H_∞ control for a class of distributed parameter systems with leakage delay and Markovian jumping

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Abstract

This paper is concerned with the problem of H_∞control for a class of distributed parameter systems with delay and Markovian jumping. The jumping parameters are generated from continue-time discrete-state Markov process, nonlinearities and leakage delay appear in the system states. An H_∞ state feedback controller is designed such that the closed-loop system is asymptotically stable in the mean square for the zero disturbance input and also achieves required H_∞ performance level. By constructing Lyapunov functional and stochastic analysis, the sufficient conditions of systems is given in terms of linear matrix inequalities. And the criteria derived are dependent on both delay and diffusion operator. Finally, a numerical example is presented to demonstrate the effectiveness of the results proposed.

Keywords: H_∞ control, distributed parameter systems, time-varying delay, leakage delay, Markovian jumping

1 Introduction

It is well known that distributed parameter systems have attracted increasing interest due to their extensively applications in solving some thermal, chemical reactor processes and population dynamics. These processes are distributed in spatial and time, which usually described by partial differential equations. Complexity of dynamical behavior has been exhibited in evolution of distributed parameter systems. Since time delays as a main source for system bad performance even instability, arise owing to finite switching speed of amplifiers and communication time. Therefore, it is extremely significant to investigate the control problem of delayed distributed parameter systems. The variable structure control scheme has been designed for stabilization of parabolic distributed parameter systems with delays in [1,2]. Time delay in the leakage term is also important on the dynamical behavior of system [3]. Recently, stability of BAM neural networks with leakage delay has been investigated in [4] and passivity analysis for neutral neural networks with Markovian jumping and delay in the leakage term has been studied in [5]. Moreover, stability, oscillation and periodicity for delayed distributed parameter systems have further been studied in [6-8]. By using meanfield theory and the auxiliary function method, the stability of stochastic distributed parameter systems has discussed in [9-10]. In particular, linear matrix inequality (LMI) technique has been introduced to study the robust control problem of delayed distributed parameter systems in [11]. After then, LMI technique has been the effective ways to analysis and control for delayed distributed parameter systems. Using LMI technique, the problem of H_∞ state feedback control for a class of distributed parameter systems with delay has been discussed in [12]. An H_∞ fuzzy observer-based control has been presented for a class of nonlinear parabolic systems with control constraints in [13]. Also, sampled-data control for fuzzy systems to achieve a required H_∞ disturbance attenuation level has been given in [14].

In recent years, increasing attention has been motivated to control of Markovian jumping systems. This class of system has switching parameters which are governed by a continuous-time discrete-state Markov chain, and has many applications such as fault-tolerant system, target tracking and so on. In [15,16], the influence of Markovian switching on stochastic neural networks with mixed time delays has been considered in the stability analysis. Since, synchronization of Markovian jumping neural networks with reaction-diffusion terms has been investigated in [17]. However, to the best of our knowledge, the H_∞ control of Markovian jumping distributed parameter systems with leakage delay and time-varying delays has never been tackled.

In this paper, the H_∞ control problem is addressed for a class of delayed distributed parameter systems with Markovian jumping parameters. Time delay include leakage delay and time-varying delays, and nonlinearities appear in the system states. Ourmain purpose is design an H_∞ state feedback control strategy such that the system is asymptotically stable in the mean square for the zero

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disturbance input and also achieves required H_∞ performance level.

Notation. Throughout this paper, $\mathbf{R}^n$ and $\mathbf{R}^{n\times m}$ denote, respectively, the n – dimensional Euclidean space and the set of $n\times m$ matrices. The superscript 'T' denotes the transpose and $X \ge Y$ (respectively, $X > Y$) means that $X - Y$ is positive semi-definite (respectively, positive definite). I is the identity matrix with compatible dimension. For $h > 0$, $C([-h,0];\mathbf{R}^n)$ denotes the family of continuous functions φ from $[-h,0]$ to $\mathbf{R}^n$ with norm $\|\varphi\| = \sup_{-h\le\theta\le 0} |\varphi(\theta)|$, where $|\cdot|$ denotes the Euclidean norm. $L^2(\Omega)$ denotes the space of real Lebesgue measurable functions. $\|\cdot\| = \left(\int_\Omega |\cdot|^2\right)^{\frac{1}{2}}$ is the norm of Banach space. Let $(S,\mathscr{F},\{\mathscr{F}_t\}_{t\ge 0},P)$ be a complete probability space with a filteration $\{\mathscr{F}_t\}_{t\ge 0}$ satisfying the usual conditions. Denote by $L^p_{\mathscr{F}_0}([-h,0],\mathbf{R}^n)$ the family of all bound, $\mathscr{F}_0$ measurable, $C([-h,0];\mathbf{R}^n)$ – valued random variables. $\mathbf{E}\{\cdot\}$ stands for the mathematical expectation operator. The asterisk * in a matrix is the term that is induced by symmetry.

2 Problem formulation

Let $r(t)$ $(t \ge 0)$ be a right-continuous Markovian process on a probability space $(S,\mathscr{F},\{\mathscr{F}_t\}_{t\ge 0},P)$ taking values in a finite state space $S = \{1,2,\cdots,N\}$ with probability transfer matrix $\Pi = (\pi_{ij})_{N\times N}$ given by

$$P\{r(t+\Delta) = j \mid r(t) = i\} = \begin{cases} \pi_{ij}\Delta + o(\Delta) & if \quad i \ne j \\ 1+\pi_{ij}\Delta + o(\Delta) & if \quad i = j \end{cases},$$

where $\Delta > 0$, and $\pi_{ij} \ge 0$ is the transition rate from i to j if $j \ne i$ while:

$$\pi_{ii} = -\sum_{j\ne i} \pi_{ij},\ i, j \in S = \{1,2,\cdots,N\}. \tag{1}$$

The distributed parameter systems with Markovian jumping and leakage delay can be described by the following partial differential equations:

$$\begin{cases} \dfrac{\partial x(t,\xi)}{\partial t} = D\Delta x(t,\xi) - A(r(t))x(t-\tau,\xi) \\ \qquad +B(r(t))f(x(t,\xi)) \\ \qquad +C(r(t))g(x(t-\sigma(t),\xi)) \\ \qquad +E(r(t))u(t,\xi) + w(t,\xi) \\ z(t,\xi) = \begin{bmatrix} F(r(t))x(t,\xi) \\ G(r(t))u(t,\xi) \end{bmatrix} \end{cases}, \tag{2}$$

for $t \in \mathbf{R}^+$ and $\xi = [\xi_1,\xi_2,\cdots,\xi_m]^T \in \Omega \subset \mathbf{R}^m$, $0 \le \xi_\varsigma \le l(\varsigma = 1,2,\cdots,m)$. Ω is a compact set with smooth boundary $\partial\Omega$ and $\mathrm{mes}\Omega > 0$ in $\mathbf{R}^m$. $x(t,\xi) = [x_1(t,\xi), x_2(t,\xi),\cdots,x_n(t,\xi)]^T \in \mathbf{R}^n$ denote the state of system in space ξ and at time t . $\Delta = \sum_{\varsigma=1}^m \frac{\partial^2}{\partial\xi_\varsigma^2}$ is Laplace operator in Ω , and transmission diffusion operator $D = \mathrm{diag}(d_1,d_2,\cdots,d_n) \ge 0$. $A(i),B(i),C(i),E(i),F(i)$ and $G(i)(i=1,2,\cdots,N)$ are $n\times n$ known matrices. $u(t,\xi)$ denotes external input vector. $w(t,\xi)$ denotes disturbance input. $z(t,\xi)$ is controlled output. The leakage delay $\tau \ge 0$ is a constant. The bounded function $\sigma(t)$ is time-varying delay, and satisfies $0 < \sigma_m \le \sigma(t) \le \sigma_M$, and $\dot\sigma(t) \le \sigma_d < 1$.

The initial conditions of Equation (2) is given by:

$$x_k(s,\xi) = \varphi_k(s,\xi), \frac{\partial x_k(s,\xi)}{\partial t} = \frac{\partial \varphi_k(s,\xi)}{\partial t},$$

$$-\bar\tau \le s \le 0,\ \bar\tau = \max\{\tau,\sigma_M,\sigma_d\},\ k = 1,2,\cdots,n.$$

and having boundary conditions:

$$\frac{\partial x_k(t,\xi)}{\partial n} = \left(\frac{\partial x_k(t,\xi)}{\partial\xi_1},\frac{\partial x_k(t,\xi)}{\partial\xi_2},\cdots,\frac{\partial x_k(t,\xi)}{\partial\xi_m}\right)^T = 0,$$

$$\forall t \ge 0,\ \xi \in \partial\Omega,\ k = 1,2,\cdots,n.$$

Throughout the paper, we make following assumption for the nonlinear function.

Assumption 1 [18]For $k \in \{1,2,\cdots,n\}$, the nonlinear function satisfy

$$l_k^- \le \frac{f_k(s_1) - f_k(s_2)}{s_1 - s_2} \le l_k^+,$$

$$m_k^- \le \frac{g_k(s_1) - g_k(s_2)}{s_1 - s_2} \le m_k^+,$$

where l_k^-, l_k^+, m_k^- and m_k^+ are fixed constants.

Assumption 2 $f(\cdot)$ and $g(\cdot)$ satisfy $f(0) = g(0) = 0$.

Remark 1: The constants l_k^-, l_k^+, m_k^- and m_k^+ are allowed to be positive, negative, or zero in Assumption 1. Hence, the resulting nonlinear functions are in the sector, and are more general than the usual Lipschitz conditions. In this paper, for the Equation (2), we consider the following state feedback controller:

$$u(t,\xi) = -Kx(t,\xi) \tag{3}$$

and the definition for asymptotically stable in the mean square as follows:

Definition 1 The controlled systems is said to be asymptotically stable in the mean square if, in case of $w(t,\xi) = 0$, for every system mode the following holds:

$\lim_{t\to\infty} \mathbf{E}\left\| x(t,\xi)\right\|^2 = 0.$

The combination of controller Equation (3) and the Equation (2) yields the following closed-loop system:

$$\begin{cases} \dfrac{\partial x(t,\xi)}{\partial t} = D\Delta x(t,\xi) - A(r(t))x(t-\tau,\xi) + \\ \qquad B(r(t))f(x(t,\xi)) + \\ \qquad C(r(t))g(x(t-\sigma(t),\xi)) - \\ \qquad E(r(t))Kx(t,\xi) + w(t,\xi) \\ z(t,\xi) = \begin{bmatrix} F(r(t)) \\ -G(r(t))K \end{bmatrix} x(t,\xi) \end{cases} \tag{4}$$

The purpose of the problem proposed in this paper is to design a stable H_∞ controller for Equation (2) via state feedback. The controller is to be designed such that:

1) The zero-solution of the closed-loop system with $w(t,\xi)=0$ is asymptotically stable in the mean square.

2) Under the zero-initial condition, the controlled output $z(t,\xi)$ satisfies $\|z\| \le \gamma \|w\|$ hold, where $\gamma > 0$ is a prescribed constant.

To obtain the main results, we introduce the following lemmas.

Lemma 1(Jensen's inequality) [19] For any positive definite matrix $M>0$, scalar $\nu>0$, and vector function $\omega:[0,\nu]\to \mathbf{R}^n$. If the integrations concerned are well defined, the following inequality holds:

$$\left(\int_0^\nu \omega(s)ds\right)^T M \left(\int_0^\nu \omega(s)ds\right) \le \nu \int_0^\nu \omega^T(s) M \omega(s)ds .$$

Lemma 2 [18] Suppose that $\mathcal{B} = \mathrm{diag}(\beta_1,\beta_2,\cdots,\beta_n)$ is a positive semi-definite matrix. Let $x=(x_1,x_2,\cdots,x_n)^T$, $\in \mathbf{R}^n$ and $\mathcal{H}(x) = (h_1(x_1),h_2(x_2),\cdots,h_n(x_n))^T$ be a continuous nonlinear function satisfying:

$$\chi^- \le \frac{h_k(s)}{s} \le \chi^+,\ s\ne 0, s\in \mathbf{R}, k=1,2,\cdots,n\ ,$$

with χ^- and χ^+ being constant scalars. Then:

$$\begin{bmatrix} x \\ \mathcal{H}(x) \end{bmatrix}^T \begin{bmatrix} \mathcal{B}\mathcal{X}_1 & -\mathcal{B}\mathcal{X}_2 \\ -\mathcal{B}\mathcal{X}_2 & \mathcal{B} \end{bmatrix} \begin{bmatrix} x \\ \mathcal{H}(x) \end{bmatrix} \le 0\ ,$$

oOr:

$$x^T\mathcal{B}\mathcal{X}_1 x - 2x^T\mathcal{B}\mathcal{X}_2\mathcal{H}(x) + \mathcal{H}^T(x)\mathcal{B}\mathcal{H}(x) \le 0$$

where $\mathcal{X}_1 = \mathrm{diag}(\chi_1^+\chi_1^-,\chi_2^+\chi_2^-,\cdots,\chi_n^+\chi_n^-)$ and $\mathcal{X}_2 = \mathrm{diag}\left(\frac{\chi_1^+ + \chi_1^-}{2},\frac{\chi_2^+ + \chi_2^-}{2},\cdots,\frac{\chi_n^+ + \chi_n^-}{2}\right)$.

Lemma 3(Schur complement) [19]. The following linear matrix inequality (LMI):

$$\begin{pmatrix} Q & S^T \\ S & -R \end{pmatrix} < 0,$$

is equivalent to the conditions: $R>0$, $Q+S^TR^{-1}S<0$, where $Q=Q^T$, $R=R^T$.

For presentation convenience, we denote

$$L_1 = \mathrm{diag}(l_1^+l_1^-, l_2^+l_2^-,\cdots,l_n^+l_n^-),$$

$$L_2 = \mathrm{diag}\left(\frac{l_1^+ + l_1^-}{2},\frac{l_2^+ + l_2^-}{2},\cdots,\frac{l_n^+ + l_n^-}{2}\right),$$

$$M_1 = \mathrm{diag}(m_1^+m_1^-, m_2^+m_2^-,\cdots,m_n^+m_n^-),$$

$$M_2 = \mathrm{diag}\left(\frac{m_1^+ + m_1^-}{2},\frac{m_2^+ + m_2^-}{2},\cdots,\frac{m_n^+ + m_n^-}{2}\right),$$

and $x_t(t,\xi) = \frac{\partial x(t,\xi)}{\partial t}$.

3 Stability analysis of closed-loop system

First of all, we deal with the problem of stability analysis of closed-loop Equation (4) with given controller and $w(t,\xi)=0$.

Theorem 1 Given the controller parameter K. The closed-loop Equation (4) with $w(t,\xi)=0$ is globally asymptotically stable in the mean square if there exist three matrices $Q_1>0$, $Q_2>0$ and $R>0$, three sets of matrices $P_i>0$, $P_{2i}>0$ and $P_{3i}>0 (i\in S)$, and two sets of positive-definite diagonal matrices Λ_i, $\Sigma_i (i\in S)$ such that the matrix inequality holds:

$$\Xi(i) = \begin{bmatrix} \Omega_{11}(i) & \Omega_{12}(i) & \Omega_{13}(i) & \Omega_{14}(i) & \Sigma_i M_2 & P_{2i}C(i) \\ * & \Omega_{22}(i) & -P_{3i}A(i) & P_{3i}B(i) & 0 & P_{3i}C(i) \\ * & * & \Omega_{33} & 0 & 0 & 0 \\ * & * & * & -\Lambda_i & 0 & 0 \\ * & * & * & * & \Omega_{55}(i) & 0 \\ * & * & * & * & * & \Omega_{66} \end{bmatrix} < 0\ , \tag{5}$$

where:

$$\Omega_{11}(i) = -2\frac{m}{l^2}P_{2i}D - 2P_{2i}E(i)K - Q_1 + Q_2 + \sum_{j=1}^{N}\pi_{ij}P_j - \Lambda_i L_1 - \Sigma_i M_1,$$

$$\Omega_{12}(i) = P_i - P_{2i} - K^TE^T(i)P_{3i},$$

$$\Omega_{13}(i) = Q_1 - P_{2i}A(i), \Omega_{14}(i) = P_{2i}B(i) + \Lambda_i L_2,$$

$$\Omega_{22}(i) = \tau^2 Q_1 - 2P_{3i}, \Omega_{33} = -Q_1 - Q_2,$$

$$\Omega_{55}(i) = R - \Sigma_i, \Omega_{66} = -(1-\sigma_d)R.$$

Proof: In order to obtain the stability conditions, we choose the following Lyapunov functional for each mode $i \in S$.

$$V(t,i) = V_1(t,i) + V_2(t,i) + V_3(t,i) + V_4(t,i), \tag{6}$$

where

$$V_1(t,i) = \int_\Omega x^T(t,\xi) P_i x(t,\xi) d\xi,$$

$$V_2(t,i) = \int_\Omega \Big[\tau \int_{-\tau}^{0} \int_{t+\theta}^{t} x_s^T(s,\xi) Q_1 x_s(s,\xi) ds d\theta + \int_{t-\tau}^{t} x^T(s,\xi) Q_2 x(s,\xi) ds \Big] d\xi,$$

$$V_3(t,i) = \int_\Omega \int_{t-\sigma(t)}^{t} g^T(x(s,\xi)) R g(x(s,\xi)) ds d\xi,$$

$$V_4(t,i) = \int_\Omega (\nabla x(t,\xi))^T D P_{3i} \nabla x(t,\xi) d\xi.$$

For the Equation (4), we will discuss the system mode $r(t) = i, \forall i \in S$. It is known that $\{x(t,\xi), r(t)\}$ is a Markov process. Its weak infinitesimal operator LV is defined by:

$$\mathscr{L}V(t,i) = \lim_{\Delta \to 0^+} \frac{1}{\Delta} \sup \big[\mathbf{E}\{V(t+\Delta, r(t+\Delta)) \,|\, x(t,\xi), r(t) = i\} - V(t, r(t) = i) \big], \tag{7}$$

From Equation (6), it can be seen that:

$$\mathscr{L}V(t,i) = \mathscr{L}V_1(t,i) + \mathscr{L}V_2(t,i) + \mathscr{L}V_3(t,i) + \mathscr{L}V_4(t,i), \tag{8}$$

where:

$$\mathscr{L}V_1(t,i) = \int_\Omega [2x^T(t,\xi) P_i x_t(t,\xi) + \sum_{j=1}^{N} \pi_{ij} x^T(t,\xi) P_j x(t,\xi)] d\xi, \tag{9}$$

$$\begin{aligned}
\mathscr{L}V_2(t,i) &= \int_\Omega \{\tau^2 x_t^T(t,\xi) Q_1 x_t(t,\xi) - \tau \int_{t-\tau}^{t} x_s^T(s,\xi) Q_1 x_s(s,\xi) ds + x^T(t,\xi) Q_2 x(t,\xi) - x^T(t-\tau,\xi) Q_2 x(t-\tau,\xi)\} d\xi \le \\
&\int_\Omega \{\tau^2 x_t^T(t,\xi) Q_1 x_t(t,\xi) - \left(\int_{t-\tau}^{t} x_s(s,\xi) ds \right)^T Q_1 \left(\int_{t-\tau}^{t} x_s(s,\xi) ds \right) + x^T(t,\xi) Q_2 x(t,\xi) - x^T(t-\tau,\xi) Q_2 x(t-\tau,\xi)\} d\xi = \\
&\int_\Omega \{\tau^2 x_t^T(t,\xi) Q_1 x_t(t,\xi) + x^T(t,\xi)(-Q_1 + Q_2) x(t,\xi) + 2x^T(t,\xi) Q_1 x(t-\tau,\xi) + x^T(t-\tau,\xi)(-Q_1 - Q_2) x(t-\tau,\xi)\} d\xi,
\end{aligned} \tag{10}$$

$$\mathscr{L}V_3(t,i) \le \int_\Omega \{g^T(x(t,\xi)) R g(x(t,\xi)) - (1-\sigma_d) g^T(x(t-\sigma(t),\xi)) R g(x(t-\sigma(t),\xi))\} d\xi, \tag{11}$$

$$\mathscr{L}V_4(t,i) = 2\int_\Omega (\nabla x(t,\xi))^T D P_{3i} (\nabla x(t,\xi))_t d\xi. \tag{12}$$

We utilize further the descriptor method [20] to Equation (4), where the left-hand side of:

$$0 = 2\int_\Omega x^T(t,\xi) P_{2i} [D\Delta x(t,\xi) - A(i) x(t-\tau,\xi) + B(i) f(x(t,\xi)) + C(i) g(x(t-\sigma(t),\xi)) - E(i) K x(t,\xi) - x_t(t,\xi)] d\xi, \tag{13}$$

and:

$$0 = 2\int_\Omega x_t^T(t,\xi) P_{3i} [D\Delta x(t,\xi) - A(i) x(t-\tau,\xi) + B(i) f(x(t,\xi)) + C(i) g(x(t-\sigma(t),\xi)) - E(i) K x(t,\xi) - x_t(t,\xi)] d\xi, \tag{14}$$

with a set of matrices $P_{2i} > 0$ and $P_{3i} > 0$.

From the boundary condition and Green equation, taking into account the Poincare inequality, we get:

$$\int_\Omega x^T(t,\xi) P_{2i} D \Delta x(t,\xi) d\xi = -\int_\Omega (\nabla x(t,\xi))^T P_{2i} D \nabla x(t,\xi) d\xi \le -\frac{m}{l^2} \int_\Omega x^T(t,\xi) P_{2i} D x(t,\xi) d\xi. \tag{15}$$

Integrating by parts and taking the place of boundary condition leads to:

$$\int_\Omega x_t^T(t,\xi) P_{3i} D \Delta x(t,\xi) d\xi = -2\int_\Omega (\nabla x(t,\xi))^T D P_{3i} (\nabla x(t,\xi))_t d\xi. \tag{16}$$

Therefore, by adding the right-hand side of Equations (13)-(14) to $\mathscr{L}V(t,i)$ and by taking note of Equations (15) and (16), we obtain:

$$\mathscr{L}V(t,i) \le \int_\Omega \eta^T(t,\xi) \hat{\Xi}(i) \eta(t,\xi) d\xi, \tag{17}$$

where:

$$\eta(t,\xi) = [x^T(t,\xi), x_t^T(t,\xi), x^T(t-\tau,\xi), f^T(x(t,\xi)), g^T(x(t,\xi)), g^T(x(t-\sigma(t),\xi))]^T.$$

$$\hat{\Xi}(i) = \begin{bmatrix} \hat{\Omega}_{11}(i) & \Omega_{12}(i) & \Omega_{13}(i) & P_{2i}B(i) & 0 & P_{2i}C(i) \\ * & \Omega_{22}(i) & -P_{3i}A(i) & P_{3i}B(i) & 0 & P_{3i}C(i) \\ * & * & \Omega_{33} & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & R & 0 \\ * & * & * & * & * & \Omega_{66} \end{bmatrix},$$

$$\hat{\Omega}_{11}(i) = -2\frac{m}{l^2} P_{2i} D - 2P_{2i} E(i) K - Q_1 + Q_2 + \sum_{j=1}^{N} \pi_{ij} P_j,$$

$\Omega_{12}(i)$, $\Omega_{13}(i)$, $\Omega_{22}(i)$, Ω_{33} and Ω_{66} are defined in ***Theorem 1.***

Also, by Lemma 2 and Assumption 1 and 2, we have:

$$\int_{\Omega}[x^T(t,\xi)\Lambda_i L_1 x(t,\xi)-2x^T(t,\xi)\Lambda_i L_2 f(x(t,\xi))+ f^T(x(t,\xi))\Lambda_i f(x(t,\xi))]d\xi \le 0, \tag{18}$$

$$\int_{\Omega}[x^T(t,\xi)\Sigma_i M_1 x(t,\xi)-2x^T(t,\xi)\Sigma_i M_2 g(x(t,\xi)) +g^T(x(t,\xi))\Sigma_i g(x(t,\xi))]d\xi \le 0, \tag{19}$$

From Equations (18) and (19), it follows that:

$$\mathscr{L}V(t,i)\le\int_{\Omega}\{\eta^T(t,\xi)(i)\eta(t,\xi)-[x^T(t,\xi)\Lambda_i L_1 x(t,\xi)-2x^T(t,\xi)\Lambda_i L_2 f(x(t,\xi))+f^T(x(t,\xi))\Lambda_i f(x(t,\xi))]- [x^T(t,\xi)\Sigma_i M_1 x(t,\xi)-2x^T(t,\xi)\Sigma_i M_2 g(x(t,\xi))+g^T(x(t,\xi))\Sigma_i g(x(t,\xi))]\}d\xi=\int_{\Omega}\eta^T(t,\xi)(i)\eta(t,\xi)d\xi. \tag{20}$$

where (i) are defined in Theorem 1.

Letting $\rho_0=\sup_{i\in S}\{\lambda_{\max}((i))\}$, it follows that $\rho_0<0$.

From Equation (20), it is clear that:

$$\mathscr{L}V(t,i)\le\rho_0\int_{\Omega}\eta^T(t,\xi)\eta(t,\xi)d\xi\le\rho_0\left\|x(t,\xi)\right\|^2. \tag{21}$$

Therefore, we obtain

$$\mathbf{E}V(t,r(t))=\mathbf{E}V(0,r(0))+\mathbf{E}\int_0^t\mathscr{L}V(s,r(s))ds\le \mathbf{E}V(0,r(0))+\rho_0\mathbf{E}\int_0^t\left\|x(s,\xi)\right\|^2 ds. \tag{22}$$

Note that $\rho_0<0$ and $V(t,r(t))>0$. Equation (22) implies that the integral $\int_0^t\mathbf{E}\left\|x(s,\xi)\right\|^2 ds$ is convergent when $t\to+\infty$. By Barbalat's lemma, it holds that:

$$\lim_{t\to\infty}\mathbf{E}\left\|x(t,\xi)\right\|^2=0.$$

In other words, the Equation (4) is globally asymptotically stable in the mean square.

4H∞performance analysis of closed-loop system

Next, we will analyze the H∞ performance of the closed-loop Equation (4).

Theorem 2 Let the controller parameter K be given and γ be a prescribed positive constant. The closed-loop Equation (4) is globally asymptotically stable in the mean square for $w(t,\xi)=0$ and satisfies $\left\|z\right\|^2\le\gamma^2\left\|w\right\|^2$ under the zero initial condition for any nonzero $w\in L_2[0,\infty)$, if there exist three matrices $Q_1>0$, $Q_2>0$ and $R>0$, three sets of matrices $P_i>0$, $P_{2i}>0$ and $P_{3i}>0$, $(i\in S)$ and two sets of positive-definite diagonal matrices Λ_i, $\Sigma_i(i\in S)$ such that the matrix inequality holds:

$$\varXi(i)=\begin{bmatrix}\varUpsilon_{11}(i) & \varOmega_{12}(i) & \varOmega_{13}(i) & \varOmega_{14}(i) & \varSigma_i M_2 & P_{2i}C(i) & P_{2i}\\ * & \varOmega_{22}(i) & -P_{3i}A(i) & P_{3i}B(i) & 0 & P_{3i}C(i) & P_{3i}\\ * & * & \varOmega_{33} & 0 & 0 & 0 & 0\\ * & * & * & -\varLambda_i & 0 & 0 & 0\\ * & * & * & * & \varOmega_{55}(i) & 0 & 0\\ * & * & * & * & * & \varOmega_{66} & 0\\ * & * & * & * & * & * & -\gamma^2 I\end{bmatrix}<0, \tag{23}$$

where

$$\varUpsilon_{11}(i)=-2\frac{m}{l^2}P_{2i}D-2P_{2i}E(i)K-Q_1+Q_2+\sum_{j=1}^{N}\pi_{ij}P_j-\varLambda_i L_1-\varSigma_i M_1+F^T(i)F(i)+K^TG^T(i)G(i)K,$$

$\varOmega_{12}(i), \varOmega_{13}(i), \varOmega_{14}(i), \varOmega_{22}(i), \varOmega_{33}, \varOmega_{55}(i)$ and $\varOmega_{66}$ are defined in Theorem 1.

Proof:It is easy to verify that $\varXi(i)<0$ implies $\varPsi(i)<0$. According to Theorem 1, the closed-loop Equation (4) is globally asymptotically stable in the mean square.

We now focus on the H∞ performance of the closed-loop system. Choose the same Lyapunov functional candidate $V(t,i)$ as in Theorem 1. The similar line calculation as in Theorem 1 leads to

$$\mathscr{L}V(t,i)\le\int_{\Omega}\zeta^T(t,\xi)\hat{\varPsi}(i)\zeta(t,\xi)d\xi, \tag{24}$$

where:

$$\hat{\Psi}(i) = \begin{bmatrix} \Omega_{11}(i) & \Omega_{12}(i) & \Omega_{13}(i) & \Omega_{14}(i) & \Sigma_i M_2 & P_{2i}C(i) & P_{2i} \\ * & \Omega_{22}(i) & -P_{3i}A(i) & P_{3i}B(i) & 0 & P_{3i}C(i) & P_{3i} \\ * & * & \Omega_{33} & 0 & 0 & 0 & 0 \\ * & * & * & -\Lambda_i & 0 & 0 & 0 \\ * & * & * & * & \Omega_{55}(i) & 0 & 0 \\ * & * & * & * & * & \Omega_{66} & 0 \\ * & * & * & * & * & * & 0 \end{bmatrix} < 0,$$

$\zeta(t,\xi) = [x^T(t,\xi), x_t^T(t,\xi), x^T(t-\tau,\xi), f^T(x(t,\xi)), g^T(x(t,\xi)), g^T(x(t-\sigma(t),\xi)), w^T(t,\xi)]^T$.

In order to study the H$_\infty$ performance of the Equation (4), we introduce the indicator as follow:

$$J = \mathbf{E}\int_0^{t_f} \left(\|z(t,\xi)\|^2 - \gamma^2 \|w(t,\xi)\|^2\right) dt. \quad (25)$$

Under the zero initial condition, from Equations (24) and (25), for all non-zero external disturbance $w \in L_2[0,\infty)$, we get:

$$J = \mathbf{E}\int_0^{t_f}\int_\Omega (z^T(t,\xi)z(t,\xi) - \gamma^2 w^T(t,\xi)w(t,\xi))d\xi dt \le$$
$$\mathbf{E}\int_0^{t_f}[\int_\Omega \left(z^T(t,\xi)z(t,\xi) - \gamma^2 w^T(t,\xi)w(t,\xi)\right)d\xi + \quad (26)$$
$$\mathrm{L}V(t,i)]dt = \mathbf{E}\int_0^{t_f}\int_\Omega \zeta^T(t,\xi)\Psi(i)\zeta(t,\xi)d\xi dt,$$

where $\Psi(i)$ is defined in Theorem 2.

By Lemma 3, Equations (27) and (28) can be rewritten as the form of Equation (23). Along the similar line as in the proof of Theorem 1, we can derive that $J < 0$. Letting $t_f \to \infty$, we obtain that $\|z(t,\xi)\| \le \gamma \|w(t,\xi)\|$. This completes the proof.

Accordingly, an approach to design a stable H$_\infty$ controller for Equation (2) is provided in Theorem 2. The result in Theorem 2 can be solved efficiently by LMI algorithms.

5 Two special cases

In fact, the Equation (2) is rather general. Two special cases are considered in the following.

Case 1 InEquation (2) has no Markovian jumping parameters and the model can be reduced to:

$$\begin{cases} \dfrac{\partial x(t,\xi)}{\partial t} = D\Delta x(t,\xi) - Ax(t-\tau,\xi) + Bf(x(t,\xi)) \\ \qquad +Cg(x(t-\sigma(t),\xi)) \\ \qquad +Eu(t,\xi) + w(t,\xi) \\ z(t,\xi) = \begin{bmatrix} Fx(t,\xi) \\ Gu(t,\xi) \end{bmatrix} \end{cases}. \quad (27)$$

The following corollary is easy obtained from Theorem 2.

Corollary 1 Let the controller parameter K be given and γ be a prescribed positive constant. The closed-loop system of Equation (27) is globally asymptotically stable for $w(t,\xi) = 0$ and satisfies $\|z\|^2 \le \gamma^2 \|w\|^2$ under the zero initial condition for any nonzero $w \in L_2[0,\infty)$, if there exist six matrices $P > 0$, $P_2 > 0$, $P_3 > 0$, $Q_1 > 0$, $Q_2 > 0$ and $R > 0$, and two positive-definite diagonal matrices Λ, Σ such that the matrix inequality holds:

$$\Psi_1 = \begin{bmatrix} \Upsilon_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Sigma M_2 & P_2 C & P_2 \\ * & \Omega_{22} & -P_3 A & P_3 B & 0 & P_3 C & P_3 \\ * & * & \Omega_{33} & 0 & 0 & 0 & 0 \\ * & * & * & -\Lambda & 0 & 0 & 0 \\ * & * & * & * & \Omega_{55} & 0 & 0 \\ * & * & * & * & * & \Omega_{66} & 0 \\ * & * & * & * & * & * & -\gamma^2 I \end{bmatrix} < 0, \quad (28)$$

where:

$$\Upsilon_{11} = -2\frac{m}{l^2}P_2 D - 2P_2 EK - Q_1 + Q_2 - \Lambda L_1 - \Sigma M_1 +$$
$$F^T F + K^T G^T GK,$$
$$\Omega_{12} = P - P_2 - K^T E^T P_3,$$
$$\Omega_{13} = Q_1 - P_2 A, \; \Omega_{14} = P_2 B + \Lambda L_2,$$
$$\Omega_{22} = \tau^2 Q_1 - 2P_3, \; \Omega_{33} = -Q_1 - Q_2,$$
$$\Omega_{55} = R - \Sigma, \; \Omega_{66} = -(1-\sigma_d)R.$$

Case 2 In second cases, by taking $\tau = 0$ and $C = 0$ in Equation (27), i.e., the Equation (27) has no leakage delay and time-varying delay. Equation (27) becomes:

$$\begin{cases} \dfrac{\partial x(t,\xi)}{\partial t} = D\Delta x(t,\xi) - Ax(t,\xi) + Bf(x(t,\xi)) + \\ \qquad Eu(t,\xi) + w(t,\xi) \\ z(t,\xi) = \begin{bmatrix} Fx(t,\xi) \\ Gu(t,\xi) \end{bmatrix} \end{cases}, \quad (29)$$

and we have the following corollary.

Corollary 2 Let the controller parameter K be given and γ be a prescribed positive constant. The closed-loop system of Equation (29) is globally asymptotically stable for $w(t,\xi) = 0$ and satisfies $\|z\|^2 \le \gamma^2 \|w\|^2$ under the zero initial condition for any nonzero $w \in L_2[0,\infty)$, if there exist three matrices $P>0$, $P_2>0$ and $P_3>0$, and a positive-definite diagonal matrices Λ such that the matrix inequality holds:

$$\Psi_2=\begin{bmatrix}\tilde{\Upsilon}_{11} & \tilde{\Omega}_{12} & \tilde{\Omega}_{13} & P_2\\ * & -2P_3 & P_3B & P_3\\ * & * & -\Lambda & 0\\ * & * & * & -\gamma^2 I\end{bmatrix}<0, \quad (30)$$

where:

$$\tilde{\Upsilon}_{11}=-2\frac{m}{l^2}P_2D-2P_2EK-2P_2A-\Lambda L_1+F^TF+K^TG^TGK,$$

$$\tilde{\Omega}_{12}=P-P_2-A^TP_3-K^TE^TP_3,\ \tilde{\Omega}_{13}=P_2B+\Lambda L_2.$$

6 A numerical example

In this section, an example is given to show the effectiveness of our results. Consider the closed-loop Equation (4) with the following parameters:

$$l=1,\ m=1,\ D=\begin{bmatrix}5 & 0\\ 0 & 3\end{bmatrix},\ \tau=0.1,\ \sigma_d=0.4.$$

$$A(1)=\begin{bmatrix}4 & 0\\ 0 & 3\end{bmatrix},\ B(1)=\begin{bmatrix}1.5 & 1.4\\ -1.7 & -1.3\end{bmatrix},\ C(1)=\begin{bmatrix}1.7 & 1.1\\ -1.8 & 1.6\end{bmatrix},$$

$$E(1)=\begin{bmatrix}0.6\\ 1\end{bmatrix},\ F(1)=\begin{bmatrix}0.7\\ 0.8\end{bmatrix},\ G(1)=\begin{bmatrix}1\\ 0.6\end{bmatrix};$$

$$A(2)=\begin{bmatrix}3.5 & 0\\ 0 & 2.5\end{bmatrix},\ B(2)=\begin{bmatrix}1.7 & 1.9\\ -1.6 & -1.1\end{bmatrix},\ C(2)=\begin{bmatrix}1.5 & 1.3\\ -1.4 & 1.7\end{bmatrix},$$

$$E(2)=\begin{bmatrix}0.7\\ 1\end{bmatrix},\ F(2)=\begin{bmatrix}0.8\\ 1\end{bmatrix},\ G(2)=\begin{bmatrix}1\\ 0.7\end{bmatrix}.$$

The probability transfer matrix of the Markov process governing the mode switching is $\Pi=\begin{bmatrix}-2 & 2\\ 4 & -4\end{bmatrix}$.

In this example, the H∞performance level is given as $\gamma=1.2$ and state feedback controller is designed by $u(t,\xi)=-[8\quad 6]x(t,\xi)$.

Nonlinear function are described by: $f_1(s)=g_1(s)=\tanh(-0.8s),\ f_2(s)=g_2(s)=\tanh(s)$ and we have:

$$L_1=M_1=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix},\ L_2=M_2=\begin{bmatrix}-0.4 & 0\\ 0 & 0.5\end{bmatrix}.$$

With the above parameters, we can solve Equation (23) by using the MATLAB LMI toolbox.

$$P_1=\begin{bmatrix}0.1318 & -0.1464\\ -0.1464 & 0.3123\end{bmatrix},\ P_2=\begin{bmatrix}0.0797 & -0.0531\\ -0.0531 & 0.3051\end{bmatrix},$$

$$P_{21}=\begin{bmatrix}0.8813 & 0.3635\\ 0.3635 & 0.7294\end{bmatrix},\ P_{22}=\begin{bmatrix}0.9014 & 0.3223\\ 0.3223 & 0.6515\end{bmatrix},$$

$$P_{31}=\begin{bmatrix}0.0572 & 0.0335\\ 0.0335 & 0.0377\end{bmatrix},\ P_{32}=\begin{bmatrix}0.0687 & 0.0218\\ 0.0218 & 0.0426\end{bmatrix};$$

$$Q_1=\begin{bmatrix}1.4745 & -0.1608\\ -0.1608 & 1.2787\end{bmatrix},\ Q_2=\begin{bmatrix}0.9150 & 0.3651\\ 0.3651 & 0.1818\end{bmatrix};$$

$$R=\begin{bmatrix}1.2197 & 0.3036\\ 0.3036 & 2.2693\end{bmatrix},\ \Lambda_1=\begin{bmatrix}1.5833 & 0\\ 0 & 1.3644\end{bmatrix},$$

$$\Lambda_2=\begin{bmatrix}2.2490 & 0\\ 0 & 2.0964\end{bmatrix},\ \Sigma_1=\begin{bmatrix}2.2955 & 0\\ 0 & 4.3833\end{bmatrix},$$

$$\Sigma_2=\begin{bmatrix}2.7588 & 0\\ 0 & 4.5153\end{bmatrix}.$$

Therefore, the Equation (2) is globally asymptotically stable in the mean square when $w(t,\xi)=0$ and the effect of the disturbance input on the controlled output is constrained to the required level (Figure 1-6).

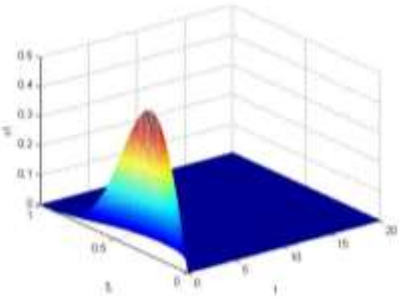

FIGURE 1 Mode 1:x_1 State Evolution

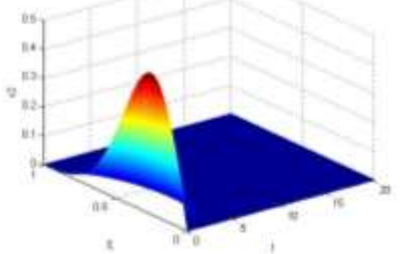

FIGURE 2Mode 1:x_2 State Evolution

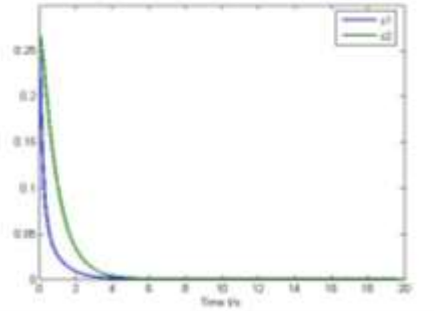

FIGURE 3Mode 1: Evolution of state L_2 norm

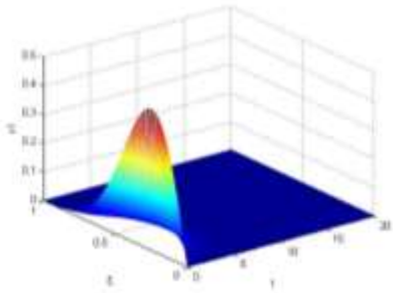

FIGURE 4Mode 2:x_1 State Evolution

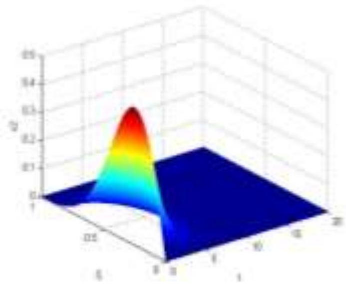

FIGURE 5Mode 2:x_2 State Evolution

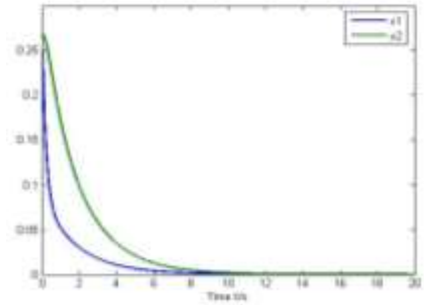

FIGURE 6Mode 2: Evolution of state L_2 norm

7 Conclusions

In this paper, H∞ control problem for a class of delayed distributed parameter systems with leakage delay and Markovian jumping parameters has been studied. A state feedback controller has been proposed such that the resulting closed-loop system is asymptotically stable in the mean square for zero disturbance input and also achieves a required H∞ performance level. The criterion obtained is delay-dependent and LMI-based, which can be solved by MATLAB easily. It is worth noting that sector condition of nonlinear function is considered reducing the conclusion conservative. A numerical example has been provided to demonstrate the effectiveness of our results.

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