Finite element analysis of fluid conveying pipeline of nonlinear vibration response

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Abstract

Fluid filled pipe system was widely used in the city water supply and drainage, water power, chemical machinery, aerospace, marine engineering and the nuclear industry and other fields, it was play an important role for improving the living standards of the nation and the national economic strength. Pipe conveying fluid was easy to design and manufacture, according to the characteristics of fluid conveying pipe, transformed the axial vibration mathematical model of the fluid conveying pipe, which considerate the fluid solid coupling to the beam element model for two nodes. Using Lagrangian interpolation function, the first order Hermit interpolation function and the Ritz method to obtain the element standard equation, and then integrated a global matrix equation, obtained the response of conveying fluid pipe with the Newmark method and Matlab. With the Matlab to simulate the axial motion equation of the conveying fluid pipe, study the response of the system in two aspect of fluid pressure disturbance and the fluid velocity disturbance, and the simulation results are analysed, which provides theoretical support for the work of fluid conveying pipes.

Keywords: response, MATLAB, Numerical simulation, nonlinear vibration

1 Introduction

Study of the fluid solid coupling vibration of pipeline in our country started relatively late, there is no paper of this aspect until the mid80's in last century, and compared with the international level there are still large differences [1-2]. In recent years, with the development of China's modern industry and city modernization, the domestic scholars made a lot of significance research for the fluid solid coupling phenomenon on the long distance oil pipeline, water pipeline, large-scale city heat supply system and nuclear power plant water circulation system, which puts forward the many methods to control pipe vibration. In the aspect of modelling of the fluid conveying pipe, Yang Ke make the pipeline axial vibration of fluid solid coupling four equation model as the foundation, introduced the two order differential equations group with symmetrical "rigidity", "damping" and "quality" matrix, which regard the displacement as basic variables, both considerate the Poisson coupling and coupling of friction and damping of pipeline [3-5]. Fei Wenping established the fluid solid coupling model of a complex pipeline system by the complex modal theory, and studied theoretically. Chen Guiqing pointed out and corrected many error equation for the current mathematical modelling of the pipeline system vibration, and reclassify the pipeline according to the ground of linear pipeline, the ground of nonlinear linear pipeline and buried pipeline, last come out the most commonly used pipeline vibration differential equation. In this paper, considering the infusion pipe liquid vibration condition of small deformation, take the mathematical model of axial vibration as plane beam element, obtained the standard equation with first-order Hermit interpolation function and Ritz method. Then obtained matrix equation through made each unit assembled into the global mass matrix, global damping matrix, the global stiffness matrix and the global load matrix. Apply the finite element method to get the numerical solution for partial differential equation of higher order, and obtained the response of pipe conveying fluid system with Newmark method [11-13].

2 The establish of mathematical model for the output response of the fluid conveying pipe

Taking into account the pipe ratio for length to diameter is relatively large, the deformation of radial is the same, just only exists a certain angle difference, which can be regarded the pipe as plane beam element to consider, using two node element, as shown in Figure 1, the node number of I and J. The conveying fluid pipe is only affected by the lateral force, no axial force, so analysis with the two node element, the nodal displacement model can be defined.

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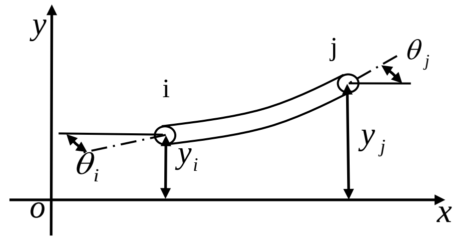

FIGURE 1 An example. Good quality with clear lettering

2.1 A SUBSECTION THE ESTABLISH OF MATHEMATICAL MODEL

The node number of I and J. The conveying fluid pipe is only affected by the lateral force, no axial force, so analysis with the two node element, the nodal displacement model can be defined [6-8]:

$$y(t) = [y_i, \theta_i, y_j, \theta_j]^T . \quad (1)$$

In the node parameter of unit, in addition to the node value of field function, also contains node value for a derivative of the field function. In order to maintain the continuity of field function derivative between the public node element, and in the end nodes to keep the derivative order is first for the field function, so the first-order Hermit interpolation polynomial is used:

$$N(\xi) = [N_1(\xi), N_2(\xi), N_3(\xi), N_4(\xi)], \quad (2)$$

where $N_1(\xi) = 1 - 3\xi^2 + 2\xi^3$, $N_2(\xi) = \xi - 2\xi^2 + \xi^3$, $N_3(\xi) = 3\xi^2 - 2\xi^3$, $N_4(\xi) = \xi^3 - \xi^2$, ξ is the local dimensionless coordinate ($0 \le \xi \le 1$).

Taken the Ritz method interpolation functions to establish standard unit equation of the approximate solution of the lateral vibration after determine the interpolation function:

$$[M^e]\{\ddot{y}_j\} + [C^e]\{\dot{y}_j\} + [K^e]\{y_j\} = [Q^e], \quad (3)$$

where $[M^e] = [M^e_{ij}]$, $[C^e] = [C1^e_{ij}] + [C2^e_{ij}]$, $[K^e] = [K1^e_{ij}] + [K2^e_{ij}] + [K3^e_{ij}]$, $[Q^e] = [Q1^e_{ij}] + [Q2^e_{ij}] + [Q3^e_{ij}]$,

$$[M^e_{ij}] = \int_l N_i (m_p + m_f) N_j dx ,$$

$$[C1^e_{ij}] = \int_l N_i (2 m_f v_f) \frac{\partial N_j}{\partial x} dx ,$$

$$[C2^e_{ij}] = \int_l N_i (\frac{A_f}{c^2} \frac{\partial P}{\partial t}) N_j dx ,$$

$$[K1^e_{ij}] = \int_l \frac{\partial^2 N_i}{\partial^2 x} (EI) \frac{\partial^2 N_j}{\partial^2 x} dx ,$$

$$[K2^e_{ij}] = -\int_l \frac{\partial N_i}{\partial x} (m_f v_f^2 + (1-2\gamma) A_f P) \frac{\partial N_j}{\partial x} dx ,$$

$$[K3^e_{ij}] = \int_l N_i (m_f \frac{\partial v_f}{\partial t}) \frac{\partial N_j}{\partial x} dx ,$$

$$[Q2^e_{ij}] = \int_l (-\frac{v_f A_f}{c^2} \frac{\partial P}{\partial t}) N_j dx ,$$

$$[Q3^e_{ij}] = \int_l (m_p g + m_f g) N_j dx .$$

2.2 THE ENTIRETY MATRIX

There are some matrix must be appropriately expanded rewrite when the unit matrix integrated to the entirety matrix so that the matrix of all elements with uniform format, then according to the superposition to assembly.

The usually study boundary constraint conditions include fixed to hinge and fixed to fixed constraints, the mathematical expression of its boundary is given below, respectively (I) and (II).

$$\begin{cases} y(0,t) = 0, \left.\frac{\partial y}{\partial x}\right|_{x=0} = 0 \\ y(L,t) = 0 \end{cases} , \quad (I)$$

$$y(0,t) = 0, \left.\frac{\partial y}{\partial x}\right|_{x=0} = 0 . \quad (II)$$

The four kinds of boundary conditions of above given all belong to the first class constraint conditions, for this kind of constraint conditions can usually use "row column method" and "multiplied with bigger number method". The "multiplied with bigger number method" is make the main diagonal element about the specified node displacement in the overall stiffness matrix with multiply by the large number λ, at the same time, give the specified value of node displacement to the corresponding element of load matrix, then multiply by the same number as well as the main diagonal elements. Using the “multiplied with bigger number method "to deal with the boundary constraint condition by", finally forms the whole matrix:

$$[M]\{\ddot{y}\} + [C]\{\dot{y}\} + [K]\{y\} = [Q] . \quad (4)$$

The Newmark method is used to solving the flow pipeline vibration response, the Newmark method is a step-by-step integration method, the key is to establish the recurrence relations of state vector from t to $t + \Delta t$, assume at the moment of $t + \Delta t$, the $\ddot{y}_{t+\Delta t}$, $\dot{y}_{t+\Delta t}$ and $y_{t+\Delta t}$ satisfy the dynamics equation:

$$[M]\{\ddot{y}_{t+\Delta t}\} + [C]\{\dot{y}_{t+\Delta t}\} + [K]\{y_{t+\Delta t}\} = [Q_{t+\Delta t}] . \quad (5)$$

In addition to the Newmark method assume the velocity and displacement satisfy the follow equations at the same moment:

$$\dot{y}_{t+\Delta t} = \dot{y}_t + [(1-\alpha)\ddot{y}_t + \alpha\ddot{y}_{t+\Delta t}]\Delta t,\ 0 \le \alpha \le 1, \tag{6}$$

$$y_{t+\Delta t} = y_t + \dot{y}_t\Delta t + [(\frac{1}{2}-\beta)\ddot{y}_t + \beta\ddot{y}_{t+\Delta t}]\Delta t^2,\ 0 \le 2\beta \le 1. \tag{7}$$

According to the analysis results of the algorithm stability when the $\alpha \ge 0.5$, $\beta \ge (\frac{1}{2}+\alpha)^2 \Big/ 4$, the Newmark method is unconditionally stable.

The calculation steps of the Newmark method can be summarized as follows.

Step 1: forming the stiffness matrix *K*, mass matrix *M*, damping matrix *C*.

Step 2: obtaining the initial state vector $\ddot{y}_0$, $\dot{y}_0$ and y_0.

Step 3: choosing the time step Δt as well as the parameter α and β, then calculated the constant: $\gamma_0 = \frac{1}{\beta\Delta t^2}$, $\gamma_1 = \frac{\alpha}{\beta\Delta t}$, $\gamma_2 = \frac{1}{\beta\Delta t}$, $\gamma_3 = \frac{1}{2\beta} - 1$, $\gamma_4 = \frac{\alpha}{\beta} - 1$, $\gamma_5 = \frac{\Delta t}{2}(\frac{\alpha}{\beta} - 2)$, $\gamma_6 = \Delta t(1-\alpha)$, $\gamma_7 = \alpha\Delta t$.

Step 4: calculation of the effective stiffness matrix:

$$\tilde{K} = K + \gamma_0 M + \gamma_1 C, \tag{8}$$

Step 5: calculation of the effective load vector at the moment of $t + \Delta t$:

$$\hat{Q}_{t+\Delta t} = Q_{t+\Delta t} + M(\gamma_0 y_t + \gamma_2\dot{y}_t + \gamma_3\ddot{y}_t) + C(\gamma_1 y_t + \gamma_4\dot{y}_t + \gamma_5\ddot{y}_t), \tag{9}$$

Step 6: calculation of the displacement at the moment of:

$$\tilde{K}y_{t+\Delta t} = \hat{Q}_{t+\Delta t}, \tag{10}$$

Step 7: calculation of the acceleration and velocity at the moment of

$$\ddot{y}_{t+\Delta t} = \gamma_0(y_{t+\Delta t} - y_t) - \gamma_2\dot{y}_t - \gamma_3\ddot{y}_t, \tag{11}$$

$$\dot{y}_{t+\Delta t} = \dot{y}_t + \gamma_6\ddot{y}_t + \gamma_7\ddot{y}_{t+\Delta t}. \tag{12}$$

3 Simulation and analysis of the axial vibration

For the axial vibration, the constraint of the fixed to hinge and constraint of the fixed - overhanging belonging to the same constraints, which is decided by its displacement and load form, While the boundary constraint of the hinge to hinge, because the axial without any restraint, this pipeline system is in an unstable state, the modal and response could not be calculated. So the axial vibration simulation of the fluid conveying pipeline was the main considerate the constraint of the fixed to hinged and constraint of the fixed - overhanging. In the simulation process, the two end points of pipe as the supporting point, and assumed to be rigid constrain. Then make the pipe length divided into 100 equal parts, a total of 101 nodes, in the process of analysis with room temperature water as the fluid and rolling copper as pipe material [9-10].

3.1 A SUBSECTION THE VIBRATION RESPONSE OF THE FLUID CONVEYING PIPELINE

The four order mode of vibration in two kinds of boundary conditions. The first four order mode of vibration of the constraint of fixed to hinge as shown in Figure 2. The first four order mode of vibration of the constraint of fixed to fixed hinge as shown in Figure 3.

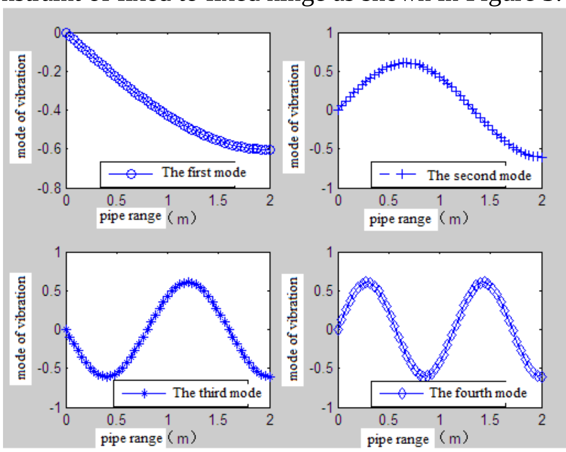

FIGURE 2 The first four order mode of vibration of the constraint of fixed to hinge

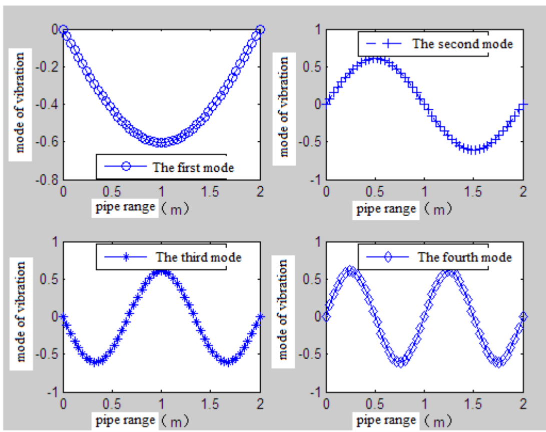

FIGURE 3 The first four order mode of vibration of the constraint of fixed to fixed to hinge

3.2 THE TWO BOUNDARY CONSTRAINTS VIBRATION RESPONSE

As shown in Figure 4 is the vibration response of same node in different time for the constraint of the fixed to hinge, we can get the vibration cycle of the tenth nodes, fifty-first nodes and hundred and first nodes from the figure shows, the amplitude becomes larger from tenth nodes, fifty-first nodes, hundred and first nodes. At the end of pipe the hinge hundred and first nodes is the maximum amplitude with match the actual situation. As shown in Figure 5 is the vibration response of same node in different time for the constraint of the fixed to fixed, compared with the fixed to hinge node, the node vibration graphs is same, but for the vibration amplitude the latter is small, the vibration cycle is small, in addition to the fifty-first node is the maximum vibration amplitude of the constraint of the fixed to fixed, that match with the actual situation.

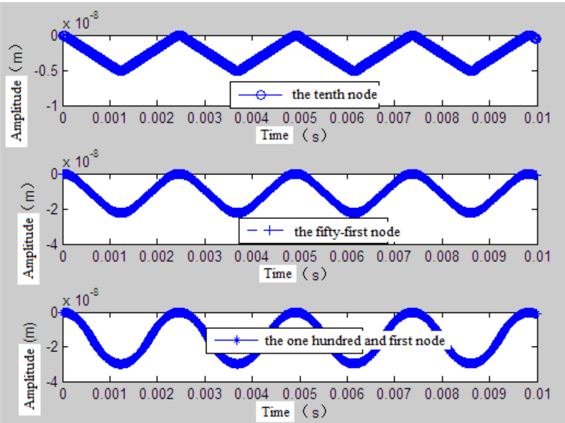

FIGURE 4 The vibration response of same node in different time for the constraint of the fixed to hinge

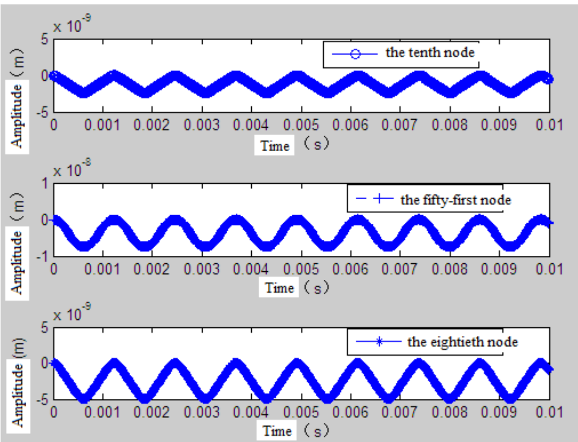

FIGURE 5 The vibration response of same node in different time for the constraint of the fix to fix

3.3 THE EFFECT OF VELOCITY AND PRESSURE FOR THE VIBRATION RESPONSE

The effect of fluid velocity perturbation for response of the fluid conveying pipeline The effect of fluid velocity perturbation for system response of the constraint of the fixed to hinge as shown Figure 6.

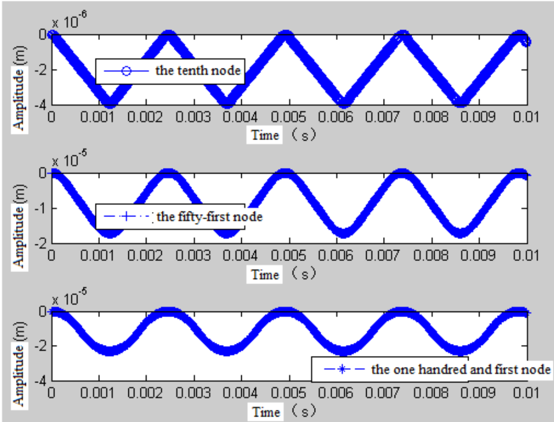

FIGURE 6 The effect of fluid velocity perturbation for response of the constraint of the fix to fix

From the Figure 6 we can see the response of node periodic change in velocity perturbation, when the fluid velocity and pressure of nodes is a constant value, the vibration amplitude increases about 100 times with the time increased. The boundary conditions for the constraint of the fix to fix, the response of nodes has a cycle changes under the disturbance velocity, which the vibration amplitude increased about 100 times.

The effect of the fluid pressure disturbance is for response of the fluid conveying pipeline. From the Figure 7 we can see the response of node periodic change in velocity perturbation in the constraint of the fix to hinge, when the fluid velocity and pressure of nodes is a constant value, the amplitude of vibration is increased, but not very strong, the response is much smaller than the velocity perturbation. The boundary conditions for the response of the constraint of fix to fix constraint nodes is almost to the same with fix to hinge.

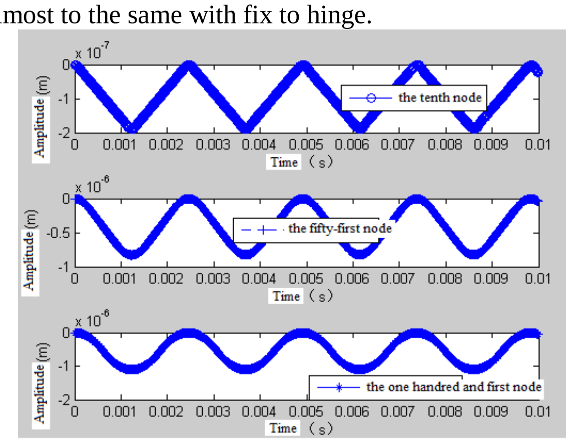

FIGURE 7 The effect of the fluid pressure disturbance for response of the constraint of the fix to hinge

3.4 CHARACTERISTICS COMPARATIVE ANALYSIS OF AXIAL VIBRATION OF PIPELINE

The Table 1 gives the first four order natural frequency for response of the fluid conveying pipeline of the constraint of the fix to hinge and the fix to fix. Can see from the table, the axial vibration natural frequency of the fluid conveying pipeline system is bigger, the first-order natural frequency of the constraint of the fix to fix is two times that of the fix to hinge.

TABLE 1 The axial vibration of pipeline system in two kinds of boundary characteristics

Frequency	**Fixed hinge constraint**	**Fixed constraint**
The first-order natural frequency (Hz)	407.0157	814.0566
The second-order natural frequency (Hz)	1221.1476	1628.314
The third-order natural frequency (Hz)	2035.5808	2442.9732
The fourth-order natural frequency (Hz)	2850.5163	3258.2352

The effect for response of the fluid conveying pipeline of velocity and pressure for the vibration response is bigger, in a general it will increase 100 times.

4 Conclusion

The simulation for the response of the fluid conveying pipe axial motion summarized and analysed. In two kinds of boundary conditions, analysed the effect for the vibration response of the fluid conveying pipeline of velocity and pressure disturbance, verified the correctness of the established vibration model of the fluid conveying pipeline, and obtained the vibration characteristics of the fluid conveying pipe of velocity and pressure disturbance in the constraint of the fix to fix and fix to hinge, which provides theoretical support for the work of fluid conveying pipes.

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