The nonlinear vibration analysis of the fluid conveying pipe based on finite element method

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Abstract

A Coupling between the fluid and the structure existed almost in all industrial areas the vibration of fluid solid coupling for fluid conveying pipe was called the "dynamics of typical"[1], Because of the physical model and mathematical description for the fluid conveying pipe was simple, especially it was easy to design and manufacturing, according to the characteristics of fluid conveying pipe, transformed the transverse vibration of the fluid conveying pipe to the beam element model of two nodes. Using Lagrangian interpolation function, the first order Hermite interpolation function and the Ritz method to obtain the element standard equation, and then integrated a global matrix equation. Used the mode decomposition method, obtained the vibration modal of the fluid conveying pipe with Matlab programming. The vibration modal of the fluid conveying pipe in four kinds of boundary conditions was analysed. The characteristics of pipes conveying fluid was obtained which the pipeline system parameters under different boundary constraints. To provide the theoretical support for the research of vibration attenuation of fluid conveying pipes.

Keywords: Fluid solid coupling, Nonlinear vibration, Modal analysis, Interpolation, The finite element algorithm

1 Introduction

A system of conveying fluid pipe was widely used in the city water supply and drainage, water power, chemical machinery, aerospace, marine engineering and the nuclear industry and other fields, it was play an important role for improving the living standards of the nation and the national economic strength. However, according to statistics, in the industrial production, the damage of the water hammer in pipeline interface and pipeline rupture accounted for over 75% in the total system failure rates, seriously affected the normal production and operation, resulting in huge economic losses. Coupling between the fluid and the structure is almost exist in all industrial areas, the fluid and solid coupling vibration of pipeline flow is called the typical "dynamic" [2], because of its simple physical model and mathematical description, especially the pipeline is easy to design and manufacture, which provides convenience to the coordinated development of the theoretical and experimental research. But the pipeline, as the application extremely widely, of the coupling vibration of pressure flow is the most representative in this field, which has a broad background in engineering applications, and a very high theoretical research value and practical significance, but also has many challenges [3].

2 The establish of mathematical model for the output response of the fluid conveying pipe

Taking into account the pipe ratio for length to diameter is relatively large, the deformation of radial is the same, just only exists a certain angle difference, which can be regarded the pipe as plane beam element to consider, using two node element, as shown in Figure 1, the node number of I and J. The conveying fluid pipe is only affected by the lateral force, no axial force, so analysis with the two node element, the nodal displacement model can be defined as [4]:

$$y(t) = [y_i, \theta_i, y_j, \theta_j]^T \, . \tag{1}$$

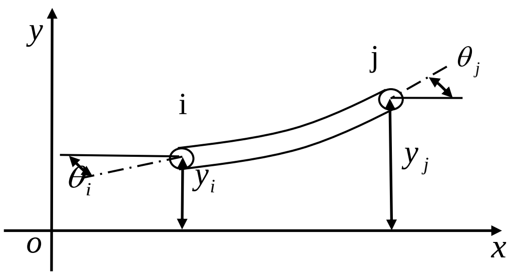

FIGURE 1 The deformation of two node element

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In the node parameter of unit, in addition to the node value of field function, also contains node value for a derivative $\partial y/\partial x$ of the field function. In order to maintain the continuity of field function derivative between the public node element, and in the end nodes to keep the derivative order is first for the field function, so the first-order Hermite interpolation polynomial is used [5]:

$$N(\xi)=[N_1(\xi),N_2(\xi),N_3(\xi),N_4(\xi)]\,, \tag{2}$$

where

$$N_1(\xi)=1-3\xi^2+2\xi^3,$$
$$N_2(\xi)=\xi-2\xi^2+\xi^3,$$
$$N_3(\xi)=3\xi^2-2\xi^3,$$
$$N_4(\xi)=\xi^3-\xi^2.$$

When the local dimensionless coordinate is taken, the ξ is get $0\le\xi\le1$.

Using the Ritz method interpolation function to establish standard unit equation of the approximate solution of the lateral vibration we find the interpolation function:

$$[M^e]\{\ddot{y}_j\}+[C^e]\{\dot{y}_j\}+[K^e]\{y_j\}=[Q^e]\,, \tag{3}$$

where

$$[M^e]=[M_{ij}^e]\,,[C^e]=[C1_{ij}^e]+[C2_{ij}^e]\,,$$
$$[K^e]=[K1_{ij}^e]+[K2_{ij}^e]+[K3_{ij}^e]\,,$$
$$[Q^e]=[Q1_{ij}^e]+[Q2_{ij}^e]+[Q3_{ij}^e]\,,$$
$$[M_{ij}^e]=\int_l N_i(m_p+m_f)N_j dx\,,$$
$$[C1_{ij}^e]=\int_l N_i(2m_f v_f)\frac{\partial N_j}{\partial x}dx\,,$$
$$[C2_{ij}^e]=\int_l N_i(\frac{A_f}{c^2}\frac{\partial P}{\partial t})N_j dx\,,$$
$$[K1_{ij}^e]=\int_l \frac{\partial^2 N_i}{\partial^2 x}(EI)\frac{\partial^2 N_j}{\partial^2 x}dx\,,$$
$$[K2_{ij}^e]=-\int_l \frac{\partial N_i}{\partial x}(m_f v_f^2+(1-2\gamma)A_f P)\frac{\partial N_j}{\partial x}dx\,,$$
$$[Q1_{ij}^e]=\int_l (-m_f\frac{\partial v_f}{\partial t})N_j dx\,,$$
$$[Q2_{ij}^e]=\int_l (-\frac{v_f A_f}{c^2}\frac{\partial P}{\partial t})N_j dx\,,$$
$$[Q3_{ij}^e]=\int_l (m_p g+m_f g)N_j dx\,.$$

2.1 THE ENTIRETY MATRIX

There are some matrix must be appropriately expanded rewrite when the unit matrix integrated to the entirety matrix so that the matrix of all elements with uniform format, then according to the superposition to assembly [6].

The usually study boundary constraint conditions include fix to hinge and fix to fix constraints, the mathematical expression of its boundary is given below, respectively: (I), (II), (III), (IV):

$$\begin{cases} y(0,t)=0,\left.\frac{\partial y}{\partial x}\right|_{x=0}=0 \\ y(L,t)=0 \end{cases}, \tag{I}$$

$$y(0,t)=0,\left.\frac{\partial y}{\partial x}\right|_{x=0}=0\,, \tag{II}$$

$$y(0,t)=0,\left.\frac{\partial y}{\partial x}\right|_{x=0}=0\,, \tag{III}$$

$$\begin{cases} y(0,t)=0,\left.\frac{\partial y}{\partial x}\right|_{x=0}=0 \\ y(L,t)=0,\left.\frac{\partial y}{\partial x}\right|_{x=L}=0 \end{cases}. \tag{IV}$$

The four kinds of boundary conditions of above given all belong to the first class constraint conditions, for this kind of constraint conditions can usually use "row row column method" and "multiplied with bigger number method". The "multiplied with bigger number method" is make the main diagonal element about the specified node displacement in the overall stiffness matrix with multiply by the large number λ, at the same time, give the specified value of node displacement to the corresponding element of load matrix [7], then multiply by the same number as well as the main diagonal elements. Using the “multiplied with bigger number method ‘to deal with the boundary constraint condition by’, finally forms the whole matrix:

$$[M]\{\ddot{y}\}+[C]\{\dot{y}\}+[K]\{y\}=[Q]\,. \tag{4}$$

2.2 THE MODAL SOLUTION OF FLUID CONVEYING PIPES BASED ON MODE-SUPERPOSITION METHOD

Solving the modal of the system when the movement of the global matrix is obtained, we select the mode-superposition method for the model. Solving the inherent frequency and vibration type of fluid conveying pipes it is usually divided into two cases about damped and non-damped. The non-damping case is solved in the real domain, and the damping case is solved in the complex domain.

1) The free vibration equation without consider the damping:

$$M\ddot{y}(t)+Ky(t)=0\,, \tag{5}$$

using a solution:

$$y=\phi\sin\omega(t-t_0)\,, \tag{6}$$

where ϕ is the n order vector, ω is the vibration ϕ frequency vector, t is the time variable, t_0 is the time constant that determined by the initial conditions.

Using (6) and (5) we can get the generalized eigenvalue, i.e.:

$$K\phi-\omega^2M\phi=0\,. \tag{7}$$

According to the general solution of eigenvalues and eigenvectors to the identified ϕ and ω, the results can be obtained as n characteristic solutions $(\omega_1^2\,,\varphi_1)$, $(\omega_2^2\,,\varphi_2)$,…, (ω_n^2,φ_n), which the characteristic values $\omega_1\,,\omega_2 \ldots \omega_n$ for the natural frequency of the conveying fluid pipes system take place. There is also the ordering $0\le\omega_1<\omega_2<\cdots<\omega_n$ and the eigenvectors $\phi_1,\phi_2,\cdots,\phi_n$ for N inherent vibration feature vector, which corresponds to the inherent natural frequency.

2) The free vibration equation with a damping:

$$M\ddot{y}(t)+C\dot{y}(t)+Ky(t)=0\,. \tag{8}$$

A solution of the natural frequency and vibration type equations in the damped, early obtained the natural frequency and vibration model, which corresponds to the free damped vibration equation, gives:

$$\Phi=[\phi_1,\phi_2,\cdots,\phi_n]\,, \tag{9}$$

$$\Omega=diag[\omega_1,\omega_2,\cdots,\omega_n]\,, \tag{10}$$

where the Φ is the vector based on the n natural modes without damping, the Ω is the vector based on the n natural frequency without damping. Then, using the method of truncated model for the ratios of model damping, we obtain: $\xi_1,\xi_2,\cdots,\xi_n$,

$$\Xi=diag[\xi_1,\xi_2,\cdots,\xi_n]\,. \tag{11}$$

Using the vibration equation of the generalized coordinates and the equation (8) we obtain:

$$[\Phi]^T[M][\Phi][\Phi]^{-1}[\ddot{y}(t)]+[\Phi]^T[C][\Phi][\Phi]^{-1}[\dot{y}(t)] +[\Phi]^T[K][\Phi][\Phi]^{-1}[y(t)]=0. \tag{12}$$

The generalized mass matrix, the generalized damping matrix and generalized stiffness matrix, respectively are: $[M]_z=[\Phi]^T[M][\Phi]$, $[C]_z=[\Phi]^T[C][\Phi]$, $[K]_z=[\Phi]^T[K][\Phi]$.

After the generalized coordinate transformation $\{q\}=[\Phi]^{-1}\{y\}$ we obtain:

$$\ddot{q}+2\Xi\Omega\dot{q}+\Omega^2q=0\,. \tag{13}$$

The equation (13) is a group of uncoupled equations in component form:

$$\ddot{q}_i+2\xi_i\omega_i\dot{q}_i+\omega_i^2q_i=0\,,\ (i=1,2,\ldots n) \tag{14}$$

It is easy to obtain the complex eigenvalue and complex eigenvectors using the method of the generalized eigenvalues and then to obtain the natural vibration modes and natural frequencies in the damping case.

3 The effects of the system parameters on the conveying fluid pipe

The developed equation of transverse motion for conveying fluid pipes with the Matlab simulation software and all of the parameters in the simulation process are given in Table 1. During the simulation process, we make two end points of pipe as the supporting points assuming a rigid constrain with a room temperature water of the fluid and rolling copper as pipe material.

3.1 FOUR KINDS OF BOUNDARY CONDITIONS OF THE FIRST FOUR ORDER VIBRATION MODE

The first four order mode of vibration of the constraint of fix to hinge as shown in Figure 2. From Figure 2, the relative position of all nodes in the vibration pipes that reflect the inherent form vibration of the conveying fluid pipe in the constraint of fix to hinge. The first four-order vibration mode that corresponding to the first four order natural frequency of the conveying fluid pipe, the natural frequency of the first order vibration mode is the minimum and two order mode, three order mode, four order mode is increasing [8]. The constraints of fix to suspension extension and fix to hinge does not belong to the same class, so the vibration mode is also different, as shown in Figure 3. For the same boundary, damped and no damped vibration mode is basically the same.

3.2 FOUR KINDS OF BOUNDARY OF VIBRATION MODE OF DEFLECTION

For different boundary constraints, a vibration mode of deflection for conveying fluid pipe is not identical [9]. The constraint of fix to hinge is shown in Figure 4, the constraint of hinge to hinge is shown in Figure 5, the constraint of fix to fix is shown in Figure 6, the constraint of fix to suspension extension is shown in Figure 7.

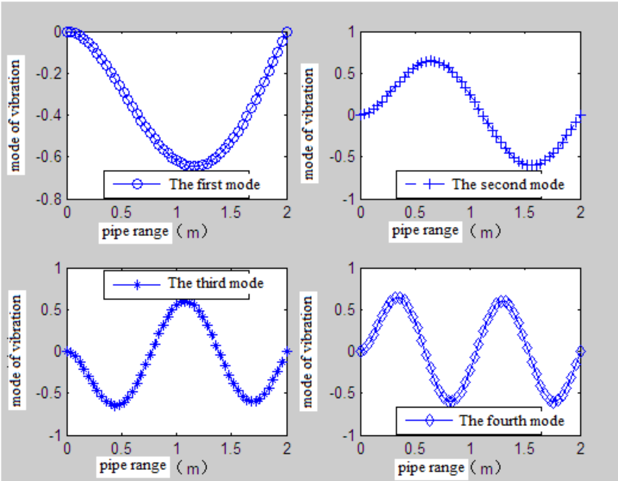

FIGURE 2 The first four order mode of vibration of the constraint of fix to hinge

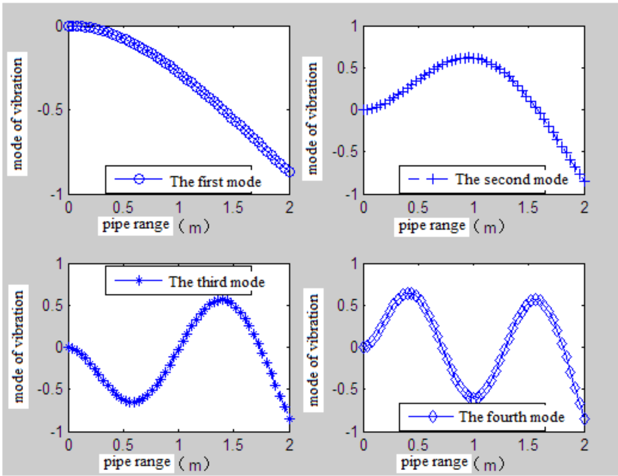

FIGURE 3 The first four order mode of vibration of the constraint of fix to suspension extension

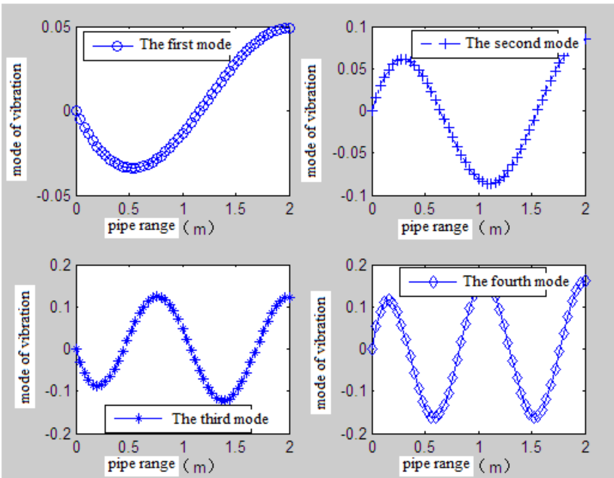

FIGURE 4 The first four order mode of deflection vibration mode for the constraint of fix to hinge

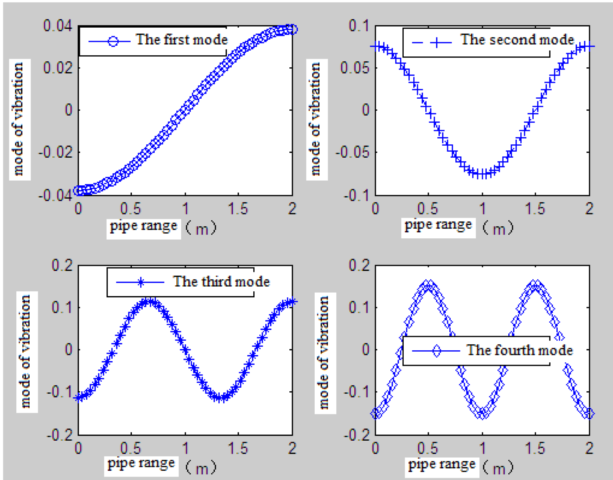

FIGURE 5 The first four order mode of deflection vibration mode for the constraint of hinge to hinge

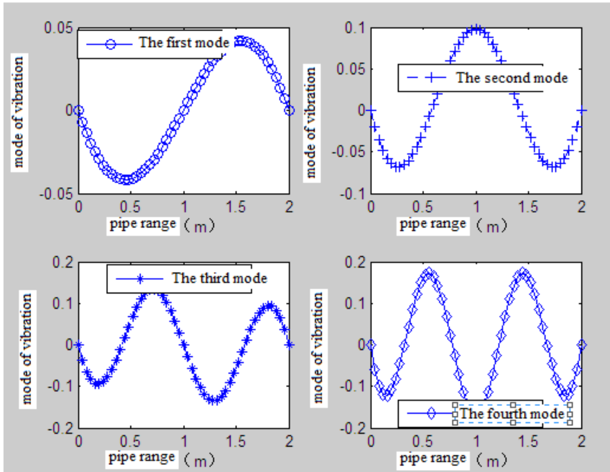

FIGURE 6 The first four order mode of deflection vibration mode for the constraint of fix to fix

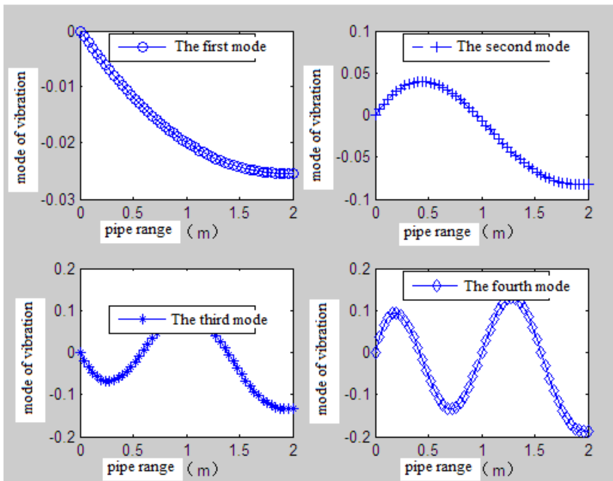

FIGURE 7 The first four order mode of deflection vibration mode for the constraint of fix to suspension extension

From the simulation results, the boundary constraints have great influence on the vibration characteristics of the conveying fluid pipe. Compare with vibration characteristics under the various boundary conditions, it can be concluded that the constraint of fix to hinge, hinge to hinge, fix to fix, fix to suspension extension:

a) The first-order natural frequency arrange as follows: the constraint of fix to suspension extension, hinge to hinge, fix to hinge, fix to fix;

b) Node vibration period arrange as follows: the constraint of fix to fix, fix to hinge, hinge to hinge, fix to suspension extension;

c) Node vibration amplitude arrange as follows: the constraint of fix to fix, fix to hinge, hinge to hinge, fix to suspension extension.

TABLE 1 The vibration characteristics in four kinds of boundary conditions

Name		Boundary conditions			
		Fix to hinge	Hinge to hinge	Fix to fix	Fix to suspension extension
Natural frequency (Hz)	First order	23.3534	13.9358	34.8789	1.8728
	Second order	78.5668	61.4928	97.5413	32.5998
	Third order	165.5251	140.6239	192.3795	96.666
	Fourth order	284.1623	251.3922	318.8951	191.7252

4 Conclusions

This paper transformed the fluid conveying pipe to the beam element model for two nodes, and with the Matlab to simulate the transverse motion equation of the conveying fluid pipe, summarized the simulation results and analyzed. In four kinds of boundary conditions, analysis the affect factors of pipeline system parameters for the vibration modal of the fluid conveying pipe, verified the correctness of the established vibration model of the fluid conveying pipeline. The characteristics of pipes conveying fluid is obtained which the pipeline system parameters under different boundary constraints. To provide the theoretical support for the research of vibration attenuation of fluid conveying pipes.

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