | 0.1 | |3
 Adhesive (glue-d) joint strength (concrete/concrete) | N/mm² | + | 0.0052 | 1.3 | 1 | 1.2 | 1 | 1 | 1.6 | 1.2 | 1.3 | |4
 Thermal conductivity of thermal insulating board | W/mK | - | 0.004 | 0.038 | 0.041 | 0.039 | 0.041 | 0.039 | 0.038 | 0.039 | 0.038 | |5
 Compressive strength of thermal insulating board | kPa | + | 0.013 | 40 | 50 | 40 | 50 | 40 | 50 | 40 | 40 | |6
 Tensile strength of thermal insulating board perpendicular to the surface | kPa | + | 0.016 | 15 | 80 | 15 | 80 | 15 | 15 | 15 | 80 | |7
 Density of thermal insulating board | kg/m³ | - | 0.007 | 150 | 80 | 80 | 80 | 140 | 140 | 150 | 90 | |

Figure 4. The fragment of Knowledge Based Decision Support System for Buildings Refurbishment

The presented system can make up to 100,000 building refurbishment alternative versions, perform their multiple criteria analysis, determine the utility degree, market value and select the most beneficial alternative without human interference.

Basing oneself on the collected information and the BR-DSS it is possible to perform a multiple criteria analysis of the refurbishment project's components (walls, windows, roof, floors, volumetric planning and engineering services, etc.) and select the most efficient versions. After this, the received compatible and rational components of a refurbishment are joined into the projects. Having performed a multiple criteria analysis of the projects in this way, one can select the most efficient projects [15].

Authors believe that these advantages can be achieved also in management of the construction projects of other types.

Conclusions

Knowledge management is the key factor of the successful implementation of construction projects and tasks achievement of interested bodies as well as organizations. Indeed there is no universal concept of knowledge management in construction. It must be developed by each organization individually.

In order to systemize knowledge management for construction projects, authors developed the model consisting by four main stages: project information and knowledge gathering, knowledge acquisition, best

practice knowledge data base creation and knowledge based decision support for other projects implementation. Model shows the integrated view to construction projects life-cycle as well as IT usage. Basing on this knowledge management model for construction projects, the architecture of computerized Knowledge Based Decision Support System for Construction Projects (DSS-CP) has been created.

The presented concept has been already used to create Knowledge Based Decision Support System for Refurbishment projects. The system is based on Multiply Criteria Analysis in applying COPRAS method. Authors believe that system's advantages can be achieved in management of construction projects of other type.

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MODELLING OF THE MULTIPRODUCT INVENTORY PROBLEM

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The given paper considers a multiproduct inventory control model with random demands for goods. The authors propose the model with a fixed period of time between the moments of placing neighbouring orders. The order quantity is determined as the difference between the fixed stock level and the quantity of goods at the moment of ordering. It is assumed that each order may enclose only fixed kinds of products. The considered model is realized by the combination of the analytical and simulation approaches in packages MS Excel and ExtendSim correspondingly. The numerical example of the problem solving is presented. The considered model is of interest because it illustrates a real business situation.

Keywords: inventory control, random demand, analytical method, optimisation, simulation

1. Introduction

The inventory control problems are complex in practice.

Decision-makers need to understand that this complexity depends on their role in business and the way they choose to solve the problems. Modelling of inventory control has always been accepted by the tools, which can provide the description of those business situations, which are difficult to examine in any other way. The appropriate technique must be employed to model a different type of the inventory control system: deterministic or stochastic, single- or multi-period, single- or multi-product, and so on. There must be different ways in which the company can place and receive an order, for example, using direct delivery or using the chain “producer – wholesaler – customer”.

The economic, social and technical characteristics of the problem determine the structure of the constructed model. We have to investigate the stochastic models for different situations characterized by the inventory control systems and there is a set of stochastic models available for solving the inventory control problem [1, 2].

In the given paper a multiproduct inventory control model for n products with random demand and known distributions is considered. We investigate the ordering strategy when each order encloses only fixed kinds of products. In the considered problem the lead time L is constant. As it was shown in many works, the analytical stochastic inventory control model is rather complex. As an alternative to the analytical approach, the authors have used simulation models realized in the simulation package Extend [3]. *The aim* of the given research is to construct the optimal ordering policy for the group of n products building a complex method, which uses the combination of analytical and simulation approaches.

We have to answer three basic questions: the quantity of each product ordering, the frequency of each product ordering, and the structure of the orders.

In the considered approach the authors propose to divide the process of the problem solving into the following stages:

1) search of the problem investigation starting point:

a) Task 1 – approximate solution of an inventory control deterministic problem (deterministic analytical approach);

b) Task 2 – correction of the Task 1 solution with the account of the losses from the possible goods deficit and random demand for the goods distributions (stochastic analytical approach);

2) Task 3 – optimal solution of the inventory problem on the basis of the Task 1 and Task 2 results with the use of the simulation approach; note that the criteria of optimisation is minimum of the average total cost of the goods ordering, holding and losses from the deficit per a unit of time.

The matter of each task is considered in the given paper. The numerical results of problem solving are obtained accordingly in packages MS Excel (analytical calculations) and ExtendSim (simulation).

2. Approximate Search of the Starting Point

Task 1. In a real situation, different strategies of multiproduct ordering are used. In the simplest variant, all kinds of products can be included in each order. With the account of a possible considerable difference in demands for different goods, it may be more effective to use the strategy when each order encloses only fixed kinds of products. Let us denote the vector of the order as $Q = (Q_1, Q_2, \dots, Q_n)$. In consideration of the second variant, we suppose that for the given order some products cannot be ordered, that means:

$(\forall i \notin Z) \rightarrow Q_i = 0$, where $Z \subseteq \{1, 2, \dots, n\}$ is the set of the numbers of products, which can be included in the order. The last variant is the basic one in this paper.

Let D_i be the demand for the i -th product within a time unit (in our case one year). At the initial stage of the task solution we suppose that $D_i, i = 1, 2, \dots, n$ are deterministic and equal to their average values. Denote by T time period between the moments of placing neighbouring orders; in considered task it is constant. We suppose that for each product there exists its own cycle of ordering and the duration of this cycle for i -th product is proportional to T with the coefficient of proportionality k_i . Time period T and coefficients $k_i, i = 1, 2, \dots, n$ are *control parameters* of the suggested model. We suppose that at the beginning of the considered process all n products are ordered. There might be a situation when at the moment of ordering $m \times T$ nothing is ordered, as it is illustrated in Table 1. Note the following: plus denotes that the i -th product can be included in the order; minus denotes that the i -th product cannot be included in the order.

Table 1. Rules for including the products in the order

i	k_i	Period of time													
		0	T	$2T$	$3T$	$4T$	$5T$	$6T$	$7T$	$8T$	$9T$	$10T$	$11T$	$12T$...
1	2	+	-	+	-	+	-	+	-	+	-	+	-	+	...
2	3	+	-	-	+	-	-	+	-	-	+	-	-	+	...
3	2	+	-	+	-	+	-	+	-	+	-	+	-	+	...

In the considered problem we suppose that the following economic parameters are known:

- the ordering cost E_{OD} is a known function of the order $Q = (Q_1, Q_2, \dots, Q_n)$, i.e. $E_{OD} = f(Q)$;
- the holding cost of the i -th product is proportional to its quantity in the stock and the holding time with the coefficient of proportionality C_{H_i} ;
- the losses from the deficit of the i -th product are proportional to the quantity of its deficit with the coefficient of proportionality C_{SH_i} .

We denote l as the number of orders during a unit of time, i.e. $l = 1/T$. Let us suppose for simplicity that $1/T$ is an integer. Let the delivery at the nil-point of time has number 0, the delivery at time T has number 1, and the delivery at the time moment jT has number j . As we suppose, at the nil-point of time all products are ordered, then at the moment of time jT the product with number i has to be delivered if $[j/k_i] = j/k_i$, where $[a]$ is an integer part of number a .

Let $Q_j = (Q_{1,j}, Q_{2,j}, \dots, Q_{n,j})$ be the delivery with number j , where $Q_{i,j}$ is the quantity of the i -th product in this delivery:

$$Q_{i,j} = \begin{cases} D_i k_i T, & \text{if } [j/k_i] = j/k_i; \\ 0, & \text{if } [j/k_i] \neq j/k_i. \end{cases} \quad (1)$$

The holding cost of the i -th product during the year equals $C_{H_i} \frac{Q_i}{2}$ (see, for example [4], where $Q_i = D_i k_i T$), and the holding cost of n products E_H during the year is

$$E_H = \sum_{i=1}^n C_{H_i} \frac{Q_i}{2} = \frac{T}{2} \sum_{i=1}^n C_{H_i} k_i D_i. \quad (2)$$

The ordering cost E_{OD} is the sum of the orders forming the administrative expenditure E_{AD} and the products transportation expenditure E_{TR} during the year. If C_F is the expenditure for one order forming, then $E_{AD} = l C_F = \frac{1}{T} C_F$ and

$$E_{OD} = E_{AD} + E_{TR} = \frac{1}{T} C_F + \sum_{j=0}^{l-1} C_{TR_j}, \quad (3)$$

where C_{TR_j} is the transportation expenditure of the j -th order.

From (2) and (3) we can find the total holding and ordering cost during a year:

$$E = E_H + E_{OD} = \frac{T}{2} \sum_{i=1}^n C_{H_i} k_i D_i + E_{AD} + \sum_{j=0}^{l-1} C_{TR_j}. \quad (4)$$

The cost of the i -th product transportation depends on the order quantity x , i.e. $C_{TR}^{(i)} = \varphi_i(x)$, and we suppose that $\varphi_i(0) = 0$. We shall consider the situation when the dependence of the transportation expenditure on the non-zero quantity of the i -th product is a linear one:

$$C_{TR}^{(i)} = \varphi_i(x) = \begin{cases} a_i + b_i x, & \text{if } x \neq 0; \\ 0, & \text{if } x = 0 \end{cases}, \quad (5)$$

where a_i and b_i are familiar coefficients.

Then the transportation expenditure of the j -th order C_{TR_j} can be calculated by the following formula:

$$C_{TR_j} = \sum_{i=1}^n (a_i + b_i Q_{i,j}) y_{i,j}, \quad (6)$$

$$\text{where } y_{i,j} = \begin{cases} 1, & \text{if } Q_{i,j} \neq 0; \\ 0, & \text{if } Q_{i,j} = 0. \end{cases} \quad (7)$$

Taking into account (3) we have

$$E_{OD} = \frac{1}{T} C_F + \sum_{j=1}^l \sum_{i=1}^n (a_i + b_i Q_{i,j}) y_{i,j}. \quad (8)$$

The total holding and ordering cost E in this situation will be

$$E = \frac{T}{2} \sum_{i=1}^n C_{H_i} k_i D_i + \frac{1}{T} C_F + \sum_{j=1}^l \sum_{i=1}^n (a_i + b_i Q_{i,j}) y_{i,j}, \quad (9)$$

where $Q_{i,j}$ is defined by (1) and $y_{i,j}$ by (7).

In the previous consideration, the transportation expenditure was calculated separately for every order (see (6)–(8)) and it will be used for the Task 3 solving.

Now we shall consider another approach when the transportation expenditure is calculated separately for each product during one year. This approach is more convenient for using it in Solver package, which is included in MS Excel.

Let $k_i T$ be the time period between the neighbouring orders of the i -th product, then $Q_i = D_i k_i T$ is the quantity of the i -th product in the corresponding delivery and the cost of this cargo transportation for one delivery is

$$a_i + b_i Q_i = a_i + b_i D_i k_i T. \quad (10)$$

The cost of the i -th product transportation for one year is

$$(a_i + b_i Q_i) \frac{1}{k_i T} = a_i \frac{1}{k_i T} + b_i D_i. \quad (11)$$

The total holding and ordering cost for all products during the year is

$$E^{year} = \frac{1}{2} \sum_{i=1}^n C_{H_i} Q_i + \frac{1}{T} C_F + \sum_{i=1}^n \left(\frac{a_i}{k_i T} + b_i D_i \right) = \frac{1}{2} \sum_{i=1}^n C_{H_i} D_i k_i T + \frac{1}{T} C_F + \sum_{i=1}^n \left(\frac{a_i}{k_i T} + b_i D_i \right). \quad (12)$$

The formula (12) determines the objective function of the model. So we can formulate the considered model:

$$E^{year} \rightarrow \min_{(k_i, i=1, \dots, n; T)} \quad (13)$$

$$k_i \geq 1, i = 1, 2, \dots, n;$$

$$k_i \text{ is integer, } i = 1, 2, \dots, n;$$

$$T > 0,$$

where E^{year} is calculated by formula (12) and $k_i, i = 1, 2, \dots, n$ and T are control parameters of the model.

Solving the optimisation problem (13) and finding the corresponding values of the control parameters, and using (6), it is possible to calculate the transportation cost for each order.

Task 2. The next stage of considered problem solving is improving of the Task 1 solution taking into account losses from the possible deficit. It is reasonable to increase the goods quantity until the expected losses from the deficit of the product unit are more then its holding cost. Let C_{SH_i} be the losses from the deficit of the i -th product unit, q_i be the quantity of the goods ordered for the period $\tau = k_i T$, and $P(D_i^{(\tau)} > q_i)$ be the probability that demand, $D_i^{(\tau)}$ for the i -th product during the period τ exceeds the quantity of goods in the stock. Then the expected losses from the deficit of the goods unit will be $P(D_i^{(\tau)} > q_i) C_{SH_i}$. The holding cost of the unit of the i -th product during the period τ is $C_{H_i} k_i T$.

From the condition $P(D_i^{(\tau)} > q_i) C_{SH_i} \geq C_{H_i} k_i T$ we have

$$P(D_i^{(\tau)} > q_i) \geq \frac{C_{H_i} k_i T}{C_{SH_i}}. \quad (14)$$

If the distribution of the i -th product demand for the period $\tau = k_i T$ is known, it is possible to find the goods quantity required for this period.

If the rest of the i -th product at the ordering moment is R_i and the lead time of the order is L , the order quantity for this product will be

$$Q_i = \begin{cases} q_i - (R_i - \bar{D}_i^{(L)}), & \text{if } R_i - \bar{D}_i^{(L)} > 0; \\ q_i, & \text{if } R_i - \bar{D}_i^{(L)} \leq 0 \end{cases}, \quad (15)$$

where $\bar{D}_i^{(L)}$ is the average demand during the lead time.

3. Finding an Optimal Solution

Task 3. The principal aim of the given task is to define the optimal variant of the control parameters T and $k_i, i = 1, 2, \dots, n$, that minimizes the average total cost for the goods ordering, holding and losses from the deficit per year. In a general case we have to investigate the inventory control model using the set of values of parameters T and $k_i, i = 1, 2, \dots, n$, which are located in the neighbourhood of the corresponding "starting point".

For the considered problem solving, the authors have used a simulation model realized in the package ExtendSim, which is the most powerful and flexible simulation tool for analysing, designing, and operating complex systems in the market. It enables the researcher to test the hypotheses without

having to carry them out. ExtendSim has repeatedly proven its being capable of modelling large complex systems [5].

ExtendSim’s design facilitates every phase of the simulation project, from creating, validating, and verifying the model, to the construction of the user interface, which allows others to analyze the system. ExtendSim products allow the researcher to build models of any size, to save changes for the existing models and to save the models themselves, print the model worksheets, and to use scripting for the automatic model. ExtendSim is a modular application. Its model is constructed with library-based iconic blocks. Each block describes a calculation or a step in the process.

The created simulation inventory model for multiproduct inventory control system for 5 products is shown on Figure 1. The model consists of four parts presented correspondingly in figures named as a, b, c, d.

Let us consider the main blocks of the simulation subschema shown on Figure 1a. Block #1 is the transaction creation block; block #2 is the orders counter; block #3 is the delivery simulation block; block #4 produces the order allocation by the goods type; blocks #5 – #9 are the warehouse simulation custom blocks; separate exit for goods and deficit simulation transactions are made in block #10. The purposes of blocks shown on Figures 1b-1d are given in captions.

Using the created simulation model, we can find an average total cost for the goods ordering, holding and losses from the deficit per year in inventory system for different variants of the control parameters (coefficients of proportionality $k_i, i = 1, 2, \dots, n$ and the time period between the neighbouring orders T). The optimal solution of the problem is selected from the set of considered variants using the optimization criteria – the minimum of the average total cost (see example in the next section). As a starting point for $k_i, i = 1, 2, \dots, n$ and T we shall take the results of the task (13); and the order quantity for each product will be calculated by using formula (15).

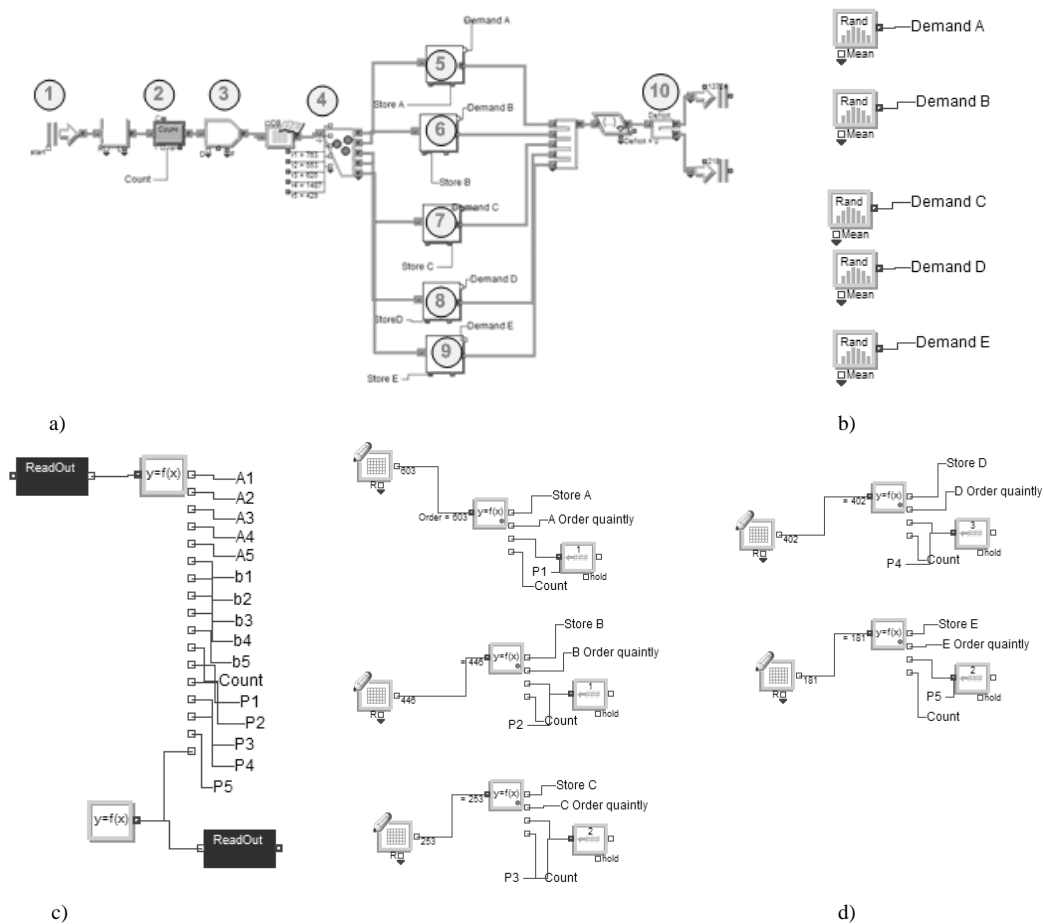


Figure 1. Inventory control simulation model realized in ExtendSim

a – main part; b – demand simulation blocks; c – cost calculation blocks; d – order quantity calculation blocks

4. Example

Let us consider an inventory control system with 5 different products. Demands for these products are random and have normal distribution with the known parameters as the mean μ_i and as the standard deviation σ_i (see Table 2). Shortage C_{SH_i} and holding C_{H_i} costs, coefficients a_i and b_i for the transportation expenditure calculation are given in Table 2, too. Lead time L is constant and equals 7 days. Administrative expenditure for one order C_F is equal to 210 EUR.

Table 2. Basic data

Parameters	Numbers of product, i				
	1	2	3	4	5
μ_i , units/day	530,61	382,26	437,18	1073,0	331,53
σ_i , units/day	148,24	105,12	92,16	206,14	148,15
C_{SH_i} , EUR/unit	10,50	9,80	5,50	9,10	6,00
C_{H_i} , EUR/year	5,50	6,50	9,45	4,00	4,50
a_i , EUR	200	300	500	900	300
b_i , EUR/unit	0,80	1,20	0,95	1,30	0,90

The numerical results of Task 1 solving are obtained in the package MS Excel and are presented in Table 3. Note that in considered case time period between neighbouring orders is $T = [62,08] = 62$ days.

Table 3. Results of Task 1

Parameters	Numbers of product, i				
	1	2	3	4	5
k_i	1	1	2	3	2
$k_i \cdot T$, days	62	62	124	186	124

The values of left and right parts expressions of inequality (14) for every type of the goods and the corresponding values of their quantities q_i are given in Table 4.

Table 4. Results of Task 2

Parameters	Numbers of product, i				
	1	2	3	4	5
$\frac{C_{H_i} k_i T}{C_{SH_i}}$	0,0089	0,011	0,021	0,022	0,254
$P(D_i^{(\tau)} > q_i)$	0,9911	0,989	0,979	0,978	0,746
q_i , units	881,5	622,8	624,9	1487	429,0

Using results of Task 1 and Task 2 presented in Tables 3 and 4, we can form Table 5, which encloses basic data for Task 3.

Table 5. Basic data for Task 3

Parameters	Numbers of product, i				
	1	2	3	4	5
k_i	1	1	2	3	2
$k_i T$, days	62	62	124	186	124
q_i , units/ cycle	882	623	625	1487	429

Let us consider the solution of Task 3 using the created simulation model. The period of simulation is one year and the number of realization is 100.

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An example of the inventory control process simulation (one realization) is shown on Figure 2. Note, that each figure presents up three plots. So, two figures describe one realization. As it can be easily observed, five products have different multiple cycles of ordering. There are cases of deficit for three products.

In the given example we have investigated different variants T_0, T_1, \dots, T_g of control parameter T , starting from point $T = 62$ days and using the initial data from Table 5, and we have suggested that the control parameters $k_i, i = 1, 2, \dots, 5$ are permanent in all the variants considered herein. The optimal decision T^{opt} is selected from the set of $\{T_0, T_1, \dots, T_g\}$ using the optimisation criteria – the minimum of the average total cost for ordering the goods, holding and losses from the deficit per a unit of time, i.e.

$$\overline{E^{year}}(T^{opt}) = \min_{T_i, i=0,1,\dots,g} \left[\overline{E^{year}}(T_i) \right], \quad (16)$$

where $\overline{E^{year}}(T_i)$ is the average total cost (expenses) for ordering the goods, holding and losses from the deficit per a year for the variant with time period T_i .

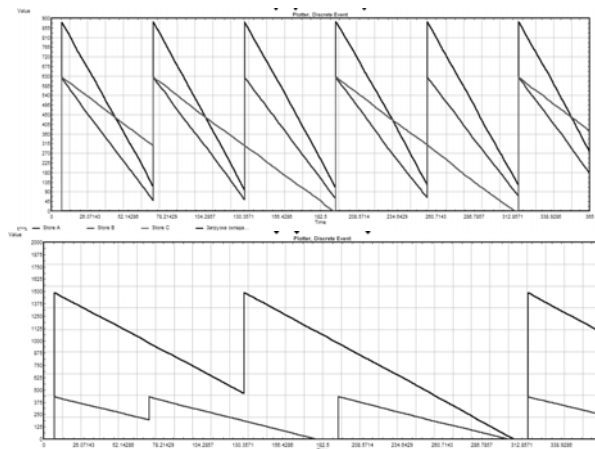


Figure 2. Example of simulation of the inventory control process

The results of the simulation are shown in Table 6. For the given steps of the control parameter T changing, the best result is achieved at point $T = 61$ units, where for 100 realizations the average total cost of one year period equals 27880 EUR.

Table 6. Average expenses for the goods holding, ordering and losses from the deficit per a year

Time cycle T , day	59	60	61
Total cost $\overline{YC}(T)$, EUR	29618	28620	27880
Time cycle T , day	62	63	64
Total cost $\overline{YC}(T)$, EUR	28433	28982	30054

Conclusions

1. The considered model is suggested for the systems with a fixed moment of placing orders. The principal aim of the task is to define the structure of each multi-product order and the moments of orders to achieve the minimum expenses for the goods holding, ordering and losses from the deficit per a time unit. Each order may enclose only fixed kinds of products; we assume that for each product there exists a cycle of ordering; and the coefficient of the ordering ratio must be determined for each kind of products, this coefficient is also a control parameter.

2. The process of the task solving is divided into two stages:

- search of the starting point (an approximate solution of the inventory control deterministic problem and the correction of the received solution with the account of losses from the possible goods deficit and the random demand distributions);

- finding an optimal solution of the inventory problem using the simulation approach.

3. Further guidelines of the current research are the following: to consider the case with random lead time for the goods; to investigate the multi-product inventory control problem with certain constraints.

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RANKING METHOD BASED ON THE DIFFERENCE BETWEEN WEIGHTED OUTPUT AND INPUT

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In this paper we developed a new method to establish common weights for measuring the efficiency score of Decision-Making Units (DMUs), based on multiple inputs and multiple outputs. In the new method, these common weights are estimated according to the difference between a weighted sum of outputs and a weighted sum of inputs of the DMUs. We suggest two approaches to rank the DMUs: the first method ranks the DMUs according to the absolute "net profit" (the difference between a weighted sum of outputs and a weighted sum of inputs), while the second method ranks the DMUs according to the relative efficiency score of the ratio between a weighted sum of outputs and a weighted sum of inputs. In addition we present a method to fix objective bounds for the weights of the variables in the Data Envelopment Analysis (DEA), which is based on the above ranking method. We proved that these bounds are feasible solutions for the DEA methodology. The ranking methods are illustrated on a case study of 24 hospitals in Israel.

Keywords: Data Envelopment Analysis (DEA); Common Weights; Hospitals

1. Introduction

The purpose of the paper is to rank units (as cites, hospitals, schools, banks, etc.) according to multiple inputs and multiple outputs in the Data Envelopment Analysis (DEA) context.

The Data Envelopment Analysis (DEA) was first introduced by Charnes, Cooper and Rhodes (CCR) in 1978. The DEA is a non-parametric method to evaluate the relative efficiency of Decision-Making Units (DMU) based on multiple inputs and multiple outputs. The efficiency score is measured as a ratio between a weighted sum of outputs and a weighted sum of inputs, even if the production function is unknown. The weights are chosen so as to find the best advantage for each unit to maximize its relative efficiency, under the restriction that this score is bound by 100% efficiency. If a unit with its optimal weights receives the score efficiency of 100%, it is efficient, and for a score smaller than 100% it is inefficient. These optimal weights differ from unit to unit. There are DEA researchers that emphasize the difficulty to rank all the units on one scale, claiming that DEA provides only a dichotomy classification into two groups: efficient and inefficient. If the number of units is small relative to the number of inputs and outputs, most of the units will be efficient.

Sometimes there is a need to fix common weights to the inputs and outputs for all the units, in contrast to the DEA. The idea of Common Set of Weights (CSW) was first published by Cook, Roll and Kazakov (1989), and later tested and reformulated by Roll, Cook and Golany (1991). There are several aims for the use of these common weights; the first one boils down to the fact that for the normalized data, the common weights indicate the importance of each factor (input/output) to determine the efficiency of the units. One can utilize these common weights to disregard factors (inputs/outputs) with very small

importance. The idea of the importance of the factors has been discussed by Sengupta (1990), Arnold et al. (1996). Especially in the cases when weights are negative, one can examine if this factor was chosen in the right set, namely if it is output (input) instead of input (output). The second aim of common weights is to help us to define the bounds for the weights of inputs/outputs for the DEA method. The common weights may be the mid point of the range of the bounds of each weight. The third aim is to rank all the units with the common weights on one scale and not as in DEA where weights vary from unit to unit.

There are several methods in the literature for establishing the common weights while each method implements another objective function for this purpose. We shall present a few methods:

1) The Canonical Correlation Analysis (CCA) method, {Friedman and Sinuany-Stern (1997)}. In this method, the objective function for finding the common weights for the inputs and outputs is to maximize the correlation between a weighted sum of outputs and a weighted sum of inputs.

2) The Discriminant Analysis of Ratio (DR/DEA) method, {Sinuany-Stern and Friedman (1998)}. In this method, the discrimination into two groups: efficient and inefficient sets from the DEA, is first carried out. The objective function in this method is to maximize the discrimination between the means of these two groups.

3) The Global Efficiency (GE) method, {Ganley and Cubbin (1992)}; this method focuses on the measurement of aggregate technical efficiency by using common weights for all units. The objective function is to maximize the sum of the efficiency scores of all the units, where each efficiency score is the ratio between a weighted sum of outputs and a weighted sum of inputs. This is done by obtaining the optimal solution by one programming. It can be well-recognized that the latter comes in contrast to the DEA which advocates the solution of the linear programming n times, generating a separate set of optimal weight for each unit. The disadvantage of the GE method boils down to the fact that its objective function is nonlinear; therefore the solution is not obviously optimal. Some more methods for determining common weights are given in the literature review of Adler et al. (2002).

The purpose of our paper is to develop a new ranking method designated as SDEA that is based on multiple inputs and multiple outputs, with common weights for all units. Our method focuses on the difference between a weighted sum of outputs and a weighted sum of inputs, namely the “net profit”. The objective function of this method is to maximize the sum of the “net profits” of all the units. The idea of maximizing the difference between the weighted outputs to the weighted inputs, for each unit, with weights that vary from unit to unit has been suggested in the past as the Additive model {see Charnes et al. (1985)}, and some of its ideas were further developed by Ali and Lerne (1997).

In addition we present a method to fix objective bounds for the weights of the variables in the DEA, which is based on the above ranking method. We proved that these bounds are feasible solutions for the DEA methodology. The ranking methods are illustrated on a case study of 24 hospitals in Israel, in order to single out the most efficient one. We choose 2 inputs: the number of standardized beds in the end of 2003, the number of standardized beds in day care (ambulatory), as well as 3 outputs: the number of total discharges in 2003, the number of hospitalisation days during 2003 and the same in ambulatory care.

Our paper is presented as follows: Section 2 introduces the SDEA method; a numerical example is outlined in Section 3. Section 4 contains the summary and conclusions.

2. Essentials of SDEA

Our new ranking method finds the common set of weights of the inputs/outputs where the objective function is to maximize the sum of all the “net profits” of the DMUs under two types of constraints. The first one is that for each DMU, its net profit can't be positive as the DEA methodology and the Global Efficiency (GE) method (see *Appendix*). The second one is that the common weights are bonded from below by some value ε suggested by Sueyoshi (1999).

Consider n Decision-Making Units (DMUs), when each DMU j ($j=1, \dots, n$) utilizes m inputs $x_j = (x_{1j}, x_{2j}, \dots, x_{mj})^T > 0$ for producing s outputs $Y_j = (Y_{1j}, Y_{2j}, \dots, Y_{sj})^T > 0$. In this case, $\sum_{r=1}^s U_r Y_{rj}$ represents the weighted sum of outputs of DMU j , $\sum_{i=1}^m V_i X_{ij}$ stands for the weighted sum of inputs of DMU j . The net profit of DMU j may be therefore calculated as $S_j = \sum_{r=1}^s U_r Y_{rj} - \sum_{i=1}^m V_i X_{ij}$.

The objective function is: $Z = \max \sum_{j=1}^n \left(\sum_{r=1}^s U_r Y_{rj} - \sum_{i=1}^m V_i X_{ij} \right) = \max \sum_{j=1}^n S_j$. The constraints are:

the net profit is less or equal to zero, namely $S_j = \sum_{r=1}^s U_r Y_{rj} - \sum_{i=1}^m V_i X_{ij} \leq 0$.

We obtain thus the linear problem

$$Z = \max \sum_{j=1}^n \left(\sum_{r=1}^s U_r Y_{rj} - \sum_{i=1}^m V_i X_{ij} \right) = \max \sum_{j=1}^n S_j$$

s.t.

$$S_j = \sum_{r=1}^s U_r Y_{rj} - \sum_{i=1}^m V_i X_{ij} \leq 0 \quad j=1,2,\dots,n \quad (1)$$

$$U_r \geq \varepsilon_r^+ > 0 \quad r=1,2,\dots,s$$

$$V_i \geq \varepsilon_i^- > 0 \quad i=1,2,\dots,m$$

Our method differs from GE in two characteristics: first, our method can be solved by means of linear programming so that the optimal solution may be obtained, and second, our method bases on the difference between the weighted sum of outputs and weighted sum of inputs and not on the ratio between the weighted sum of outputs and the weighted sum of inputs.

The Dual Problem

Let us define a dual variable λ_j that is fitted to the constraint of each unit j and the dual variables L_r^+ and L_i^- that are fitted to the constraints of each output/input, respectively. The dual problem will be defined as follows:

$$V = \min \left\{ \sum_{r=1}^s \varepsilon_r^+ L_r^+ + \sum_{i=1}^m \varepsilon_i^- L_i^- \right\}$$

s.t.

$$\sum_{j=1}^n \lambda_j Y_{rj} + L_r^+ \geq \sum_{j=1}^n Y_{rj} \quad r=1,2,\dots,s$$

$$-\sum_{j=1}^n \lambda_j X_{ij} + L_i^- \geq -\sum_{j=1}^n X_{ij} \quad i=1,2,\dots,m \quad (2)$$

$$\lambda_j \geq 0 \quad j=1,2,\dots,n$$

$$L_r^+ \leq 0 \quad r=1,2,\dots,s$$

$$L_i^- \leq 0 \quad i=1,2,\dots,m$$

Lemma 1. If $S_j^* = \sum_{r=1}^s U_r^* Y_{rj} - \sum_{i=1}^m V_i^* X_{ij} = 0 \quad \forall j=1,2,\dots,n$ then $\frac{1}{n} \sum_{j=1}^n \lambda_j^* = 1$.

Proof: If $S_j^* = \sum_{r=1}^s U_r^* Y_{rj} - \sum_{i=1}^m V_i^* X_{ij} = 0 \quad \forall j=1,2,\dots,n$ then $Z^* = 0$.

According to the strong duality property [23] there exist $Z^* = V^* = 0$ therefore $\min \left\{ \sum_{r=1}^s \varepsilon_r^+ L_r^+ + \sum_{i=1}^m \varepsilon_i^- L_i^- \right\} = 0$ and because $\varepsilon_r^+ > 0 \quad r=1,2,\dots,s$ $\varepsilon_i^- > 0 \quad i=1,2,\dots,m$ and $L_r^+ \leq 0 \quad r=1,2,\dots,s$ $L_i^- \leq 0 \quad i=1,2,\dots,m$ then $L_r^+ = 0 \quad \forall r=1,2,\dots,s$ $L_i^- = 0 \quad \forall i=1,2,\dots,m$

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In addition all the variables of the primal problem exist in the basis, therefore according to the complementary slackness theory $\sum_{j=1}^n \lambda_j^* Y_{rj} = \sum_{j=1}^n Y_{rj} \quad r = 1, 2, \dots, s$. This constraint can be written

as: $\sum_{j=1}^n (\lambda_j^* - 1) Y_{rj} = 0 \quad r = 1, 2, \dots, s$ the same for the constraints on the inputs. The solution of these

constraints gives the following solution $\lambda_j^* = 1 \quad \forall j = 1, 2, \dots, n$, therefore $\frac{1}{n} \sum_{j=1}^n \lambda_j^* = 1$.

Ranking according to SDEA

We suggest two methods to rank the DMUs, the first method ranking the DMUs according to the absolute "net profit" (the difference between a weighted sum of outputs and a weighted sum of inputs), and the second method ranking the DMUs according to the relative efficiency score of the ratio between the weighted sum of outputs and the weighted sum of inputs.

Let us define the "net profit" S_j^* of DMU j as: $S_j^* = \sum_{r=1}^S U_r^* Y_{rj} - \sum_{i=1}^m V_i^* X_{ij}$,

where:

U_r^* and V_i^* are the optimal common weights from the SDEA method, and they are the same for the two ranking approaches;

if $S_j = 0$ the DMU j is efficient;

if $S_j < 0$ the DMU j is inefficient.

First method to rank the DMUs

The ranking is based on the "net profit" of each unit. The score is determined as $S_j^* = \sum_{r=1}^s U_r^* Y_{rj} - \sum_{i=1}^m V_i^* X_{ij}$. The DMU that received the highest S_j^* is ranked in the first place.

In this method the ranking is dependent on the size of the DMU's.

Second method to rank the DMUs

In this case, the DMUs are ranked according to the relative efficiency score of the ratio between the weighted sum of outputs and a weighted sum of inputs. The ranking score is determined by:

$$T_k = 1 + \frac{S_k^*}{\sum_{i=1}^m V_i^* X_{ik}} = \frac{\sum_{i=1}^m V_i^* X_{ik} + \sum_{i=1}^m U_r^* Y_{rk} - \sum_{i=1}^m V_i^* X_{ik}}{\sum_{i=1}^m V_i^* X_{ik}} = \frac{\sum_{i=1}^m U_r^* Y_{rk}}{\sum_{i=1}^m V_i^* X_{ik}}.$$

Bounded weights in SDEA

In order to prevent the trivial solution where all weights are zero in the SDEA method $U_r = 0 \quad \forall r = 1, 2, \dots, S$ and $V_i = 0 \quad \forall i = 1, 2, \dots, m$ and therefore $S_j = 0 \quad \forall j = 1, 2, \dots, n$, it is necessary to set a lower bound to the weights.

This issue has been extensively dealt with in scientific literature.

The first research on bounds on the weights by a "Non Archimedean Quantity" (NAQ) was carried out by Charnes et al. (1979, 1984). Thompson et al. (1990) established the so-called assurance region (AR) in order to outline a region for the possible values of the weights. The information on the assurance region originates from experts' opinions, common sense, or previous experience.

Sueyoshi (1999) suggested restricting the weights only by a lower bound which will be a function of the number of inputs and outputs. In the literature there are many papers on methods for determining the bounds on the weights: Cooper et al. (1999), Thompson et al. (1995), Roll et al. (1991), Dyson et al. (1988). The main purpose of the constraints on the weights is to reduce the number of efficient units and also to avoid the problems of extreme values of the weights.

In our research, we adopt the Sueyoshi approach. It can be well-recognized that the primary and dual problems in the SDEA method will appear therefore as:

Primary problem

$$Z = \max \sum_{j=1}^n \left(\sum_{r=1}^s U_r Y_{rj} - \sum_{i=1}^m V_i X_{ij} \right) = \max \sum_{j=1}^n S_j$$

s.t.

$$S_j = \sum_{r=1}^s U_r Y_{rj} - \sum_{i=1}^m V_i X_{ij} \leq 0 \quad j = 1, 2, \dots, n \quad (3)$$

$$U_r \geq \frac{1}{(m+S) \max_j \{Y_{rj}\}} \quad r = 1, 2, \dots, s$$

$$V_i \geq \frac{1}{(m+S) \max_j \{X_{ij}\}} \quad i = 1, 2, \dots, m$$

Dual problem

$$V = \min \left\{ \sum_{r=1}^s \frac{1}{(m+S) \max_j \{Y_{rj}\}} L_r^+ + \sum_{i=1}^m \frac{1}{(m+S) \max_j \{X_{ij}\}} L_i^- \right\}$$

s.t.

$$\sum_{j=1}^n \lambda_j Y_{rj} + L_r^+ \geq \sum_{j=1}^n Y_{rj} \quad r = 1, 2, \dots, s \quad (4)$$

$$-\sum_{j=1}^n \lambda_j X_{ij} + L_i^- \geq -\sum_{j=1}^n X_{ij} \quad i = 1, 2, \dots, m$$

$$\lambda_j \geq 0 \quad j = 1, 2, \dots, n$$

$$L_r^+ \leq 0 \quad r = 1, 2, \dots, s$$

$$L_i^- \leq 0 \quad i = 1, 2, \dots, m$$

Lower bound on weights in SDEA

The constraint that normalizes the weighted sum of inputs $\left(\sum_{i=1}^m V_i^k X_{ik} = 1 \right)$ in the classical DEA approach may result in difficulties to attain a feasible solution, or, in certain cases, in creating non-realistic solutions. We suggest a new method for implementing bounds on the weights in the DEA.

The procedure that is capable of preventing the above mentioned difficulties may be presented in the following steps:

Step 1. Solve the optimisation problem of SDEA of Eq. (1) and find out from it the common weights of all the inputs and outputs, $V_i \quad i = 1, 2, \dots, m \quad U_r = 1, 2, \dots, s$.

Step 2. Calculate the maximal value over all the n units, of the weighted inputs from the SDEA model in

Step 1, namely $UL = \max_j \left\{ \sum_{i=1}^m V_i X_{ij} \right\}$.

Step 3. Calculate the ratio of the common weights V_i and U_r to the maximal value UL calculated in

Step 2, that is $\frac{V_i}{UL} \quad i = 1, 2, \dots, m$ and $\frac{U_r}{UL} \quad r = 1, 2, \dots, s$. These are the fitted bounds for the weights that will solve all the problems and will give a suitable solution.

Lemma 2. The efficiency score based on common weights (SDEA method) is always less than the efficiency score in the DEA.

Proof: The DEA score is the maximal score that a unit can receive. If we restrict the problem to find only common weights, these weights produce a score smaller than the one with the varying weights, as defined in DEA methodology.

Lemma 3. Let us denote $UL = \max_j \left(\sum_{i=1}^m V_{i(CW)} X_{ij} \right)$ then $V_{i(DEA) \geq \varepsilon_i^-} = \frac{V_{i(CW)}}{UL}$ $i = 1, 2, \dots, m$ and

$U_{r(DEA) \geq \varepsilon_r^+} = \frac{U_{r(CW)}}{UL}$ $r = 1, 2, \dots, s$, are a feasible solution for unit j by the DEA model.

Proof:

$$(1) S_j = \sum_{r=1}^s U_{r(CW)} Y_{rj} - \sum_{i=1}^m V_{i(CW)} X_{ij} \leq 0 \quad \forall j = 1, 2, \dots, n \Rightarrow E_j = \frac{\sum_{r=1}^s U_{r(CW)} Y_{rj}}{\sum_{i=1}^m V_{i(CW)} X_{ij}} \leq 1 \quad \forall j = 1, 2, \dots, n$$

$$(2) UL = \max_j \left\{ \sum_{i=1}^m V_{i(CW)} X_{ij} \right\}$$

$$\text{from (2) : (3) } \frac{\sum_{i=1}^m V_{i(CW)} X_{ij}}{UL} \leq 1 \quad \forall j = 1, 2, \dots, n$$

$$\text{from (1) and (3) : (4) } \frac{\sum_{r=1}^s U_{r(CW)} Y_{rj}}{UL} \leq \frac{\sum_{i=1}^m V_{i(CW)} X_{ij}}{UL} \leq 1 \Rightarrow \frac{\sum_{r=1}^s U_{r(CW)} Y_{rj}}{UL} \leq 1$$

$$(5) \sum_{i=1}^m V_{i(DEA)} X_{ij} = 1 \Rightarrow \sum_{i=1}^m V_{i(DEA)} X_{ij} \geq \frac{\sum_{i=1}^m V_{i(CW)} X_{ij}}{UL}$$

$$(6) \sum_{r=1}^s U_{r(DEA)} Y_{rj} \leq 1$$

Therefore the solutions that were defined by the above bounds $U_{r(DEA)} \geq \frac{U_{r(CW)}}{UL}$ $r = 1, 2, \dots, s$,

represent a feasible solution for the DEA model.

3. The Case Study ON Israeli Hospitals

Hospitals account for about 40% of Israel's national health expenditure in 2003. This is the largest category of spending on health; community clinics, including those providing preventive medicine, account for about 38% of this expenditure [Central Bureau of Statistics (CBS) 2003].

3.1. Output and Input Measures

The main problem that appears in hospitals research concerning the DEA is that there are many types of inputs and outputs. The choice of the inputs/outputs influences the results of the efficiency of the hospitals. Therefore a literature review on DEA efficiency of hospitals was done in our research. A total of 42 publications on the regarded issue was studied, out of which we shall outline the most important and relevant to the discussed subject: O'Neill (1988), reported on the efficiency of 27 hospitals in USA. Al-Shammari (1999) reported on the efficiency of 15 government hospitals in Jordan. Hofmarcher et al. (2002) reported on the efficiency of 15 hospitals in Austria. Kirigia et al. (2002) reported

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on the efficiency of 54 government hospitals in Kenya, Gruca and Nath (2000) reported on the efficiency of 168 hospitals in Canada. Grosskof et al. (2001) reported on the efficiency of 236 teaching hospitals and 556 non-teaching hospitals. Hao and Pegels (1994) reported on the efficiency of 93 medical centers.

The data reported in the references summarize the main inputs and outputs measures used in different DEA hospital studies. Evidently the most used input is no. of beds (52% of studies). Cost is used in 48% of the studies, supplies in 44% and employees in 41% of the studies.

Due to the lack of data on these additional inputs we did not include them in our study. However they all are reported in the literature as related to the number of beds, since the budget is largely derived by number of beds.

From the database available to us from the Health Ministry we used 2 inputs and 3 outputs. The inputs are: the number of standardized beds in the end of 2003 (X_1), the number of standard beds in day care (ambulatory) (X_2). The outputs are: the number of total discharges in 2003 (Y_1), the number of hospitalization days during 2003 (Y_2) and the same in ambulatory care (Y_3).

Day care (ambulatory) has been a venue for increasing the efficiency of hospitals in Israel. Thus, we included it in our input and output variables. Overall the input/output variables we have used are those common in the literature.

The list of hospitals in Israel includes 45 hospitals. We deleted hospitals that did not have internal care and outpatient clinics (day care) units. Consequently, 24 hospitals were left for our study. The data is presented in *Table 1*, which includes information on the 24 hospitals with 3 outputs and 2 inputs.

Table 1. The numerical data

No.	DMU	INPUT		OUTPUT		
		X_1	X_2	Y_1	Y_2	Y_3
1	Asaf Harofe – Zrifin	675	72	53,017	220,017	16,535
2	Asuta – Tel Aviv	148	12	24,127	35,748	905
3	Bikur Holim – Jerusalem	193	8	16,116	62,865	465
4	Bney Zion – Haifa	366	16	32,411	124,478	1,483
5	Barzilai – Ashkelon	448	40	33,577	145,554	9,433
6	E.M.M.S – Nazaret	108	4	11,056	37,252	1,398
7	Emek – Afula	415	38	33,492	131,009	5,753
8	Hadassa (Ein Karem) – Jerusalem	618	70	49,631	226,015	23,849
9	Hadassa (Har Hatzofim) – Jerusalem	215	23	24,239	85,459	7,265
10	Hagalil Hamaaravi – Naharia	499	17	54,379	225,010	2,765
11	Hilel Yafe – Hadera	394	14	32,234	129,435	1,393
12	Kaplan – Rehovot	535	75	51,688	189,197	9,403
13	Lady Davis – Haifa	403	23	32,030	153,140	6,435
14	Laniado – Natania	207	32	24,631	96,605	5,069
15	Meir – Kfar Saba	639	55	56,016	213,304	7,416
16	Rabin (Belinson) – Petah Tikva	949	71	71,766	304,311	28,735
17	Rabin (Golda) – Petah Tikva	321	15	32,358	110,128	1,193
18	Rambam – Haifa	858	40	68,113	287,528	48,391
19	Rivka Ziv – Tzfat	315	10	22,136	91,151	3,763
20	Shearay Tsedek – Jerusalem	378	26	31,661	155,705	2,726
21	Shiba – Tel Hashomer, Ramat Gan	984	100	72,187	323,055	18,932
22	Soraski – Tel Aviv, Jaffa	984	76	82,527	354,925	21,532
23	Soroka – Beer Sheva	911	46	75,698	306,468	24,056
24	Wolffson – Holon	618	30	58,338	230,186	9,647

Table 2. The scores and the ranking of the DMUs

DMU	METHOD 1		METHOD 2	
	SCORE	RANK	SCORE	RANK
Asaf Harofe – Zrifin	-0.1759	22	0.6983	20
Asuta – Tel Aviv	0.0000	2	1.0000	2
Bikur Holim – Jerusalem	-0.0390	7	0.7247	17
Bney Zion – Haifa	-0.0627	10	0.7680	13
Barzilai – Ashkelon	-0.1142	17	0.6925	22
E.M.M.S – Nazaret	-0.0069	4	0.9123	5
Emek – Afula	-0.1128	16	0.6740	23
Hadassa (Ein Karem) – Jerusalem	-0.1145	18	0.7887	11
Hadassa (Har Hatzofim) – Jerusalem	-0.0098	5	0.9475	4
Hagalil Hamaaravi – Naharia	0.0000	2	1.0000	2
Hilel Yafe – Hadera	-0.0751	14	0.7360	16
Kaplan – Rehovot	-0.1436	19	0.7117	19
Lady Davis – Haifa	-0.0650	11	0.7890	10
Laniado – Natania	-0.0236	6	0.8814	6
Meir – Kfar Saba	-0.1482	20	0.7181	18
Rabin (Belinson) – Petah Tikva	-0.1781	23	0.7655	14
Rabin (Golda) – Petah Tikva	-0.0412	8	0.8275	9
Rambam – Haifa	0.0000	2	1.0000	2
Rivka Ziv – Tzfat	-0.0683	12	0.6963	21
Shearay Tsedek – Jerusalem	-0.0701	13	0.7649	15
Shiba – Tel Hashomer, Ramat Gan	-0.2868	24	0.6587	24
Soraski – Tel Aviv, Jaffa	-0.1684	21	0.7875	12
Soroka – Beer Sheva	-0.1059	15	0.8453	8
Wolffson – Holon	-0.0563	9	0.8782	7

Table 3. Common weights values

	FOR NORMAL VALUES	FOR REGULAR VALUES
U1	0.327141	0.00065074
U2	0.207787	0.002
U3	0.2	3.964E-06
V1	0.640326	5.8544E-07
V2	0.2	4.133E-06

It turns out that $\max_j \{V_i X_{ij}\} = 0.8403$ and the bounds in the DEA method may be estimated therefore as listed in Table 4:

Table 4. Bounds in the DEA method

	FOR NORMAL VALUES	FOR REGULAR VALUES
U1	0.00065074	0.000774387
U2	0.002	0.002380028
U3	3.964E-06	4.71727E-06
V1	5.8544E-07	6.9668E-07
V2	4.133E-06	4.91833E-06

4. Conclusions

In this paper we developed a new method to establish common weights for measuring the efficiency score of DMUs, based on multiple inputs and multiple outputs. In the new method, these common weights are estimated according to the difference between a weighted sum of outputs and a weighted sum of inputs of the DMUs. We suggest two approaches to rank the DMUs: the first method ranks the DMUs according to the absolute "net profit" (the difference between a weighted sum of outputs and a weighted sum of inputs), while the second method ranks the DMUs according to the relative efficiency score of the ratio between the a weighted sum of outputs and a weighted sum of inputs. If the DMUs have similar size of inputs and outputs, the ranking is similar. If not, the ranking is different. Which method is better? There is no ultimate answer, it depends on the context.

We have proved that weights obtained by the suggested procedure when divided by the weighted sum of inputs, can effectively serve as a lower bound for the DEA method. We have implemented this procedure for quality ranking of 24 hospitals in Israel. It appears that 3 hospitals (Asuta-Tel Aviv, Hagalil Hamaaravi-Naharia and Rambam-Haifa) out of the total amount proved to be efficient (their score was equal to 1), while the rest occupied positions 4–24. The ranking of the efficient hospitals proves to be the same for both suggested DEA approaches, while the inefficient ones tended to display significant differences for both types of ranking. One may appreciate the importance of the weights for normalized values of the scores as well.

Should we sub-divide the total amount of hospitals into 3 groups according to their accommodating capacity (small ones – up to 420 beds, intermediate – from 420 to 700 beds, and big ones – more than 700 beds), it can be well-recognized that small hospitals mostly tend to obtain higher scores by the first type of ranking, while the big ones, on the contrary, are ranked higher by the second DEA approach (e.g., the Bikur Holim-Jerusalem hospital which is a small one was ranked 7th by the first procedure and 17th by the second one, while the Rabin-Petach Tikva hospital which is a big one – 23rd and 14th, respectively, and the same for Soraski-Tel-Aviv hospital – 21st and 12th).

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Appendix. The global efficiency method (GE)

The global efficiency (GE) focuses on the measurement of aggregate technical efficiency by using the same set of weights (common weights) for the efficiency score for all the units. This is in contrast to the DEA efficiency score, which advocates the solution of the linear programming n times, generating a separate set of optimal weight for each unit. The GE involves an optimal solution by one programming for the common set of weights, which maximizes the sum of efficiency scores of all the units. Each efficiency score of the GE with the common weights has the same structure as the DEA efficiency score with the weights that vary from unit to unit; i.e. it is the ratio of total weighted output to total weighted input, bounded by 1. The formulation of the GE is as follows:

$$\begin{aligned}
 Z &= \max \sum_{j=1}^n E_j^* \\
 \text{s.t.} \\
 E_j^* &= \frac{\sum_{r=1}^s U_r Y_{rj}}{\sum_{i=1}^m V_i X_{ij}} \leq 1 \quad j = 1, 2, \dots, n \\
 V_i, U_r &\geq 0 \quad i = 1, 2, \dots, m \quad r = 1, 2, \dots, s
 \end{aligned}$$

The objective function helps the efficiency score with the common weights to reach the DEA scores E_k globally, because by the definition of the DEA, the sum of the DEA scores is the maximum value that any score can reach. The advantage of the GE is its simplicity, the ease with which the average manager may interpret results, and its ranking capabilities according to the common weights that are in the DEA context.

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1994-1999 Head of the Department of Aircraft Structure & Strength Analysis, Riga Aviation University
1974-1994 Design and Development of automated control systems: black box data processing (for Ilyushin Aircraft Design Bureau, IL-86, IL-96); planning of aircraft structures periodic inspection (for Ilyushin and Yakovlev Aircraft Design Bureaus), Head of the Department of Economical-Mathematical Research; Deputy Director for Science, Central Scientific Research Institute of Automated Control Systems of Civil Aviation, Riga
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- **Professional interests:** mathematical theory of reliability, fatigue life, mathematical statistics, aircraft structure and strength analysis

CUMULATIVE INDEX

COMPUTER MODELLING and NEW TECHNOLOGIES, volume 12, No. 3, 2008 (Abstracts)

Yu. Paramonov. Bayes-Fiducial Approach for Mathematical Statistics Problem Solution, *Computer Modelling and New Technologies*, vol. 12, No 3, 2008, pp. 7–15.

Review of using of Bayes-Fiducial (BF) approach for solution of some important statistical problem is given. BF decision is always a function of sufficient statistics (even in case when sufficient statistics coincides with the sample itself (for example, for Weibull's distribution)). By contrast with maximum likelihood method BF decision is based on the use of specific loss function. By contrast with Bayes decision it does not need a priori distribution of unknown parameters.

For the distributions with location and/or scale parameter the solutions of the following problems are considered: point estimates of location and scale parameters, p-bound (prediction limit) calculation and specified life nomination with specific loss function. Numerical examples are given.

Keywords: *parameter estimation, testing statistical hypotheses, prediction limit*

V. Lyumkis, A. Tarasov. Estimation of Risk Insurance in Practice of Work of Actuaries, *Computer Modelling and New Technologies*, vol. 12, No 3, 2008, pp. 16–21.

The problem of estimating various probabilistic characteristics of risk in activity of actuaries of the insurance company is considered in the article. The model of collective risk is discussed more detailed. As well as in models of individual risk, in models of collective risk ruin is defined by total payments S of the insurance company of kind $S = Y_1 + Y_2 + Y_3 + \dots + Y_v$, and the probability of ruin of the company are defined as $P(Y_1 + Y_2 + Y_3 + \dots + Y_v > U)$, where U – size of actives of the company. Exact analytical estimation of this probability for any distributions of random variables v and Y_v are far not always possible to receive. In the article is offered and numerically realized the algorithm of an estimation of probability of ruin of the insurance company under conditions of model of collective risk at use practically any distribution of size of damage and trivial discrete (non-negative integer) distribution of the number of insurance cases.

Keywords: *statistics, insurance, estimation, collective risk model*

E. K. Zavadskas, Z. Turskis, J. Tamosaitiene, V. Marina. Selection of Construction Project Managers by Applying COPRAS-G Method, *Computer Modelling and New Technologies*, vol. 12, No 3, 2008, pp. 22–28.

There are a number of attributes and associated sub-factors influencing the matching of managers to construction projects. Attributes and sub-attributes were identified based on a thorough review of the related literature and interviews of management personnel involved in the selection of project managers. Project managers' characteristics are considered to be less important for effective project management. The model is based on multi-attribute evaluation of project managers. The evaluation embraces the identified attributes influencing the process of construction project manager selection. This paper considers the application of grey relations methodology for defining the utility of alternatives, and a multiple criteria method of COMplex PROportional ASsessment of alternatives with Grey relations (COPRAS-G) is offered. In this model, the parameters of the alternatives are determined by the grey relational grade and expressed in intervals. A case study presents the selection of construction project manager. The results obtained show that this method may be used as an effective decision aid in multi-attribute selection.

Keywords: *COPRAS, grey relations, manager, selection*

L. Ustinovichius, G. Shevchenko. Risk Level Evaluation of Construction Investments Projects, *Computer Modelling and New Technologies*, vol. 12, No 3, 2008, pp. 29–37.

Management of investment risk is a usual practice of any investment project or business. The estimation of risk must be carried out at various project stages. In practice it is impossible to avoid not exhaustive and inaccurate information, therefore, unfavourable risky situations occur,

the consequences of which can be very damaging to the project or business. Due to close cooperation of the participants of the project the risk occurring in one stage of the project can transfer to other stages and one type of risk can change into another one. This means that chain reaction is characteristic to the risk and it decreases efficiency and safety of any project or business. Various types of risk can be caused by different factors. Classification of risk is determined by efficiency of risk management organisation. Risk classification is understood as risk allotment according to certain features into concrete groups for reaching the set objectives. Conceptually reasoned risk classification allows defining the role of each risk in the total system of all risks. This paper presents methods of multi-attribute comparative analysis (CLARA and SAW methods) of variants of investment classified risks in construction. A practical case of illustrating the methods work is presented.

Keywords: *investment, project, risk, multi-attribute analysis*

L. Tupenaite, L. Kanapeckiene, J. Naimaviciene. Knowledge Management Model for Construction Projects, *Computer Modelling and New Technologies*, vol. 12, No 3, 2008, pp. 38–46.

In the past there has been no structured approach to learning from construction projects once they are completed. At present the construction industry is adapting concepts of knowledge management to improve the situation. In this paper knowledge management benefits to construction industry organizations and projects are discussed.

The main purpose of this paper is to present knowledge management model for construction projects. Paper consists of three parts. In the first part the concept of knowledge management in construction is discussed. In the second part different knowledge management models presented in scientific literature are discussed and compared as well as the new model, developed by the authors, is presented. In the third part, basing on the proposed model, the architecture of Knowledge Based Decision Support System for Construction Projects Management has been created as well as Multiply Criteria Analysis Method COPRAS application for construction decisions support have been discussed.

Keywords: *knowledge management, construction projects, knowledge based decision support system, multiply criteria analysis*

E. Kopytov, L. Greenglaz, A. Muravyov. Modelling of the Multiproduct Inventory Problem, *Computer Modelling and New Technologies*, vol. 12, No 3, 2008, pp. 47–54.

The given paper considers a multiproduct inventory control model with random demands for goods. The authors propose the model with a fixed period of time between the moments of placing neighbouring orders. The order quantity is determined as the difference between the fixed stock level and the quantity of goods at the moment of ordering. It is assumed that each order may enclose only fixed kinds of products. The considered model is realized by the combination of the analytical and simulation approaches in packages MS Excel and ExtendSim correspondingly. The numerical example of the problem solving is presented. The considered model is of interest because it illustrates a real business situation.

Keywords: *inventory control, random demand, analytical method, optimisation, simulation*

Y. Hadad, L. Friedman, Z. Sinuany-Stern, A. Ben-Yair. Ranking Method Based on the Difference Between Weighted Output and Input, *Computer Modelling and New Technologies*, vol. 12, No 3, 2008, pp. 55–65.

In this paper we developed a new method to establish common weights for measuring the efficiency score of Decision-Making Units (DMUs), based on multiple inputs and multiple outputs. In the new method, these common weights are estimated according to the difference between a weighted sum of outputs and a weighted sum of inputs of the DMUs. We suggest two approaches to rank the DMUs: the first method ranks the DMUs according to the absolute “net profit” (the difference between a weighted sum of outputs and a weighted sum of inputs), while the second method ranks the DMUs according to the relative efficiency score of the ratio between a weighted sum of outputs and a weighted sum of inputs. In addition we present a method to fix objective bounds for the weights of the variables in the Data Envelopment Analysis (DEA), which is based on the above ranking method. We proved that these bounds are feasible solutions for the DEA methodology. The ranking methods are illustrated on a case study of 24 hospitals in Israel.

Keywords: *Data Envelopment Analysis (DEA), Common Weights, Hospitals*

COMPUTER MODELLING and NEW TECHNOLOGIES, 12.sējums, Nr. 3, 2008

(Anotācijas)

J. Paramonovs. *Bayes-Fiducial* (BF) pieeja matemātiskās statistikas problēmu risināšanā, *Computer Modelling and New Technologies*, 12.sēj., Nr.3, 2008, 7.–15. lpp.

Rakstā tiek dots pārskats par BF pieejas izmantošanu dažu svarīgu statistisku problēmu risināšanā BF lēmums vienmēr ir pietiekamās statistikas funkcija (pat tanī gadījumā, kad pietiekamā statistika saplūst ar pašu paraugu (piemēram, Veibula sadalījuma gadījumā)). Pretēji maksimuma varbūtības metodei BF lēmums pamatojas ar specifiskās zuduma funkcijas lietošanu. Pretēji *Bayes* lēmumam, tas neprasa nezināmo parametru *a priori* sadalījumu.

Sadalījumiem ar dislokācijas un/vai mēroga parametriem sekojošu problēmu risinājumi tiek uzskatīti par dislokācijas un mēroga parametru mērķa novērtējumu, p-robežas (prognozes limits) aprēķins un specifiska dzīves nominācija ar specifisku zuduma funkciju. Rakstā tiek doti skaitliski piemēri.

Atslēgvārdi: *parametra novērtējums, statistisko hipotēžu testēšana, prognozes limits*

V. Ļumkis, A. Tarasovs. Riska apdrošināšanas novērtējums praksē statistikas jomas darbā, *Computer Modelling and New Technologies*, 12.sēj., Nr.3, 2008, 16.–21. lpp.

Rakstā tiek apskatītas dažādu riska varbūtības raksturojumu novērtēšanas problēmas apdrošināšanas kompānijas aktuāru darbībā. Kolektīvā riska modeļi tiek apskatīti sīkāk. Kā individuālajos riskos, tā arī modeļos ar kolektīvo risku krahs tiek definēts ar apdrošināšanas kompāniju veida $S = Y_1 + Y_2 + Y_3 + \dots + Y_v$ kopējiem maksājumiem S , un kompānijas kraha varbūtība tiek definēta kā $P(Y_1 + Y_2 + Y_3 + \dots + Y_v > U)$, kur U ir kompānijas aktīvu lielums. Rakstā tiek piedāvāts un skaitliski parādīts apdrošināšanas kompānijas kraha varbūtības novērtēšanas algoritms.

Atslēgvārdi: *statistika, apdrošināšana, novērtējums, kolektīvā riska modelis*

E. K. Zavadskas, Z. Turksis, J. Tamosaitiene, V. Marina. Būvprojektu vadītāju izvēle, pielietojot Copras-G metodi, *Computer Modelling and New Technologies*, 12.sēj., Nr.3, 2008, 22.–28. lpp.

Pastāv vesela rinda iespēju un papildfaktoru, kas ietekmē vadītāju piesaisti būvprojektiem. Šīs raksturīpašības un tām pakļautās raksturīpašības tiek noteiktas, pamatojoties uz pilnīgu atbilstošās literatūras pārskatu un vadības personāla intervijām, kuri piedalās projekta vadītāju izvēles konkursā. Projekta vadītāju raksturojumi tiek uzskatīti par mazāk svarīgiem efektīvai projektu vadībai. Modelis ir bāzēts uz projektu vadītāju multi-raksturīpašību izvērtēšanu.

Šajā rakstā tiek izskatīta pelēko attiecību metodoloģija, lai noteiktu alternatīvu lietderību, un tiek piedāvāta arī multi-kritēriju metode – kompleksā proporcionālā alternatīvu novērtēšana (COMplex PROportional Assessment) ar Greija attiecībām (COPRAS-G).

Atslēgvārdi: *COPRAS, pelēkās attiecības, vadītājs, izvēle*

L. Ustinovičius, G. Ševčenko. Būvniecības investīciju projektu riska līmeņa izvērtēšana, *Computer Modelling and New Technologies*, 12.sēj., Nr.3, 2008, 29.–37. lpp.

Investīciju riska pārvaldīšana ir ikviena investīciju projekta vai biznesa parasta prakse. Riska izvērtēšana ir jāveic dažādās projekta stadijās. Praksē tas ir neiespējami izvairīties no neprecīzas informācijas, tādējādi nepatīkamās riska situācijas rodas un tā konsekvences var būt ļoti bīstamas kā projektam, tā arī biznesam kopumā.

Rakstā tiek iztirzāti dažādi riska paveidi. Risku klasifikācija tiek noteikta ar riska pārvaldīšanas efektivitāti. Autori rakstā piedāvā multi raksturīpašību salīdzinājuma analīzes metodi (CLARA un SAW metodi). Lai ilustrētu metodes darbībā, rakstā autori sniedz praktiskus piemērus.

Atslēgvārdi: *investīciju projekts, risks, multi raksturīpašību analīze*

L. Tupenaite, L. Kanapeckiene, J. Naimaviciene. Zināšanu pārvaldīšanas modelis būvniecības projektos, *Computer Modelling and New Technologies*, 12.sēj., Nr.3, 2008, 38.–46. lpp.

Pagātnē pastāvēja nestrukturēta pieeja mācīties pēc būvniecības projektiem, kad tie ir pabeigti. Pašlaik būvniecības industrija adaptē zināšanu vadības konceptus, lai uzlabotu situāciju. Šajā rakstā tiek iztirzāti zināšanu vadības labumi, kas noder būvniecības industrijas organizācijām un projektiem.

Parādīt zināšanu vadības modeli ir šī raksta galvenais mērķis. Raksts sastāv no trīs daļām. Pirmajā daļā tiek apskatīts zināšanu vadības konceptus būvniecībā. Otrajā daļā tiek iztirzāti dažādi

zināšanu vadības modeļi, kuri ir apskatīti zinātniskajās publikācijās. Trešajā daļā, pamatojoties uz doto modeli, būvniecības projektu vadībai pielietotās sistēmas (*Knowledge Based Decision Support System*) uzbūve un (*Multiply Criteria Analysis Method COPRAS*) metodes pielietojums būvniecības lēmumu atbalstam tiek iztirzāti.

Atslēgvārdi: *zināšanu pārvaldīšana, būvniecības projekti, multiplo kritēriju analīze*

J. Kopitovs, L. Gringlāz, A. Muravjovs. Multiproduktu inventarizācijas problēmas modelēšana, *Computer Modelling and New Technologies*, 12.sēj., Nr.3, 2008, 47.–54. lpp.

Autori rakstā apskata multiproduktu inventāra kontroles modeli ar preču nejaušu pieprasījumu. Tiek piedāvāts modelis ar fiksētu laika periodu starp momentiem, izvietojot blakusesošos pasūtījumus. Pasūtījumu daudzums tiek noteikts kā starpība starp fiksēto krājumu līmeni un preču daudzumu pasūtījuma momentā. Tiek pieņemts, ka katrs pasūtījums drīkst ietvert tikai fiksētos produktu veidus. Apskatītais modelis tiek realizēts, kombinējot analītisko un simulācijas pieeju attiecīgi MS Excel un ExtendSim paketēs. Apskatītais modelis ir aktuāls, jo tas parāda reālo biznesa situāciju.

Atslēgvārdi: *inventāra kontrole, nejaušs pieprasījums, analītiskā metode, optimizācija, simulācija*

A. Ben-Jards, Z. Sinuani-Sterns, L. Fridmans, J. Hadads. Rindu metode, kas pamatota uz atšķirību starp svērtiem preču izlaidumiem un ieguldījumiem, *Computer Modelling and New Technologies*, 12.sēj., Nr.3, 2008, 55.–65. lpp.

Šajā rakstā autori izstrādā jaunu metodi, lai parādītu kopējo svaru lēmumu pieņemšanas vienību – Decision-Making Units (DMUs) – efektivitātes punktu izvērtēšanai, kas pamatojas uz daudzkārtīgām investīcijām un daudzkārtīgiem preču izlaidumiem. Jaunajā metodē, šis kopējais svars tiek izvērtēts saskaņā ar atšķirību starp *DMUs* preču izlaidumu svērto summu un investīciju svērto summu. Autori piedāvā divas pieejas *DMUs* sarindošanai; pirmā metode sarindo *DMUs* saskaņā ar absolūto „tīro peļņu” (atšķirība starp preču izlaidumu svērto summu un investīciju svērto summu), kamēr otrā metode sarindo *DMUs* saskaņā ar attiecības relatīvās efektivitātes punktiem starp preču izlaidumu svērto summu un investīciju svērto summu. Pielikumā autori prezentē metodi fiksēt objektīvās saiknes mainīgo lielumu svāriem datu apiešanas analīzē – Data Envelopment Analysis (DEA), kas pamatojas uz iepriekšminēto sarindošanas metodi. Autori pierāda, ka šīs saiknes ir iespējamie risinājumi *DEA* metodoloģijai. Sarindošanas metodes tiek ilustrētas ar piemēriem, izpētot 24 slimnīcas Izraēlā.

Atslēgvārdi: datu apiešanas analīze – Data Envelopment Analysis (DEA), kopējais svars, slimnīcas

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19. **Authors Index**

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20. **Acknowledgements**

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The registration fees will be **Euro 100** before 10 September 2009, and **Euro 150** after this date. This fee will cover the participation in the sessions, coffee breaks, daily launch, hard copy of the conference proceedings.

VENUE

Riga is the capital of the Republic of Latvia. Thanks to its geographical location, Riga has wonderful trade, cultural and tourist facilities. Whilst able to offer all the benefits of a modern city, Riga has preserved its historical charm. It's especially famous for its medieval part – Old Riga.

Old Riga still preserves many mute witnesses of bygone times. Its old narrow streets, historical monuments, organ music at one of the oldest organ halls in Europe attract guests of our city. In 1998 Old Riga was included into the UNESCO list of world cultural heritage.

ACCOMMODATION

A wide range of hotels will be at the disposal of participants of the conference and accompanying persons (http://eng.meeting.lv/hotels/latvia_hotels.php).

FURTHER INFORMATION

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