The application of MuPAD in the plotting of analytic geometry

Nada Wu*

Department of mathematics and statistics, Hanshan Normal University, South Dongxing Road, Chaozhou, China

Abstract

MuPAD is a toolbox in the well-known software MATLAB. The paper mainly talks about how to use MuPAD to plot all kinds of Graphics in Analytic Geometry, including cylinders, cones, surfaces of revolution and all kinds of surfaces and curves with equations.

Keywords: analytic geometry, mupad, surface, space curve, command

1 Introduction

MuPAD is a computer algebra system. Originally developed by the MuPAD research group at the University of Paderborn, Germany, development was taken over by the company SciFace Software GmbH & Co. KG in cooperation with the MuPAD research group and partners from some other universities starting in 1997. In September 2008, SciFace was purchased by MathWorks and the MuPAD code was included in the Symbolic Math Toolbox add–on for MATLAB [1].

The college course Analytic Geometry is basic and helpful for further studying of the courses Advanced Geometry, Differential Geometry and Topology. Its task is to help students know geometrical spaces in reality and develop space concept, developing the students' ability of intuition thinking and spatial imagination, teaching the thinking method of symbolic-graphic combination. However, in traditional teaching, it is not convenient to show curves and surfaces of equations.

MuPAD has a perfect plotting function [2]. Type a function and select parameters, after putting the Enter key to run, you get the graphics or animation demonstration of graphics. Now we can apply it to plot all kinds of graphics and animation demonstration easily in the teaching of Analytic Geometry Course. Moreover, MuPAD allows us to Drag and rotating its output graphics, so that students can watch the geometric objects from different angles. No doubt, this is very helpful for the training of Spatial thinking ability. In a word, MuPAD increases the teaching efficiency greatly.

There is an easy way to open the tool box MuPAD as follows [2-4].

Setp 1. Open the software MATLAB;
Setp 2. Input "mupad" in the command window;
Setp 3. Press the 'Enter 'key.

After opening MuPAD, just type corresponding statements in its one document window, and then you can get graphics in the same window. In what follows, we talk about how to plot all kinds of graphics in the college course Analytic Geometry by MuPAD.

2 Plotting suffaces expressed in equations

Surface can be expressed in three ways: explicit function, implicit function and parameter equation[5,6]. For these three ways, MuPAD provides three corresponding drawing commands.

2.1 SURFACES EXPRESSED IN EXPLICIT FUNCTION

Surface expressed in z=f(x,y) can be plotted by the following command.
plot::Function3d(f, x = xmin .. xmax, y = ymin .. ymax, <a = amin .. amax>, attributes)
Description [3]:
plot::Function3d creates the 3D graph of a function in 2 variables.

The expression f(x, y) is evaluated at finitely many points x, y in the plot range. There may be singularities. Although a heuristics is used to find a reasonable z range when singularities are present, it is highly recommended to specify a z range via ViewingBoxZRange = `z_{min}` .. `z_{max}` with suitable numerical real values zmin, zmax.

Animations are triggered by specifying a range a = `a_{min}` .. `a_{max}` for a parameter a that is different from the indedependent variables x, y. Thus, in animations, the x–range x = `x_{min}` .. `x_{max}`, the y–range y = `y_{min}` .. `y_{max}` as well as the animation range a = `a_{min}` .. `a_{max}` must be specified.

The function f is evaluated on a regular equidistant mesh of sample points determined by the attributes XMesh and YMesh (or the shorthand–notation for both, Mesh). By default, the attribute AdaptiveMesh = 0 is set, i.e., no adaptive refinement of the equidistant mesh is used.

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* *Corresponding author's* e-mail: 85157607@qq.com

If the standard mesh does not suffice to produce a sufficiently detailed plot, one may either increase the value of XMesh and YMesh(default numbers 25, 25) or set AdaptiveMesh = n with some (small) positive integer n. This may result in up to 4n times as many triangles as used with AdaptiveMesh = 0, potentially more when f has non–isolated singularities.

There are default settings for attributes. If you really need to change some attributes, you can see the help document.

Example 1 Graphics of z= $(x^2+y^2)^{1/2}$.

Mupad Statements:

plot(plot::Function3d(sqrt(x^2+y^2), x = –1.. 1, y = –1 ..1), ViewingBox=[–1..1, –1..1, 0..1, Color=RGB::Grey], #C);
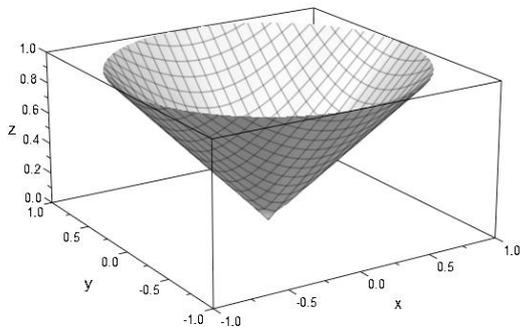
The results are shown in Figure 1.

FIGURE 1 Graphics of z= $(x^2+y^2)^{1/2}$

2.2 SURFACES EXPRESSED IN IMPLICIT FUNCTION

Surface expressed in f(x,y,z)=0 can be plotted by the following command.

plot::Implicit3d(f, x = xmin .. xmax, y = ymin .. ymax, z = zmin .. zmax, <a = amin .. amax>, attributes)

Description [3]

This command plots the surfaces where the smooth function f is zero.

It assumes that f is regular almost everywhere on this surface, which means that f must be differentiable and at least two of its partial derivatives must be nonzero. The statement"x = xmin .. xmax, y = ymin .. ymax, z = zmin .. zmax"gives the range of value x,y,z.

Example 2 Graphics of $x^2+y^2–z^2=0$.

Mupad code:

plot(plot::Implicit3d(x^2+y^2–z^2, x =–1.. 1, y = –1 ..1, z=–1..1, Color=RGB::Grey), Mesh=[50,50], #C);

The results are shown in Figure 2.
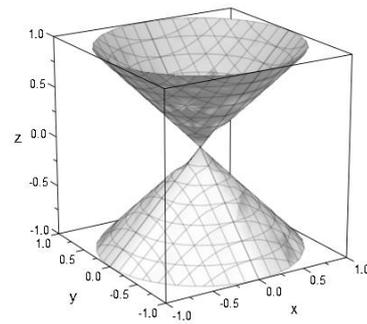

FIGURE 2 Graphics of $x^2+y^2–z^2=0$

The surfaces of ternary quadratic equation only have 17 different classes of graphics. Most of them are pair of plans, virtue surfaces, cylindrical surfaces and conical surfaces, and they are called degenerated surface. 5 classes of surfaces are called non–degenerated, which are ellipsoid, unparted hyperboloid, parted hyperboloid, elliptic paraboloid and hyperbolic paraboloid [5–6]. Non–degenerated surfaces are important in the research of Analytic Geometry, and can be plotted by the MuPAD command "Implicit3d".

Example 3 Ellipsoid $x^2/16+y^2/9+z^2/25=1$.

MuPAD code: a:=4: b:=3: c:=5:

plot(plot::Implicit3d(x^2/a^2+y^2/b^2+z^2/c^2–1, x =–a.. a, y =–b ..b,z=–c..c, Color=RGB::Grey), #C);
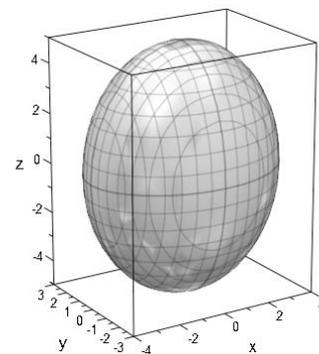
Result is shown in Figure 3.

FIGURE 3 Ellipsoid

Example 4 Unparted hyperboloid $x^2/16+y^2/9–z^2/25=1$.

MuPAD code:

a:=4: b:=3: c:=5: z1:=25: r1:=sqrt(1+z1^2/c^2):
plot(plot::Implicit3d(x^2/a^2+y^2/b^2–z^2/c^2–1,   x=–a*r1..a*r1,y=–b*r1..b*r1,z=–z1..z1,
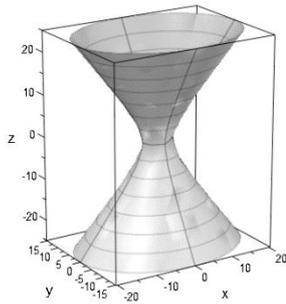*Color=RGB::Grey),#C);*

The results are shown in Figure 4.

FIGURE 4 Unparted hyperboloid

Example 5 Parted hyperboloid $x^2/16 - y^2/9 - z^2/25 = 1$

MuPAD code:

```
a:=4: b:=3: c:=5: z1:=25: r1:=sqrt(1+z1^2/c^2):
plot(plot::Implicit3d(x^2/a^2–y^2/b^2–z^2/c^2–1,x=–
a*r1..a*r1,y=–b*r1..b*r1,z=–z1..z1, Color=RGB::Grey),
#C);
```
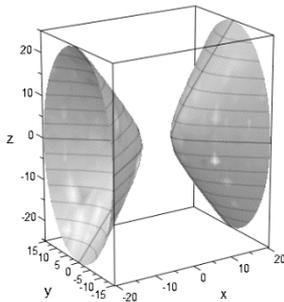
The results are shown in Figure 5.

FIGURE 5 Parted hyperboloid

Example 6 Elliptic paraboloid $x^2/16 + y^2/9 - z = 0$

MuPAD code:

```
a:=4: b:=3: z1:=49: r1:=sqrt(z1):
plot(plot::Implicit3d(x^2/a^2+y^2/b^2–z,x=–
*r1..a*r1,y=–b*r1..b*r1,z=–0..z1,    Color=RGB::Grey),
#C);
```
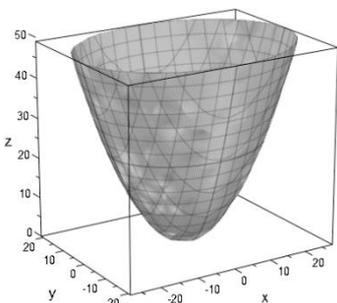
The results are shown in Figure 6.

FIGURE 6 Elliptic paraboloid

Example 7 Hyperbolic paraboloid $x^2/16 - y^2/9 - z = 0$

MuPAD Code:

```
a:=4: b:=3: z1:=49: r1:=sqrt(z1):
plot(plot::Implicit3d(x^2/a^2–y^2/b^2–z,x=–
a*r1..a*r1,y=–b*r1..b*r1,z=–z1..z1,Color=RGB::Grey),
#C);
```

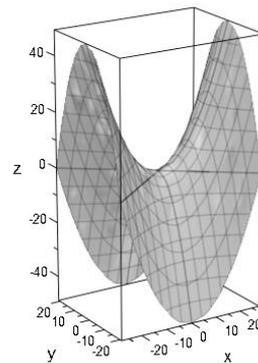The results are shown in Figure 7-8.
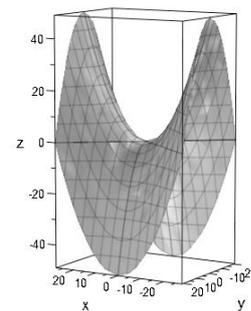

FIGURE 7 Hyperbolic paraboloid

FIGURE 8 Anther angle to show

2.3 SURFACES EXPRESSED IN PARAMETER EQUATIONS

Surface expressed as $\begin{cases} x = x(u,v) \\ y = y(u,v) \\ z = z(u,v) \end{cases}$ can be plotted by the following command.

plot::Surface([x, y, z], u = umin .. umax, v = vmin .. vmax, <a = amin .. amax>, attributes)

Example 8 Surface of $\begin{cases} x = u\cos v \\ y = u\sin v \\ z = u \end{cases}$ $\begin{array}{l} (u \in R, \\ v \in [0,2\pi)) \end{array}$.

Mupad code:

```
plot(plot::Surface([u*cos(v), u*sin(v),u], u = –1 .. 1, v =
0 .. 2*PI, Color=RGB::Grey),#C);
```
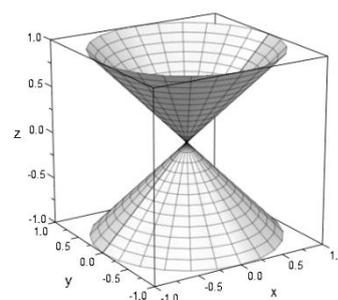
The results are shown in Figure 9.

FIGURE 9 Surface in Example 8

3 Plotting space curves

3.1 SPACE CURVES EXPRESSED IN PARAMETER EQUATIONS

Surface expressed in $\begin{cases} x = x(t) \\ y = y(t) \\ z = z(t) \end{cases}$ can be plotted by the following command.

plot::Curve3d([x, y, z], t = tmin .. tmax, <a = amin .. amax>, attributes).

Example 9 Graphics of $\begin{cases} x = 9\cos t \\ y = 9\sin t \\ z = 2t \end{cases}$, $t \in [0, +\infty)$.

Mupad codes:
*plot(plot::Curve3d([9*cos(t), 9*sin(t),2*t], t=0..20, Color=RGB::Black),#C);*

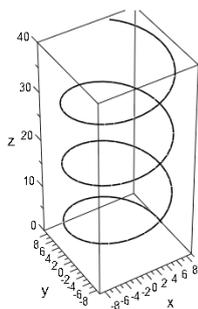The results are shown in Figure 10

FIGURE 10 Conical helix

3.2 SPACE CURVES EXPRESSED IN GENERAL EQUATIONS

For a space curves expressed in general equations

$$\begin{cases} f(x, y, z) = 0 \\ g(x, y, z) = 0 \end{cases},$$

we can first use the command "plot::Implicit3d" to plot the surfaces and by different colors, respectively, $f(x, y, z) = 0$ and $g(x, y, z) = 0$ then watch the intersection. We also can use the command "solve" to turn it as a parameter equation, and then use the command "plot::Curve3d" to finish it [4].

4 Plotting surface without an equation

4.1 CYLINDRICAL SURFACE

Definition 1 [5]. Let Γ be a space curve and V(a,b,c) be a direction. All straight lines with the direction V(a,b,c) and intersecting Γ generate a surface which is called a cylindrical surface. Γ is called a directrix of the cylindrical surface.

Let $\Gamma : \begin{cases} x = x(t) \\ y = y(t) \\ z = z(t) \end{cases}$ be a directrix and V(a,b,c) be the direction. Please plot the corresponding cylindrical surface.

For any A $(x_1, y_1, z_1) \in \Gamma$, the generatrix who pass the point A is $\dfrac{x - x_1}{a} = \dfrac{y - y_1}{b} = \dfrac{z - z_1}{c}$.

Let the above equation equal to parameter v, then $x = x_1 + va$, $y = y_1 + vb$, $z = z_1 + vc$, i.e:

$$\begin{cases} x = x(t) + va \\ y = y(t) + vb \\ z = z(t) + vc \end{cases} \tag{1}$$

Equation (1) is a parametric equation of surface, so that the cylindrical surface can be plotted by the command of "plot:: Surface".

Example 10 Let $\Gamma : \begin{cases} x = 9\cos t \\ y = 9\sin t \\ z = \sin t \end{cases}$ be a directrix, V(3,2,5) be the direction, please plot the corresponding cylindrical surface.

Using Equation (1), the parametric equation of conical surface is:

$$\begin{cases} x = 9\cos t + 3v \\ y = 9\sin t + 2v \\ z = \sin t + 5v \end{cases}.$$

Mupad codes:
*plot(plot::Surface([9*cos(t)+3*v,9*sin(t)+2*v, sin(t)+5*v], t=0..7, v=−5..5, Color=RGB::Grey),#C);*
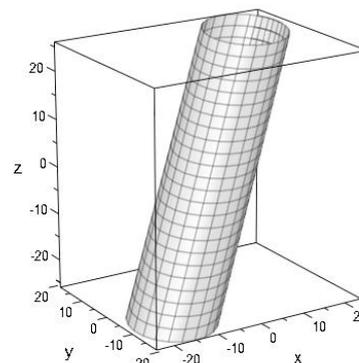
The results are shown in Figure 11.

FIGURE 11 A conical surface

4.2 CYLINDRICAL SURFACE

Definition 2 [5]. Let Γ be a space curve and P(a,b,c) be a point. All straight lines passing through the point P and intersecting Γ generate a surface which is called a conical surface. Γ and P are called a directrix and the vertex of the conical surface, respectively. Those straight lines are called generatrixes.

Let $\Gamma : \begin{cases} x = x(t) \\ y = y(t) \\ z = z(t) \end{cases}$ be a directrix and P(a,b,c) be the vertex,

please plot the corresponding conical surface.

For any $A\,(x_1, y_1, z_1) \in \Gamma$, the generatrix passing through the point A is $\dfrac{x - x_1}{x_1 - a} = \dfrac{y - y_1}{y_1 - b} = \dfrac{z - z_1}{z_1 - c}$.

Let the above formula equal to parameter v, then
$x = (v+1)x_1 - va$,
$y = (v+1)y_1 - vb$,
$z = (v+1)z_1 - vc$, i.e

$$\begin{cases} x = (v+1) \cdot x(t) - va \\ y = (v+1) \cdot y(t) - vb \\ z = (v+1) \cdot y(t) - vc \end{cases} \qquad (2)$$

The conical surface can be plotted by the command of "plot:: Surface".

Example 11 Let $\Gamma : \begin{cases} x = 9\cos t \\ y = 9\sin t \\ z = 2t \end{cases}$ be a directrix, P(1,2,3) be

the vertex, please plot the corresponding conical surface.

By Equation (2), the parameter equation of conical surface is

$$\begin{cases} x = 9(v+1)\cos t - v \\ y = 9(v+1)\sin t - 2v \\ z = 2(v+1)t - 3v \end{cases}.$$

Mupad codes:
plot(plot::Surface([9(v+1)*cos(t)–v,9*(v+1)*sin(t)–2*v,2*(v+1)*t–3*v],t=0..10,v=–10..10,
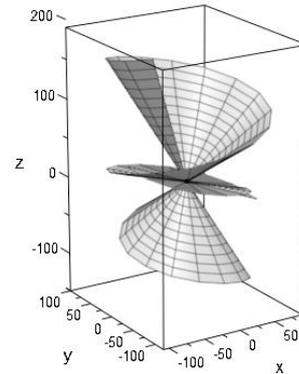Color=RGB::Grey),#C);*
The results are shown in Figure 12.

FIGURE 12 A cylindrical surface

4.3 ROTATING SURFACE

Definition 3 A surface of revolution is a surface created by rotating a curve (the generatrix) around a straight line in its plane (the axis) [7].
Mupad commands:
Plot::Rotate3d(angle,<[cx,cy,cz],[dx,dy,dz]>,obj1,<obj2,...>,<a=a1..a2>)
Description [4]
[cx,cy,cz]: a point in the rotation axis.
[dx,dy,dz]: direction of the rotation axis.
obj1,<obj2,…>: 3D–graphic object, one or more.
The following command can be used to plot a group of 3D Graphics.
plot::Group3d(obj1,obj2,…,<attributes>).

Example 12 Let $l_1 : \dfrac{x}{2} = \dfrac{y}{1} = \dfrac{z-1}{0}$ be a directrix and

$l_2 : x = y = z$ be the rotation axis, plot the rotating surface.
Mupad code:
L1:=plot::Curve3d([2*t, t, 1], t=–7..7, Color=[0,0,1,0.9]):
L2:=plot::Curve3d([t, t, t], t=–7..7, LineStyle=Dashed):
gL:=plot::Group3d(plot::Rotate3d(a, [0,0,0], [1,1,1], L1) $ a=0..2*PI step PI/32):
plot(L1, L2, gL, ViewingBox=[–7..7, –7..7, –12..12]);
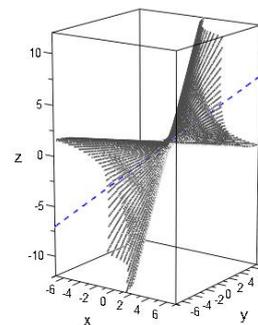The results are shown in Figure 13.

FIGURE 13 A rotating surface

5 Conclusions

Plotting is very important for the study or research of Analytic Geometry. In the traditional way, the drawing is both time consuming and not easy to get good effect. Considering MuPAD has a series of plotting commands, this paper mainly presents and summarizes methods to plot all kinds of shapes in Analytic Geometry by MuPAD. This paper only enumerates the ten several examples, but each case is typical. Using similar methods people almost can

plot all graphics in the research of Analytic Geometry, including cylinders, cones, surfaces of revolution and all kinds of surfaces and curves with equations.

Acknowledgments

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Author

Nada Wu, May 1981, Chaozhou, China

Current position, grades: PhD, lecturer.
University studies: master's degree and a doctor's degree in mathematics at the Department of Mathematics, Shantou University.
Scientific interests: topology, geometry, digital image
Publications: 9