The weakening of the energy flow of surface waves due to scattering by the roughness

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Abstract

The stream of elastic energy in a wave extending on a rough surface is calculated. For enough smooth surface attenuation of a superficial wave is defined by the transport time of a relaxation considering of non-essentially processes of scattering on a small corner.

Keywords: *energy flow, physics*

1 Introduction

In work [1] considered the spread of Rayleigh waves on a rough surface of an elastic isotropic half-space. The surface of the elastic medium assumed to be random, and the deviation of the random surface of the plane χ=0 was believed to be small compared with the length of spreading over the surface of the Rayleigh wave. Calculates the displacement vector of the medium averaged over the ensemble of random surfaces. Appeared attenuation τ was due to scattering Rayleigh waves into secondary Rayleigh waves and body waves. Similar problems have been considered earlier in work [2] and [3] by a different method. However, taking into account only the scattering of Rayleigh waves in the body waves. In some cases, the attenuation due to the excitation secondary Rayleigh waves, as shown in [1], is the main. Considered in [1] attenuation was calculated as the displacement pole averaged roughness of the Green's function of elasticity equations with appropriate boundary conditions. For this reason, the τ was determined by the scattering amplitude waves on the roughness and an increase in the correlation length of roughness *d* does not tend to zero.

On the other hand, an increase in *d* means an approximation to a perfectly flat surface, and in this case, the damping would be to decrease.

To answer this question in the present work calculated physical observable quantity - the energy flux density averaged over the rough. It is shown that in the limit of large *d* damping is determined by the transport time τ_{tr}, decreasing for large *d*. The reason for replacing the τ to τ_{tr} related to the fact that the scattering at zero angle does not contribute to attenuation. In the calculation of the flow along with the Green's function is required to consider vertex diagrams, which leads to the appearance. The problem is being solved is close to the problem of skin-effect on the rough surface [4]. Another example - the resistance of alloys [5].

The energy flux density $\mathbf{q}(x,\ s,\ t)$ in the sound wave determined based on the continuity equation

$$\frac{d\varepsilon(x,s,t)}{dt} = -div\,\mathbf{q}(x,s,t),$$

where the energy density is determined by the expression

$$\varepsilon = \rho c_t^2 u_{ik}^2 + \frac{\rho}{2}\left(c_l^2 - 2c_t^2\right)u_{jj}^2 + p_i\frac{u_i^2}{2}, \tag{1}$$

u_{ik} - strain tensor, S - a two-dimensional vector with coordinates *y,z*. Using the equations of motion

$$\ddot{u}_i = c_t^2\frac{\partial^2 u_i}{\partial x_j^2} + \left(c_l^2 - 2c_t^2\right)\frac{\partial^2 u_j}{\partial x_i \partial x_j}. \tag{2}$$

We obtain for $q(x,s,t)$ expression

$$q_j(x,s,t) = -\left[2\rho c_t^2 \dot{u}_i u_{ij} + \rho\left(c_l^2 - 2c_t^2\right)\dot{u}_j u_{ii}\right]. \tag{3}$$

From equation (2) and the boundary conditions

$$\sigma_{\alpha\beta} n_\beta = P_\alpha(x,s,t), \tag{4}$$

that must be done on a random surface $x = \xi(s)$,($\sigma_{\alpha\beta}$ - stress tensor), determined components of displacement $u_\alpha(x,s,t)$. We assume that the force $P_\alpha(x,s,t)$ is different from zero in a small area on the surface and calculate the flow at large distances from the point of excitation.

The solution of equations (2), decomposed in *t* and *s* as a Fourier integral, write in the form

$$u_\alpha(x,\mathbf{p},\omega) = \sum_\gamma u_\alpha^{(\gamma)}(\mathbf{p},\omega) C_\gamma(\mathbf{p},\omega) e^{ip_x^\gamma x}, \tag{5}$$

where $u_\alpha^{(\gamma)}(\mathbf{p},\omega)$ - own solution of the equations (2),

$$u_\alpha^{(\gamma)}(\mathbf{p},\omega) = \begin{pmatrix} p_x^{(x)} & ip_y & ip_z \\ -ip_y & -p_x^{(y)} & 0 \\ -ip_z & 0 & p_x^{(z)} \end{pmatrix},\ p_x^{(\gamma)} = \sqrt{\frac{\omega^2}{c_\gamma^2} - p^2}. \tag{6}$$

The γ index of a column of a matrix (6) enumerates three independent decisions, the speed of spreading of c_γ coincides with the speed of transverse waves at $\gamma = y, z$ and with a speed of longitudinal waves of $\gamma = x$. $\mathbf{C}(\mathbf{p},\omega)$ vector is determined from the boundary condition (4), which, after decomposition in a Fourier integral and substituting (5) into (4) takes the form (drop the vector indices).

$$H^{(0)}(\mathbf{p},\omega)\mathbf{C}(\mathbf{p},\omega)+\int\frac{d^2q}{(2\pi)^2}\left[\mathbf{V}^{(1)}(\mathbf{p},\mathbf{q})+\mathbf{V}^{(2)}(\mathbf{p},\mathbf{q})\right]\mathbf{C}(\mathbf{q},\omega)=\mathbf{P}(\mathbf{p},\omega), \quad (7)$$

where

$$H^{(0)}_{\alpha\gamma}(\mathbf{p},\omega)=i\left(p_x^{(\gamma)}u_\alpha^{(\gamma)}(\mathbf{p},\omega)+p_\alpha u_x^{(\gamma)}(\mathbf{p},\omega)+\delta_{\alpha x}\left(\frac{c_l^2}{c_t^2}-2\right)p_\beta u_\beta^{(\gamma)}(\mathbf{p},\omega)\right)$$

$$\left.\begin{aligned}
&\mathbf{V}^{(1)}_\alpha(p,q)=\xi(\mathbf{p}-\mathbf{q})\Big[\delta_{\alpha\beta}(\mathbf{p}-\mathbf{q})\mathbf{q}u_\beta^{(\gamma)}(\mathbf{q},\omega)+(p-q)_\beta q_\alpha u_\beta^{(\gamma)}(\mathbf{q},\omega)+\\
&+\left(\frac{c_l^2}{c_t^2}-2\right)(p-q)_\alpha q_\beta u_\beta^{(\gamma)}(\mathbf{q},\omega)-q_x^{(\gamma)^2}u_\alpha^{(\gamma)}(\mathbf{q},\omega)+\\
&q_\alpha q_x^{(\gamma)}u_x^{(\gamma)}(\mathbf{q},\omega)+\left(\frac{c_l^2}{c_t^2}-2\right)\delta_{\alpha x}q_x^{(\gamma)}q_\beta u_\beta^{(\gamma)}(\mathbf{q},\omega)\Big],\\
&\mathbf{V}^{(2)}_{\alpha\gamma}(\mathbf{p},\mathbf{q})=i\int\frac{d^2q'}{(2\pi)^2}\xi(\mathbf{p}-\mathbf{q}-\mathbf{q}')\xi(\mathbf{q}')q_x^{(\gamma)}\Bigg[\left(qq'-\frac{q_x^{(\gamma)^2}}{2}\right)u_\alpha^{(\gamma)}(\mathbf{q},\omega)+\\
&+\left(q'_{\beta\delta}-\delta_{\beta x}\frac{q_x^{(\gamma)}}{2}\right)q_\alpha^{(\gamma)}u_\beta^{(\gamma)}(\mathbf{q},\omega)+\left(\frac{c_l^2}{c_t^2}-2\right)\left(q'_{\alpha\delta}-\delta_{\alpha x}\frac{q_x^{(\gamma)}}{2}\right)q_\beta^{(\gamma)}u_\beta^{(\gamma)}(\mathbf{q},\omega)\Bigg].
\end{aligned}\right\}. \quad (8)$$

In the formulas (8) $p_{\alpha\delta}=0$ at $\alpha = x$, $p_\alpha = p_y, p_z$ at $\alpha = y, z$. As in [1], we consider the amplitude of the roughness is small compared to the wavelength of sound ad, so the derivation of (7) the left side of the boundary condition (4) is decomposed by ~ ξ accurate to the second order.

To calculate the flow required substitute (5) into (3)

$$\begin{aligned}
q_s(x,\mathbf{p},\omega)=-\frac{\rho c_t^2}{(2\pi)^3}\int\omega' d\omega' d^2p'\Big\{(p-p')_{s'}\Big[u_k^{(\gamma)}(\mathbf{p}',\omega')u_k^{(\beta)}(\mathbf{p}-\mathbf{p}',\omega-\omega')\delta_{s's}+\\
+u_{s'}^{(\gamma)}(p',\omega')u_s^{(\beta)}(\mathbf{p}-\mathbf{p}',\omega-\omega')+\left(\frac{c_l^2}{c_t^2}-2\right)u_s^{(\gamma)}(\mathbf{p}',\omega')u_{s'}^{(\beta)}(\mathbf{p}-\mathbf{p}',\omega-\omega')\Big]+\\
+p_x^{(\beta)}(p-p')u_x^{(\gamma)}(\mathbf{p}',\omega')u_s^{(\beta)}(\mathbf{p}-\mathbf{p}',\omega-\omega')+\\
+\left(\frac{c_l^2}{c_t^2}-2\right)p_x^{(\beta)}(p-p')u_s^{(\gamma)}(\mathbf{p}',\omega')u_x^{(\beta)}(\mathbf{p}-\mathbf{p}',\omega-\omega')\Big\}\times\\
\times C_\gamma(\mathbf{p}',\omega')C_\beta(\mathbf{p}-\mathbf{p}',\omega-\omega')\exp\left\{i\left(p_x^{(\gamma)}(p')+p_x^{(\beta)}(p-p')\right)x\right\}
\end{aligned}. \quad (9)$$

In the flow interesting us at large distances from the point of excitation term with $p_x^{(\beta)}(p-p')$ using conditions (4) can be reduced to the form containing $(p-p')_s$, then the expression (9) takes the form

$$\begin{aligned}
q_s(x,\mathbf{p},\omega)=-\frac{\rho c_t^2}{(2\pi)^3}\int\omega' d\omega' d^2p'(p-p')_{s'}C_\gamma(\mathbf{p}',\omega')C_\beta(\mathbf{p}-\mathbf{p}',\omega-\omega')\times\\
\times T_{ss'}^{(\gamma\beta)}(\mathbf{p}',\omega',\mathbf{p}-\mathbf{p}',\omega-\omega')\exp\left\{i\left(p_x^{(\gamma)}(p')+p_x^{(\beta)}(p-p')\right)x\right\}
\end{aligned}, \quad (10)$$

where

$$\begin{aligned}
T_{ss'}^{(\gamma\beta)}(\mathbf{p}',\omega',\mathbf{p}-\mathbf{p}',\omega-\omega')=u_k^{(\gamma)}(\mathbf{p}',\omega')u_k^{(\beta)}(\mathbf{p}-\mathbf{p}',\omega-\omega')\delta_{s's}+\\
+u_{s'}^{(\gamma)}(\mathbf{p}',\omega')u_s^{(\beta)}(\mathbf{p}-\mathbf{p}',\omega-\omega')-2u_s^{(\gamma)}(\mathbf{p}',\omega')u_{s'}^{(\beta)}(\mathbf{p}-\mathbf{p}',\omega-\omega')-.\\
-u_x^{(\gamma)}(\mathbf{p}',\omega')u_x^{(\beta)}(\mathbf{p}-\mathbf{p}',\omega-\omega')\delta_{s's}
\end{aligned}$$

To calculate **C** need to solve the equation (7). Considering $\xi(s)$ small, we solve equation (7) iterations on $\mathbf{V}^{(1)}$ and $\mathbf{V}^{(2)}$.

In the zero approximation

$$\mathbf{C}^{(0)}(\mathbf{p},\omega)=H^{(0)-1}(\mathbf{p},\omega)P(\mathbf{p},\omega)$$

Substituting $\mathbf{C}^{(0)}(\mathbf{p},\omega)$ in (5), we obtain the solution on a perfectly flat surface

$$u_\alpha(x,\mathbf{p},\omega)=u_\alpha^{(\gamma)}(\mathbf{p},\omega)H_{\gamma\beta}^{(0)-1}(\mathbf{p},\omega)P_\beta(\mathbf{p},\omega)e^{ip_x^{(\gamma)}x}. \quad (11)$$

The poles of $u_\alpha(x,p,\omega)$ (11) defined from a condition

$$\det H^{(0)}(p,\omega)=\left(2p^2-\frac{\omega^2}{c_t^2}\right)^2+4p^2p_x^yp_x^x=0$$

give the known spectrum of Rayleigh waves on the ideal surface

$$p=p_0(\omega)=\frac{\omega}{c_R}.$$

The time-averaged flow of energy density, determined by integrating (10) with respect to p multiplier $e^{i\mathbf{ps}}$, must be calculated at $\omega=0$. On the ideal surface the result of integration at large distances determined by the poles and branch points of the expression (11), as well as the saddle point exponent. Recent determine the contribution of body waves in the flow of energy. In the integration over the angle between p and s are two saddle points corresponding to the two directions of spreading. We find the contribution of the poles, i.e Rayleigh waves, in the flow of energy. Writing the inverse matrix $\mathbf{H}^{(0)-1}(\mathbf{p},\omega)$ as near the pole in the form of

$$\mathbf{H}_{\gamma\beta}^{(0)-1}(\mathbf{p},\omega)=\left(p^2-p_0^2(\omega)+i\delta\right)^{-1}R_{\gamma\beta}(\mathbf{p}), \quad (12)$$

where $R_{\gamma\beta}(\mathbf{p})$ has no singularities, we obtain for the contribution of Rayleigh waves in the flow of energy the following estimate

$$q^{(s)}(0,s,0)\sim\frac{\rho}{s}\int d\omega'\omega'^3p^2(\omega'). \quad (13)$$

The contribution of the saddle point $p_\gamma=\frac{\omega}{c_\gamma}\left(s\big/\sqrt{\left(x^2+s^2\right)}\right)$, available in the integral modulo the p, is estimated as follows

$$q^s(x,s,0)\sim\frac{\rho c_t x^3}{\left(x^2+s^2\right)^{5/2}}\int d\omega'\omega'^3p^2(\omega'). \quad (14)$$

It is easy to see that the input branch at large distances proportional $1\big/\left(x^2+s^2\right)^{5/4}$, that is small compared to (14). Returning to the case of a rough surface on the n step iterations, we find

$$C^{(n)}(\mathbf{p},\omega)=-\int\frac{d^2q}{(2\pi)^2}H^{(0)-1}(\mathbf{p},\omega)\left[\mathbf{V}^{(1)}(\mathbf{p},\mathbf{q})+\mathbf{V}^{(2)}(\mathbf{p},\mathbf{q})\right]C^{(n-1)}(\mathbf{p},\omega). \quad (15)$$

Substituting (15) into (10) and averaging over a rough, we see that there are two types of terms. In terms of the first type $C_\gamma(\mathbf{p}',\omega')C_\beta(\mathbf{p}-\mathbf{p}',\omega-\omega')$ average of the product into a product of averages. Calculate the average of $C_\gamma(\mathbf{p}',\omega')$, described in detail in [1], leads to the following result $\left\langle C_\gamma(p,\omega)\right\rangle=H_{\gamma\alpha}^{-1}(p,\omega)P_\alpha(p,\omega)$

$$H_{\gamma\alpha}(\mathbf{p},\omega)=H_{\gamma\alpha}^{(0)}(\mathbf{p},\omega)-\int\frac{d^2q}{(2\pi)^2}\left(\left\langle\mathbf{V}_{\gamma\beta}^{(1)}(\mathbf{p},\mathbf{q})H_{\beta\delta}^{(0)-1}(\mathbf{q},\omega)\mathbf{V}_{\delta\alpha}^{(1)}(\mathbf{q},\mathbf{p})\right\rangle+\left\langle\mathbf{V}_{\gamma\alpha}^{(2)}(\mathbf{p},\mathbf{p})\right\rangle\right). \quad (16)$$

Members with **V** change spectrum of Rayleigh waves. Near the pole H^{-1} can be represented in a form similar to (12)

$$\mathbf{H}^{-1}(\mathbf{p},\omega)=\left(p^2-p_0^2(\omega)+i/\tau c_R\right)^{-1}R(\mathbf{p}). \quad (17)$$

Asymptotic expressions for $(\tau c_R)^{-1}$ are given in [1]. Note that formula (16) allows us to calculate the attenuation of a second order in the ξ/λ, in which it is determined by the binary correlation function

$$\left\langle V^{(1)}V^{(1)}\right\rangle\sim\xi_2\sim\left\langle\xi(\mathbf{p})\xi(\mathbf{p}')\right\rangle=(2\pi)^2\delta(p+p')\xi_2(p)$$

Easy to see that while $\mathbf{V}^{(1)}$ defines as attenuation and the displacement the ownl frequencies, $\left\langle\mathbf{V}^{(2)}(\mathbf{p},\mathbf{p})\right\rangle$ - does not contribute to attenuation. For this reason, further $\left\langle\mathbf{V}^{(2)}(\mathbf{p},\mathbf{p})\right\rangle$ won't be considered.

Does not reduce to the product of the average expression of the second order in shown in Figure 1.

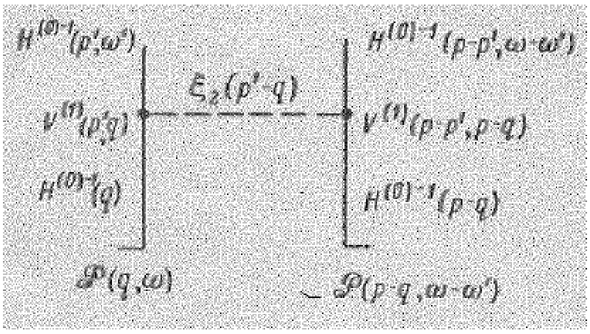

FIGURE 1 The diagram of the second order

Thin line begins with $H^{(0)-1}(\mathbf{p}',\omega')$ and ends with $P(\mathbf{q},\omega')$, or begins with $H^{(0)-1}(\mathbf{p}-\mathbf{p}',\omega-\omega')$ and ends with $P(\mathbf{p}-\mathbf{q},\omega-\omega')$, and the dotted line corresponds to the binary correlation function ξ_2, at the top there is a multiplier $\mathbf{V}^{(1)}$. Performing the summation of the iterative series for the average

$$\left\langle C_\gamma(\mathbf{p}',\omega')C_\beta^*(\mathbf{p}'-\mathbf{p},\omega'-\omega)(p'-p)_s\right\rangle \equiv \prod\nolimits_{\gamma\beta s}(\mathbf{p}',\omega',\mathbf{p}'-\mathbf{p},\omega'-\omega)$$

in the ladder approximation we obtain the equation

$$\begin{gathered}\prod\nolimits_{\gamma\beta s}(\mathbf{p}',\omega',\mathbf{p}'-\mathbf{p},\omega'-\omega)=H_{\gamma\alpha}^{-1}(\mathbf{p}',\omega')H_{\beta\delta}^{-1*}(\mathbf{p}'-\mathbf{p},\omega'-\omega)\times\\ \times\left[P_\alpha(\mathbf{p}',\omega')P_\delta^*(\mathbf{p}'-\mathbf{p},\omega'-\omega)\right](p'-p)_s+\\ +\int\frac{d^2q}{(2\pi)^2}\left\langle \mathbf{V}_{\alpha\mu}^{(1)}(\mathbf{p}',\mathbf{q})\mathbf{V}_{\delta\alpha}^{(1)*}(\mathbf{p}'-\mathbf{p},\mathbf{q}-\mathbf{p})\right\rangle\prod\nolimits_{\mu\alpha s}(\mathbf{q},\omega',\mathbf{q}-\mathbf{p},\omega'-\omega)\end{gathered}. \quad (18)$$

Later on we will be interested in the case when the correlation length d (an area in which ξ_2 significantly different from zero) is large in comparison with the characteristic length of Rayleigh waves. In deriving (18) we have neglected terms of order λ/d. The time-averaged flow $\mathbf{q}(x,\mathbf{p},\omega)$ at large distances is determined by p and ω, are small in comparison with the characteristic frequency and length of Rayleigh waves. For this reason, the integral

$$\int\frac{d^2t}{(2\pi)^2}\left\langle \mathbf{V}_{\alpha\mu}^{(1)}(\mathbf{p}',\mathbf{t})\mathbf{V}_{\beta\nu}^{(1)*}(\mathbf{p}'-\mathbf{p},\mathbf{t}-\mathbf{p})\right\rangle\prod\nolimits_{\mu\nu s}(\mathbf{t},\omega',\mathbf{t}-\mathbf{p},\omega'-\omega)=p_s'\mathbf{K}_{\alpha\beta}(\omega,\omega'). \quad (19)$$

We will calculate at small p and ω, p' corresponds to the length of the excited Rayleigh waves. To solve equation (18) multiply it by $\left\langle \mathbf{V}_{\alpha\gamma}^{(1)}(\mathbf{l},\mathbf{p}')\mathbf{V}_{\delta\beta}^{(1)*}(\mathbf{l}-\mathbf{p},\mathbf{p}'-\mathbf{p})\right\rangle$ and integrate over p'. We obtain

$$\begin{gathered}\mathbf{l}_s\mathbf{K}_{\alpha\delta}(\omega,\omega')=\int\frac{d^2p'}{(2\pi)^2}\left\langle \mathbf{V}_{\alpha\gamma}^{(1)}(\mathbf{l},\mathbf{p}')\mathbf{V}_{\delta\beta}^{(1)*}(\mathbf{l}-\mathbf{p},\mathbf{p}'-\mathbf{p},)\right\rangle\times\\ \times H_{\gamma\mu}^{-1}(\mathbf{p}',\omega')H_{\beta\gamma}^{-1*}(\mathbf{p}'-\mathbf{p},\omega'-\omega)\left[P_\mu(\mathbf{p}',\omega')P_\gamma^*(\mathbf{p}'-\mathbf{p},\omega'-\omega)+\mathbf{K}_{\mu\gamma}(\omega,\omega')\right]p_s'\end{gathered}. \quad (20)$$

The character of the solutions of (20) is determined by which of the functions $\xi_2(\mathbf{l}-\mathbf{p}')\sim\left\langle \mathbf{V}^{(1)}(\mathbf{l},\mathbf{p}')\mathbf{V}^{(1)*}(\mathbf{l}-\mathbf{p},\mathbf{p}'-\mathbf{p})\right\rangle$ or $H^{-1}(\mathbf{p}',\omega')H^{-1*}(\mathbf{p}'-\mathbf{p},\omega'-\omega)$ are in the integration over p' more acute. The first significantly changes at modification of the module in the range of $1/d$ order, and the second – in the range of $1/\tau c_R$. Appropriate intervals of the angle of the vector $\mathbf{p}'$: $1/dp_0(\omega')$ for $\xi_2(\mathbf{l}-\mathbf{p}')$ and $1/\sqrt{\tau c_R p}$ for $H^{-1}(\mathbf{p}',\omega')H^{-1*}(\mathbf{p}'-\mathbf{p},\omega'-\omega)$. Multiplying (20) on the scalar $\mathbf{l}$ and using (18) we see that the following limiting cases.

The correlation radius d is large in comparison with the τc_R. Integration with respect to the vector $\mathbf{p}'$ in (20) is performed using a sharp function $\xi_2(\mathbf{l}-\mathbf{p}')$, and pole denominators H^{-1} may be submitted at $\mathbf{p}'=\mathbf{l}$. In this case, used the ladder approximation is inapplicable because it omitted the derivation of (18) diagrams are large in parameter $(ap')^{-2}$

At lower values of the correlation length $d<\tau c_R$ integration modulo $\mathbf{p}$ performed using pole denominators, and the integration over the angle θ' - between $\mathbf{p}'$ and $\mathbf{l}$ and using the function ξ_2 on condition.

$$\frac{P}{(P_0(\omega')d)^2}\ll\frac{1}{\tau c_R}. \quad (21)$$

Allocating polar as denominator $b\equiv\left(p\cos\theta+p_0(\omega)-i/\tau c_R\right)^{-1}$, where θ - a angle between $\mathbf{p}'$ and $\mathbf{l}$, and entering the matrix which does not have features $M_{\alpha\beta}(\mathbf{l},\mathbf{p}')=\dfrac{\mathbf{V}_{\alpha\delta}^{(1)}(\mathbf{l},\mathbf{p}')R_{\delta\beta}(\mathbf{p}')}{2p_0(\omega')}$, we obtain

$$K_{\alpha\delta}(\omega,\omega')=i\pi b\left[\left\langle-\int M_{\alpha\mu}M_{\delta\gamma}^*\cos\theta'd\theta'\right\rangle P_\mu P_\gamma^*+\left\langle-\int M_{\alpha\mu}M_{\delta\gamma}^*\cos\theta'd\theta'\right\rangle K_{\mu\gamma}(\omega,\omega')\right]. \quad (22)$$

Solving (22), we find

$$K_{\mu\gamma}(\omega,\omega')=\left[\delta_{\mu\beta}\delta_{\nu\gamma}-i\pi b\left\langle-\int M_{\mu\beta}M_{\nu\gamma}^*\cos\theta'd\theta'\right\rangle\right]^{-1}\times\left\langle-\int M_{\beta\alpha}M_{\gamma\delta}^*\cos\theta'd\theta'\right\rangle i\pi bP_\alpha P_\delta^*.$$

Substituting the value $K_{\alpha\delta}$ of (22) into the expression for the polarized operator $\prod_{\gamma\beta s}$ (18), we obtain

$$\prod\nolimits_{\gamma\beta s}=H_{\gamma\alpha}^{-1}H_{\beta\delta}^{-1*}p_s'\left\{P_\alpha P_\delta^*+\left[\delta_{\alpha\beta}\delta_{\delta\gamma}-i\pi b\left\langle-\int M_{\alpha\beta}M_{\delta\gamma}^*\cos\theta'd\theta'\right\rangle\right]^{-1}\times\ i\pi b\left\langle-\int M_{\beta\mu}M_{\nu\nu}^*\cos\theta'd\theta'\right\rangle P_\mu P_\gamma^*\right\}.$$

Presenting the inverse matrix as

$$\left[\delta_{\mu\beta}\delta_{\nu\gamma} - i\pi b\left\langle -\int M_{\mu\beta}M^*_{\nu\gamma}\cos\theta' d\theta'\right\rangle\right]^{-1} = \\ = \det\left[\delta_{\mu\beta}\delta_{\nu\gamma} - i\pi b\left\langle -\int M_{\mu\beta}M^*_{\nu\gamma}\cos\theta' d\theta'\right\rangle\right] N_{\mu\beta,\nu\gamma},$$

where $N_{\mu\beta,\nu\gamma}$ has no features, we find

$$\prod\nolimits_{\gamma\beta s}(\mathbf{p}',\omega',\mathbf{p}'-\mathbf{p},\omega'-\omega) = \\ = H^{-1}_{\gamma\alpha}(\mathbf{p}',\omega')H^{-1*}_{\beta\delta}(\mathbf{p}'-\mathbf{p},\omega'-\omega)\,p'_s\left\{\frac{-p\cos\theta - p_0(\omega)+\dfrac{i}{\tau r_R}}{-p\cos\theta - p_0(\omega)+\dfrac{i}{\tau_{tr}c_R}}\right\} N_{\mu\alpha,\nu\delta}P_\mu P^*_\nu, \quad (23)$$

where

$$\frac{1}{\tau_{tr}} = \pi c_R\left\langle -\int Sp\mathbf{M}\; Sp\mathbf{M}^*\left(\cos\theta' - 1\right)d\theta'\right\rangle$$

The last integral is determined by the range of angles $|\theta'| < 1/p_0(\omega')d$, and we obtain the following estimate

$$|\theta'| < 1/p_0(\omega')d. \quad (24)$$

From formula (24) we see that with increasing d time τ_{tr} increases. Asymptotic of the flow of energy in this case is given by

$$q(0,s,0) \sim \exp\left\{-\frac{s}{\tau_{tr}c_R}\right\}S^{-1}p\int \omega'^3|P(\omega')|^2 d\omega', \\ (\tau c_R)^{-1} < d^{-1} < \frac{p_0(\omega')}{\sqrt{\tau c_R p}}. \quad (25)$$

In the case of $p/(p_0(\omega')d)^2 \gg (\tau c_R)^{-1}$ estimate solution of equation (18) is given by the first term on the right side. In this case, damping is determined by the same order of magnitude as the displacement of the poles of the Green functions $H^{-1}(\mathbf{p},\omega)$. Corresponding asymptotic formulas are given in [1].

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