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THE VOICE OF THE ANCIENT BARD

Youth of delight, come hither,
And see the opening morn,
Image of truth new-born.

Doubt is fled, and clouds of reason,
Dark disputes and artful teasing.
Folly is an endless maze,
Tangled roots perplex her ways.

How many have fallen there!
They stumble all night over bones of the dead,
And feel they know not what but care,
And wish to lead others, when they should be led.

****

SONGS OF EXPERIENCE, **William Blake**, 1794

This 13th volume No.4 is devoted to various questions of **Applied Statistics and Operation Research, Mathematical Physics and Computer Modelling, Logistics, Solid State Physics and Innovative Technologies**. In particular, we present actual papers from Israel, USA, India, Byelorussia and Latvia.

Our journal policy is directed on the fundamental and applied sciences researches, which are the basement of a full-scale modelling in practice.

This edition is the continuation of our publishing activities. We hope our journal will be interesting for research community, and we are open for collaboration both in research and publishing. This number continues the current 2009 year of our publishing work. We hope that journal's contributors will consider the collaboration with the Editorial Board as useful and constructive.

EDITORS

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Computer Modelling and New Technologies, 2009, Vol.13, No.4, 7–14
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TIME FACTOR IN THE AGE OF GLOBALISATION

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This article considers time factor as a physical category and its interrelation with financial processes in economy by the example of classical market economy of the USA. There is performed a regression and correlation analysis of proposed mathematical models for interrelations of financial indicators during 45 years of relatively stable development.

Keywords: *time, rate of return, Dow Jones Index, regression, correlation, mathematical model*

Nowadays, strategy of companies and whole countries should consider post-industrial factors of development, such as toughening of competition in world markets, globalisation, prevalence of information technologies, acceleration of changes etc. Today processes of globalisation cover more than one third of all countries in the world. It must be emphasized that in the age of globalisation socio-economic events are of non-Markovian type [1]. Globalisation does not solve problems of counties, which are behind in their development: industrially advanced countries dominate in the world economy, and their breakaway from developing states does not get shorter. In the foreseeable future the forthcoming of integrated open world market of technologies, goods and services is not intended [2]. The global leader is the USA that provides control of world innovations as the main source of income. The U.S. produces about one fourth of the gross world product and about one third of world added value.

Globalisation became reality owing to the development of modern communications, providing instant connection among people all over the world. Psychologically people observe a sort of contraction of space and acceleration of time flow. Progress of globalisation provides time saving, but then the deficiency of individual's free time increases. The rate of changes in human life is constantly growing; new technologies are rapidly spreading across the world; the periods of mastering of new product releases are reduced. For instance, today the time needed for elaboration and start-up of production of new cars decreased by 1.5 times, during 5 years, starting from the year 2000, the number of mobile phone network subscribers increased by 10 times etc.

Acceleration of development rates and changes in economy of the countries result in that standard management solutions do not work in global economy anymore. According to Pareto principle (80/20) 80% of the efforts give 20% of the results, the rest 80% are provided by 20% of resources and time. According to experts' opinion in the age of globalisation this principle tightens to 99/1. When making decisions in global economy, the key point is not the amount of efforts but the correct application thereof. Moreover, there are not standard algorithms for choosing application point and direction of the applied efforts, as it was earlier. Increasing rates of changes and time deficiency require elaboration of new ideas in business, creative solutions and new direction of thinking. Nowadays intuition of managers has paramount importance when quickly making executive decisions. But unfortunately educational institutions are not able to teach graduating students creative thinking and develop their intuition, such teaching methods do not exist. In the last few years lots of business projects are based on intuition, not on calculations, because it is often impossible to estimate the situation and decisions should be made quickly.

There is not any clear scientific definition of human’s intuition. Researches of the last few years show that there is a connection between human's intuition and subjective change of time flow, in particular, with the “slow time” phenomenon [3]. The researches, conducted with people, who happened to be involved in extreme situations (test pilots, astronauts), when their life depended on quick arrival at the right decision, showed the following. With positive emotions people experienced acceleration of subjective time flow. Even for ordinary people pleasant pastime passes quickly, as one day, for example, relaxation at a resort during vacation. On the other hand, with negative emotions under severe stress (air crashes, road traffic accidents etc.), when human life hangs on a thread, subjective time flow slows down. This gives a possibility for an experienced and organized individual to evaluate the situation, make the right decision and perform the necessary actions to save own life within a few split seconds. In the last few years there were some well-known cases, when during demonstration flights planes impacted in the air. However, owing to their intuition and experience, pilots managed to eject themselves and survived. Then, recalling the accident in details, they noted that at the moment of the accident time almost stopped, and owing to that they could save their life and often other people life as well. Undoubtedly, similar situations of severe stress and time deficiency take place also in business, when managers have to make executive decisions, which affect the fate of thousands of people. This all indicates the important role of the time phenomenon in contemporary business conditions of development.

The nature of interactions between economic and social systems and the environment changes in post-industrial economy. There appear such new forms of interaction as individual approach to clients, the use of crucial moments in the development of society, anticipation of changes in the needs for new goods in the markets etc. The significant importance has high velocity of innovative changes, continuous development, concentration of workforces and means on breakthrough directions and flexible strategies. At the end of the 20th century becomes actual the synergetic market hypothesis [4]. It is a market of goods, which have the major part of their cost composed of innovations, information and intellect. A specific thing here is non-linear nature of development of economic systems, as well as their multi-variance and alternativity. The state of linear development of systems is only a single particular case, usually on short time intervals. Actually, during the process of development systems are subjected to exponential growth, pass through various bifurcation points, transitions to attractors etc. As a result there is a field of multi-probabilistic states, but not a single finite state of the system. Significant become quantum leaps of economic systems from one level to another, caused by accumulation of fluctuations, or catastrophes [5]. Here it is necessary to determine the controlling interactions, which are able to initiate or prevent from such leaps.

In such complicated economic systems, which develop in non-linear way, time is the backbone factor. Functional efficiency of economic and social systems in many ways will depend on the proper consideration and application of time phenomenon. Formerly used classical target functions, directed to profit maximisation and cost minimisation in economy, do no longer quite adequately correspond to successful development of society. A contemporary alternative is acquisition of positive synergetic effect, which is the result of mutual actions of different elements of economic systems and which leads to changes of properties of systems, their development paths [4]. Synergetic effect is achieved under conditions of bifurcations in a system, when determinism and randomness of development become equally probable. As it is shown below the significant role in achieving or not achieving the synergetic effect belongs to the phenomenon of time density decrease – to a singular bifurcation point.

It is known that such properties as space and time are common to any system. Although “time” is a fundamental notion, its conception and even definition have not been elaborated and given in full until now. There are only clear definitions of different ways for measuring time spans. There are two opposite approaches to the nature of time. According to the first approach (relational approach) there is no time by itself in the nature, it is merely a property of physical objects. The second approach (substantial approach) considers time as independent natural phenomenon, which interacts with surrounding objects. In classical Newton’s physics time is not directly connected to the physical world, it flows with a constant velocity and is not subjected to external influence. However, early in the 20th century A. Einstein elaborated the theory of relativity, where the main difference from the Newton’s theory was negation of unified time flow. In Einstein’s theory time is closely connected to space and both they form a four dimensional space-time continuum. According to the theory of relativity observations in space and time are always relative, they do not have absolute nature. A. Einstein refuses absolute nature of time: time can accelerate and slow down. The speed of time flow depends on the speed of object movement. On the basis of the known formula for object velocity it is possible to write that $dt = dS/v$, where S is path, t is time and v is object velocity, d is the corresponding variation. If we assume that there are infinitely large object movement velocities v, then time variation dt will tend to zero. This means that if an object moves with infinitely large velocity, time flow slows down. According to Einstein’s general theory of relativity time flow is affected by the properties of space itself, namely by the gravity force, which is able to distort space. This implies that variation of the common time flow in the system can lead to variation of properties and structure of space, where the system is located.

Astronomer N. Kozyrev suggested a hypothesis that time has physical properties [6]. Therefore it can have an effect on substance. N. Kozyrev thought that action of time depends on its density, which is non-uniformly distributed in space. The effect of time density decreases the entropy. In addition, time is a flow, which carries information. For economy it is an important factor. Dilation and stop of time break the transfer of information from the past to the future, further development of the system no longer depends on the past, and it becomes non-linear and undetermined. As the result the order of the system is broken, and a singular bifurcation point is reached. In economic systems in such situation there is a possibility that chaos occurs that is the time of making wrong decisions with all the crisis sequences [7] that come out with it. Time periods, which are described by minimal dispersions of costs and profit in economy correspond to bifurcation points, dilation of time and the maximal uncertainty of the further economic development [8]. It is practically impossible to forecast at this time. Statistical data, by the example of the USA, show that the least values in dynamics of variation of rate of profit quite exactly correspond to the onsets of crisis developments in the stock market (fall of stocks) [9]. It is known that the stock market is a “barometer”, which responds to the difference in values of the rate of profit between various market objects. Along with dilation of time there occurs uncertainty that leads investors to wrong decisions, and generally – to the chaos in the market, panic and fall of stocks.

For confirmation of interrelation between the rate of profit and the change in stock price statistical data of classical American economy were analysed over a period of 45 years with respect to its stable development (1947 through 1991). In the diagrams for Dow Jones Industrial Index (1) and variation coefficient of rate of profit (2) it is possible to clearly distinguish several domains (A, B, C,…), where a certain conformity can be observed (Figure 1). With decrease of variation coefficient of rate of profit after some time, or sometimes at once, there is a fall of stocks in the exchange market. Then, after variation of rate of profit starts increasing, stock price grows as well, usually with a delay.

In order to study dependencies between the given data, correlation and regression analysis is made. As the resultant attribute (y) is taken the stock price (Dow Jones Index), factor attributes are variation coefficient of rate of profit (x_1) and net profit margin (x_2). Originally a possibility of application of a linear dependence between y and attributes x_1, x_2 is considered. For this purpose were calculated the best approximation functions for the considered attributes $y = f(x_i)$ and corresponding determination indexes R^2:

$$R^2 = 1 - \frac{\varepsilon^2}{\sigma^2}\ , \tag{1}$$

where ε^2 – residual (unexplained) dispersion of attribute y; σ^2 – total dispersion of the resultant attribute y.

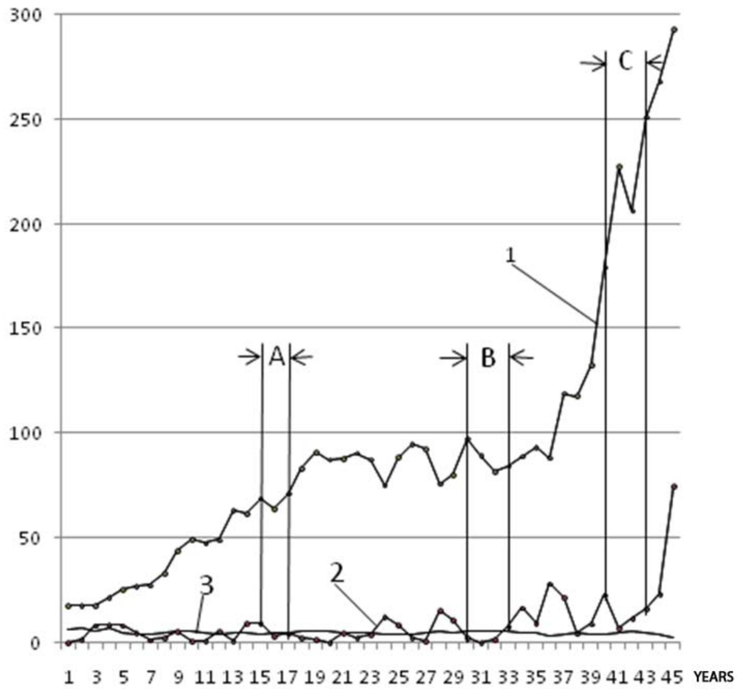

Figure 1. Dynamics of stock price in the stock market (Dow Jones Index/10, curve 1), variation coefficient of rate of profit (%%, curve 2) and net profit margin (%%, curve 3) in industry [9]

Linear regression models $\bar{y}_x = f(x)$ were constructed and linear correlation coefficients r were calculated:

$$r = \frac{\sum_{1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{n \cdot S_x \cdot S_y}\ , \tag{2}$$

where $\bar{x}$, $\bar{y}$ are mean values of arrays , Y; S_x, S_y X are standard deviations of arrays X, Y; n is the number of observations in arrays.

The results of calculations are shown in Table 1. It is known that the larger is curvature of regression line, the lesser is coefficient of determination r^2 than index of determination R^2. According to data computed in Table 1, for both dependencies $y = f(x_i)$ the difference $(R^2 - r^2)$ does not exceed 0.1. Therefore, there is no need to complicate the form of regression line and it is possible to use a linear function.

Table 1. Results of calculating dependencies

Dependence	Approximating function, index of determination R^2	Linear regression, coefficient of determination r^2
$y = f(x_1)$	Polynomial $y = -0.2042\,x_1^2 + 46.976x_1 + 554.37$ $R^2 = 0.4139$	$y = 34.121x_1 + 622.68$ $r^2 = 0.403$
$y = f(x_2)$	Exponential $y = 5223.5\exp(-0.403x_2)$ $R^2 = 0.2563$	$y = -325.28x_2 + 2504.6$ $r^2 = 0.1918$

To define the structure of interrelations of attributes a correlation matrix is calculated and analysed (Table 2).

Table 2. Correlation matrix

	y	x_1	x_2
y	1		
x_1	0.634838	1	
x_2	–0.43790	–0.57078	1

In order to evaluate overall quality of the acquired models null-hypotheses were checked for Student's t – distribution and F – Fisher statistics: $H_0 - (\rho = 0)$ and $H_0 - (R^2 = 0)$. Here ρ is the verified correlation coefficient of general population. Standard error of a correlation coefficient:

$$S_r = \sqrt{\frac{1-r^2}{n-2}}, \tag{3}$$

where n is the total number of observations.

The actual value of t – criterion was found by using formula:

$$t_{act} = \frac{r-\rho}{S_r}. \tag{4}$$

To check the main hypothesis about statistical insignificance of regression equations F – Fisher test was used:

$$F_{\mathbf{act}} = \frac{R^2/k}{(1-R^2)/(n-k-1)}, \tag{5}$$

where k is the number of factor variables, n is the number of observations.

Results of calculation of criterions are given in Table 3.

Table 3. Observed values of criterions

Dependence	S_r	t_{act}	F_{act}
$y = f(x_1)$	0.1178	5.390	29.034
$y = f(x_2)$	0.1371	–3.194	10.202
$x_1 = f(x_2)$	0.1252	–4.559	20.784

Assuming that for both criterions significance level is $\alpha = 0.05$, number of degrees of freedom for factor sum of squares is $k_1 = h - 1 = 2 - 1 = 1$ and residual sum of squares is $k_2 = n - h = 45 - 2 = 43$ (h is the number of parameters in a regression equation) and using tables of Student's and Fisher's distributions critical values of criterions are found: $t_{cr} = 2.02$ and $F_{cr} = 4.07$. With significance level $\alpha = 0.01$ critical values are: $t_{cr} = 2.7$ and $F_{cr} = 7.3$. In both cases ($\alpha = 0.05$ and $\alpha = 0.01$) it is obtained that absolute values of actual values of criterions exceed their critical levels ($t_{act} > t_{cr}$ и $F_{act} > F_{cr}$), therefore, both null-hypotheses are rejected. With probability of 99% it is possible to make conclusion that in all three cases of attribute dependencies there is a linear correlation and the quality of linear models is good enough.

Computation of multiple correlation coefficients $r_{y(x_1,x_2)}$ and determination coefficients $R^2_{y(x_1,x_2)}$ is done according to the data of the correlation matrix:

$$r_{y(x_1,x_2)} = \sqrt{\frac{r^2_{yx_1} + r^2_{yx_2} - 2r_{yx_1} \cdot r_{yx_2} \cdot r_{x_1x_2}}{1 - r^2_{x_1x_2}}}\ , \tag{6}$$

$r_{y(x_1,x_2)} = 0.6416;$ $\qquad R^2_{y(x_1,x_2)} = 0.4116.$

In order to compare values of determination coefficients R^2 of various models, numbers of independent variables in models were taken in consideration – corrected coefficient of determination R^2_{corr} were calculated:

$$R^2_{corr} = 1 - (1 - R^2) \cdot \frac{n-1}{n-h}\ . \tag{7}$$

It is obtained that corrected coefficient of determination for a one-factor model ($R^2_{corr.y(x_1)} = 0.3891$) is larger than this for a two-factor model ($R^2_{corr.y(x_1x_2)} = 0.3836$). Therefore, in terms of determination coefficient R^2 one-factor regression model is preferable to a two-factor model.

Regression of time series can have a problem of multicollinearity – a linear dependence between factor attributes, which can cause errors in analysis. In the considered case independent factor variables x_1 and x_2 have a defined time-trend and are correlated with each other. Linear correlation coefficient is $r_{x_1(x_2)} = -0.57078$ (according to the correlation matrix), it states that there is a moderate negative linear association between x_1 and x_2. Taking into account the abovementioned and that attributes of x_1 and x_2 are similar by implication, one-factor model of linear regression is finally chosen:

$$y = 34.121x_1 + 622.68; \tag{8}$$

$r_{y(x_1)} = 0.634838.$

In this equation regression coefficient and the equation itself are significant when $\alpha = 0.05$ as well as when $\alpha = 0.01$, because actual F – Fisher criterions are larger than their critical levels. The correlation coefficient $r_{y(x_1)} = 0.634838$ states that there is a moderate linear dependence $y = f(x_1)$.

To check the quality of regression model, analysis of residuals e_i was carried out, it made possible to determine models adequacy to empirical data. Here residual is deviation of the actual value of the dependent variable y from the value of the same variable, calculated from regression equation: $e_i = y_i - y'_i$ ($i = 1; n$). Root-mean-square error of the regression equation was calculated:

$$S_e = \sqrt{\frac{\sum_{i=1}^{n} e_i^2}{n-h}}\ . \tag{9}$$

It is obtained that $S_e = 511.78$. This is less than the calculated root mean square deviation of resultant attribute: $\sigma_y = 654.803$. Therefore it is possible to state that regression model corresponds to empirical data and is appropriate for application.

Since analysis of dynamics time series is fulfilled, autocorrelation test in residuals according to Durbin-Watson criterion was performed. This method is used to determine autocorrelation of a random component u_i in a regression equation, which is subjected to autoregressive process of the first order: $u_t = \rho \cdot u_{t-1} + e_t$ ($t = 1$; n). Check of the main hypothesis H_0: $\rho = 0$ is performed. For the check is used statistics of Durbin-Watson criterion:

$$DW = \frac{\sum_{t=2}^{n} (e_t - e_{t-1})^2}{\sum_{t=1}^{n} e_t^2}. \tag{10}$$

It was obtained that $DW_{\text{obs.}} = 0.561382$. Critical values of Durbin-Watson statistics d_L and d_U with significance level of 5% are $d_L = 1.475$; $d_U = 1.566$ (number of regressors $k = 1$, $n = 45$). Because of it is obtained that $DW_{\text{obs.}} < d_L$, an alternative hypothesis $H_1 : \rho > 0$ is assumed. This is the case of positive autocorrelation in residuals of the first order. Taking into account that sample volume is quite large, it is possible to assume that $DW \approx 2\,(1 - r_{e_t}, r_{e_{t-1}})$, then sample autocorrelation coefficient of the first order will be $r_{e_t, e_{t-1}} = 0.72$.

Regression coefficient of x_1 in the equation of model (8), which is equal to 34.121, fixes the connection force of y and x_1. This means that with increase of variation of rate of profit by a 1% point, Dow Jones Index increases on average by 34.121 points, on condition of invariant trend.

Since autocorrelation is found in residuals and both considered dynamics series have a common tendency to growth, in order to include this tendency into the regression model time t is included as an independent factor. In general, the model looks like this:

$$y = a + b \cdot x_1 + c \cdot t, \tag{11}$$

where a, b, c are coefficients.

Use of the least squares method gives a system of normal equations:

$$\begin{cases} a \cdot n + b \cdot \sum x_1 + c \cdot \sum t = \sum y \\ a \cdot \sum x_1 + b \cdot \sum x_1^2 + c \cdot \sum x_1 t = \sum x_1 y \\ a \cdot \sum t + b \cdot \sum x_1 t + c \cdot \sum t^2 = \sum yt \end{cases}, \tag{12}$$

Solution of the system (12) gives: $a = -13.938$; $b = 14.653$; $c = 35.184$.

The regression equation, taking into account the time factor, looks like this:

$$y = -13.938 + 14.653 \cdot x_1 + 35.184 \cdot t. \tag{13}$$

Parameter $b = 14.653$ determines the constraint force of y и x_1: with increase of variation of rate of profit by a 1% point and on condition of invariant trend, Dow Jones Index increases on average by 14.653 points. Parameter $c = 35.184$ describes absolute average annual increase of Dow Jones Index under the influence of various factors, but on condition that variation of rate of profit is fixed. The corrected index of multiple determinations with regard to the number of independent variables in the model is $R^2_{corr.} = 0.66$, respectively, the corrected index of multiple correlations is $r_{yx_1t} = 0.81$. Therefore, 66% of Dow Jones Index variation are determined by this multiple regression equation and depend on variation of rate of profit and factor of time. The index of multiple correlation is larger than 0.8 and it is the evidence of close relationship between factors x_1, t and the result y.

Taking into account that these are time series, multi-collinearity of independent factors x_1 and t, that is their linear connectedness, is checked. As a result such equation is obtained:

$$x_1 = 0.4761 \cdot t - 2.0833, \tag{14}$$

$R^2 = 0.2634$; $r_{x_1t} = 0.51$.

Since linear dependence of factors is expressed weakly (correlation coefficient $r_{x_1t} = 0.51$), collinearity of variables can be neglected.

Statistical significance of multiple regression equation is checked with the help of F – Fisher criterion. With the assumed significance levels $\alpha_1 = 0.05$ and $\alpha_2 = 0.01$ critical values of F – distribution points are $F_{1cr} = 3.2$ and $F_{2cr} = 5.16$ respectively. The actual calculated value of the criterion ($F_{act.} = 40.765$) exceeds critical levels, null-hypothesis is rejected and an alternative hypothesis is assumed, which states that with probability of 99% the regression equation is significant overall.

Estimation of the observed value of Durbin-Watson criterion (10) and its comparison with critical table values with significance level of 5% showed the presence of positive autocorrelation in residuals, as it was with the first model (8): $DW_{obs.} = 0.438366$; $\alpha = 0.05$; $k = 2$; $n = 45$; $d_L = 1.430$; $DW_{obs.} < d_L$. An alternative hypothesis $H_1: \rho > 0$ is assumed. Usually, when observation period and time series of dynamics are quite long, as it is in this case, DW statistics is close to zero. Since time series with annual data are analysed, it is hard to interpret regularities of behaviour of consequent residuals, because they do not have seasonal pattern. Taking into account the presence of residuals in the autocorrelation model, it is not recommended to use it for making forecasts. The corrected index of multiple correlation ($r_{yx_1t} = 0.81$) is relatively large, which tells about strong degree of connection $y = f(x_1, t)$, and the multiple regression equation adequately describes the general trend of Dow Jones Index, but does not reflect all deviations from this trend (Figure 2).

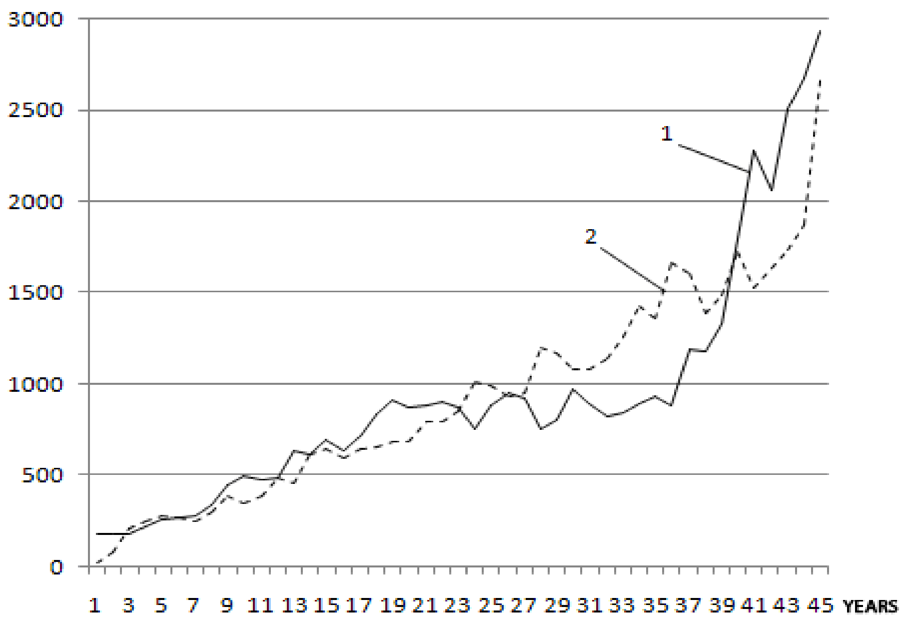

Figure 2. Change of Dow Jones Index. 1 – actual data, 2 – data from the model

Thus, multiple regression equation, which includes time factor, in conditions of stable development of the economy is more preferable than the model of paired regression without considering time. The time factor exercises a significant influence on the trend of Dow Jones Index.

An important feature of financial and economic systems in the age of globalisation is the fact that entry of even a piece of external information, which may seem unimportant at the first sight, can cause dynamic change of the parameters. Short-term and medium-term trends appear and disappear quickly. With decrease of information flows or time dilation a system approaches the state of equilibrium, when it is difficult to determine its further direction of development. The incoming economic and other information contains a stable component and a stochastic one. The stable information of time series determines trends of changes, for example, Dow Jones Index, and the stochastic information causes variation of factors, determining the phenomenon of instability. In the proposed multiple regression model $y = f(x_1, t)$ variation of Dow Jones Index is caused mainly by the effect of stochastic information of different type. The main tool to guarantee order in economic systems and to avoid slipping down to the state of crisis is timely regulation from the direction of the state. One of regulation method is, for example, change of short-term accounting interest rates. This is an impulse for financial and economic systems to exit from the state of uncertainty. This conclusion is confirmed by opinions of many experts about the global financial and economic crisis, which started in 2008.

Conclusions

In this work the connection of time phenomenon with processes in economy, in particular, with changes of indexes in the stock market is shown. It is established that the mathematical model in the form of multiple regression equation, which includes time factor, quite sufficiently describes industrial Dow Jones Index trend in conditions of economy development. This model is more preferable than the model of paired regression without considering time. An important factor for maintenance of stability in financial and economic systems and prevention of crisis developments is the regulating role of the state.

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EFFECT OF TEMPERATURE DEPENDENT PROPERTIES ON MHD FREE CONVECTION FLOW AND HEAT TRANSFER NEAR THE LOWER STAGNATION POINT OF A POROUS ISOTHERMAL CYLINDER

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The objective of the present study is to investigate the influence of temperature dependent viscosity and temperature dependent thermal conductivity on MHD free convection flow near the lower stagnation point of porous circular cylinder. The cylinder was assumed to be isothermal. The governing equations of motion and energy have been transformed into a system of non-dimensional coupled non-linear ordinary differential equations by using similarity transformations. These equations are solved by Runge-Kutta and shooting method. The numerical results are presented for velocity profiles, temperature distributions as well as heat transfer coefficient and skin friction coefficients for a wide range of viscosity variation parameter, thermal conductivity variation parameter, magnetic parameter, porosity parameter and Prandtl number.

It has been observed that with the increase in thermal conductivity parameter the velocity as well as temperature increases. Also, it is seen that the velocity increases with the increase in variable viscosity parameter, but there is a negligible decrease in temperature.

Keywords: *MHD free convection, lower stagnation point, isothermal circular cylinder, variable thermal conductivity, variable viscosity, Runge-Kutta shooting technique*

1. Introduction

MHD free convection and heat transfer process occurs in many industrial applications such as, the cooling of nuclear reactor, the geothermal systems, aerodynamic processes and heat exchanger design. The effect of temperature dependent viscosity on natural convection of fluid from heated vertical wavy surface was studied by Hossain et al. [3]. In case of vertical cone, this effect was studied by Hossain et al. [5]. Nazar et al. [12] studied the free convection boundary layer on an isothermal horizontal circular cylinder in a micro-polar fluid. In case of horizontal cylinder the radiation-conduction interaction on mixed convection was investigated by Hossain et al. [4]. Kafoussius et al. [6] studied the combined free and force convection laminar boundary layer past a vertical isothermal flat plate with temperature dependent viscosity. In porous media the effect of viscosity variation was considered by Gray et al. [1] and Mehta et al. [7]. Free convection boundary layer on cylinders of elliptic cross section was studied by Merkin et al. [10]. Harris et al. [2] studied the transient free convection near the lower stagnation point of a cylindrical surface subjected to a sudden change in surface temperature. Effect of aligned magnetic field on steady viscous flow past a circular cylinder was studied by sekhar et al. [13]. Free convection and mixed convection boundary layer flow past a horizontal cylinder was studied by Merkin [8] and [9] respectively. The effect of variable viscosity on the fluid flow past a horizontal cylinder was also investigated by Molla et al. [11].

It is observed that the effect of temperature dependent thermal conductivity on MHD free convection flow near the lower stagnation point of an isothermal horizontal circular cylinder has received a little attention. The aim of the present study therefore, is to investigate the effect of temperature dependent thermal conductivity and temperature dependent viscosity on MHD free convection flow near the lower stagnation point of a porous, isothermal horizontal circular cylinder.

2. Formulation

Consider a two dimensional MHD free convection flow of a viscous, incompressible, electrically conducting fluid over a uniformly heated circular cylinder of radius “r”. It is assumed that the surface temperature of the porous cylinder is T_w and T_∞ is the ambient temperature of the fluid. A uniform radial magnetic field of strength B_0 is applied perpendicular to the surface of the cylinder.

A locally orthogonal co-ordinate system is chosen with origin O, at lower stagnation point and X and Y denoting the distances measured along and perpendicular to the surface respectively. If “a” is the curvature of the body surface, then by the choice of axes, "a" is the principal curvature at O. The physical model and coordinate system is shown on Figure 1.

Under the usual Bousinesq approximation, the equations that governing the flow are as follows:

Equation of continuity:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0. \tag{1}$$

Equation of momentum:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta(T-T_\infty)ax+\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu(T)\frac{\partial u}{\partial y}\right)-\frac{\sigma B_0^2 u}{\rho}-\frac{\mu}{\rho}\frac{u}{k'}. \tag{2}$$

Equation of energy:

$$\rho C_\rho\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(k(T)\frac{\partial T}{\partial y}\right), \tag{3}$$

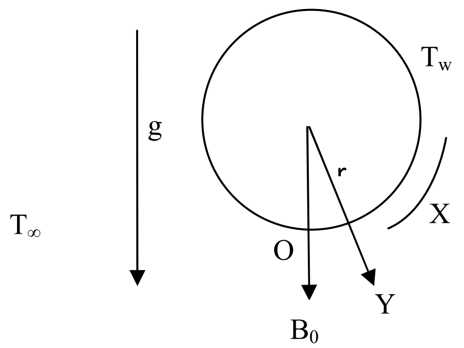

Figure 1. Physical model and coordinate system

where u and v denote the fluid velocity components in the x and y directions respectively, T is the fluid temperature, g is the magnitude of acceleration due to gravity, β is the coefficient of thermal expansion, ρ is the density of the fluid, σ is the fluid electrical conductivity, B_0 is the strength of applied magnetic field, k′ is the permeability of porous medium, C_p is specific heat at constant pressure, μ(T) is the temperature dependent viscosity of the fluid and k(T) is the temperature dependent thermal conductivity . The term $g\beta(T - T_\infty)ax$ arises from the component of buoyancy force in the x direction in the vicinity of O.

The initial and boundary conditions are:

$$u = 0,\quad v = 0,\quad T = T_w \text{ at } y = 0 \tag{4a}$$

$$u\to 0,\quad T\to T_\infty \text{ as } y\to\infty, \tag{4b}$$

it is assumed that the viscosity μ(T) and thermal conductivity k(T) varies with temperature as follows:

$$\mu(T)=\frac{\mu_\infty}{1+\gamma(T-T_\infty)} \tag{5}$$

$$k(T)=k_\infty(1+b(T-T_\infty)). \tag{6}$$

The system of partial differential equations (1)–(4) after introducing equations (5) and (6) can be reduced to a system of semi-similar equations by employing the following transformations:

$$\psi=Gr^{\frac{1}{4}}avf(\eta),\qquad \eta=Gr^{\frac{1}{4}}ay,\qquad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\qquad Gr=\frac{g\beta(T_w-T_\infty)}{a^3\ v^2}, \tag{7}$$

where ψ is the stream function, f is non-dimensional reduced stream function, θ is non-dimensional reduced temperature, Gr is Grashoff number.

Thus reduced non-dimensional equations are:

$$f'''=(1+\varepsilon\theta)\left[f'^2-ff'+\frac{\varepsilon}{(1+\varepsilon\theta)^2}\theta'f'+\frac{1}{\sqrt{Gr}}M+\left(\frac{1}{(1+\varepsilon\theta)K}\right)f'-\theta\right] \tag{8}$$

$$(1+\omega\theta)\theta''=-\left[Pr.f.\theta'+\omega\theta'^{\,2}\right]. \tag{9}$$

Here $\varepsilon=\gamma(T_w-T_\infty)$ is variable viscosity parameter, $\omega=b(T_w-T_\infty)$ is variable thermal conductivity parameter, $M=\dfrac{\sigma B_0^2a^2}{\mu_\infty}$ is magnetic parameter, $K=\dfrac{k'}{a^2}$ is porosity parameter, Prandtl number is $Pr=\dfrac{k_\infty}{\mu_\infty c_p}$ and prime (′) denote the differentiation with respect to η.

The corresponding initial and boundary conditions are:

$$f(0)=0,\qquad f'(0)=0,\qquad \theta(0)=1,\qquad f'(\infty)\to 0,\qquad \theta'(\infty)\to 0 \tag{10}$$

keeping in view of engineering aspects, the most important characteristics of the flow are rate of heat transfer and skin friction coefficients, which can be written as

$$Nu=\frac{Gr^{-1/4}}{ak_\infty(T_w-T_\infty)}q_w,\qquad C_f=\frac{Gr^{-3/4}}{a^3xv_\infty}\tau_w, \tag{11}$$

where $$q_w=-\left(k\frac{\partial T}{\partial y}\right)_{y=0},\qquad \tau_w=\left(\mu\frac{\partial u}{\partial y}\right)_{y=0}. \tag{12}$$

Using the variables equations (5)–(7) and initial and boundary conditions (10), we get the following expressions for the local Nusselt number and friction factor:

$$Nu=-(1+\omega)\theta'(0),\qquad C_f=\frac{1}{(1+\varepsilon)}f''(0). \tag{13}$$

3. Results and Discussion

The equations (8) and (9) with initial and boundary conditions (10) have been solved using Runge-Kutta and Shooting method by taking $\Delta\eta = 0.05$ and applied shooting technique for missing boundary conditions. In this numerical technique, the missing boundary conditions at the initial point being guessed first suitably.

The value of dependent variable is then calculated at the terminal point by adopting fourth-order Runge-Kutta method. The values of the dependent variable may overshoot or undershoot the given value at the terminal point. The process is repeated in each case by changing the value for the missing boundary condition at the initial point until the calculated value of the dependent variable match with the given value at the terminal point within an admissible tolerance viz., 10^{-6}.

For several values of the dimensionless parameters, values of dimensionless velocity f′(η) and dimensionless temperature θ(η) have been computed numerically and are presented on Figures 2–6. Figure 2 shows the effect of variable viscosity parameter on velocity and temperature respectively. It is seen that the velocity increases with the increase in variable viscosity parameter, but there is a negligible decrease in temperature.

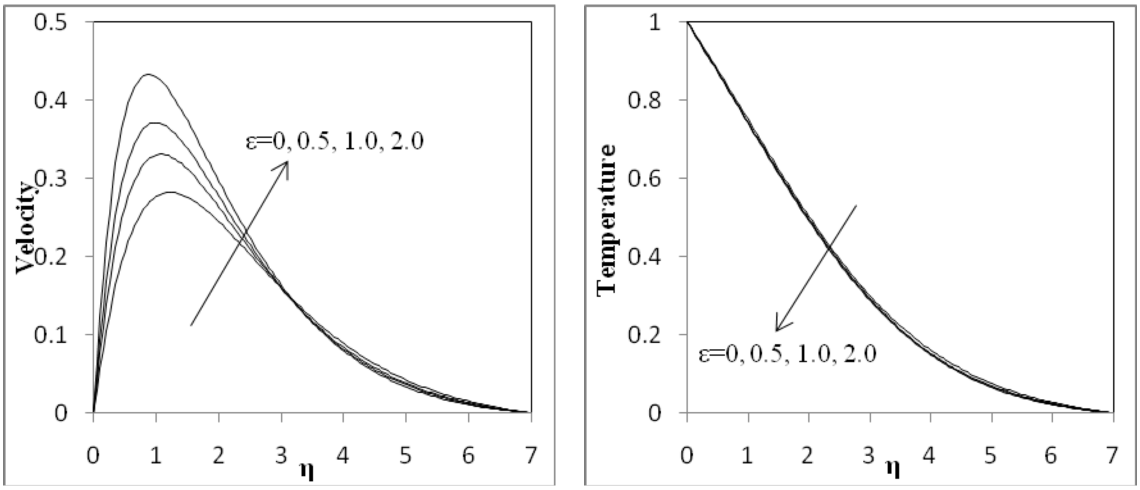

Figure 2. Velocity profile and Temperature distribution for various values of ε at Gr = 1.0, M = 0.5, K = 1.0, ω = 0.5 and Pr = 0.71

The effect of Magnetic parameter and porosity parameter on velocity and temperature are shown on Figures 3 and 4.

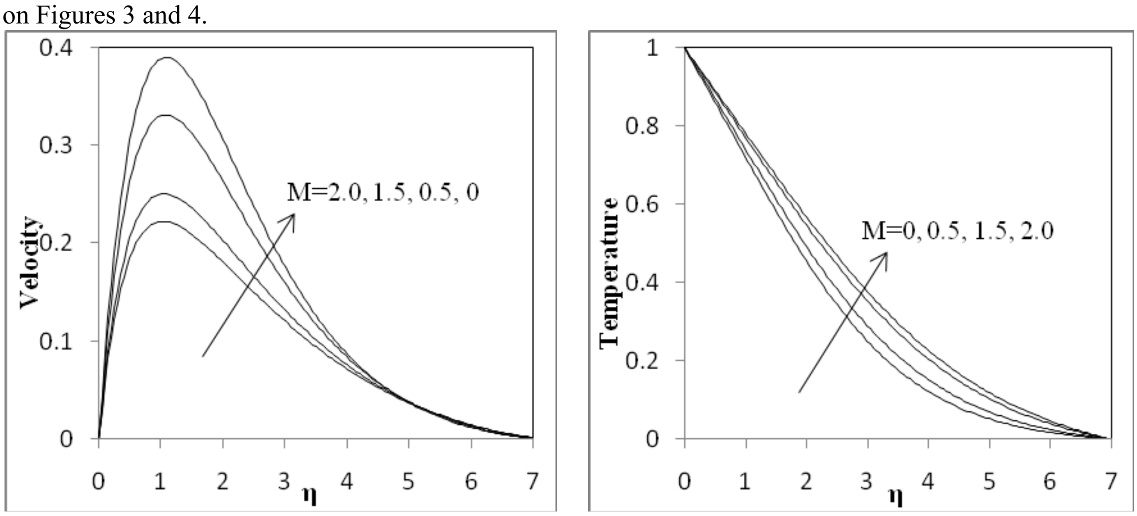

Figure 3. Velocity profile and Temperature distribution for various values of M at Gr = 1.0, ε = 0.5, K = 1.0, ω = 0.5 and Pr = 0.71

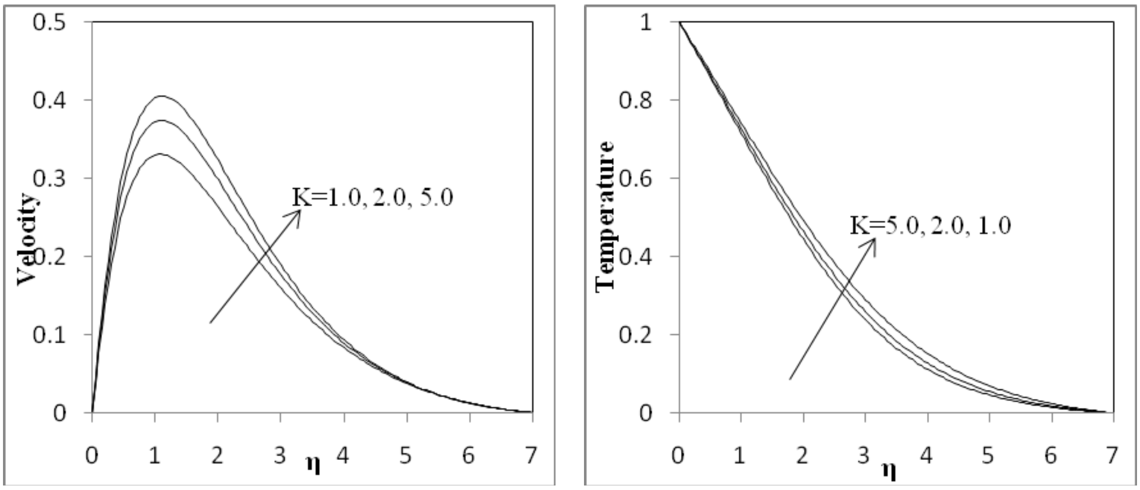

Figure 4. Velocity profile and Temperature distribution for various values of K at Gr = 1.0, ε = 0.5, M = 0.5, ω = 0.5 and Pr = 0.71

From these figures it is noticed that velocity decreases with the increase in magnetic parameter, while temperature increases. On increasing porosity parameter the velocity increases but temperature decreases.

Figure 5 depicts the effect of thermal conductivity. It is seen that velocity and temperature both increases with the increase in thermal conductivity parameter.

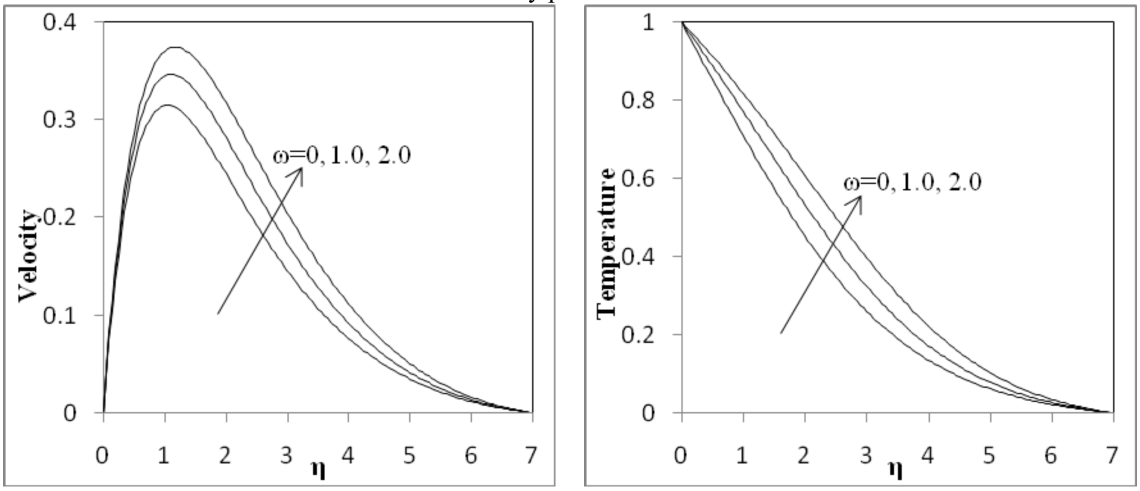

Figure 5. Velocity profile and Temperature distribution for various values of ω at Gr = 1.0, ε = 0.5, M = 0.5, K = 1.0 and Pr = 0.71

This is because as thermal conductivity parameter ω increases, the thermal conductivity of the fluid increases. This increase in the fluid thermal conductivity increases the fluid temperature and accordingly its velocity. Moreover, it is obvious that neglecting the variation of fluid thermal conductivity for high temperature differences introduces a substantial error. This error has been shown by plotting the dimensionless velocity and temperature for $\omega = 0$. The effects of Prandtl number on dimensionless velocity and temperature are shown on Figure 6.

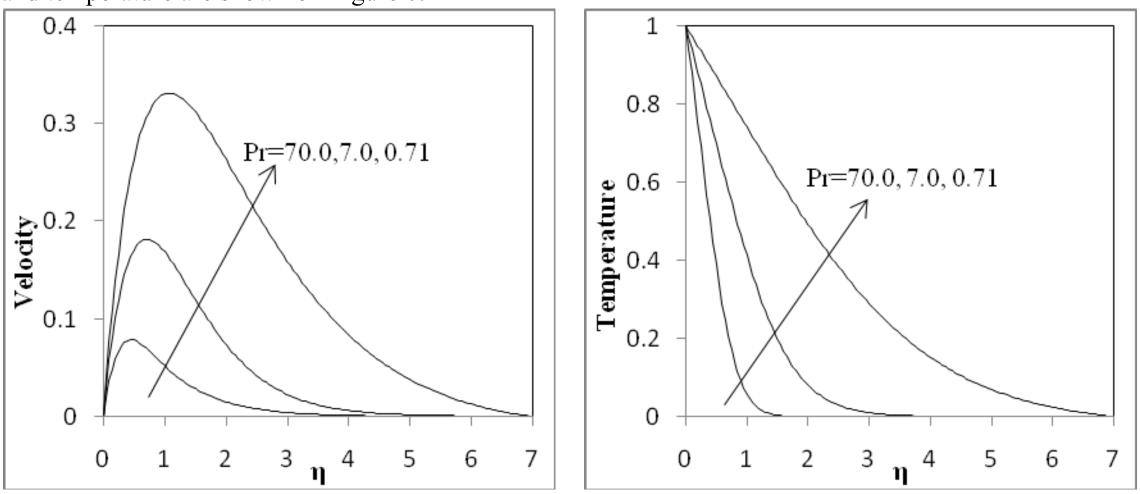

Figure 6. Velocity profile and Temperature distribution for various values of Pr at Gr = 1.0, ε = 0.5, M = 0.5, K = 1.0 and ω = 0.5

It is clear that thermal boundary layer thickness and momentum boundary layer thickness both decreases sharply with the increase in Prandl number. The numerical values of $f''(0)$ and $\theta'(0)$ are presented in table 1 for different values of dimensionless parameters ε, M, K, ω and Pr. The study for $\varepsilon = 0$, $M = 0$, $\omega = 0$ and $Pr = 1.0$ the value of $-\theta'(0) = 0.4214$ shows the good agreement with the results of by Merkin [8], Nazar et al. [12] and Molla et al. [11]. It is obvious from Table 1 that dimensionless wall velocity gradient $f''(0)$ increases as ε, K and ω increase, while it decreases with the increase in M and Pr. Moreover, the dimensionless wall temperature gradient decreases with the increase in ε and K, while it decreases with the increase in M, ω and Pr.

Table 1. Numerical values of $f''(0)$ and $\theta'(0)$ for different values of non dimensional parameters at Gr = 1.0

ε	**M**	**K**	**ω**	**Pr**	**f″(0)**	**θ′(0)**
0.0	0.0	10000	0.0	1.00	0.816500	–0.421400
0	0.5	1.0	0.5	0.71	0.625536	–0.234300
0.5	0.5	1.0	0.5	0.71	0.842870	–0.249200
2.0	0.5	1.0	0.5	0.71	1.361233	–0.274800
0.5	2.0	1.0	0.5	0.71	0.653744	–0.210446
0.5	0.5	2.0	0.5	0.71	0.912910	–0.264000
0.5	0.5	5.0	0.5	0.71	0.963130	–0.274400
0.5	0.5	1.0	0.0	0.71	0.814417	–0.312500
0.5	0.5	1.0	2.0	0.71	0.885513	–0.174900
0.5	0.5	1.0	0.5	7.00	0.655314	–0.576700
0.5	0.5	1.0	0.5	70.0	0.442264	–1.183880

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APPROXIMATION AND CONSTRUCTION OF COMPOSITE BÉZIER SURFACES USING MINIMIZATION AND FINITE-ELEMENT METHODS

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This paper presents a global method for approximation and construction of surfaces. The method is based on a minimization of a functional that describes approximation and differential geometric characteristics. The functional includes weighting factors, which are used to control the approximation process. The numerical solution of the problem takes full advantage of the Finite-Elements Method with Bézier-Bernstein shape functions and uses the *p-method* in FEM in order to improve the approximation.

Keywords: *Bézier surface; surface fitting; approximate conversion; variational problem formulation; Finite Element Method (FEM)*

1. Introduction

Approximation of surfaces has a variety of applications. For example:

- Approximation to a set of scattered points in three-dimensional space originated from scientific experiments, earth terrain description, or data from satellites.
- Exchanging format of formal data. It is required in geometric modeling systems for free form surfaces, as they use different mathematical representations and different polynomial bases for curves and surface representation.
- Conversion between non-polynomial representations (such as rational surfaces) to polynomial ones.

Additional motivations for approximation are the ability of merging curves and surfaces in order to reduce information.

Finite element methods are an essential tool for the approximation of a solution of a partial differential equation and are based on the weak variational formulation of boundary and initial value problems. The importance of this property is twofold:

- It provides the proper setting for the existence of a very irregular solution to differential equations.
- The solution appears in the integral of a quantity over a domain, which can be broken up into the sum of integrals over an arbitrary collection of almost disjoint sub-domains whose union is the original domain.

These properties allow analysis to be done locally on a typical sufficiently small sub-domain, so that polynomial functions of various degrees are adequate for representing the local behavior of the solution (see [27]). In order to arrive at a global approximation of a solution of a partial differential equation in the finite element method, their contributions of local approximation over individual elements are assembled together in a systematic way. This leads to schemes which are robust in appropriate norms and insensitive to distortions and singularities of the mesh.

1.1. HP-Method in FEM

The h-method (or the *H-Version*) is the classical form of the finite element method, in which piecewise polynomials of fixed degree p are used to approximate the solution of given partial differential equations and the size of the element mesh *h* is refined to increase accuracy. The *p-method* (or the *P-Version*) of FEM uses a fixed mesh size, but increases polynomial degree to increase ac-curacy. In the hp-method (or the H-P Version), both approaches are combined; the mesh is refined and the polynomial degree is increased to improve accuracy. An adaptive hp-method combines the h-method and p-method by refining the mesh locally in some parts of the domain and enlarging the local function spaces in other parts, or in the same parts (see [1, 2]).

1.2. Previous Work

There are several approaches for approximation of curves, surfaces, or points in three-dimensional space. Among the early important works in this field for the approximate conversion of curves and surfaces, we would like to signify the works of [11, 26] and [16]–[24]. Among the recent works there is the work of Weiss et al. [30] that attempts to provide practical solutions to overcome problems of irregular distribution of data points which are over topologically irregular domains. The Weiss et al. method includes algorithms to compute a good initial parameterisation, a procedure for handling weakly defined control points, a shape dependent knot refinement, and a fitting strategy to maintain tight tolerances and smoothness simultaneously. Their method achieves a high accuracy relative to the published 'standard' solutions.

Borges and Pastva [9] deal with the problem of fitting a single Bézier curve segment to a set of ordered data so that the error is minimized in the total least squares sense. They developed an algorithm for applying the Gauss-Newton method to this problem with a direct method for evaluating the Jacobian based on implicitly differentiating a pseudo-inverse. Chen Guo-Dong and Wang Guo-Jin [10] consider simultaneous fitting of multiple curves and surfaces to 3D measured data captured as part of a reverse engineering process, where constraints exist between the parameters of the curves or surfaces. Enforcing such constraints may be necessary.

There are several works in the CAGD field that use Bernstein-Bézier finite elements in the context of approximation. One of the earliest works on actual approximate conversion is Bercovier and Jacobi [3, 4] and Luscher [25]. Examples of later works that use FEM in CAGD are hierarchical methods for linear spline approximation and construction of surface triangulations or quadrangulations by adaptively subdividing a surface to a form of tree. It is used in an approximation to a set of scattered points in three-dimensional space using hierarchical spline and surface approximation methods such as in [8], or for approximation over irregular domain as introduced in [5, 6, 7, 31]. An implementation of cubic tetrahedral Bernstein-Bézier finite elements and their application in the context of facial surgery simulation is presented in [28].

1.3. Our Approach and Its Strategy

Our approach uses the combination of the finite element method with the Bernstein-Bézier representation, introducing a valuable finite element due to the many advantages of the Bernstein-Bézier shape functions [14]. It introduces the construction of surfaces given by piecewise definitions of their parameter range and allows surface editing, both in the direction of several (lower order) surfaces approximating a single given one, or, conversely replacing several by a single one. This approach exploits the p-method in FEM in order to improve approximation. This is done by using elements of higher degrees to overcome areas that are di cult to approximate and by using elements with lower degrees for approximating the rest of surface.

The strategy we use (see [3, 4]) is a global and continuous method for the approximation and construction of parametric surfaces. The method is based on a variational formulation of the squared integrals of the zeroth, first, and second derivative (semi) norms of the approximation and approximated surfaces. A weighting factor is related to each derivative (semi) norm. These weighting factors allow one to control the approximation of the related norm. The solution of this variational problem is done by the Finite Element Method (FEM) over Bernstein basis functions.

1.4. Outline of This Paper

The outline of this paper is as follows. Introduction with a survey on previous works and our strategy is introduced in Section 1. Presentation of the problem and its solution is in sections 2–3.6. In section 4 a survey of the methods for estimating the approximation errors is given. We briefly describe the methods we use for the initial subdivision of the problem parametric domain into finite elements in section 5. A number of examples for surface degree reduction and surface merging are shown in section 6.

2. The Problem

2.1. Problem Statement

We will first define the problem without constraints. Given a parametric surface:

$\mathbf{f}(u,v) = \mathbf{f}(f_1(u,v), f_2(u,v), f_3(u,v)), \quad u \in [a,b],\ v \in [c,d]$, find the unknown vector function $\mathbf{x}(u,v) = \mathbf{x}(x_1(u,v), x_2(u,v), x_3(u,v)), \quad u \in [a,b],\ v \in [c,d]$,

which minimizes the functional:

$$J^l(\mathbf{x}) = \begin{cases} J^0(\mathbf{x}) = E(\mathbf{x}) \\ J^1(\mathbf{x}) = E(\mathbf{x}) + \bar{E}(\mathbf{x}) \\ J^2(\mathbf{x}) = E(\mathbf{x}) + \bar{E}(\mathbf{x}) + \hat{E}(\mathbf{x}) \end{cases}, \tag{1}$$

where

$$E(x) = \alpha \iint_\Omega (\mathbf{x}(u,v) - \mathbf{f}(u,v))^2 \, dudv, \tag{2}$$

$$\bar{E}(x) = \beta \iint_\Omega \left(\frac{\partial}{\partial u}\mathbf{x}(u,v) - \frac{\partial}{\partial u}\mathbf{f}(u,v)\right)^2 + \left(\frac{\partial}{\partial v}\mathbf{x}(u,v) - \frac{\partial}{\partial v}\mathbf{f}(u,v)\right)^2 dudv, \tag{3}$$

and

$$\hat{E}(x) = \gamma \iint_\Omega \left(\frac{\partial^2}{\partial u^2}\mathbf{x}(u,v) - \frac{\partial^2}{\partial u^2}\mathbf{f}(u,v)\right)^2 + \left(\frac{\partial^2}{\partial v^2}\mathbf{x}(u,v) - \frac{\partial^2}{\partial v^2}\mathbf{f}(u,v)\right)^2 + \\ + \left(\frac{\partial^2}{\partial u \partial v}\mathbf{x}(u,v) - \frac{\partial^2}{\partial u \partial v}\mathbf{f}(u,v)\right)^2 dudv, \tag{4}$$

are the zeroth, first, and second error (semi) norms, respectively, and $\alpha, \beta, \text{and} \gamma$ positive modules, which are used as weighting factors.

3. Solution for the Problem Using the FEM Technique

In the following section we present the solution of the problem stated in section 2.1 using the FEM technique. The solution process includes: the partition of the problem's domain into two-dimensional elements, the calculation of a stiffness matrix $\mathbf{M}_e$ and load vector $\mathbf{m}_e$ for a given element e, the assembly of the elements' stiffness matrices and load vectors into the main stiffness matrix and load vector, and the calculation of the approximation error. The solution to problem (1), follows the Galerkin-Ritz solution scheme (a computational example for the Rayleigh-Ritz and Galerkin methods, using the strong form of Poisson's equation can be seen in [29]).

3.1. The Approximation FEM Space

Given the partition:

$$a = u_0 < u_1 < u_2 \cdots < u_m = b, \quad c = v_0 < v_1 < v_2 \cdots < v_n = d, \tag{5}$$

of the rectangular range

$$\bar{\mathbf{\Omega}} = [a \le u \le b \ ; \ c \le v \le d], \tag{6}$$

each sub-range

$$\Delta_e = [u_i \le u \le u_{i+1} \ ; \ v_j \le v \le v_{j+1}] \tag{7}$$

for $i = 0, \ldots, m-1,\ \ j = 0, \ldots, n-1$, is the global parameter range of an element e, where $e = 0, \ldots, L-1$ and L is the number of elements.

We use the following linear transformation to establish the relation between the global parameters $u, v \in \Delta_e$, and the local parameters $r, s \in [0,1]$:

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} u_i \\ v_j \end{bmatrix} + r\mathbf{a} + s\mathbf{b}, \tag{8}$$

where

$$\mathbf{a} = \begin{bmatrix} u_{i+1} \\ v_j \end{bmatrix} - \begin{bmatrix} u_i \\ v_j \end{bmatrix}, \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} u_i \\ v_{j+1} \end{bmatrix} - \begin{bmatrix} u_i \\ v_j \end{bmatrix}. \tag{9}$$

We introduce an $n \times m$ dimensional approximation space $V^{l,m,n}$, consisting of functions which are piecewise C^0 Bézier patches over the range $\bar{\mathbf{\Omega}}$.

Let

$$\begin{array}{lll} V^{l,m,n} = \{\mathbf{x}(u,v) & : & \mathbf{x}(a,c) = \mathbf{f}(a,c), \quad \mathbf{x}(b,c) = \mathbf{f}(b,c), \\ & & \mathbf{x}(a,d) = \mathbf{f}(a,d), \quad \mathbf{x}(b,d) = \mathbf{f}(b,d) \\ \text{such that there are} & & p \le m \text{ and } q \le n \text{ with} \\ & & \mathbf{x}(u,v) \mid_{\Delta_e} = \mathbf{S}_{p,q}(r(u), s(v)) \\ & & \text{a Bézier patch, for all } 0 \le e \le L-1 \\ & & r(u), s(v) \ \text{ derived from equation (18)}\}. \end{array} \tag{10}$$

We define $V^l \equiv \overline{\bigcup_{m,n=1}^{\infty} V^{l,m,n}}^{J^l}$, to be the *minimization space*, where $V^{l,m,n}$ is the finite dimensional subspace (10) of V^l, and J^l is functional (1).

3.2. Problem Description for a Given Element *e*

Using partition (5) and given element *e*, where $e = 0, \ldots, L-1$ and L is the number of elements, we set:

$$E_e(\mathbf{x}) = \alpha \iint_{\Delta_e} (\mathbf{x}(u,v) - \mathbf{f}(u,v))^2 \, dudv, \tag{11}$$

$$\bar{E}_e(\mathbf{x}) = \beta \iint_{\Delta_e} \left(\frac{\partial}{\partial u}\mathbf{x}(u,v) - \frac{\partial}{\partial u}\mathbf{f}(u,v) \right)^2 + \left(\frac{\partial}{\partial v}\mathbf{x}(u,v) - \frac{\partial}{\partial v}\mathbf{f}(u,v) \right)^2 dudv, \tag{12}$$

and

$$\hat{E}_e(\mathbf{x}) = \gamma \iint_{\Delta_e} \left(\frac{\partial^2}{\partial u^2}\mathbf{x}(u,v) - \frac{\partial^2}{\partial u^2}\mathbf{f}(u,v) \right)^2 + \left(\frac{\partial^2}{\partial v^2}\mathbf{x}(u,v) - \frac{\partial^2}{\partial v^2}\mathbf{f}(u,v) \right)^2 + \left(\frac{\partial^2}{\partial u \partial v}\mathbf{x}(u,v) - \frac{\partial^2}{\partial u \partial v}\mathbf{f}(u,v) \right)^2 dudv, \tag{13}$$

to be the zeroth, first, and second error (semi) norms for the element e, and for the element sub-range Δ_e (as in (7)), respectively.

Let

$$J^l(\mathbf{x}(u,v)) = \sum_{e=0}^{L-1} J_e^l(\mathbf{x}(u,v)), \tag{14}$$

where

$$J_e^l(\mathbf{x}(u,v)) = J^l(\mathbf{x}(u,v))\ |_{u,v\in\Delta_e}, \tag{15}$$

or,

$$J_e^l(\mathbf{x}(u,v)) = \{\frac{1}{2}\mathbf{b}_e^T\mathbf{M}_e\mathbf{b}_e - \mathbf{M}_e^T\mathbf{b}_e\} \quad for \;\; 0 \le e \le L-1, \tag{16}$$

where for the e-th element, $\mathbf{b}_e \equiv [\mathbf{b}_{e_{0,0}}, \ldots, \mathbf{b}_{e_{m,n}}]^T$, is the vector of the unknown Bézier points, $\mathbf{M}_e$ is the element load vector and $\mathbf{M}_e$ is the element stiffness matrix.

Our objective is to find for all m, n the function $\mathbf{x}(u,v)$ which is taken over the space $V^{l,m,n}$, in order to approximate $\mathbf{f}(u,v)$ in some sense to be defined later.

3.3. Creation of the Element Stiffness Matrix for the Bernstein-Bézier Basis

In the following section we present the calculation of the element stiffness matrix $\mathbf{M}_e$ for a given element ***e***. For each of the functionals (11–13), the construction of the stiffness matrix is influenced only by the square integrals of $\mathbf{x}(u,v)$. Therefore, after expanding functionals (11–13) and extracting the square integrals of $\mathbf{x}(u,v)$ we obtain for the L^2 norm (11):

$$\alpha \iint_{\Delta_e} \mathbf{x}(u,v)^2 dudv, \tag{17}$$

for the first error (semi) norm (12):

$$\beta \iint_{\Delta_e} \left(\frac{\partial}{\partial u}\mathbf{x}(u,v)\right)^2 dudv \quad \text{and} \quad \beta \iint_{\Delta_e} \left(\frac{\partial}{\partial v}\mathbf{x}(u,v)\right)^2 dudv, \tag{18}$$

and for the second error (semi) norm (13):

$$\gamma \iint_{\Delta_e} \left(\frac{\partial^2}{\partial u^2}\mathbf{x}(u,v)\right)^2 dudv, \quad \gamma \iint_{\Delta_e} \left(\frac{\partial^2}{\partial v^2}\mathbf{x}(u,v)\right)^2 dudv, \quad \text{and} \quad \gamma \iint_{\Delta_e} \left(\frac{\partial^2}{\partial u \partial v}\mathbf{x}(u,v)\right)^2 dudv. \tag{19}$$

In order to solve (17–19), we present the following results.
Using

$$\int_0^1 \sum_{j=0}^{n} B_j^n(t)dt = \frac{1}{n+1}, \tag{20}$$

and

$$B_i^m(t)B_j^n(t) = \frac{\binom{m}{i}\binom{n}{j}}{\binom{m+n}{i+j}} B_{i+j}^{m+n}(t), \qquad i = 0,\ldots,m; \quad j = 0,\ldots,n, \tag{21}$$

the matrix

$$\begin{pmatrix} \int_0^1 B_0^n(t)B_0^n(t)dt \ldots \int_0^1 B_0^n(t)B_n^n(t)dt \\ \vdots \ddots \vdots \\ \int_0^1 B_n^n(t)B_0^n(t)dt \ldots \int_0^1 B_n^n(t)B_n^n(t)dt \end{pmatrix} \tag{22}$$

equals

$$\frac{1}{2n+1}\left(\begin{array}{c} A^n_{0,0} \quad A^n_{0,1}\ldots A^n_{0,n} \\ A^n_{1,0} \quad A^n_{1,1}\ldots A^n_{1,n} \\ \vdots\ddots\ddots\vdots \\ A^n_{n,0} \quad A^n_{n,1}\ldots A^n_{n,n} \end{array}\right) = \frac{1}{2n+1}\mathbf{a}^n, \tag{23}$$

with

$$\mathbf{a}^n_{i,j} \equiv \frac{\binom{n}{i}\binom{n}{j}}{\binom{2n}{i+j}}, \qquad i,j = 0,\ldots,n, \tag{24}$$

which is a square coefficient matrix ($n \times n$) We similarly construct the integral

$$\int_0^1\int_0^1 \mathbf{b}^{m,n}\mathbf{b}^{m,n^T} drds, \tag{25}$$

with

$$(\mathbf{b}^{m,n})^T = \left[B^m_0(r)B^n_0(s)\ldots B^m_0(r)B^n_n(s),\ B^m_1(r)B^n_0(s)\ldots B^m_m(r)B^n_n(s)\right]^T, \tag{26}$$

as the following matrix

$$\left(\begin{array}{c} \int_0^1 B^m_0(r)B^n_0(s)B^m_0(r)B^n_0(s)drds\ldots\int_0^1 B^m_0(r)B^n_0(s)B^m_m(r)B^n_n(s)drds \\ \int_0^1 B^m_0(r)B^n_1(s)B^m_0(r)B^n_0(s)drds\ldots\int_0^1 B^m_0(r)B^n_1(s)B^m_m(r)B^n_n(s)drds \\ \vdots\ddots\ddots\vdots \\ \int_0^1 B^m_m(r)B^n_n(s)B^m_0(r)B^n_0(s)drds\ldots\int_0^1 B^m_m(r)B^n_n(s)B^m_m(r)B^n_n(s)drds \end{array}\right), \tag{27}$$

which equals to:

$$\frac{1}{(2m+1)(2n+1)}\mathbf{a}^{m,n}, \tag{28}$$

where

$$\mathbf{a}^{m,n} = \left(\begin{array}{c} \frac{\binom{m}{0}\binom{m}{0}}{\binom{2m}{0+0}}\mathbf{a}^n \ldots \frac{\binom{m}{0}\binom{m}{m}}{\binom{2m}{0+m}}\mathbf{a}^n \\ \vdots\ddots\vdots \\ \frac{\binom{m}{m}\binom{m}{0}}{\binom{2m}{m+0}}\mathbf{a}^n \ldots \frac{\binom{m}{m}\binom{m}{m}}{\binom{2m}{m+m}}\mathbf{a}^n \end{array}\right). \tag{29}$$

We denote that

$$\iint_{\Delta_e} \mathbf{x}(u,v)dudv = \mathbf{J_e}\int_\mathbf{0}^\mathbf{1}\int_\mathbf{0}^\mathbf{1} \mathbf{x(r,s)drds}, \tag{30}$$

where

$$\mathbf{J_e} = \left|\begin{array}{cc} u & v \\ & \\ u & v \end{array}\right| \tag{31}$$

is the Jacobian of the parameter transformation for the element e and $u, v \in$ and $r, s \in [0,1]$. Using the derivatives of u and v:

$$u = \Delta_{u_e}, \quad v = 0, \quad u = 0, \quad v = \Delta_{v_e}, \tag{32}$$

therefore, we have,

$$\mathbf{J_e} = \mathbf{\Delta_{u_e}} \mathbf{\Delta_{v_e}}. \tag{33}$$

Using (22–33), we can rewrite $\iint_{\Delta_e} \mathbf{x}(u,v)dudv$ as

$$\frac{\Delta_{u_e}\Delta_{v_e}}{(2m+1)(2n+1)}\ \mathbf{a}^{m,n}. \tag{34}$$

Using patch derivatives and the results (22–34), we can evaluate the integrals (17–18) which influence the stiffness matrix construction.

3.3.1. The J^0 Stiffness Matrix for a Given Element e

Writing the square integral (17) for the range $[0,1]\times[0,1]$ yields: $\alpha\mathbf{J_e}\int_0^1\int_0^1 \mathbf{x(r,s)^2 drds}$, in vector form: $\alpha\mathbf{J_e}\ \mathbf{b_e^T}\int_0^1\int_0^1 \mathbf{b^{m,n}b^{m,n^T} drds\ b_e}$, and by the substitution of (34) we obtain:

$$\mathbf{b}_e^T\ \frac{\alpha\Delta_{u_e}\Delta_{v_e}}{(2m+1)(2n+1)}\ \mathbf{a}^{m,n}\ \mathbf{b}_e. \tag{35}$$

We denote $\mathbf{C^0} \equiv \dfrac{\alpha\mathbf{\Delta_{u_e}\Delta_{v_e}}}{\mathbf{(2m+1)(2n+1)}}\ \mathbf{a^{m,n}}$ as the $\mathbf{J^0}$ stiffness matrix for the element e.

3.3.2. The $\mathbf{J^1}$ Stiffness Matrix for a Given Element e

Using the following derivatives of r and s:

$$\tfrac{\partial}{\partial u}r = \tfrac{1}{\Delta_{u_e}},\quad \tfrac{\partial}{\partial u}s = 0,\quad \tfrac{\partial}{\partial v}r = 0,\quad \tfrac{\partial}{\partial v}s = \tfrac{1}{\Delta_{v_e}}, \tag{36}$$

we can write the square integrals (18) for the range [0, 1] × [0, 1] as follows:

$$\beta\mathbf{J_e}\int_0^1\int_0^1 \frac{\partial}{\partial\mathbf{u}}\mathbf{x(r,s)^2drds}\quad \text{and}\quad \beta\mathbf{J_e}\int_0^1\int_0^1 \frac{\partial}{\partial\mathbf{v}}\mathbf{x(r,s)^2drds},\quad \text{respectively.}$$

By the expansion of the preceding terms using derivatives (36) we obtain:

$$\beta\mathbf{J_e}\int_0^1\int_0^1 \left(\mathbf{x(r,s)}\frac{\partial}{\partial\mathbf{u}}\mathbf{r} + \mathbf{x(r,s)}\frac{\partial}{\partial\mathbf{u}}\mathbf{s}\right)^2 \mathbf{drds} = \frac{\beta\mathbf{\Delta_{v_e}}}{\mathbf{\Delta_{u_e}}}\int_0^1\int_0^1 (\mathbf{x(r,s)})^2\,\mathbf{drds} \tag{37}$$

and

$$\beta\mathbf{J_e}\int_0^1\int_0^1 \left(\mathbf{x(r,s)}\frac{\partial}{\partial\mathbf{v}}\mathbf{r} + \mathbf{x(r,s)}\frac{\partial}{\partial\mathbf{v}}\mathbf{s}\right)^2 \mathbf{drds} = \frac{\beta\mathbf{\Delta_{u_e}}}{\mathbf{\Delta_{v_e}}}\int_0^1\int_0^1 (\mathbf{x(r,s)})^2\,\mathbf{drds}, \tag{38}$$

respectively. Writing (37–38) using Bézier patch partial derivatives in vector form (see [14]) yields:

$$\frac{\beta\Delta_{v_e}}{\Delta_{u_e}}\int_0^1\int_0^1 (\mathbf{x(r,s)})^2\,drds = \frac{\beta\Delta_{v_e}}{\Delta_{u_e}} m\mathbf{b}^{m,n^T}\int_0^1\int_0^1 \mathbf{\dot{B}_r^{mn}}\ \mathbf{\dot{B}_r^{mn^T} drdsb^{m,n}} \tag{39}$$

and

$$\frac{\beta\Delta_{u_e}}{\Delta_{v_e}}\int_0^1\int_0^1 (\mathbf{x}(r,s))^2\,drds = \frac{\beta\Delta_{u_e}}{\Delta_{v_e}} n\mathbf{b}^{m,n^T}\int_0^1\int_0^1 \mathbf{\dot{B}_s^{mn}}\ \mathbf{\dot{B}_s^{mn^T} drdsb^{m,n}},$$

respectively, which are the $\mathbf{J^1}$ stiffness matrices for the element e, and $\dot{\mathbf{B}}_{\mathbf{r}}^{\mathbf{mn}}$ and $\dot{\mathbf{B}}_{\mathbf{s}}^{\mathbf{mn}}$ are the first order partial derivative of Bézier patch in vector form (see 14). We denote the stiffness matrices of the first semi error norm as:

$$\mathbf{C_r^1} \equiv \frac{\beta\mathbf{\Delta_{v_e}}}{\mathbf{\Delta_{u_e}}}\mathbf{m}\int_\mathbf{0}^\mathbf{1}\int_\mathbf{0}^\mathbf{1} \dot{\mathbf{B}}_\mathbf{r}^\mathbf{mn}\ \ \dot{\mathbf{B}}_\mathbf{r}^{\mathbf{mn}^\mathbf{T}}\mathbf{drds}$$

and

$$\mathbf{C_s^1} \equiv \frac{\beta\mathbf{\Delta_{u_e}}}{\mathbf{\Delta_{v_e}}}\mathbf{n}\int_\mathbf{0}^\mathbf{1}\int_\mathbf{0}^\mathbf{1} \dot{\mathbf{B}}_\mathbf{s}^\mathbf{mn}\ \ \dot{\mathbf{B}}_\mathbf{s}^{\mathbf{mn}^\mathbf{T}}\mathbf{drds}.$$

3.3.3. The J^2 Stiffness Matrix for a Given Element e

Expanding the following square integrals of (19):

$$\gamma\iint_{\Delta_e}\frac{\partial^2}{\partial u^2}\mathbf{x(u,v)^2dudv},\quad \gamma\iint_{\mathbf{\Delta_e}}\frac{\partial^\mathbf{2}}{\partial\mathbf{v^2}}\mathbf{x(u,v)^2dudv}\quad \text{and}$$
$$\gamma\iint_{\Delta_e}\frac{\partial^2}{\partial u\partial v}\mathbf{x(u,v)^2dudv} \tag{40}$$

for the range $[0,1]\times[0,1]$, and by using derivatives (36), we obtain:

$$\gamma\mathbf{J_e}\int_\mathbf{0}^\mathbf{1}\int_\mathbf{0}^\mathbf{1}\left(\left(\mathbf{x(r,s)}\frac{\partial}{\partial\mathbf{u}}\mathbf{r}+\mathbf{x(r,s)}\frac{\partial}{\partial\mathbf{u}}\mathbf{s}\right)\frac{\partial}{\partial\mathbf{u}}\mathbf{r}+\ \left(\mathbf{x(r,s)}\frac{\partial}{\partial\mathbf{u}}\mathbf{s}+\mathbf{x(r,s)}\frac{\partial}{\partial\mathbf{u}}\mathbf{r}\right)\frac{\partial}{\partial\mathbf{u}}\mathbf{s}\right)^\mathbf{2}\mathbf{drds}=$$
$$\frac{\gamma\Delta v_e}{\Delta_{u_e}^3}\int_0^1\int_0^1\mathbf{x(r,s)^2drds},$$

$$\gamma\mathbf{J_e}\int_\mathbf{0}^\mathbf{1}\int_\mathbf{0}^\mathbf{1}\left(\left(\mathbf{x(r,s)}\frac{\partial}{\partial\mathbf{v}}\mathbf{r}+\mathbf{x(r,s)}\frac{\partial}{\partial\mathbf{v}}\mathbf{s}\right)\frac{\partial}{\partial\mathbf{v}}\mathbf{r}+\ \left(\mathbf{x(r,s)}\frac{\partial}{\partial\mathbf{v}}\mathbf{s}+\mathbf{x(r,s)}\frac{\partial}{\partial\mathbf{v}}\mathbf{r}\right)\frac{\partial}{\partial\mathbf{v}}\mathbf{s}\right)^\mathbf{2}\mathbf{drds}$$
$$=\frac{\gamma\Delta u_e}{\Delta_{v_e}^3}\int_0^1\int_0^1\mathbf{x(r,s)^2drds},$$

and

$$\gamma\mathbf{J_e}\int_\mathbf{0}^\mathbf{1}\int_\mathbf{0}^\mathbf{1}\left(\left(\mathbf{x(r,s)}\frac{\partial}{\partial\mathbf{v}}\mathbf{r}+\mathbf{x(r,s)}\frac{\partial}{\partial\mathbf{v}}\mathbf{s}\right)\frac{\partial}{\partial\mathbf{u}}\mathbf{r}+\ \left(\mathbf{x(r,s)}\frac{\partial}{\partial\mathbf{v}}\mathbf{s}+\mathbf{x(r,s)}\frac{\partial}{\partial\mathbf{v}}\mathbf{r}\right)\frac{\partial}{\partial\mathbf{u}}\mathbf{s}\right)^\mathbf{2}\mathbf{drds}$$
$$=\frac{\gamma}{\Delta_{u_e}\Delta_{v_e}}\int_0^1\int_0^1\mathbf{x(r,s)^2drds},$$

respectively.

Writing the preceding equations in vector form using the partial derivatives of a Bézier patch yields:

$$\frac{\gamma\Delta v_e}{\Delta_{u_e}^3}\int_0^1\int_0^1\mathbf{x(r,s)^2drds}=\frac{\gamma\mathbf{m(m-1)\Delta v_e}}{\mathbf{\Delta_{u_e}^3}}\mathbf{b^{m,n^T}}\int_\mathbf{0}^\mathbf{1}\int_\mathbf{0}^\mathbf{1}\ddot{\mathbf{B}}_\mathbf{rr}^\mathbf{mn}\ \ \ddot{\mathbf{B}}_\mathbf{rr}^{\mathbf{mn}^\mathbf{T}}\mathbf{drds}\ \ \mathbf{b^{m,n}},$$

$$\frac{\gamma\Delta u_e}{\Delta_{v_e}^3}\int_0^1\int_0^1\mathbf{x(r,s)^2drds}=\frac{\gamma\mathbf{n(n-1)\Delta u_e}}{\mathbf{\Delta_{v_e}^3}}\mathbf{b^{m,n^T}}\int_\mathbf{0}^\mathbf{1}\int_\mathbf{0}^\mathbf{1}\ddot{\mathbf{B}}_\mathbf{ss}^\mathbf{mn}\ \ \ddot{\mathbf{B}}_\mathbf{ss}^{\mathbf{mn}^\mathbf{T}}\mathbf{drds}\ \ \mathbf{b^{m,n}},$$

$$\frac{\gamma}{\Delta_{u_e}\Delta_{v_e}}\int_0^1\int_0^1\mathbf{x(r,s)^2drds}=\frac{\gamma\mathbf{mn}}{\mathbf{\Delta_{u_e}\Delta_{v_e}}}\mathbf{b^{m,n^T}}\int_\mathbf{0}^\mathbf{1}\int_\mathbf{0}^\mathbf{1}\ddot{\mathbf{B}}_\mathbf{rs}^\mathbf{mn}\ \ \ddot{\mathbf{B}}_\mathbf{rs}^{\mathbf{mn}^\mathbf{T}}\mathbf{drds}\ \ \mathbf{b^{m,n}}, \tag{41}$$

respectively, which are the $\mathbf{J^2}$ stiffness matrices for the element e. We denote the stiffness matrices of the second semi error norm as:

$$\mathbf{C^2_{rr}} \equiv \frac{\gamma \mathbf{m(m-1)\Delta v_e}}{\mathbf{\Delta^3_{u_e}}} \int_0^1 \int_0^1 \ddot{\mathbf{B}}^{\mathbf{mn}}_{\mathbf{rr}} \ \ddot{\mathbf{B}}^{\mathbf{mn}^T}_{\mathbf{rr}} \mathbf{drds},$$

$$\mathbf{C^2_{ss}} \equiv \frac{\gamma \mathbf{n(n-1)\Delta u_e}}{\mathbf{\Delta^3_{v_e}}} \int_0^1 \int_0^1 \ddot{\mathbf{B}}^{\mathbf{mn}}_{\mathbf{ss}} \ \ddot{\mathbf{B}}^{\mathbf{mn}^T}_{\mathbf{ss}} \mathbf{drds},$$

$$\mathbf{C^2_{rs}} \equiv \frac{\gamma \mathbf{mn}}{\mathbf{\Delta_{u_e}\Delta_{v_e}}} \int_0^1 \int_0^1 \ddot{\mathbf{B}}^{\mathbf{mn}}_{\mathbf{rs}} \ \ddot{\mathbf{B}}^{\mathbf{mn}^T}_{\mathbf{rs}} \mathbf{drds}.$$

3.3.4. General Element Stiffness Matrix

From results (35, 39, 41), one obtains the general stiffness matrix for the functionals $\mathbf{J^0}$, $\mathbf{J^1}$ and $\mathbf{J^2}$ as follows:

$$\mathbf{M}^0_e = C^0, \tag{42}$$

$$\mathbf{M}^1_e = C^0 + C^1_r + C^1_s, \tag{43}$$

$$\mathbf{M}^2_e = C^0 + C^1_r + C^1_s + C^2_{rr} + C^2_{ss} + C^2_{rs}. \tag{44}$$

3.3.5. Properties of the Element Stiffness Matrix in Relation to the Bernstein-Bézier Basis

The Bézier element mass matrix $\mathbf{C^0}$ is a symmetrical (both diagonals), positive definite matrix. All the elements are positive, and the largest element per row or column is in the main diagonal. The sum of all elements of a row (or column) is a constant for each n, and is influenced by the lengths of the element range Δ_e.

The Bézier stiffness element matrices $C^1_r, C^1_s, C^2_{rr}, C^2_{ss}$ and C^2_{rs}, are symmetrical (both diagonals), positive semi-definite matrices, with the elements in the main diagonal greater than zero. The largest absolute value per row or column is in the main diagonal and the sum of all elements of a row (or column) is zero.

Since the stiffness element matrix $\mathbf{M}_e$ is a constant matrix for every m, n, most of its calculation can be done in advance for the available degrees of approximation surfaces. The parametric components ($\Delta_{u_e}, \Delta_{v_e}$) are dependent upon the problem range partition, therefore, they cannot be calculated in advance, and only their product with the pre-calculated stiffness matrices is needed. This provides an efficient and a fast calculation of the element stiffness matrix.

3.4. Creation of the Element Load Vector for the Bernstein-Bézier Basis

In the following section, we introduce the construction of the load vector $\mathbf{M}_e$ for an element e. For each of the functionals (11–13), the construction of the load vector is influenced only by the square integrals of $\mathbf{x(u,v)}$ and $\mathbf{f(u,v)}$. This means, that after expanding the functionals (11–13) and extracting the square integrals of $\mathbf{x(u,v)}$ and $\mathbf{f(u,v)}$ we obtain:

$$\mathbf{M}^0_e(\mathbf{x}) \quad = \quad \alpha \iint_{\Delta_e} (\mathbf{x(u,v)} \cdot \mathbf{f(u,v)})\, dudv, \tag{45}$$

$$\begin{aligned} \mathbf{M}^1_e(\mathbf{x}) \quad = \quad & \alpha \iint_{\Delta_e} (\mathbf{x(u,v)} \cdot \mathbf{f(u,v)})\, dudv \quad + \\ & \beta \iint_{\Delta_e} \left(\frac{\partial}{\partial u}\mathbf{x(u,v)} \cdot \frac{\partial}{\partial \mathbf{u}}\mathbf{f(u,v)}\right) + \left(\frac{\partial}{\partial v}\mathbf{x(u,v)} \cdot \frac{\partial}{\partial \mathbf{v}}\mathbf{f(u,v)}\right) dudv, \end{aligned} \tag{46}$$

and

$$\begin{aligned}\mathbf{M}_e^2(\mathbf{x}) &= \alpha \iint_{\Delta_e} (\mathbf{x}(\mathbf{u},\mathbf{v})\cdot\mathbf{f}(\mathbf{u},\mathbf{v}))\,dudv \;+ \\ &\beta \iint_{\Delta_e} \left(\frac{\partial}{\partial u}\mathbf{x}(\mathbf{u},\mathbf{v})\cdot\frac{\partial}{\partial \mathbf{u}}\mathbf{f}(\mathbf{u},\mathbf{v})\right) + \left(\frac{\partial}{\partial v}\mathbf{x}(\mathbf{u},\mathbf{v})\cdot\frac{\partial}{\partial \mathbf{v}}\mathbf{f}(\mathbf{u},\mathbf{v})\right) dudv \;+ \\ &\gamma \iint_{\Delta_e} \left(\frac{\partial^2}{\partial u^2}\mathbf{x}(\mathbf{u},\mathbf{v})\cdot\frac{\partial^{\mathbf{2}}}{\partial \mathbf{u^2}}\mathbf{f}(\mathbf{u},\mathbf{v})\right) + \left(\frac{\partial^2}{\partial v^2}\mathbf{x}(\mathbf{u},\mathbf{v})\cdot\frac{\partial^{\mathbf{2}}}{\partial \mathbf{v^2}}\mathbf{f}(\mathbf{u},\mathbf{v})\right) + \\ &\left(\frac{\partial^2}{\partial u\partial v}\mathbf{x}(\mathbf{u},\mathbf{v})\cdot\frac{\partial^{\mathbf{2}}}{\partial \mathbf{u}\partial \mathbf{v}}\mathbf{f}(\mathbf{u},\mathbf{v})\right) dudv, \end{aligned} \tag{47}$$

which are the load vectors of $\mathbf{J^0}$, $\mathbf{J^1}$ and $\mathbf{J^2}$ respectively. The evaluation of the integrals in (46–48) is done by Gaussian quadrature, adapted to the degrees of the element.

3.5. Creation of the Global Stiffness Matrix and Load Vector

After integration element by element, we obtain:

$$\sum_{e=0}^{L-1} J_e^l(\mathbf{x}_e) = \{\frac{1}{2}\mathbf{b}^T\mathbf{M}^l\mathbf{b} - \mathbf{M}^{l^T}\mathbf{b}\} \text{ for l=0,1,2.} \tag{48}$$

The elements' stiffness matrices $\mathbf{M}_e^l$, and load vectors $\mathbf{M}_e^l$, $(e = 0, \ldots, L-1)$ are assembled into the global stiffness matrix $\mathbf{M}$ and load vector $\mathbf{M}$.

3.6. Properties of the Global Stiffness Matrix

The global stiffness matrix is sparse, square banded, symmetric and positive definite. As for the element stiffness matrix, all of the elements are positive, the largest element per row or column is in the main diagonal, and the sum of all elements of a row (or column) is a constant for each n. Its graph's shape is determined by the numbering of each of the degrees of freedom involved in the problem.
Let,

- $s = 3L(m+1)(n+1)$ be the sum of degrees of freedom for all the elements,
- ElementsAlongU, ElementsAlongV be the number of elements along u and v parameter lines respectively,
- NodesAlongU, NodesAlongV be the number of nodes along u and v parameter lines respectively (every node contains 3 degrees of freedom).

For each element of degrees $m \times n$, there are $3(m+1)(n+1)$ degrees of freedom.. Every boundary curve which shares two neighbouring elements (patches) decreases the total number of degrees of freedom by:

$$s_b = NodesAlongU * (ElementsAlongV - 1) + NodesAlongV * (ElementsAlongU - 1).$$

Every cross section of such two boundary curves (meaning, at a cross section of four neighbouring elements) decreases the total number of degrees of freedom by 9. The total sum of the of degrees of freedom at the cross sections is: $s_c = (ElementsAlongV - 1) + (ElementsAlongU - 1)$.

Therefore, the total number of the problem's degrees of freedom and the order of the global stiffness matrix is at most: $s_c = (ElementsAlongV - 1) * (ElementsAlongU - 1)$.

The creation of the global stiffness matrix is fast and efficient, since it involves only the assembly of the element stiffness matrices, which are pre-calculated and small (as described in section 3.3.5). The minimum of (49) is given by the approximation surface $\mathbf{x}(\mathbf{u},\mathbf{v})$, where $\mathbf{b}$ is the solution of the system

$$\nabla_b J^l(\mathbf{x}) = \mathbf{Mb} + \mathbf{M} = \tilde{\mathbf{0}}. \tag{49}$$

The system (50) is linear symmetric positive definite, and we use the LDL^T algorithm or the Conjugate-Gradient [32] to solve it. The coordinate components of $J^l(\mathbf{x})$ are decoupled, and the solution of the system refers to each coordinate component by itself.

4. Error Estimation

Since the approximation depends on the parameterisation of $\mathbf{x}(u,v)$ and $\mathbf{f}(u,v)$, it does not necessarily yield orthogonal error vectors between corresponding values of parameters. The absolute Euclidean minimum (or maximum) is at the point where two normals are collinear. Therefore a reparametrization is needed so that the correction of the parameterisation will direct the error vectors to be as orthogonal as possible to the tangent plane of $\mathbf{x}$ at (u,v). This will result in better error estimation [23].

We use two types of discrete error estimators:

- the largest error Euclidean distance δ:

$$\delta = \max\{\| \mathbf{x}(u_i,v_j) - \mathbf{f}(u_i,v_j) \|, \quad u_i \in [a,b], \quad v_j \in [c,d]\}. \tag{50}$$

- the maximal angle deviation ϑ, between the normals:

$$\vartheta = \max\left\{\arccos\left(\frac{\mathbf{n}_x(u_i,v_j)\cdot\mathbf{n}_f(u_i,v_j)}{\| \mathbf{n}_x(u_i,v_j) \| \| \mathbf{n}_f(u_i,v_j) \|}\right), \quad u_i \in [a,b], \quad v_j \in [c,d]\right\}. \tag{51}$$

In order to use error measurements which are not dependent on the parameterisation, we use the zeroth, first and second derivative error (semi) norms (2–4) for error estimation. The zero derivative error norm (squared error integral) is the L^2 norm. The first and second derivative error (semi) norms are used to estimate the error in the first and second partial derivatives displacements. We also use the error (semi) norms $E(x)/S^2$, $\bar{E}(x)/S^2$ and $\hat{E}(x)/S^2$, which measure the mean error displacement per unit area, where S is an approximated area of $\mathbf{f}(u)$. For error in curvature, we use the Gaussian curvature L^2 error norm:

$$E^K = \int_a^b\int_c^d (\tilde{K}-K)^2 dudv, \int_a^b\int_c^d, \tag{52}$$

where $\tilde{K}$ is the Gaussian curvature of the approximated surface $\mathbf{x}(u,v)$, and the Gaussian curvature mean deviation error: $E_m^K = \dfrac{\int_a^b\int_c^d(\tilde{K}-K)^2dudv}{\int_a^b\int_c^d(K)^2dudv}$.

We also use the mean curvature L^2 error norm:

$$E^H = \int_a^b\int_c^d (\tilde{H}-H)^2 dudv, \tag{53}$$

where $\tilde{H}$ is the mean curvature of the approximated surface $\mathbf{x}(u,v)$, and the mean curvature mean deviation error: $E_m^H = \dfrac{\int_a^b\int_c^d(\tilde{H}-H)^2dudv}{\int_a^b\int_c^d(H)^2dudv}$.

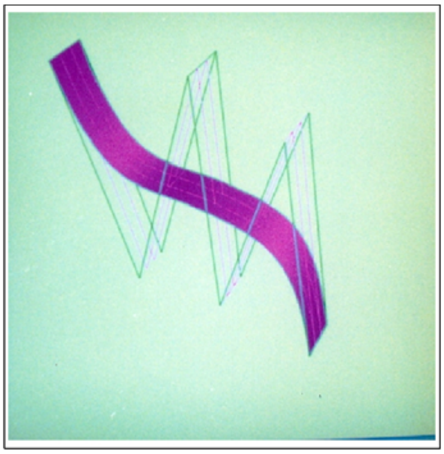

Figure 1. Bézier patch of degrees (5 x 3), with sizes of 3 x 28.5 mm and approximated area of S = 85.36 mm^2

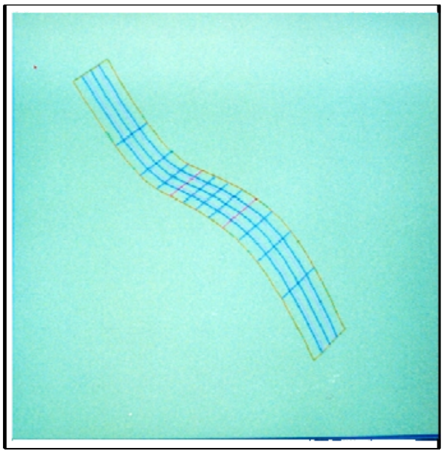

Figure 2. Reduction of the Bézier surface in Figure 1 to a (C^2, C^3) Bézier surface with three patches of degrees (4 x 3)

Error estimations of all types presented in this section are presented in Tables 1, 2, for the approximation of a Bézier patch of degrees 5 x 3, with sizes of 3 x 28.5 mm and approximated area of $S = 85.36\ mm^2$ (see Figures 1, 2), by a Bézier patch with different degrees and continuity orders between elements.

5. Methods for Initial Subdivision into Elements

We use three kinds of strategies for the initial division of the problem's parametric domain into smaller rectangular domains – each belongs to an element. After the partition of the parametric domain, each element represents an approximation patch segment which is a Bézier patch of any given degrees (elements may have different degrees as in the *p-method*, see [1].)

The strategies are:

- The division is determined arbitrarily, according to the operator's under-standing of the problem.
- The division is calculated according to an algorithm developed by Hoschek [20], which creates a subdivision into generic patches, using minimal absolute values of the isoparametric spline curvature (vertices, inflection points).

The idea behind the algorithm is to separate the minimum points of curvatures of the iso-parametric curve into distinct surface patches. This geometric criterion is then used to reduce the number of surface patches to a minimum, resulting with surface patches that do not have more than one minimum of curvature on every discretized parameter line.

Using the notion of generic curves presented in [20], the segmentation procedure is:

a) The minimal absolute values of curvature on the u and v iso-parametric lines are calculated.
b) New separating points are generated by using the mean values of the parameter values of the points in step (a), two in the cubic case and at least four in the quintic case.
c) Determination of a new separating line along the direction with the smallest number of minimal points, such that the most minimal points are separated.

This algorithm has been refined by Hoschek and presented in [24].

Table 1. Various error estimations by the derivative error (semi) norms, for the approximation of a Bézier patch of degrees 5 x 3, with sizes of 3 x 28.5 mm and approximated area of S = 85.36 mm^2, by a Bézier patch with different degrees. See Figures 1, 2.

Degrees	*Segments*	δ	ϑ	$E(x)/S^2$	$\bar{E}(x)/S^2$	$\hat{E}(x)/S^2$
2 x 3	1 x 1	1.479	0.348	2.348264e-04	2.523826e-05	5.107362e-05
3 x 3	1 x 1	0.380	0.240	1.465209e-05	1.818131e-06	5.128979e-05
4 x 3	1 x 1	0.365	0.220	1.395186e-05	1.595004e-06	1.254311e-05
5 x 3	1 x 1	0.0	1.13e-07	4.447024e-19	2.911940e-18	1.115636e-04
3 x 3	2 x 1	0.450	0.245	1.396280e-05	2.934349e-06	1.763063e-04
4 x 3	2 x 1	0.041	0.039	1.395732e-07	7.531847e-08	1.712148e-04
2 x 3	3 x 1	0.540	0.276	1.530292e-05	2.978221e-06	1.713536e-04
3 x 3	3 x 1	0.051	0.058	1.651474e-07	1.487066e-07	1.769908e-04
4 x 3	3 x 1	0.002	0.006	5.137242e-10	2.253008e-09	1.929809e-04
4 x 3	3 x 1	0.002	0.005	4.885186e-10	1.619946e-09	1.930424e-04
4 x 3	3 x 1	0.011	0.001	8.110637e-09	8.997212e-09	1.935975e-04

Table 2. Various error estimations by the curvature error norms, for the same given Bézier patch and approximation surfacesas in Table 1

Degrees	E^K	E^K_m	E^H	E^H_m
2 x 3	3.339768e+00	9.764867e-01	5.592338e+00	8.735588e-01
3 x 3	2.059826e+00	5.233216e-01	3.087451e-01	5.079714e-02
4 x 3	6.194731e-01	1.573840e-01	2.562805e-01	4.216526e-02
5 x 3	2.012408e-14	5.084107e-15	1.561175e-14	2.442805e-15
3 x 3	8.055888e-01	3.363193e-01	3.795821e-01	5.976159e-02
4 x 3	8.473989e+00	3.537743e+00	8.030403e-02	1.264311e-02
2 x 3	4.354274e-01	5.014103e-01	4.724982e-01	7.439097e-02
3 x 3	1.259487e+01	1.449829e+01	1.270752e-01	1.998752e-02
4 x 3	8.319730e+01	9.577063e+01	6.819476e-01	1.072628e-01
4 x 3	8.309422e+01	9.565197e+01	6.816075e-01	1.072093e-01
4 x 3	8.281638e+01	9.533215e+01	6.837922e-01	1.075529e-01

6. Examples

Approximation of surfaces can be used in the applications of: degree reduction of surfaces and merging of surfaces with large number of patches. Among the surfaces that we used in the section, there are the following three surfaces:

1) The surface **BRODE** (Fig. 3(a)), which is a 9 x 9 Bézier patch with sizes of 100 x 140 mm.
2) The surface **SEITE1**(Fig. 3(b)), which is a collection of 9 x 7 Bézier patches, each of degrees 3 x 3 and sizes about 500 x 2200 mm.
3) The surface **SURFB** (Fig. 3(c)), which is a collection of 17 x 63 Bézier patches, each of degrees 5 x 5 and sizes about 450 x 1800 mm.

These surfaces were used as test examples (bench-mark) for the comparison between spline conversion methods by: Eck [13], Goult [15], Hoschek and Schneider [24], Lyche and Dokken [12], and Patrikalakis and Walter [24]. The bench-mark specifications were to convert these surfaces to a 3 x 3 or a 5 x 5 Bézier or B-Spline surface with a maximal error tolerance of 0.1 and 0.01 mm. For each of the surfaces involved in the bench-mark a table was presented, in which the degrees and the segment number of the result approximation surface are listed in the first two columns. In the third column the inner continuities in both parameter directions are entered. The fourth column contains the compression factor. The compression factor for the conversion to Bézier surface is given by the quotient:

$$\frac{\text{number of Bézier points of the given patches}}{\text{number of Bézier points of the converted patches}},$$

and the compression factor for the conversion to B-Spline surface is given by the quotient:

$$\frac{3(\text{number of Bézier points of the given patches})}{3(\text{number of B-Spline control points}) + \text{knots in the knot vectors}}.$$

In the fifth column the approximation presents the prescribed approximation tolerance, and in the sixth column the largest error Euclidean distance

$$\delta = \max\{\| \mathbf{x}(u_i, v_j) - \mathbf{f}(u_i, v_j) \|, \quad u_i \in [a, b], \quad v_j \in [c, d]\},$$

is given. We will use the same format of table for our examples.

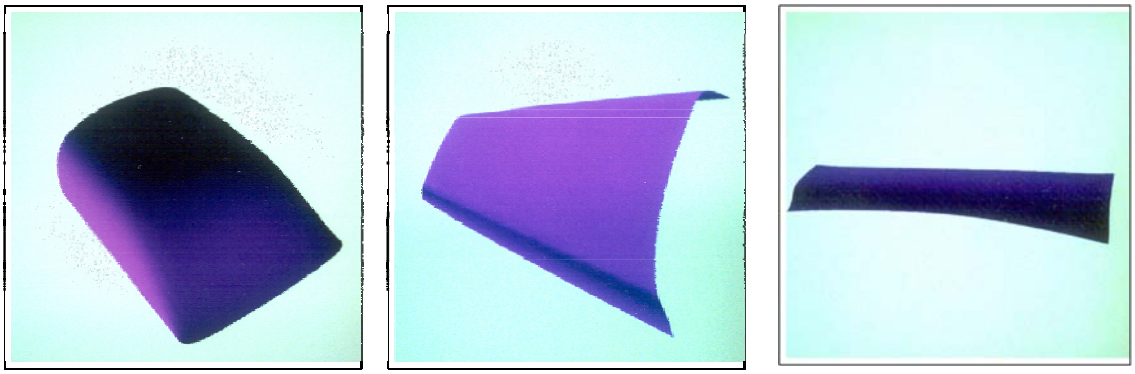

Figure 3. Benchmark Surfaces:

(a) The **BRODE** surface (b) The **SEITE1** surface (c) The **SURFB** surface

6.1. Degree Reduction

Reduction of degree of high order polynomial surfaces to polynomials of a lower order. The degree reduction approximation might reduce or increase the number of surface patches. Tables 3 and 4 present degree reduction for **BRODE** and **SURFB**. The last two lines of Table 3 shows that a surface can be approximated by giving each row and column of patches in the lattice that defines their topology different degrees, this is an important property which allows greater flexibility in the choice of the elements degrees, a more economical approximation result. Other approximation methods do not provide this property. Figure 4 presents an approximation C^0, C^0 surface with (3 x 2) patches and degrees: (3, 5, 3 x 3, 4).

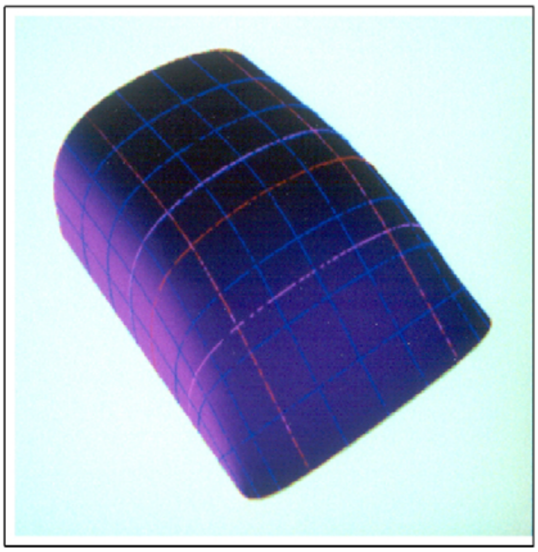

Figure 4. Reduction of the surface **BRODE** in Figure: 3(a) to a (C^0, C^0) Bézier surface with (3 x 2) Bézier patches of degrees (3,5,3 x 3,4)

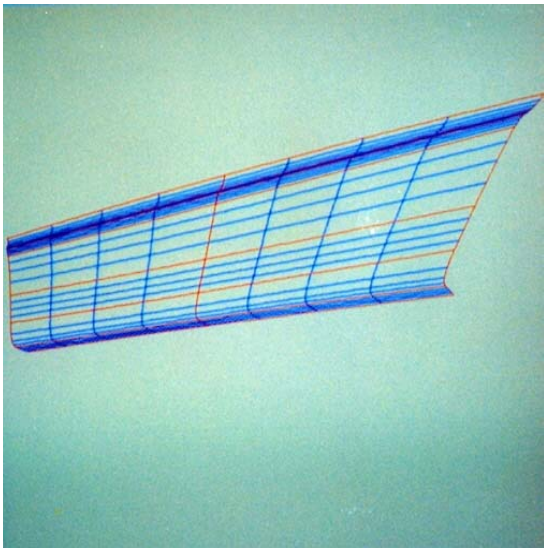

Figure 5. Reduction of the surface **SEITE1** in Figure: 3(b) to a (C^0, C^0) Bézier surface with 132 (12 x 11) Bézier patches of degrees (5 x 5)

Figure 5 presents a reduction of the surface **SEITE1** (Figure 3(b)) to a C^0, C^0 Bézier surface with 132 (12 x 11) Bézier patches of degrees (5 x 5).

6.2. Merging of Surface Patches

It is sometimes needed to merge a surface which is constructed from many small patches into a surface with less patches of the same degrees, or even with higher degrees. Table 5 presents approximation surfaces to **SEITE1** (with 63 patches), that have the same degrees as **SEITE1** or higher degrees, and with less number of patches (7 up to 21 patches). Also Table 4 presents the merging (with or without degree reduction) of **SURFB** which has 1071 patches into approximation surfaces with 36 up to 408 patches.

Table 3. Degree Reduction Results for the Surface: **BRODE**:

surface: **BRODE**: degree: 9 x 9, segments: 1 x 1					
degree	segments	minimal continuity	compression factor	error tolerance	measured error
3 x 3	3 x 2 2 x 2	C^0, C^0 C^0, C^0	1.04 1.56	0.01 0.1	0.010003 0.041476
4 x 3	2 x 2	C^0, C^0	1.25	0.1	0.010259
4 x 4	2 x 2 2 x 1	C^0, C^0 C^0, C^4	1.0 2.0	0.01 0.1	0.003822 0.025107
4 x 5	2 x 1	C^0, C^5	1.66	0.1	0.015462
5 x 3	1 x 2	C^5, C^0	2.08	0.1	0.019293
5 x 5	2 x 2 2 x 1 1 x 1	C^0, C^0 C^0, C^5	0.69 1.39 2.78	0.01 0.1 0.1	0.001641 0.015 0.022
7 x 6	1 x 1		1.79	0.1	0.004
7 x 7	1 x 1		1.56	0.1	0.001
3,5,3 x 3,4	3 x 2	C^0, C^0	1.04	0.01	0.008425
3,3,2,2,3,3 x 3,4	6 x 2	C^0, C^0	0.52	0.01	0.009057

Table 4. Degree Reduction Results for the Surface: **SURFB**

surface: SURFB: degree: 5 x 5, segments: 1071 (17 x 63)					
degree	segments	minimal continuity	compression factor	error tolerance	measured error
3 x 3	408 (17 x 24)	C^0, C^0	5.90	0.01	0.005273
	272 (17 x 16)	C^0, C^0	8.86	0.1	0.022636
	187 (17 x 11)	C^0, C^0	12.88	0.1	0.049658
3 x 5	187 (17 x 11)	C^0, C^0	8.59	0.01	0.007725
5 x 5	132 (12 x 11)	C^0, C^0	8.11	0.01	0.009385
	36 (6 x 6)	C^0, C^0	29.75	0.1	0.075385
7 x 7	36 (6 x 6)	C^0, C^0	8.59	0.1	0.027889

Table 5. Merging Results for the Surface: SEIT1

surface: SEITE1: degree: 3 x 3, segments: 63 (9 x 7)					
degree	segments	minimal continuity	compression factor	error tolerance	measured error
3 x 3	21 (3 x 7)	C^0, C^0	3.0	0.01	0.007520
	14 (2 x 7)	C^0, C^0	4.5	0.1	0.055028
3 x 4	21 (3 x 7)	C^0, C^0	2.4	0.1	0.007491
4 x 3	14 (2 x 7)	C^0, C^0	2.9	0.1	0.052474
3 x 5	21 (3 x 7)	C^0, C^0	2.0	0.01	0.007520
5 x 3	21 (3 x 7)	C^0, C^0	2.0	0.01	0.002506
5 x 3	7 (1 x 7)	C^5, C^0	6.0	0.1	0.107863
5 x 5	21 (3 x 7)	C^0, C^0	1.33	0.01	0.002574
	14 (2 x 7)	C^0, C^0	2.0	0.1	0.011239

Conclusions and Future Work

In this article we introduced a global and continuous method for approximation and / or construction of surfaces. As in the case for curves, it is based on a minimization of a functional which makes use of global and continuous criteria (2–4), for approximation and construction. Weighting factors were integrated in the functional to allow one to control the approximation. By the determination of the weighting factor values, one can augment a certain error norm which is included in the functional, and by that to achieve a better approximation for the first and second derivatives of the surfaces involved. This is a vital flexibility in the control of the approximation process.

The numerical solution of the functional uses FEM with the Bernstein-Bézier representation for the shape functions. And as in the case for curves [3, 4], the method presents cardinal advantages:

- Every element is treated separately, and its "influence" is added to the general stiffness matrix such that there is no limitation on the form of the general range combined from a collection of elements.
- It is possible to approximate, using different elements with different degrees of Bézier patches. Other methods use the same degrees for all the patches of the approximation surface (see for example [12, 13, 24]).
- The system of equations is linear for any degrees of the elements and any order of parametric continuity C^n between the elements. Some other methods involve non-linear system of equations for degrees higher than quintic [24].
- The use of Bézier-Bernstein representation grants good properties for the stiffness matrix
(see sections 3.3.5, 3.6), and saves much of the approximation calculation using proper solution methods (such $LDLT$, see [29]). The element matrices are prepared in advance and be used regardless of the subdivision of problem's domain.
- Segmentation of the approximation surface is natural to FEM because of the subdivision of a FEM problem's domain into elements.

Some other advantages of the approximation method are the following:

- There is not a pre-requirement on the given surface's continuity, as for some of the other methods which are dependent upon the continuity of the given surface (see [12]).
- It is possible to approximate, using different elements with different degrees of parametric or geometric continuation, including C^0.
- Many of the approximation methods are discrete (using a sample of points on the given surface, see [12, 13, 24]), as opposed to this method which operates globally on the given problem's domain.
- Some of the methods are limited only to degree reduction (see for example [12, 13]), and can not be used for other purposes such as the construction of o set surfaces.

As the next step in this research, we intend to integrate constraints, in order to achieve desired geometrical properties for the approximation surface. We will incorporate parametric (C^0–C^2) and geometric (GC^1) continuity constraints between the approximation surface patches, and other constraints, such as a constraint for the interpolation of the approximation surface end points with the end points of the given surface, or, a constraint to enforce the directions of the end edges of each boundary curve control polygon of the approximation surface to have the same directions as the tangents at the end points of the approximated surface boundary curves.

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HETEROGENEOUS FACTORS IN STOCHASTIC QUEUEING SYSTEMS

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The methods of analysing queueing systems taking into account the heterogeneousness of time-response characteristics of requests are suggested in the given paper. The designated method of analysing time-response characteristics for the queueing systems of complicated configuration is derived in this publication. The present method may be utilized with the aim of calculating the parameters of the bandwidth capacity of random configuration computer network nodes.

Keywords: queueing systems, enterprise computer network

1. Introduction

The reason for heterogeneousness of requests' processing in network nodes is the most frequently explained by the self-nature of the requests. For instance, in a computer network each request, upon being delivered to a data processing node, causes the execution of the entire series of computer programs, using various sizes of random access memory, utilizing different external devices and naturally diverse timing resources of a processor. Regretfully, in respect to application of queueing systems to the computer networks' analysis the present matter is either considered on extremely rare occasion or contemplated upon the condition of evidently occurring dependence between the principles of requests' behaviour in the queue for processing and the heterogeneousness by themselves as well as compared to other queues [1–3]. In the present publication the cases, when the types of requests do not depend on the position of the request within the queue, are considered. The method of calculating the characteristics of computer networks, based on the assumption that processing in network nodes is subordinate to Hyper-exponential functioning, and it is recommended in the present article. The approbation of this method's application, aimed at the analysis of enterprise computer networks, is represented in several works by the author [4, 5]. Alongside with the analytical research deduced below, the practical methods of estimating computer networks have been carried out by the author [6, 7], as well as the investigations, based on the simulation modelling methods [8]. The mathematical means of stochastic networks of queueing systems in this study are proposed to be considered as the basis for investigating the characteristics of an enterprise computer network, consisting of a lot of nodes [9]. Within the networks of such kind requests a network node at random may be selected. The analysis of computer networks is proposed to be carried out first and foremost on the basis of the simplest queuing system with Hyper-exponential processing rule and superimposition of Poisson streams of requests, efficient at the network logon, but further on providing the transition to a stochastic network of queueing system. That kind of method makes it possible to investigate the networks with hierarchic organization of the structure, where sub-networks might function as processing nodes.

2. Investigating the Simplest Queueing System with Heterogeneous Requests

Let us consider the single-line system with direct order of processing, in which the inbound stream of requests is divided into ***M*** groups. Conceding the average intensity of ***i***-group as λ_i**,** the moments of requests' receipt of ***i***-group form the Poisson stream. The uniting is performed by the way of superimposition of the streams of groups in such a way, that the united stream is the Poisson one as well, but its average intensity is determined by the following congruence:

$$\lambda = \sum_{i=1}^{M} \lambda_i \ . \tag{1}$$

The processing of each requests' group might be performed with an average intensity μ_i (***i*** = 1,2, …, ***M***) and is hypothesized to be distributed exponentially.

The resulting processing intensity, in case of superimposition of the streams, subordinate to Hyper-exponential functioning, or the general average intensity of processing in one node is determined by the expression:

$$\mu = \sum_{i=1}^{M} \mu_i \ . \tag{2}$$

The probability of the receipt of ***i***– type requests (q_i) for processing is determined by the relation of the average intensity of requests' incoming λ_i to resulting intensity λ, i. e. $q_i = \lambda_i / \lambda$. By reason of the performed assumption (1) we deduce the following congruence:

$$q_i = \lambda_i / \lambda \ , \tag{3}$$

where $\sum_{i=1}^{M} q_i = 1$.

The conditions of the model under consideration we will characterize with the help of the vectors $E_{n,i}(t)$, which mean, that at the moment ***t***, n requests are located in the queue and the request of ***i***–type ($n = 0, 1, 2, \ldots, \infty$, $i = 0, 1, 2, \ldots M$) is at processing.

The process $E_{n,i}(t)$ maybe considered to be the Markovian one, as the time for any one-step transition is distributed in accordance with the exponential functioning and the probability of transitions being independent variables. The simplified graph of transition of processes $E_{n,i}(t)$ is presented on Figure 1.

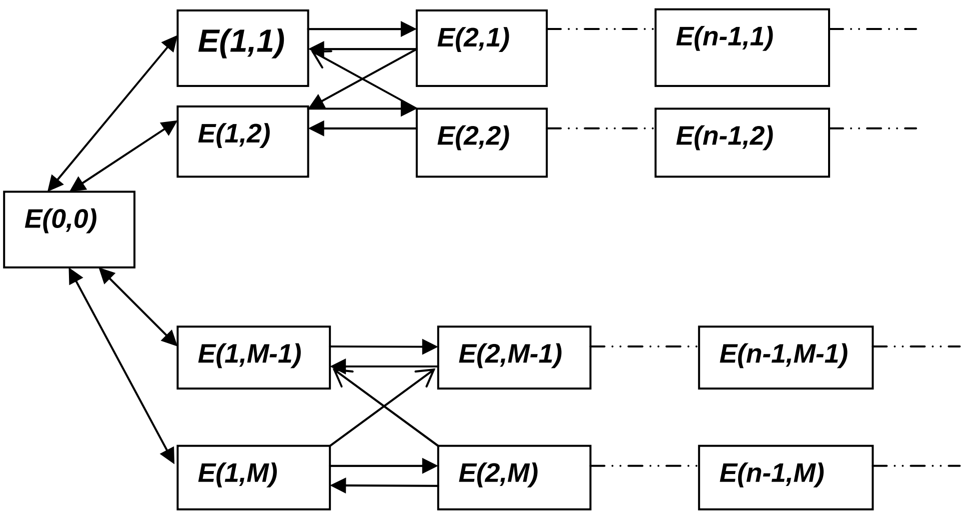

Figure 1. The simplified graph of transition of processes $E_{n,i}(t)$

Each vertical line of the graph's apex is in accord with the quantity of requests in a system's queue; each horizontal line is in accord with the type of a request, located for processing.

Separately considering each of the graph's vertical lines, excluding the first one, we come to the conclusion that transitions into any x-state $E_{n,i}$ ($n = 2, 3, \ldots, \infty$; $i = 1, 2, \ldots, M$) are possible under the following circumstances: firstly, whenever the system was in the state $E_{n-1,i}$ and one of the requests (of any type) entered the system; and secondly, whenever the system was in state $E_{n+1,j}$ ($j = 1, 2, \ldots, M$), the processing took place, and the next request, being in the queue, is of ***i***-type. On the other hand, whenever the system was in the state $E_{n,i}$, it would remain in it, if no processing was performed and no request arrived. As to the first vertical line, the above-mentioned inherence is also true, excluding only the fact, that transitions from state $E_{0,0}$ to state $E_{1,i}$ are possible in case of ***i***–type requests' receipt, but not of any other certain type. These properties allow to write the differential equation system and to recast it in a usual way into the following infinite system of homogeneous differential equations as aspects of the steady-state probabilities:

$$P_{n,i}(\lambda+\mu_i)=\lambda P_{n-1,i}+q_i\sum_{j=1}^{M}P_{n+1,j}\mu_j \ , \tag{4}$$

$$P_{n,i}(\lambda+\mu_i)=\lambda_i P_{0,0}+q_i\sum_{j=1}^{M}P_{2,j}\mu_j \ , \tag{5}$$

where, $P_{i,j}$ is the steady-state probabilities of states $E_{i,j}$, but μ_j stands for the average intensities of ***j*** –type requests' processing.

For the present system the following normalization requirement is true:

$$P_{0,0}+\sum_{n=1}^{\infty}\ \sum_{j=1}^{M}P_{1,j}\mu_j=1\,. \tag{6}$$

Analysing the possible transitions across every vertical line on Figure 1 on the basis of equilibrium condition for steady-state probabilities, it's possible to write down the following additional system of equations:

$$\lambda\sum_{j=1}^{M}P_{n-1,j}=\sum_{j=1}^{M}P_{n,j}\mu_j \ , \tag{7}$$

and for ***n*** = 1,

$$\lambda\sum_{j=1}^{M}P_{0,0}=\sum_{j=1}^{M}P_{1,j}\mu_j \ . \tag{8}$$

Having intercepted the values of steady-state probabilities with common coefficients from (4), (5), (7) and (8), we deduce the following system of equations:

$$P_{1,i}(\lambda+\mu_i)=\lambda_i P_{0,0}+\lambda_i\sum_{j=1}^{M}P_{1,j}$$

$$P_{2,i}(\lambda+\mu_i)=\lambda\ P_{1,i}+\lambda_i\sum_{j=1}^{M}P_{2,j} \tag{9}$$

. . .

$$P_{n,i}(\lambda+\mu_i)=\lambda\ P_{n,i}+\lambda_i\sum_{j=1}^{M}P_{n,j}\ .$$

While summing all equations (9) over ***n*** = 1, ∞ , we can deduce the following congruence:

$$(\lambda+\mu_i)\sum_{n=1}^{\infty}P_{n,i}=\lambda_i P_{0,0}+\sum_{n=1}^{\infty}P_{n,i}+\lambda_i\sum_{n=1}^{\infty}\ \sum_{j=1}^{M}P_{1,j}\ . \tag{10}$$

Inserting normalization requirement in (10) and performing resummation over ***i***, we determine:

$$P_{0,0} = 1 - \sum_{j=1}^{M} \lambda_i / \mu_i \, . \tag{11}$$

Further on let's embed the following generating functions:

$$Q_1(s) = \lambda_1 P_{0,0} + \sum_{n=1}^{\infty} P_{n,1} s^n$$

$$Q_2(s) = \lambda_2 P_{0,0} + \sum_{n=1}^{\infty} P_{n,2} s^n \tag{12}$$

. . .

$$Q_M(s) = \lambda_M P_{0,0} + \sum_{n=1}^{\infty} P_{n,M} s^n$$

and the expanding generating function:

$$G\ (s) = Q_1(s) + Q_2(s) + \ldots + Q_M(s) \, . \tag{13}$$

Having multiplied the left and right parts of equation (9) by s^n and summing over n we determine:

$$(\lambda + \mu_i) \sum_{n=1}^{\infty} P_{n,i} s^n = \lambda_i P_{0,0} s + \sum_{n=1}^{\infty} P_{n,i} s^{n+1} + \lambda_i \sum_{n=1}^{\infty} \ \sum_{j=1}^{M} P_{n,j} s^n \, . \tag{14}$$

Substituting the values of generating functions (12) and (13) in the received formula (14), we'll deduce:

$$(\lambda + \mu_i)[Q_i(s) - \lambda_i P_{0,0}] = sQ_i(s) + \lambda_i [G(s) - P_{0,0}] \, . \tag{15}$$

Summing once more over i for all $Q_i(s)$ we determine:

$$G\ (s) = G\ (s) \sum_{i=1}^{M} \ \lambda_i / (1 + \mu_i - s) + \sum_{i=1}^{M} \ [\lambda_i \mu_i / \lambda (1 + \mu_i - s)] P_{0,0} \, .$$

Here from the following expression for $G\ (s)$ can be deduced:

$$G\ (s) = P_{0,0} \sum_{i=1}^{M} \ [\lambda_i \mu_i / \lambda (1 + \mu_i - s)] / [1 - \sum_{i=1}^{M} \ \lambda_i / (1 + \mu_i - s)] \, . \tag{16}$$

On the other side, using (12) and (13) we can write:

$$G\ (s) = P_{0,0} + (P_{1,1} s + P_{2,1} s^2 + \ldots + P_{n,1} s^n + \ldots.) + (P_{1,2} s + P_{2,2} s^2 + \ldots + P_{n,2} s^n + \ldots.) \ldots$$

If we group the components at equal powers in the expression received, we can determine the following:

$$G\ (s) = P_{0,0} + s \sum_{j=1}^{M} P_{1,j} + s^2 \sum_{j=1}^{M} P_{2,j} + \ldots. + s^{\infty} \sum_{j=1}^{M} P_{1,\infty} \, . \tag{17}$$

Each of the summands in (17) defines the probability as to whether in our system the average number of requests equals 0, 1, 2,…, ∞ correspondingly. Because of that, performing the differentiation as to s (see (16)) and after that substituting $s = 1$, it's possible to deduce the following expression, defining E – as the average number of requests in the system:

$$E = G'(s=1) = \sum_{i=1}^{M} \lambda_i / \mu_i + [\sum_{i=1}^{M} [\lambda_i / \mu_i^2] / [\lambda(1 - \sum_{i=1}^{M} \lambda_i / \mu_i)] . \tag{18}$$

The situation, presuming that each request, being queued in the system under consideration, will expect its processing until the queue before it finishes, makes it possible to determine the following expression for the average time of a request's processing start within the system:

$$W = \sum_{i=1}^{M} [\lambda_i / \mu_i^2] / [(1 - \sum_{i=1}^{M} \lambda_i / \mu_i)] . \tag{19}$$

3. Investigating Complicated Configuration Network with Heterogeneous Requests

Herein, considering a complicated configuration network, we mean the network, consisting of the unlimited number of nodes and characterized by certain connections between the nodes. Additionally in each node the opportunities of processing different intensity requests exist: $\mu_{i,r}$ – where (i = 1,2, …, M) is the number of processing type, but (r = 1,2, …, N) is the number of a network node. Within the network the requests are supposed to be able to pass over from one node to another in the random manner. The character of these random manners is described by the transition-probability matrix ***π[i,j]*** , where i is the number of the node, from which the request leaves, but j is the number of the node, to which the request arrives (*i,j* = 1,2, …, *R*). Herewith let's suppose that the network under consideration corresponds to the closure condition of Gordon-Newell [9]; though the proposed below solutions with insignificant changes might be transposed to an open-loop network as well.

The difficulty of investigating a closed network is that, involving hyper-exponential processing, the outgoing flow of a network node might be of non- exponential nature. The error, occurring from that, is determined by the diminution between the obtained final values (18), (19), with the equivalent of final values, deduced under exponential processing. Comparing the resulting expressions (18) and (19) with equivalent solutions under exponential processing, we can prove that the tolerable error in this case doesn't exceed 25 percent from the absolute value E_r and W_r (r = 1,2, …, M).

In view of performed verbal proves, we can write down the following set of difference-differential equations for steady-state probabilities $P(n_1, n_2 \quad \ldots \quad n_R)$:

$$\sum_{i=1}^{M} \sum_{i=1}^{M} \delta(n_j)\mu_{j.r}\beta_r \, \mathbf{P}(n_1, n_2, ..., n_R) = \sum_{i=1}^{M} \sum_{j=1}^{M} \sum_{i=1}^{R} \delta(n_j)\mu_{j.r}\beta_r \, \pi_{\mathbf{i,j}} \mathbf{P}(n_1, n_2, ..., n_M) , \tag{20}$$

where $\delta(n_j)$ = 1, if $n_j \neq 0$, $\delta(n_j)$ = 0, if $n_j = 0$;

$\beta_r = m_r / N$ is the number of requests, belonging to r-type, in the system; N is the total number of requests in the system.

The solution of the present set with respect to steady-state probabilities might be determined with the help of the method of substitution.

In case we sum all probability values $P(n_1, n_2 \quad \ldots \quad n_R)$ and, having cancelled out the left and right members of the deduced equation over β_r , we can deduce the following solution:

$$\mathbf{P}(n_1, n_2, ..., n_M) = C \sum_{j=1}^{M} \mu_i / \mu \ (\prod_{i=1}^{N} X_i^{n_i}), \tag*{21)}$$

where C is the invariable of normalization for all ***i, j*** and r , provided for all space of values ***M, N*** and ***R***, that can be determined by solving simultaneous equations with respect to indeterminate X_i :

$$X_j \sum_{r=1}^{M} \mu_{j.r}\beta_r = \sum_{i=1}^{M} [\pi_{\mathbf{i,j}} X_i \sum_{i=1}^{R} (\mu_{i.r}\beta_r)] . \tag{22}$$

The solution of the system of equations (22) might be performed by any tradition approach. Thus steady-state probabilities (21) can be rated; hence we are practically able to determine the values of all required parameters of a stochastic network, such as the queues' lengths in network nodes, average standby time's values in network nodes and requests' standby time values in network queues and so forth. Hence the recommended method of analysing a complicated stochastic network might be resolved to origination of the given data – these are first and foremost $\mu_{j.r}$, $\pi_{\mathbf{i,j}}$, β_r, the posterior calculating of X_i ($\boldsymbol{i}$ = 1,2, …, $\boldsymbol{M}$), $\mathbf{P}(n_1, n_2, ..., n_M)$ and straight to the reckoning of network parameters by themselves.

Conclusions

The solution of system of equations (22) may be carried out using any traditional method. Thereby, the stationary probabilities (21) may be calculated; therefore, we can practically calculate the values of all the required parameters of the stochastic network, including the length of the queues in the network nodes, the average times of the request presence in the network nodes, and the times of the request presence in the network queues. Thus, the proposed method of analysing the complex stochastic network may be reduced to obtaining the initial data $\mu_{j.r}$, $\pi_{\mathbf{i,j}}$, β_r; the following calculation of X_i (i = 1, 2, …, $\boldsymbol{M}$) and $\boldsymbol{P}(n_1, n_2, \ldots, n_M)$; and the calculation of the network parameters.

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EAST-WEST: TRANSPORT PARADIGM OF INTERNATIONAL COLLABORATION

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The article is devoted to the development of economic and organizational aspects of international transportation, international transport corridors system particularly, which is the very important component of the successful international economic relations between countries and regions, that assists activation of international ties between West and East.

Keywords: international economy, global marketing, transcontinental transportation system, international transport corridors, international economic collaboration.

1. Introduction

One of the most important aspects of the development of international cooperation between countries is the improvement of transportation and economic interaction. Establishment of trading and manufacturing relationships between Eastern and Western countries requires more careful analyses of transport potential of economic co-operation, also theoretical grounding of the necessity of formation and use of marketing principles of functioning of the intensive transport systems, which would contribute to activation of transcontinental international contacts.

For now international trade has totally exceeded the $9037 billion, volume of Asia–Europe transit trade attains $120 billions [1, 2]. Thus, transport network should be able to serve the trade turnover. Speaking in images, it is necessary to revive the ancient route known as the “Great Silk Way” by creation of the transcontinental transport corridor Europe–Caucasus–Asia (TRACECA), which can be considered as the East–West corridor and which will cross the North–South transport corridor. Once the “Great Silk Way” had become one of the most significant achievements of the ancient civilizations, which for the first time connected different nations and countries stretching broadly from the Mediterranean Sea to the Pacific Ocean, connected material, artistic and spiritual culture of the nations. For many centuries thanks to the “Great Silk Way” the exchange of crafts, ideas, technologies, and beliefs was possible. Symbolic revival of the “Great Silk Way” nowadays means the creation of the important channel of wide and profound international cooperation in the sphere of diplomacy, culture, science, trade, and tourism [3].

Today international transportation has already become a significant factor of economic interaction between countries and regions. Effectiveness of the transport services directly determines the level of the world economic trading and manufacturing relations intensification. After the collapse of the Soviet Union all the former republics achieved their independence. Simultaneously the new independent states had to solve the numerous internal economic problems. Among others they had to create and internationalise the national transportation systems, to adjust the transport infrastructure various material and nonmaterial attributes, to ensure the appropriate level of transport system development. Especially it was important for renewal the major trade routes between Europe and Asia.

Transcontinental transport system development and its effective functioning in all levels of its members' economic interests' realization are the essential idea of the logistic chain optimisation, including the development of the international cooperation contractual principles, informational support at every stage of the material flow process, grounding the freight transportation modes and optimisation of the routes.

2. Problem’s Definition

Existence of concurrence and multi-modal transportation allows developing the effective transport road corridors and junctions, which are appropriate for providing the effective international products transportation from manufacturers to consumers, sometimes far away distant. At the same time it requires the improvement of use, maintenance and repair of existing types of transport for the minimal influence of the costs on the end product price. Low development of transportation system impedes international cooperation and yields losses in any sphere of industry. Constant supply of raw materials, products' components and spare parts as well as reliable system of transportation of the manufactured products is of great importance for manufacture. Every transportation system should have the following features:

- ✓ it should be reliable and proved; it could be used during all the year long and under any political and social and economic conditions;
- ✓ its carrying capacity should be satisfactory not only for providing just-in-time and intensive delivery but also for application of complex schemes and intermodal transportation;
- ✓ it should fit the average level of costs; according to author's research transport expenditures play significant role in economic activity of enterprises and other market players; these expenditures are one of the main parameters influencing the consumer choice while making transport decision in the process of international treaties planning.

If state has national transport infrastructure (roads, different types of transport, expeditor / forwarding services etc.) it may strengthen its economy by international transportation, simultaneously gaining profit by extending its influence in the region and in the world. Significant direction of formation and use of the transcontinental transport system is the support of economically grounded and effective economic cooperation, mutual understanding between nations and intensification of the integration processes.

Countries are seeking for international economic contacts and active international cooperation under the globalisation making attempts to use their economic potential t for effective integration into the world economy. Among such countries Iran is worth to be mentioned; the country with long history, traditions of international co-operation, different economic possibilities, which can be proposed to the foreign partners. Geographical position of Iran makes it possible to concentrate goods traffic between Asia and Europe and renew its position on the “Great Silk Way”. Attention will be paid to creation of the route from Caspian Sea to the Persian Gulf because this route is relatively cheaper (20–30% cheaper) in comparison with the route through the Suez Canal [2,8–10].

3. Results of Research

Vail Douranat wrote in his book “History of Civilization”: Persian engineers created the large main roads which connected capitals. Length of one of such roads stretching from the city Shumer to Sarde equalled two thousands four hundred kilometres. Iran was surrounded by such civilizations as China, India, North Europe and Egypt. The “Great Silk Way” was some kind of bridge connecting the East and the West (Asia and Europe) and started from Chagan (nowadays Cigna) which is situated in the East of China. It was used from the very beginning of the second century BC till the 17th century AD, thus for the period of 18 centuries. Its length exceeded 12 thousand kilometres. There were trade centers situated on the “Great Silk Way”, among them: Nishapur, Damgan, Gorgan, Ray, Hamadan and Tus. Great number of trade operations and transits were carried out in these centres. The Silk Way was of great importance till XIV century, and then several centuries later after railroads became popular and Trans Siberian Road (T.S.R.) in Russia was built it didn’t exist any more. Later the “Great Silk Way” started reviving.

With the development of international trade the GATT / WTO' requirements had been intensified, in particular the standard world economic contacts, methods and norms of co-operation between countries and regions have been significantly complicated. Thus the necessity of goods transportation and creation of the wide network of transport corridors for international cooperation appears.

Planning and development of international transport corridor is quite intensive and should engulf a lot of variety activities in a short period. Transport complexes passing through a lot of states may include roads, stations of customs control, overload, storage of the goods etc. Usually transcontinental corridors cross many countries that create numerous problems on concordance of interests, for example:

- ⇨ Pan-Asian Corridor (P.A.R.L.) which is known as “Pan Asian line” connects the majority of the continental countries: Singapore, Malaysia, Thailand, Cambodia, Laos, and Vietnam. After modernization of this line and addition of such countries as Bangladesh, India, Pakistan, and Iran this line should gradually join the European transport infrastructure and transcontinental corridor.
- ⇨ World corridor of the XXI century (Europe–Asia–America) was created for direct connection of Europe, Asia and North America.
- ⇨ Eastern Corridor to Western Asia (China–Middle Asia–Black Sea–Europe) stretching through Middle Asia branches off in three directions:
  - Northern – this branch of the main road runs from Kazakhstan through Russia to North-Eastern Europe.
  - Central – runs from Tadjar in Turkmenistan via Caspian Sea through Caucuses and Black Sea to Eastern Europe and along the channel of European railroad.
  - Western – stretches from the borders of Saragossa to Iran and via Ray to Turkey via Istanbul through European railroad.

There are plans to be elaborated in Europe concerning the development of transport channels and improvement of their quality:

⇒ Road junction TEN (Trans European Network)

⇒ Pan European corridors (Pan European Corridors)

⇒ “Crete Corridor” (Crete) is an important international corridor in Central Eastern Europe, which consists of the broad network of transport routs:

1. Tallinn (Estonia)–Riga (Latvia)–Warsaw (Poland)
2. Berlin (Germany)– arsaw (Poland)–Minsk (Byelorussia)–Moscow (Russia)
3. Berlin / Dresden (Germany)–Lublin (Poland)–Lvov (Ukraine)–Kiev (Ukraine)
4. Berlin / Nuremberg (Germany)–Prague (Czechia)–Budapest (Hungary)–Salonika (Greece)–Istanbul (Turkey)
5. Tertcet (Italy)–Ljubljana (Slovenia)–Budapest (Hungary)–Bratislava (Slovakia)– Uzhgorod (Ukraine)–Lviv (Ukraine)
6. Gdansk (Poland)–Warsaw (Poland)–Zelena (Czechia)
7. The Danube (water-way)
8. Duris (Albania)–Tirana (Albania)–Skopje (Macedonia)–Sofia (Bulgaria)
9. Helsinki (Finland)–Saint Petersburg (Russia)–Kiev (Ukraine)–Moscow (Russia)–Odessa (Ukraine)–Kishinev (Moldova)–Bucharest (Romania)–Plovdiv (Bulgaria).

The Trans Siberian Road known as Transsib connects Russia with Far East and stretches from Moscow to the Pacific Ocean for 10000 kilometres, is the property of the Russian Federation.

The most important corridors between Europe and Asia are transport corridor Europe–Caucasus–Asia (TRASECA) and international corridor North–South.

Investment plan for development of this road for the first time was considered in May 1993 at the Brussels Conference in the presence of Ministers of Trade and Transport of five states of the Central Asia and three states of Caucasus; later Mongolia, Ukraine, Turkey, Rumania, and Moldova joined the conference. The program of technical support for creation of the transport corridor from Europe to Central Asia via Black Sea and Caucasus was elaborated at this conference. On September 8th, 1998 the International Conference in Baku took place where questions concerning the Silk Way were considered. Representatives from 32 countries of the world and 13 representatives of UN organisation participated in this conference. In the course of the conference the Heads of Bulgaria, Kyrgyz Stan, Tajikistan, Turkey, Ukraine, Azerbaijan, and the representatives of Kazakhstan, Moldova, and Uzbekistan concerning international transport and development of the corridor Europe–Caucasus–Asia ratified the bilateral agreement. In 1998 at the third conference TRACECA was added to the system of European transport and the transport system named “European Transport Area” (PETRA–Pan European Transport Area) was created. This area connected European System with transport channel of Eastern Europe and TRACECA. The TRACECA program includes also transport channels ESCATO and ECO.

Once for political reasons Iran was excluded from the transport network and instead of Iran the other route was used – Turkmenboshi via Baku. Exclusion of Iran from this union was economically disadvantageous due to the necessity of double overload while crossing the Caspian Sea. In general the existing corridor (Iran, Turkey, Bulgaria, Rumania, Hungary, Austria, Czech Republic, and Germany) is too long. Transport corridor TRACECA can be used for goods transportation to the Northern Europe via Hungary and Poland. This allows reducing expenditures and terms of transportation: Iran, Turkey, Ukraine, Poland, and Germany. For this purpose it is necessary to solve some organisational questions and simplify customs formalities, in particular:

1) Cutting of the costs on visa operations;
2) Release of road duty;
3) Equal prices for the fuel;
4) Issue of the export licenses for goods.

The treaty concerning the international corridor North–South is the most significant transport treaty between Asian and European countries: this corridor will connect more than 1.2 billion of the Earth population and its length will reach 8500 kilometres. It will be 40% shorter and 30% cheaper in comparison with the existing route via Suez Canal [1], according to which the transport connection starts from Bombay in India via Iran territory and countries of Central Asia, Russia to the North Europe. For now there are seven countries, constant members of the corridor: Iran, the Russian Federation, India, Kazakhstan, Oman, Byelorussia, and Tajikistan. Membership of Armenia, Azerbaijan, and Syria will be approved within 30 days of accepting of the membership application. Meantime Turkey, Ukraine, Kyrgyz Stan, and Bulgaria have already sent their official membership applications “Depositaries” to the state (Islamic Republic of Iran) which are being considered now [5]. Finland, Iraq, Kuwait, Korea, Malaysia, Singapore are also considering their membership in the corridor North–South.

Contributing to the development of international cooperation East–West provides the gas exporting to Northern Europe via Iran, Caspian Sea, Eastern and Western Europe. Gas will remain one of the most important energy resources for Europe in the XXI century. According to the report of International Energy Association (IEA) the world consumption of gas will increase 97% till 2030. Iran has a significant potential of gas production and transfer. Due to this fact it can became one of the main suppliers of gas to Europe [6]. Ukraine proposed two routs of gas pipeline: Iran–Armenia–Georgia–Russia–Ukraine–Europe and Iran–Armenia–Georgia–Black Sea–Ukraine–Poland–Europe [7]. In this connection the Ukrainian Government proposed to organize consortium of Ukraine, Poland, and Germany for realization of gas to Europe, excluding Russia [8].

On the other hand, functioning of such transport infrastructure gives to all the transit countries the possibility of direct utilization of resources of the Persian Gulf.

Conclusions

The conclusion can be summarized as follows: in spite of borrowings and stabilization of policy, in particular creation of the international transport corridors constructively influences the economy. Transport corridors also positively influence on the social development while new working places appear, contribute to acquaintance with the culture of other countries. They are similar to rivers due to which the neighbouring territories continue existing.

The goal of every transport transit system is the development of trade and other forms of international co-operation. Poland as one of the European countries, member of the European Union, has convenient geographical position for transit of the goods from West to East and contra verse. Three corridors among the nine chosen by the European Union run through the territory of Poland. Two countries – Iran and Poland – can with the support of each other in the membership of the corridor North–South and TRACECA develop mutually beneficial co-operation, channels to exchange fresh, modern ideas, scientific technological achievements for strengthening of the peace and friendship between two large civilizations to secure peaceful coexistence.

In connection with growth of attention to globalisation and new understanding of the importance of the rapid information and technologies exchange, the improvement of the population living standard and increasing the consumer demand, the development and broadening of internal, regional and international transport systems should be significantly intensified.

In spite of the fact that the lack of transport has never been the main problem selection of the distant and dangerous transport routs due to the political intervention makes the transport system ineffective. This all result in increase in consumer expenditures, limitation of activity on trade markets, increase in costs of manufacturing, lack of competitiveness in comparison with the other countries.

Among corridors mentioned above there are corridors TRACECA and NORTH–SOUTH running through Poland and Iran. What these countries really need is cooperation instead of opposition. Thus the use of tariffs and availability of two corridors allow cutting expenditures for transport which is of great importance in the trade relationships between two countries. Successful use of these two corridors will allow connecting such civilizations as Slavonic, Indian, Arabian, Iranian, Armenian, Turkish, and along with exchange of goods and passengers to promote exchange of thoughts and ideas, practical, economic, cultural and global achievements.

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RESEARCH OF INNOVATIVE TECHNOLOGIES INFLUENCE ON EFFICIENCY OF AGRIBUSINESS

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Agriculture is one of the major industries of economy, represented in practically every country, which is directed to provide the population with food and receipt of raw material for the different industries. Every year the volumes of consumable products of agriculture are multiplying, the requirements of users to quality of the products are multiplying as well. In order to provide demand, agriculture must improve its efficiency, and also competitiveness of the products.

Keywords: *organic fertilizers, agriculture, profitability, sapropel*

1. Introduction

Agriculture is one of the major industries of economy, represented in practically every country, which is directed to provide the population with food and receipt of raw material for the different industries. About 1,1 billion of economic active population (EAP) is involved in the world's agriculture [8]. The necessity in the products of agriculture, has existed, and will exist for always, while there is humanity. In history of humanity was noticed a transition from extensive to the intensive forms of economy, stipulated the gradual condensing of population of earth. The population can be less satisfied with that natural forces of nature gives it, and forced to add to this area more labour and capital, to extract from it the amount of products, which it needs. This common motion from extensive to the intensive forms of economy can stay too long or even get reverse direction because of various reasons. Such are historical catastrophes, diminishing the quantity of population or lowering his culture, expropriation of peasantry and development of large economy due to small. The low cultural level of population very often detains passing to the intensive forms of economy: we can see it in Russia, where the degree of intensity of peasant economy already falls short of necessities of the multiplied population.

In Western Europe a delay in development of intensity of economy and even reverse motion is noticed because of improvement of engineering, transport and appearance at its markets of cheap products from countries with extensive economy, and also because of going away of rural population to the cities, on service developing industry [3]. However, in general, all constraining moments have only a temporal and local value and can not completely stop a transition from extensive to the intensive forms of economy, because density of population and it's position is extremely different in relation to the market of various countries.

As well as any sphere of entrepreneurial activity, agriculture is oriented on receiving a profit, which depends on claimed of producible products and expenses on its production. A demand on these products is permanent, growing with the increase of welfare and quantity of population, and even at an unfavourable economic situation poorly growing and on occasion inelastic.

2. Influence of Quantity of World Population on Demand of Agricultural Products

The population of our planet with each year continues to be increased. In turn it entails increase of consumption of the foodstuffs, especially in developing economies, which population achieves 3 billion persons (China, India, Indonesia, Brazil, Mexico) [9].

The today's global demographic context is characterized by rapid growth of the population of less developed countries, and thereof also all the world. To the beginning XX century, the population explosion began to gather its force; the world's population was only 1,6 billion people. In the middle of century there were already 2,5 billion. By the end of the century the world's population had reached 6,5 billion, and growth proceeds. Now the world's population is more than 6 706 993 155 person (by July 2008), the forecast for 2050 – 9,2 billion person.

The Figure 1 presents the newest version of the forecast of a world's population up to middle of the present century; executed by experts of the United Nations in 2008 (this forecast has been specified each two years). The forecast is executed in three variants [10].

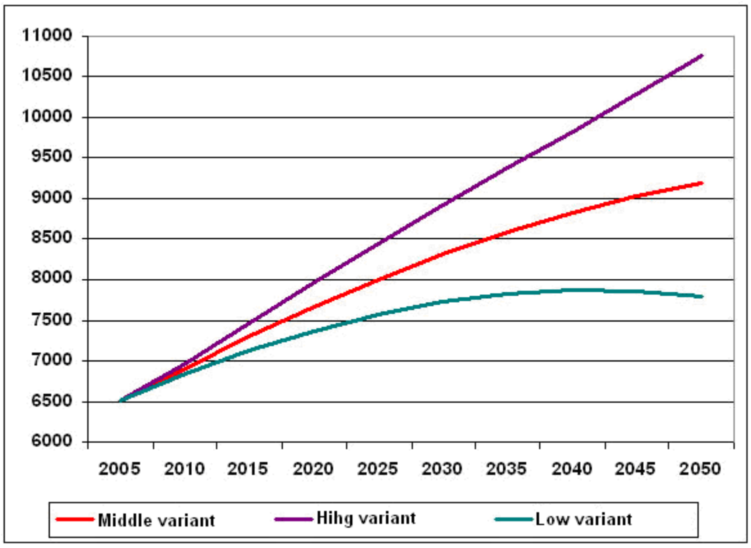

Figure 1. The world population till 2050: three variants of the forecast of the United Nations 2008, billion persons

The top variant of the forecast provides continuous growth: to the middle of century the world's population will be more than 10,5 billion persons and will continue to be increased with former speed. On middle variant to the middle of century there will be more the 9 billion person, growth also will not be stopped, but nevertheless will be appreciably slowed down. And only lower variant provides not only deceleration of the growth up to the middle of century, but also its termination, and then the beginning of reduction of the world population.

Besides the fact of growth of the world population in whole, also is very important that it grows unevenly. The population of the developed countries, already is not increased, and in some of them even is reduced. Grows, basically, the population of the developing world the countries of Asia, Africa and Latin America (see Figure 2). To be the centre of the global population explosion long time ago became Asia. In the middle of the century here will live above 5 billion persons, 60% of the world population of 2050. As a result on a planet there is very big demographic asymmetry which will grow. The population of Asia and Africa will grow especially quickly, accordingly will be increased their share in the population of a planet.

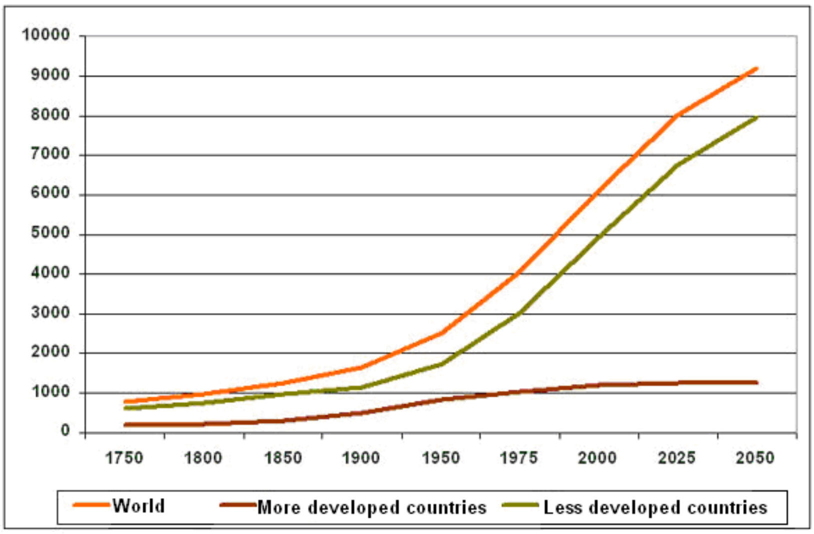

Figure 2. Quantity of world population in developed and developing countries till 2050: three variants of forecast of UNO 2008 year, million persons

Concerning the developing and least developed countries, there, where growth of the population will take place, the countries will face necessity to increase food supply due to growth of manufacture on their territory, can be in a combination with increase of import. For less developed countries there is an additional complexity in that quickly raising standard of living as a result is calculated by multiplying consumption per capita and to the changes in the feed ration.

The task of providing planet's population with food has old historical roots. The deficit of products accompanied humanity along the whole length of its history. With growth of the population very strongly grows the importance of this problem.

In this connection it is interesting to look, as the prices for the basic food cultures have grown for the last period. After continuous reduction of the world prices for the foodstuffs within many decades, the lowermost limit was achieved in 2000–2001 years, since then the prices in the world market raise.

In comparison with January 2008 the prices for food in January 2009 have mostly increased in the Great Britain, Cyprus, Lithuania and Latvia (by 10,7–11,1%). At the same time the prices for the food in Slovakia, Germany, Portugal, and Spain have increased on 0,7–1,7%. In Czech Republic was marked the reduction of prices on food on 2,2%.

In comparison with the end of the previous year in average in the countries of EU most of all have increased consumer prices of vegetables; fish and seafood; sugar, jam, honey, chocolate and sweets.

In the most degree vegetables the prices have raised in Bulgaria (on 15,6%), Slovenia (on 15%), Hungary (on 13,4%), Latvia, Cyprus, Greece, Belgium (on 10–12,8%). On fish and sea products the price have most considerable raised in Latvia (on 3,6%) and Sweden (on 2,2%). Sugar, jam, honey, chocolate and candies prices most considerable have raised in Latvia (on 2,9%), Czech Republic (on 2,2%) and United Kingdom (on 2,1%).

At the same time in the countries of EU in average have reduced month prices for oils and fats; fruit; bakery products and grouts, and also on dairy products, cheeses and eggs. Oils and fats have most essentially fallen in price in Bulgaria (on 5,5%), Spain (on 3,7%); fruit – in Poland (on 3,8%), United Kingdom (on 3,7%), Denmark (on 3,1%). Bakery products and grouts the most appreciable became cheaper in the Czech Republic (on 1,9%) and Britain (on 1,6%); dairy products, cheeses and eggs – in Estonia and United Kingdom (on 2,4% and 1,5% accordingly) [11].

One of the principal causes of increase of food prices is growing not food use of agricultural production, in particular, as raw material for manufacture of biological kinds of fuel. So, by estimations WTO, in the nearest 3 years global consumption of biological kinds of fuel will increase on 170%. Besides among the factors conducting to a rise in food prices, experts also admit changes of a climate (becoming frequent droughts owing to the process of global warming), rising prices of fertilizers [7].

The rise in prices will create the big difficulties for poor in cities of the developing and least developed with providing of feed products.

From the point of view of manufacture of food stuffs there is a common feeling, that the nearest decades approximately 80–90% of required increase of the agricultural products should be realized on the existing cultivated ground and about 10–20% – on recently restored grounds. From the point of view of steady development of rural areas, social, economic and ecological aspects play a main role. Given dynamics shows, that demand for agricultural production is high enough and with each year continues to be increased. Growth of the population of a planet should promote the development of agriculture as it will be necessary to satisfy not only pressing needs, but also tastes of people of various nationalities and age.

Thus, the agriculture is one of the stable branches of the business, having steady demand. However, it also has certain kinds to risk: natural, commercial, enterprise and countries.

- **Poor harvests** – reasons bad climatic conditions and droughts. In order to prevent occurrence of poor harvests, is needed the scientifically-grounded placing of agriculture on naturally-economic areas, conducting the profound and proof specialization of an agriculture with increase of manufacture in each zone and each area of that kind of production for which an environment is the most acceptable and which can provide the greatest profit;
- **Crises** – occurrence of various crises direct influence on the consumption of the food. Because of reduction of purchasing capacity of the population the volumes of consumption of some types of food go down;
- **Overproduction** – the big crops of fruit and vegetables. Surpluses can be used in animal industries, having lowered costs on forage to cattle. As a solution for this problem can be increased storage period of the food: a frost, conservation;
- **Economic availability** – a level of incomes, which allows the citizen to get, at least, on a minimum level of consumption irrespective of the social status and a residence;
- **Physical availability of the food** – presence of food in all territory of the country at each moment of time and in necessary assortment.

As the agriculture is strategically important branch for economy of each country and national food safety directly depends on it, the price level in agriculture is under the constant control of the state. At the planned tendency of rising market prices the state takes measures to lower them (for example, special grants for farms) or, on the contrary, at the moment of crisis and overproduction and, hence, reductions of prices, the state artificially supports their former level (for example, buys up farmers surpluses of food) [1] .

3. The Role of Agriculture in Business and Economy

The ground in agriculture is the basic means of production. Therefore efficiency of agriculture in many depends on as far as rationally it is used.

Presently is practically used all or almost all suitable for treatment earth. Ploughing up the new, less convenient areas can result in rise in price of agricultural production and to negative consequences for an environment as it has already taken place in a zone of unstable agriculture, for example in a number of the countries of Africa. Though the agricultural areas are still increased, it goes very slowly, and growth of arable lands appreciably lags behind the expansion of agricultural territories. According to information of Food and agricultural organization of UNO (FAO), the share of agricultural grounds for the last 30 years grew from 33,13 to 35,71% of all land, and a share of an arable land – from 10,41 up to 11,03%, on a half of percent. The area of the cultivated grounds for 1961–1990 years has increased from 1,3 billion hectares up to 1,4 billion hectares. There was a stabilization of the area of arable lands [12].

The role of agriculture in the economy of a country or a region shows its structure and level of development. As parameters of the role of agriculture they apply a share occupied in agriculture among economically active population, and also density of agriculture in structure of gross national product. These parameters are high enough in the majority of developing countries where in an agriculture is occupied more than half of economically active population. The agriculture there goes on an extensive way of development that is the increase of production is achieved by expansion of areas under crops, increase of a livestock of cattle is achieved by number of people occupied in agriculture. In such countries the economies of which behave to the type of agrarian, the indexes of mechanization, chemicalization, land-reclamation and other are low.

Agriculture attained the highest level in the developed countries of Europe and North America, which had entered a post-industrial stage. There in agriculture is occupied 2–6% EAP [6]. In these countries "green revolution" has taken place in the middle of XX century, the agriculture is characterized by scientifically proved organization, increase of productivity, application of new technologies, systems of agricultural machines, pesticides and mineral fertilizers, use of genetic engineering and biotechnology, robotics and electronics, that develops on an intensive way.

Similar progressive changes occurs and in the countries concerning to the type of industrial, however the level of an intensification in them is still much lower, and the share of occupied in an agriculture people is higher, than in post-industrial.

Thus, in the developed countries is observed crisis of overproduction of the food, but in agrarian on the contrary, one of the sharpest problems is the food problem (a problem of under-eating and famine).

The traced tendency of the globalisation of world market allows redistributing food resources. Thus, the developed countries with the high level of agriculture can rear agricultural cultures and to export them in countries with food problems. The role of world trade in maintenance of food safety is very great. In the Roman declaration it is spoken, that trade derivates an effective usage of resources and stimulates economic development on which depends the level of food safety. And the appropriate commercial policy promotes achievement of the purpose of steady development and food safety.

Food safety – not the same, that the food self-sufficiency. Food safety assumes maintenance of presence, availability and an opportunity of consumption of food resources for all. The given difference has the important value as many developing countries make the basic food grain crops in the volumes exceeding their own needs, however many inhabitants of these countries have no high-grade meal as they can not allow buying products of necessary quality and quantity. But in many other countries which are being importers of the foods, the share of population, which does not have a high-grade meal, is very low, as the general level of incomes in these countries is high. There is a third category of the countries: these are the countries with a low level of incomes, deficiency of the food and with a high share of chronically starving population; these countries are compelled to import the food [13].

4. Modern Tendencies of Agriculture

Authorities of the advanced countries have thought up a new way of struggle against food crisis. In their opinion, use of genetically-modified organisms (GMO) can constrain a rise in prices on food. The government of EU has given to countries – members of the EU an opportunity to make a decision about cultivation on their territories others GM products. Germany has taken this right and allowed landing of a potato and the sugar beet, containing GMO. And also the parliament of France has passed the law, allowing cultivation in the territory transgenetic products.

The world food crisis pushes agricultural ministers of the EU on the most desperate steps. To increase volumes of manufacture, they are ready to take off a number of restrictions on use of pesticides and other chemical substances in manufacturing fruit-and-vegetable and another production, while its destiny isn't solved.

But in whole the tendency is understandable – European fito-sanitary legislation becomes more liberal. Also requirements to quality of production delivered in the market of EU also have decreased. If before the requirement of them were sometimes even ridiculous, for example, the cucumber should be extremely smooth and no more than 8 cm long, and if it exceeded the data parameter or was less than it should be, such goods did not pass on the market of the EU [5]. Presently we haven't got it. Now the purpose is to increase the manufacture of agricultural products.

World production of vegetables grows faster, than world production of fruit. Rates of growth of vegetables have approximately 5% per a year, in the same time; rates of growth of fruit manufacture have 3% per a year. Because vegetables, instead of fruit are base in a meal, therefore is logically, that manufacture of vegetables grows faster. The part of global growth of manufacture is a consequence of a global increase in population in 1,87% annually.

Other part of growth occurs due to change of usual preferences of consumers, i.e. the style on a healthy way of life which now dictates USA and Europe, it conducts to that in a daily diet of the person fruits and vegetables come on change to proteins and starch.

In the world is already noticed the fallowing tendency: with growth of incomes consumption of vegetables and fruit grows. The more people earns, the more they consume healthy food. Today organic food is widely claimed in the world market [2]. In the last two decades the world market of organic food roughly develops and becomes a popular alternative to consumption of harmful and ecologically unsafe products. In 2003 its global volume has reached 25 billion dollars, annual rates of growth in the developed countries are 20–30% [14]. Principal causes of this phenomenon are covered in the following:

1) Ecological food crises of the last decade (epidemic of the cow furiousness, the bird's flu, etc.) and growth of mistrust to usual products;
2) Public excitements concerning harm of the genetically-modified components contained in food.

The governments of the developed countries actively support eco-manufacture as it promotes strengthening of a home market, increase of export and helps to solve economic and environmental problems. Today 32 countries of the world have completely authorized standards on non-polluting production, 9 countries are engaged in introduction of standardization, 15 countries – develops such standards. But global experience shows, that the state supports this or that project only then, when it starts to bring money, becomes a part of economy.

The history of formation and standardization of market EPP in USA is very indicative in this plan. The idea of organic products was born in USA in 60th years. Initially it was some kind of *anti*culture, the protest against pollution of a planet, abusing a nature. Sellers and buyers were treated like crazy. Assortment of organic products was small, the prices very high to buy them was possible only in rare specialized shops.

But gradually more and more people started to feel, that usual products not only do not benefit, but also can do much harm. The huge amount of preservatives and the chemistry in the majority of products resulted in illnesses and reduction of immunity. So "the green wave" has come to consumer masses. People have turned their looks on non-polluting products and have started to come into organic shops. Large supermarkets and the companies which earlier had rejected organic, have seen in it huge potential and profit.

5. The Necessity of Applying Innovations in Agriculture

To increase productivity in agriculture and to lower industrial expenses, it is necessary to apply the innovation technologies, which could increase efficiency of agricultural activity and improve quality of production. With each year volumes of consumed production of agriculture are increased, and with them grow requirements of consumers to the quality of buying products. To provide demand, it is necessary for agriculture to raise the efficiency, and also competitiveness of production.

Expansion of farmland meant gathering an additional crop from usage of new territories. Usage of new territories is accompanied by the high expenses for their purchase and processing which can to allow only several of farms. Moreover, many farmlands, by virtue of the arrangement, can also have restrictions in expansion.

This implies that the modern manufacturer to raise qualitative and competitive production should invest in intensive technologies.

Certainly, very important condition for maintenance of competitiveness is improvement of quality of production, favourable formation of structure of fibres, improvement of quality of sugars and vegetable oils.

The Basic ways of increase of productivity of agriculture in the world market, and also its efficiency are:

- use of GMO's;
- use of mineral fertilizers;
- use of organic fertilizers.

The use of genetically modified organisms (GMO) means use of qualitative hybrid seeds of foreign selection which show high rates of growth and efficiency. Today for a bag of seeds of usual "white" corn, which can be bought in any farmer shop, is necessary to pay 108 dollars, but for the same quantity of transgenetical corn – "bitec" – 120 dollars [15]. In view of an additional patent payment for technology the bag of "bitec" – corn is equally twice more expensive than usual corn. And all the same thousand farmers buy and sow GM – corn. Why? Transgenetical cultures are economically favourable. It is not necessary to spend money on chemistry, on spraying chemicals from the plane, spending a lot of gasoline, energy and human forces. With each year more and more farmers sow genetically changed agricultural crops, because of useful properties of them: creating a new grade is reduced from ten years, at use of usual selection, till two-three years at use of methods of genic engineering. And due to unpretentiousness transgenetical products the cost price of their cultivation is reduced in five / seven times.

However to use the widespread hybrids is possible only under condition of maintenance of full technological process of cultivation and cleaning that can allow not all manufacturers, otherwise use of expensive seeds is not favourably. For the last some years thousand farmers in India have committed suicide, others, trying to pay with duties, sell their organs. The reason is the enormous losses because of cultivation of the GM – cotton. Contrary to promises of company "Monsanto", plants were subject to a plenty of illnesses and did not give actually any crop, thus the price which farmers have paid for seeds to the companies, in average was in 4 times higher, in comparison with cost of a usual cotton. Representatives of 'Monsanto' say, that the troubles which have comprehended farmers, are connected not with bad quality of transgenetical cotton, but with infringement of technology of its cultivation [16].

There is also other problem connected to economic features of cultivation GMO. All genic inserts which are built in a gene of a plant for reception GMO, are an object of the intellectual property, hence, their use is requiring payment.

GM-cultures are aimed at use on big areas and feedback as a homogeneous mass product. The agriculture raising GM-cultures, as a rule, receive the state grants. Cultivation of transgenetical cultures is profitable in every ways only to the companies which create them under the certain marketing tasks.

In whole, cultivation of GM-plants can be accompanied by a lot of agricultural problems:

- pollution of traditional grades with transgenetical constructions;
- appearance of new steady forms of weeds and wreckers;
- appearance of new fitopatogenes (activators of illnesses);
- flashes of number of wreckers;
- transition of old wreckers to new cultures;
- oppression of useful insects;
- infringement of natural soil fertility; decrease of a variety of agricultural crops due to mass application of GMO, received from a limited number of parental grades;
- monopolization of production of a seed material by companies – developers owing to patenting genic inserts [17].

The use of GM-products can make an irreparable harm to health of the person. Scientists allocate the following basic risks of consumption GM-products:

1) Oppression of immunity, allergic reactions and metabolic frustration, as a result of direct action of transgenetical fibres. Influence of new fibres which produce built – in GMO genes, is not known. In Sweden, where transgenetics are forbidden, are sick of an allergy only 7% of the population, and in USA where they are sold even not marked, – 70,5%.

2) Various infringements of health as a result of occurrence in GMO new, unplanned fibres or toxic for the human products of a metabolism. For example, for manufacturing the food adding triptofan in USA at the end of 80th of XX century was created the GM-bacterium. However together with usual triptofan, for the unknown reason, it began to develop etilen-bis-triptofan. As a result of its use was ill 5 thousand persons, from them – 37 persons have died, 1500 became invalids. Independent experts approve, that the genetically-modified cultures of plants allocate in 1020 times more toxins, than usual organisms.
3) Occurrence of stability pathogenic microflora of the human to antibiotics. At reception GMO till now are used marker genes of stability to antibiotics which can pass in micro flora of intestines, that was shown in the appropriate experiments, and, in turn, can result in medical problems – impossibility to cure many diseases.
4) The Infringements of health connected with accumulation of herbicides in an organism of the human. The majority known transgenetical plants does not perish at mass use of agricultural chemicals and can accumulate them.
5) Reduction of reception in an organism of necessary substances.
6) The remote cancerogenic and mutagen effects. Each insert of an alien gene in an organism is a mutation; it can cause in genome undesirable consequences and in what it will result – nobody [4].

Taking into consideration the above-stated kinds of risks, connected with use of GMO by people, many countries enter a number of restrictions and interdictions on sale of GM-production which renders direct influence on its demand. The strong retentive factor is also the ecological organizations, first of all Greenpeace. The EU though allowed sale of GM-products in the countries, but under pressure of “green” their deliveries were taken to the minimum. Austria, France, Hungary and other countries try to become independent from ГМО [18].

Categorical unwillingness of Brussels to allow on the European market any goods containing GM-products, essentially limits commodity markets of production from the American agriculture, where new technologies are used with huge scope [19]. This problem has got especial acuteness after even Zambia suffering from famine, has refused from American humanitarian help, having been afraid of “contamination” of the agriculture with genetically modified grain. India has refused from use of GM-mustard plant.

Zones, completely free from GMO, became Switzerland where in 2007 the national referendum was carried out, Greece, in which authority of all provinces have declared their zones, free from GMO, Poland where is entered the moratorium on GMO. Free from GMO was proclaimed Serbia, Albania and Venezuela. Today separate “pure” territories are present practically in all countries. Even in three states of USA the interdiction on use of products of genic engineering is fixed legislatively [20].

The use of mineral fertilizers allows achieving a gain of productivity, depending on a kind of cultures, in average are 9–15%. Mineral fertilizers are nitric, phosphoric, potassium, complex and micro fertilizers. They are a production of chemical manufacture and comprise one or several nutrients in high concentration. It is considered, that from the general increase of a crop approximately 50% of it is provided with fertilizers, 25% is advantage of a grade and 25% by technology of cultivation. Each kg of the mineral fertilizers applied at a proper correlation, gives in average 10 kg of a grain or equivalent amount of another agricultural production [21]. In the last years the prices for mineral fertilizers became so high, that their use, frequently, becomes not profitable. The minimal growth of the prices was 40%, maximal – 100%. The world market of mineral fertilizers promptly develops in the last years. The basic driving force of demand is the increase of the world population, essentially outstripping expansion of Arabic countries. Without use of fertilizers the present amount of the ground already now only would suffice to support no more than 3 billion persons. The areas of arable lands will be reduced per capita: instead of 22,8 dam^2 in 2000 to 2020 year will remain 18,3 dam^2, and to 2050 year – only seven dam^2. Increase of intensity of agriculture, including with use of fertilizers, is represented by the unique decision of the problem of maintenance with the food. One more reason of growth of demand for mineral fertilizers – rough development of the industry of bio fuel, as raw material for which serve agricultural crops. While for manufacture of bio fuel is used about 1% of the world arable land, however, according to the World food organization, by 2050 it will demand about 20% of the cultivated grounds [22].

However, an application of mineral fertilizers is effective only at their correct use, considering type of ground, dozes of fertilizers and their forms. The contents of high dozes of mineral fertilizers causes explosive decomposition of mineral oil and accumulation in ground for a short period of time plenty of destructions – the so-called petroleum acids distinguished with very high level of toxicity for micro flora of ground, and for plants.

The use of organic fertilizers means conducting agriculture within the framework of which there is a conscious minimization of use of synthetic fertilizers, pesticides, regulators of growth of plants, the fodder additives, genetically modified organisms. From organic fertilizers it is possible to list: manure, dung water, humus, lake silt, ashes, peat, green fertilizer, waste products of a city municipal services, sapropel, straw, guano etc. the most perspective kind of organic fertilizers are non-polluting organic fertilizers from sapropel. Sapropel is the adjournment of fresh-water reservoirs consisting of organic substances and mineral impurity which age is some tens and even hundreds thousand years. Organic fertilizers from sapropel are the ecologically pure product, which is not rendering negative influence on ground, they are applicable for all its types, have a number of additional benefits from their application (term of their action is traced till 14 years), are more effective (as a minimum twice) in comparison with mineral fertilizers.

As a result of the carried out researches of efficiency of organic fertilizers from sapropel at cultivation of agricultural crops, it was established, that organic fertilizers from sapropel have a wide complex of vitamins and mineral substances. After use of organic fertilizers from sapropel productivity of cultures grows from 40 up to 70% [23]. Their use excludes overfeeding plants and reduces accumulation of nitrates, providing the high productivity on various types of grounds. Use of sapropelic fertilizers brings the additional benefits expressing in improvement of mechanical structure of soils, their humidity and an aeroration. The given fertilizers promote mobilization of soil structure, as a result of auto purification from pathogenic plants, mushrooms and harmful micro organisms. Sapropelic fertilizers have a number of conclusive competitive advantages not only before mineral fertilizers, but also before organic fertilizers:

- In comparison with compost of animal origin sapropel does not contain a stock of seeds of weed plants, is not infected with pathogenic bacteria and flora. At long storage in packing losses of nitrogen it is not observed.
- In comparison with peat – contains more extensive list of the organic substances necessary for plants. Sapropels are extremely rich with nitrogen. Any mineral, neither peat, nor slates, oil, have such high contents of nitrogen, as sapropel. Sapropels differ with higher thermal capacity, than peat (up to 0,95 cal / g deg.).
- Unlike many chemical fertilizers, organic sapropel does not render harmful toxic action on people and animals, and vice versa, reduces the contents of nitrates, nitrites, salts, heavy metals.
- Terms of bringing and ways of doing up sapropel in soil do not differ from terms and ways of bringing of other organic fertilizers, but there are also advantages: at bringing sapropel "randomly" on the area before ploughing the losses of nitrogen are not observed even at a long delay of ploughing. Validity of fertilizer is traced till 14 years.
- At bringing in soil it improves its mechanical structure, water-absorbing and moisture-holding ability, gives an increase of humus in soil, and activates the processes of soils. Due to slow solubility of substances operating in a product is provided the balanced feed of plants with all elements of feed.

Furthermore, sapropelic fertilizers can be used as preservative at storage of root crops (tubers are poured the thin layer of sapropel).Unlike many chemical fertilizers, organic sapropel does not render harmful toxic action on people and animals. Sensational property of sapropel was recently opened – to hinder an accumulation in the agricultural product of radionuclids that has an enormous value, in particular, for areas, injured of Chernobyl accident [24]. It is obvious that the foregoing analysis shows that, application of sapropelic fertilizers gives a number of advantages in comparison with other kinds of fertilizers, raising productivity of different cultures and as consequence productivity of agriculture.

Sapropel can be used as fertilizers, as in granules, as in the pure state, not losing efficiency from its application. Its cost price can be considerably reduced, due to minimization of technological process, being limited only with alluvium and transportation to a place of application. The major task of any agricultural enterprise is to provide cost-effective work and promote its level, leaning against growth of the labour productivity, cutting prime costs and upgrading products.

Let's consider one of the variants of solving the given task, on an example of cultivation of tomatoes. The average norm of bringing sapropel in a wet kind for cultivation of tomatoes is 60 tons on 1 ha. The cost price of 1 ton is 10 EUR. That is, expenses for fertilizers on 1 ha are 600 EUR. Average productivity of tomatoes is 35 tons from 1 ha. The gain from application of sapropel averages 45% (35 tons * 45% = 15,75 tons). Average cost of tomatoes is 1,3 EUR for 1 kg (1300 EUR for 1 ton); (15,75 tons * 1300 EUR = 20475 EUR). Apparently from the above-stated example, processing the soil with sapropel enables to receive 19875 EUR of additional income from 1 ha.

Thus, sapropelic fertilizers help fully to solve the set task. Moreover, they allow to raise competitive organic production which besides on 20–60% is more expensive in comparison with usual and not having any restrictions on the markets of its selling [25].

Conclusions

During the conducted research, it was exposed, that agriculture is one of the most stable industries of business, possessing steady demand, because demand on its products had existed, and will exist always, while there is humanity. The major task of any agricultural enterprise is to provide cost-effective work and promote its level, leaning against growth of the labour productivity, cutting prime costs and upgrading products. Fertilizers from sapropel can fully solve this problem. Moreover, they allow producing competitive environmentally clean products.

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CUMULATIVE INDEX

COMPUTER MODELLING and NEW TECHNOLOGIES, volume 13, No. 4, 2009

(Abstracts)

Yu. Kochetkov. Time Factor in the Age of Globalisation, *Computer Modelling and New Technologies*, vol. 13, No 4, 2009, pp. 7–14.

This article considers time factor as a physical category and its interrelation with financial processes in economy by the example of classical market economy of the USA. There is performed a regression and correlation analysis of proposed mathematical models for interrelations of financial indicators during 45 years of relatively stable development.

Keywords: *time, rate of return, Dow Jones Index, regression, correlation, mathematical model*

Z. Uddin, M. Kumar. Effect of Temperature Dependent Properties on MHD Free Convection Flow and Heat Transfer Near the Lower Stagnation Point of a Porous Isothermal Cylinder, *Computer Modelling and New Technologies*, vol. 13, No 4, 2009, pp. 15–20.

The objective of the present study is to investigate the influence of temperature dependent viscosity and temperature dependent thermal conductivity on MHD free convection flow near the lower stagnation point of porous circular cylinder. The cylinder was assumed to be isothermal. The governing equations of motion and energy have been transformed into a system of non-dimensional coupled non-linear ordinary differential equations by using similarity transformations. These equations are solved by Runge-Kutta and shooting method. The numerical results are presented for velocity profiles, temperature distributions as well as heat transfer coefficient and skin friction coefficients for a wide range of viscosity variation parameter, thermal conductivity variation parameter, magnetic parameter, porosity parameter and Prandtl number.

It has been observed that with the increase in thermal conductivity parameter the velocity as well as temperature increases. Also, it is seen that the velocity increases with the increase in variable viscosity parameter, but there is a negligible decrease in temperature.

Keywords: *MHD free convection, lower stagnation point, isothermal circular cylinder, variable thermal conductivity, variable viscosity, Runge-Kutta shooting technique*

A. Jacobi. Approximation and Construction of Composite Bézier Surfaces Using Minimization and Finite-Element Methods, *Computer Modelling and New Technologies*, vol. 13, No 4, 2009, pp. 21–37.

This paper presents a global method for approximation and construction of surfaces. The method is based on a minimization of a functional that describes approximation and differential geometric characteristics. The functional includes weighting factors, which are used to control the approximation process. The numerical solution of the problem takes full advantage of the Finite-Elements Method with Bézier-Bernstein shape functions and uses the *p-method* in FEM in order to improve the approximation.

Keywords: *Bézier surface; surface fitting; approximate conversion; variational problem formulation; Finite Element Method (FEM)*

V. N. Boicov. Heterogeneous Factors in Stochastic Queueing Systems, *Computer Modelling and New Technologies*, vol. 13, No 4, 2009, pp. 38–43.

The methods of analysing queueing systems taking into account the heterogeneousness of time-response characteristics of requests are suggested in the given paper. The designated method of analysing time-response characteristics for the queueing systems of complicated configuration is derived in this publication. The present method may be utilized with the aim of calculating the parameters of the bandwidth capacity of random configuration computer network nodes.

Keywords: *queueing systems, enterprise computer network*

COMPUTER MODELLING and NEW TECHNOLOGIES, 13.sējums, Nr. 4, 2009 (Anotācijas)

J. Kočetkovs. Laika factors globalizācijas laikmetā, *Computer Modelling and New Technologies,* 13.sēj., Nr.4, 2009, 7.–14. lpp.

Šajā rakstā laika faktors tiek uzskatīts kā fizikāla kategorija un tā kopsakarības ar finansiālajiem procesiem ekonomikā ar piemēriem ASV klasiskajā tirgus ekonomikā. Tiek veikta matemātisko modeļu regresijas un korelācijas analīze finansiālo rādītāju savstarpējām sakarībām 45 gadu laikā relatīvi stabilā attīstībā.

Atslēgvārdi: laiks, ienākuma likme, *Dow Jones* indekss, regresija, korelācija, matemātiskais modelis

Z. Udins, M. Kumars. No temperatūras atkarīgo īpašību ietekme uz MHD brīvo konvekcijas plūsmu un siltuma pārveidi poraina izotermiska cilindra zemākā kritiskā punkta tuvumā, *Computer Modelling and New Technologies,* 13.sēj., Nr.4, 2009, 15.–20. lpp.

Dotā pētījuma mērķis ir izpētīt no temperatūras atkarīgās viskozitātes un no temperatūras atkarīgās vadāmības ietekmi uz MHD brīvās konvekcijas plūsmu poraina izotermiska cilindra zemākā kritiskā punkta tuvumā. Tiek pieņemts, ka cilindrs ir izotermisks. Galvenie kustības un enerģijas vienādojumi tika pārveidoti nelineārā vienkāršu diferenciālvienādojumu nedimensionālā savienojuma sistēmā, pielietojot līdzības transformācijas. Šie vienādojumi tiek atrisināti ar Runge-Kutta un uzņemšanas metodi. Skaitliskie rezultāti tiek parādīti ātruma profilam, temperatūras sadalēm, kā arī siltuma pārveides koeficients un apvalka frikcijas koeficienti priekš plaša viskozitātes diapazona pie parametru izmaiņas, siltuma vadāmība pie parametru izmaiņām, magnētiska parametra, porozitātes parametra un Prandtla skaitlim.

Tika novērots, ka, palielinoties siltuma vadāmības parametram, ātrums un temperatūra palielinās. Bez tam, tiek novērots, ka ātrums palielinās līdz ar mainīgo viskozitātes parametru, bet tur ir nenozīmīgs temperatūras kritums.

Atslēgvārdi: *MHD brīvā konvekcija, zemākais kritiskais punkts, izotermisks apļveida cilindrs, mainīgais viskozitātes parametrs, mainīgā siltuma vadāmība, Runge-Kutta uzņemšanas metode*

A. Džekobi. Bézier kompozīta virsmu aproksimācija un uzbūve, lietojot minimizācijas un finītā-elementa metodes, *Computer Modelling and New Technologies,* 13.sēj., Nr.4, 2009, 21.–37 lpp.

Šis raksts parāda globālu metodi virsmu aproksimācijā un uzbūvē. Metode pamatojas uz funkcionāla minimizāciju, kurš apraksta aproksimācijas un diferenciālās ģeometrijas īpašības. Funkcionāls ietver svēršanas faktorus, kuri tiek lietoti, lai kontrolētu aproksimācijas procesu. Uzdevuma skaitliskais risinājums pilnā mērā izmanto finīto-elementu metodi (*Finite-Elements Method – FFM*) ar Bézier-Bernstein formas funkcijām un finīto-elementu metodē (*FFM*) pielieto *p-metodi* , lai uzlabotu aproksimāciju.

Atslēgvārdi: *Bézier virsma, virsmas piemērotība, aptuvena konversija, variāciju problēmas formulēšana, finīto-elementu metode*

V. N. Boicovs. Heterogēnie faktori stohastiskajās masu apkalpošanas sistēmās, *Computer Modelling and New Technologies,* 13.sēj., Nr.4, 2009, 38.–43 lpp.

Masu apkalpošanas sistēmu analīzes metodes, ņemot vērā pieprasījumu laika-atbildes īpašības heterogenitāti, tiek izskatīta dotajā rakstā. Analizējamo laika-atbildes īpašību noteiktā metode sarežģītas konfigurācijas masu apkalpošanas sistēmām tiek veidota šajā pētījumā. Dotā metode var būt izmantota, lai aprēķinātu nejaušas konfigurācijas datora tīkla mezglu frekvenču joslas platuma kapacitātes parametrus.

Atslēgvārdi: *masu apkalpošanas sistēmas, uzņēmuma datoru tīkls*

F. Parands. Austrumi-rietumi: starptautiskās sadarbības transporta paradigma, *Computer Modelling and New Technologies,* 13.sēj., Nr.4, 2009, 44–47 lpp.

Raksts paredzēts starptautiskās transportēšanas jautājumiem, starptautiskās transportēšanas koridoram īpaši, ekonomisko un organizatorisko aspektu attīstībai, kas ir ļoti svarīga sastāvdaļa starptautisko ekonomisko attiecību sekmīgai attīstībai starp valstīm un reģioniem, kas palīdz aktivizēt starptautiskās saiknes starp rietumiem un austrumiem.

Atslēgvārdi: *starptautiskā ekonomika, globālais tirgus, starpkontinentālā transportēšanas sistēma, starptautiskie transportēšanas koridori, starptautiskā ekonomiskā sadarbība*

A. Zaleski, V. Čladze. Inovatīvu tehnoloģiju ietekmes izpēte uz agrobiznesa efektivitāti, *Computer Modelling and New Technologies,* 13.sēj., Nr.4, 2009, 48–56 lpp.

Lauksaimniecība ir viena no lielākajām ražošanām ekonomikā, kas pastāv gandrīz ikvienā valstī un kas nodrošina savas valsts iedzīvotājus ar pārtiku, kā arī dod izejmateriālu daudzām pārtikas ražotnēm. Ar katru gadu lauksaimniecības pārtikas preču daudzums palielinās, un arī prasības pēc kvalitātes arvien pieaug. Lai nodrošinātu pieprasījumu, lauksaimniecībai ir jāuzlabo tās efektivitāte un produktu konkurētspēja.

Atslēgvārdi: *organiskais mēslojums, lauksaimniecība, ienesīgums, sapropelis*

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