Structure performance analysis of vehicular *ad hoc* networks based on complex network theory

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Abstract

A very promising direction in intelligent transportation system is the applications based on vehicular ad-hoc networks (VANET). VANET of a country is one of the most importation indicators of its economic growth. In this paper, we analysed the characteristics of VANET network topology using complex network theory. The author contribute VANET model, analyse the statistical properties of complex network based on the degree distribution, the clustering coefficient, the average path length and find that the network has scale-free and small world features. The structure and properties of VANET have great implications for traffic congestion and urban planning. The robustness analysis indicates that VANET is robust to random attack when considering static robustness, but somewhat vulnerable to intentional attack.

Keywords: complex network, VANET, topological analysis, robustness.

1 Introduction

If we put the elements in a system as a node, and relation between the nodes as a connection, then the whole system form a network, such as computer network form interacting systems that can be viewed as a computer through cable, twisted-pair cable. There are a large number of complex systems in real-world which can be describe as network. These topology nodes abstracted from real network, known as complex network. At the end of the 1960s Paul Erdos and Alfred Renyi proposed a completely random network model (ER). However, ER random networks are theoretical constructs and cannot describe the properties of the most real-world systems. Watts and Strogatz as well as Barabási and Albert et al. proposed small world and scale-free networks [1-3]. With future development of complex network theory, several systems have been investigated using various concepts of statistical physics for complex networks, such as the average path length, the clustering coefficient, the degree distribution and betweenness. According to the existing complex network research, the high degree node is not high betweenness and vice verse. At home Li et al [4] study complex network community structure division algorithm, Gan et al [5] investigate the global adaptive synchronization problem for a class of supply chain complex networks that have nonlinearly coupled identical nodes and an asymmetrical coupling matrix. Wang et al [6] prove that a single controller can pin a coupled complex network to a homogenous solution, which is investigated for both continuous-time and discrete-time cases. Sufficient conditions are presented to guarantee the convergence of the pinning process locally and globally. The efficiency of the derived-results is illustrated by numerical simulation. Hu et al [7] constructs China's port shipping network structure chart, and analyses the degree distribution, average path length, network density and network clustering coefficient of the network. The results show that China's port shipping network has scale-free characteristic with short average path length, high network density, and its degree distribution follows a power law. Through the computer simulation, the higher correlation coefficient also shows that data fitting is satisfactory, which further validates that China's port shipping network is a scale-free network. Complex networks have attracted increasing attention from various fields of science and engineering today. Many scholars thought the most influential nodes in communication network is more connected nodes (hubs) which is often high betweenness of nodes, but other scholars disagree with this statement [8]. Other scholars (such as Husdal, Bell, Jenelius) contacted the consequences of an event and the probability as well as taking into account the probability and consequences of a propagation event. As shown in Figure 1, the III quadrant describes the greatest probability and the most serious consequences while the I quadrant is on the contrary.

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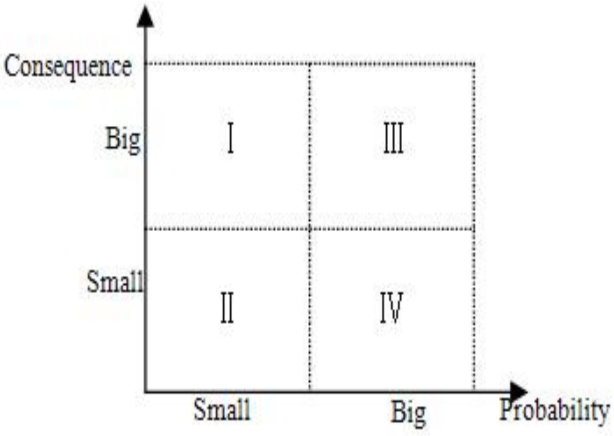

FIGURE 1 Probability-consequences matrix diagram

The vehicular ad-hoc networks (VANET) consist of a mass of nodes and connections between nodes are seriously complex, so it has the common characteristic of complex network. However, many different properties are found in VANET, such as autonomous and selective behaviours. A very promising direction in intelligent transportation system is the applications based on VANET. Through information, sharing between vehicles via VANET, the efficiency and safety of transportation system can be improved and the existing road network can give full play of its role. The high mobility of vehicles, the complex distribution of road networks and the time property of traffic peak hours lead to the highly uneven and dynamic topology of VANET. VANET is shown in the Fig. 2.

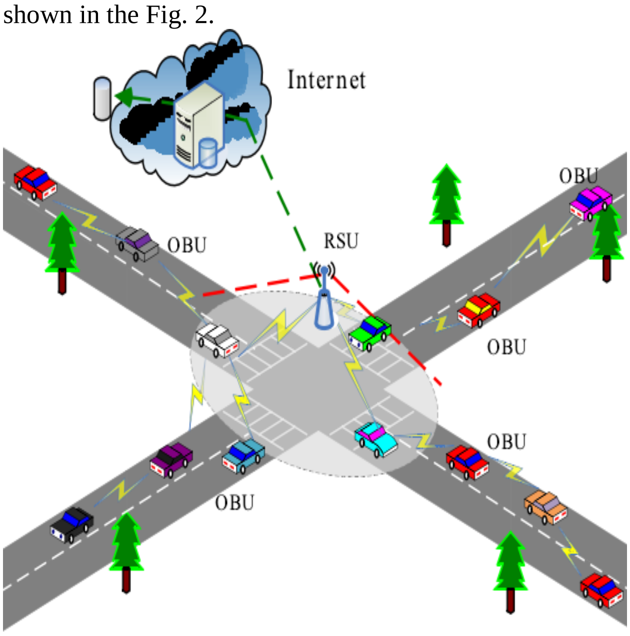

FIGURE 2 VANET network

The key problem of complex networks is to analyse their structure complexity including the topology complexity and the characteristics in different topologies, which is also a foundational theory of VANET. Moreover, it has become a commonsense that we should study the VANET systematically. In addition, adaptability and dynamic characteristics are also importation properties, which mean that their topology structures are not fixed and unchangeable, and they will evolve with time under the external and internal force drive. Firstly, the paper introduces the fundamental principle of complex network. Some fundamental complex network models and topologies are described. Then, the research background on VANET complex network is presented and finally the special characteristic different with other complex network is described in detail.

2 Topological Properties

The basic element of complex network is nodes and edges. The basic models include regular network model topology. Under the influence of the randomness, most networks between fully regular network and fully random network show small-world property significantly. Due to less nodes included, the original network generally does not show significant scale-free property. Therefore, we take small-world network as the network morphology of the new network in this paper. In the small-world model, p denotes the random rewiring probability, which can be adopted to adjust network randomness as shown in Fig. 3.

When p=0, the new module shows regular network form; when p=1, the edges of the new network are all rewired randomly which shows ER random network form; When $0 < p < 1$, the new network is a small-world network, which lies between regular network and random network. With the increase of random rewiring probability p, the randomness of the new module increases gradually.

In VANET, nodes represent the vehicles on the platform. The edges represent connection between vehicles, which show relationship of nodes that vehicles establish contact through wireless mobile network. Same as transportation complex network, network connectivity is also the most basic requirement for VANET communication. Nodes and edges form undirected-weighted networks. The edges between nodes are bidirectional, which ignore the length between nodes in wireless networks. VANET can be abstracted as network G (N, E), where N is the vehicles' number and E is the edges between the vehicles in the wireless network. If nodes represent many VANET vehicles, we use the above method to build network basic elements of nodes and the relationship between vehicles, so we get a network of VANET. The relationship between them is shown in Fig. 4.

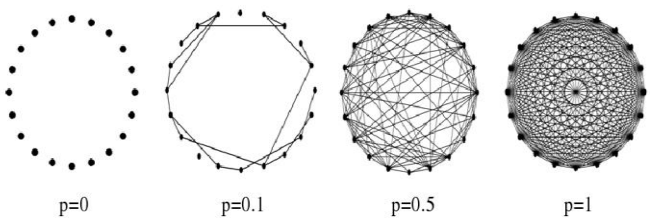

FIGURE 3 Dynamic evolvement map of regular network

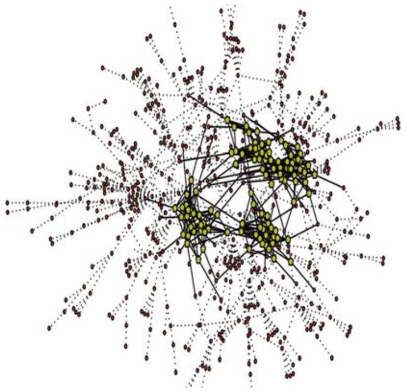

FIGURE 4 VANET network for vehicles and the relationship

Nodes represent vehicles in the VANET and the location of the nodes is not fixed. Relations between the vehicles represent whether they are wireless links. Such undirected network can use the adjacency matrix $A=(w_{ij})$. If the i and j is connected $w_{ij}=1$, otherwise $w_{ij}=0$, while the relationship of each nodes itself is 0. We get the resulting VANET network for undirected-weighted network, which the adjacency matrix is symmetric matrix. Therefore, VANET network can be expressed as a diagonal of 0 symmetrical two-dimensional matrixes. Traditionally, for network $G=[V,E]$, the adjacency matrix of the network is represented as A:

$$A=\begin{bmatrix} 0 & w_{12} & w_{13} & w_{14} \\ w_{21} & 0 & w_{23} & w_{24} \\ w_{31} & w_{32} & 0 & w_{34} \\ w_{41} & w_{42} & w_{43} & 0 \end{bmatrix}, \quad (1)$$

where w_{ij} is the weight of the network, define the edge weight as the number of vehicles' links, and the weight of node is the sum of the weights of all the edges linking with this node. In other words, the edge weight reflects the transfer convenience among network nodes. As the weighted network defined, we can investigate some vehicles, and the higher weight of node is, the more significant the vehicle is.

3 Topological Analysis of VANET

There are basic statistical parameters and degree distribution of the complex network, clustering coefficient, average shortest path length etc. [9, 10]. VANET, having complex characteristics, is a typical complex network. Based on the analysis above, we analyse the statistical properties of the network respectively.

Degree distribution: the node degrees k is the number of nodes it is directly connected to [11]. Degree of a node i is defined as $k_i=\sum_{j=1}^{N} a_{ij}$. All of the nodes average degree was found to be <k>, which presents the network's average degrees, $<k>=p(N-1)\approx pN$. In the network node, degree distribution is referred to the probability distribution function $p(k)$. Its meaning is the probability of random nodes having k edges directly connected to, also equal to the degree k number of nodes in the total number of network nodes. If the node degree distribution follows power-law distribution, it means $p(k)\propto k^{-\gamma}$. When N is large, nodes' degree distribution is approximately for the poison distribution, $p(k)\approx \frac{e^{-<k>}<k>^k}{k!}$. Power-law distribution graphics have not peak. Most of the node is only a small amount of connection, and a small number of nodes with a large number of connections. There is no characteristic scale in random networks, scholars referred to as a scale-free network. Fig. 5 and Fig. 6 show the degree distribution of regular network and Scale-Free network. They conform to Poisson distribution and power-law distribution.

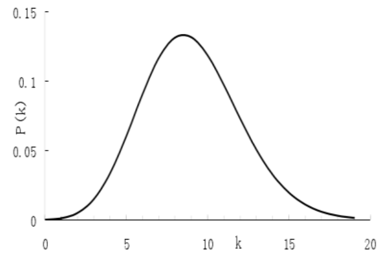

FIGURE 5 Degree distribution of regular network

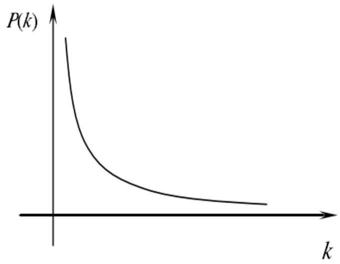

FIGURE 6 Degree distribution of Scale-Free network

Clustering coefficient: in VANET network, clustering coefficient C_i of a node is defined as the ratio of number of links shared by its neighbouring nodes to the maximum of possible links among them. For example, high clustering coefficient of the star VANET or public number VANET indicates that public pays more attention to them. The average clustering coefficient < C > reflects the VANET density relation among all nodes in the network.

$$C_i=\frac{2E_i}{k_i(k_i-1)},\ 0<C_i\le 1, \quad (2)$$

$$<C_i>=\frac{\sum_{i\in G} C_i}{N}. \quad (3)$$

Average path length: the distance between two nodes i and j in the network d_{ij} defined as the edges number of the shortest path of connecting two nodes. The diameter of the network was the maximum distance between two nodes, D=max d_{ij}. Network average path length L defined as the average distance between of all nodes:

$$L = \frac{2}{N(N-1)} \sum_{i,j \in G, i \neq j} d_{ij} \,. \quad (4)$$

Studies on complex networks have found most network is a small-world network that is often characterized by high connectivity and clustering, but so far there exist few effective approaches to evaluate small-world properties. There are comparisons of random, regular and small-world network shown in Table 1.

TABLE 1 Comparison of random, regular and small world networks

Type of network	Connectivity	Clustering
Random network	Short global separation	No clustering
Regular network	Long global separation	Highly clustered
Small-world network	Short global separation	Highly clustered

4 VANET Simulation Analyses

VANET Model Based on Complex Network. Because VANET is new, specific data acquisition is difficult. In this paper, the data obtained by JAVA programming. Starting with a small number m_0 of the original node and a small number e_0 of edges. In the lower nodes, we selected 20 nodes randomly, at every time steps, add a new module of small world network containing s nodes and n edges, which become connected to m existing nodes in the network. After t time step, the total number of nodes is m_0+st and the total number of edges is $N= e_0+(m+n)t$. [12-14] Then we structure adjacency symmetric matrix w_{ij}. Using $e = m/N$ present the ratio connection number m to the vehicles' number N. Our study found that e values between 1 and 2 even if N is different. In the regular network, such as the grid structure or tree structure in the network, e=1.In the random network $e = (N - 1) / 2$. Studies have shown that $e > 15/7$ in random ER network. Fig. 7 represents range of the e values under different sample. In VANET, e value is between 1 and 2 which shows that VANET is different from the regular networks and random networks. It has its own features.

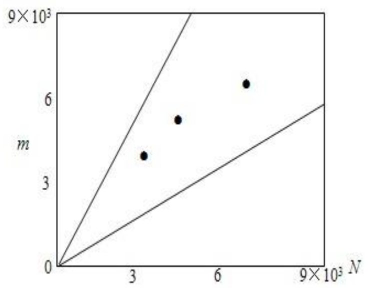

FIGURE 7 e value range under different sample scale

VANET Analysis Based on Complex Network. Degree distribution: In VANET, x present node degree, and $f(x)$ is the number of nodes. Introduce function model $f(x)=a+bx$, where a and b for the regression coefficients, and in the model a and b can be obtained by least square method and generated through the experiment data. Remove the unreasonable data by sifting, model results to $f(x) = 9.7-0.4x$. According to the power-law distribution function $p(k) \infty k^{-\gamma}$, we take logarithm, and get degree distribution equation for deformation. A power-law distribution function, its image is intuitive. When expressed in double logarithmic coordinate axis is linear. Seen from the equation, VANET has power-law distribution characteristics. The power-law index represents the level of activity through VANET, which means vehicle joined into VANET and existing vehicle can be treated as random connections. As shown in Fig. 8, it is a certain vehicle accumulation degree distribution of the VANET.

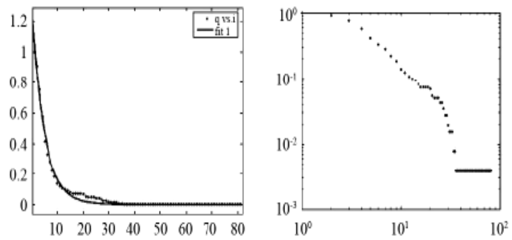

FIGURE 8 The cumulative k distribution

Clustering coefficient: VANET average aggregation coefficient $<C>$=0.245 is obtained by Matlab programming calculation. Compared with the poison distribution of random networks of the same scale, setting its density 0.05, the same as the density of VANET, we analyse it and get the random network clustering coefficient C_i=0.05 far less than the VANET C_i=0.245. Thus, VANET has larger clustering coefficient.

Average shortest path length: next we analyse the relationship matrix and get node path distribution, then the VANET average path length L = 2.45 be calculated by the above formula. For the huge network of VANET, it's smaller average shortest path. Average VANET vehicles can be connected to any other vehicles by three vehicles. Compared with the same scale of random network, average path length L = 2.16, VANET average path length is little different, as the network scale increases, the average shortest path length increase. Although the VANET number of vehicles is small, it still shows a shorter average shortest path.

The judgment of small-world characteristic: according to Watts and Strogatz analysis, to determine whether a network is small-world characteristics, It can be used an average path length L and average clustering coefficient $<C>$ comparing with ER random network with the same scale L_{rand}, $<C_{rand}>$. When meet the following formula (5), (6), it is the small world network.

$$L \geq L_{rand} = \frac{\ln N}{\ln <k>}, \quad (5)$$

$$<C> \gg <C_{rand}> = \frac{<k>}{N}. \quad (6)$$

The judgment of scale-free networks characteristic: we can judge the scale-free characteristic whether the VANET network degree distribution is power-law form. When the degree distribution is the power-law distribution, cumulative distribution function follows as:

$$P_k \propto \sum_{k'=k}^{\infty} k'^{-\gamma} \propto k^{-(\gamma-1)}. \quad (7)$$

The cumulative distribution function $P_k = \sum_{k'=k}^{\infty} P(k')$ presents the probability distribution of the node's degree greater than k or equal. Therefore, we can also judge VANET scale-free network topology characteristics by plotting the cumulative distribution curve.

5 Robustness Analysis of VANET

The robustness refers to the persistence of a system's characteristic behaviour under perturbations or conditions of uncertainty. VANET robustness is referred to the ability of the network to maintain its function when the nodes or edges in the network suffer from random or intentional attack. For random attack, each node in the network is endowed with a random probability, nodes are deleted stochastically, and the ratio of the giant connected component and average path length are calculated till all nodes in the network are not connected [17]. For selective attack, the network nodes are attacked selectively, starting from the nodes with the largest connected degree, and going on according to the descending order of nodes degree. The stability performance can be represented by the relative size of S:

$$S = \frac{N'}{N}, \quad (8)$$

where N and N' are respectively, the numbers of nodes of the network before and after the attack. The value of S is between [0, 1]. When S=1, the network is fully connected; when $0<S<1$, the network is still relatively integrated; when S=0, the network collapses. Starting with a small number m_0=20 and a small number e_0 =50. Then according to preferential mechanism of the new nodes attaching to the entire network, we choose the nodes preferentially which would connect to the nodes existing in the VANET. We assume the probability $\prod(k_i) = \frac{k_i}{\sum k_i}$ [15, 16] that node I was chosen in the module depends on its degree. Finally we obtain a VANET and N=500. Step 1: Compute the degree k of all nodes in the VANET; calculate the shortest path between all node pairs with Dijstra algorithm, the distance matrix D is obtained. Step 2: Remove the node randomly and intentional according to the descending order of node degree; if there is only one of the greatest degree node, delete it directly; if there is more than one, then remove one node of them together. Step 3: Delete the q^{th} line and qth row, compute the shortest distance of node q to all other nodes i, j; if $D(i,j)= D(i,q)+ D(q,j)$, save the node pairs in array. Read node i in turn from the array; calculate the new shortest distances after removing node q; calculate the degree k again and S. Step 4: return to step 2 and continue to carry on until the removing node rate f=1, then output the result.

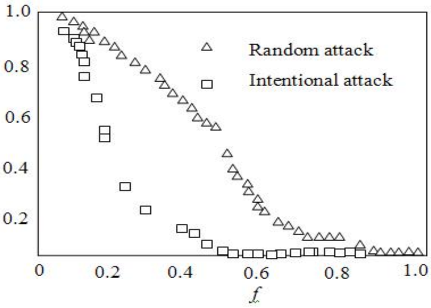

FIGURE 9 The S variation chart of VANET against random attack and intentional attack

Shown in the Fig. 9, triangle represents random attack and square is intentional attack. As seen from the analysis results, VANET shows that S drops quickly under random attack circumstance. S declines gradually, and the curve f-S has two break points when f arrives 0.5431 and 0.8160 on random attack, respectively. The entire network nearly collapses when f is at 0.8792. S drops rapidly on intentional attack when f<0.15, then changes a little, drops quickly when $0.18<f<0.35$, and falls to 0. Compared with random attack, the dropping speed of S is faster and does not appear significant fluctuation.

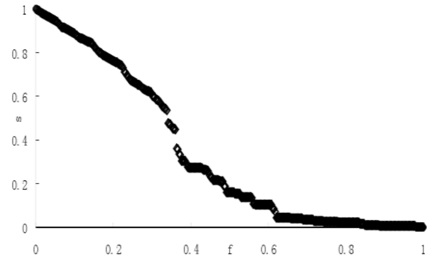

FIGURE 10 The S evolvement of VANET against random attack

Fig. 10 shows the dynamic evolvement when nodes appear random failure. It can be seen that when the network has a third of the node failure, VANET still stay connecting more than half nodes of the original network, so we can think that VANET facing random attack is robust.

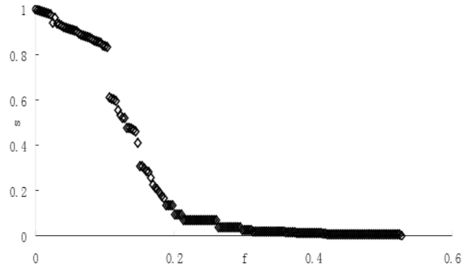

FIGURE 11 The S evolvement of VANET against intentional attack

Fig. 11 illustrates the evolution trend of the biggest connected subgraph S that attacks on node degree of value in descending order in China VANET. The relative size of maximal connected sub-graphs *S* are calculated for different removing node rates *f*, After removing a node the degree of adjacent nodes will also be changed, therefore, in this paper, in the process of removing nodes, calculate the new shortest distances after removing node to make sure removing the maximum degree of node from the global perspective.

We can seen from the Fig 11, there was an obvious two largest jumping down, the first occurred removing the 34 node, the removed node percentage is only 10.66%, and S change the size of the original 61.13%; The second occurred removing the 48 node, at this time removing 15.05% of the nodes, the relative size of network's largest connected subgraph is 30.72%; When removing the 57 node, the size of the largest network connected subgraph descented below 20%. It is clear that VANET showed a certain vulnerability to attack according to the degree descending order.

5 Conclusions

Complex network exists in many real networks. In this paper, we propose a VANET model based on complex network theory. We research the VANET topology combined with complex network. VANET model is described by the complex network method. VANET topology characteristics were analysed. Application examples show that: VANET lies between regular network and random network, and has scale-free and small-world characteristics. To find VANET network formation, understanding its development process and the way the information transmission path will provide valuable reference.

The next step we will study VANET more characteristics of complex network, and compare with other wireless complex network. There are differences between real world network and complex network. The current complex network characteristics describe the actual network, even if has two characteristics that cannot well describe the reality.

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