# New approach for ventilation network graph drawing based on Sugiyama method and GA-SA algorithm

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**Abstract**

Ventilation network graph has an important place in the management of a coal mine. In that case, aesthetics plays a major role for generating readable and understandable layouts. Besides, the drawing is required to be oval. The traditional longest path method for drawing ventilation network graph is inefficient and cannot effectively reduce the number of arc crossings because of the geometric intersection method. In this paper, we developed a new approach to draw ventilation network graph, consist of Sugiyama method framework, the longest path method and GA-SA algorithm. The longest path method was employed to rank nodes, and long arcs were removed by solving integer programming problem to minimize the sum length of ventilation network. Then genetic algorithm and simulated annealing algorithm were adopted to optimize the node order on reducing the number of arc crossings. In order to make the drawing be oval, a modified version of the longest path method was made to calculate node coordinates and arc shapes, which is called the longest parallel path method. Finally, computational experiments were carried out on two test ventilation network with our new approach.

*Keywords: v*entilation network graph, longest path method, integer programming, Sugiyama method, simulated annealing-genetic algorithm

## 1 Introduction

Mine ventilation system is a complex three-dimensional structure which is composed of crisscross and vertical overlap. So there's much inconvenience if using ventilation system graph in the ventilatory management. The coal mine safety regulation of China stipulates that mine ventilation department must draw the mine ventilation network graph [1]. The ventilation system could be transformed to a ventilation network on the basis of graph theory, which is composed of arcs, nodes and their properties. Ventilation network graph can clearly reflect the structure and flow characteristics of ventilation system, which is the basis of various ventilation calculations.

Li Husheng was the first one to propose the longest path method (LPM) for drawing ventilation network graph [2], searching all the longest paths corresponding to the fans, and lay outing the long paths on both sides and the short paths in the centre. Some minor fixes were contributed to LPM by Wu Bing, et al [3]. Actually, LPM is the most popular method to draw ventilation network graph automatically.

Ventilation network graph is also a kind of digraphs. Most approaches for drawing directed graphs used in practice follow the framework developed by Sugiyama et al. [4], which produces layered layouts. This framework consists of four phases: In the first phase, called Node Rank, the nodes are assigned to horizontal layers. During the second phase, called Node Order (also called Crossing Reducing); an optimal order of the nodes within a layer is computed such that the number of arc crossings is reduced. The third phase, called Node Coordinate Assignment, calculates an x-coordinate for each node. The fourth phase, called Arc Drawing, calculates the arc shapes of the digraph.

Some minor changes were made to Sugiyama framework for drawing ventilation network graph. In the first phase of the original framework, long arcs between nodes of non-adjacent layers are replaced by chains of dummy nodes and arcs between the corresponding adjacent layers. This work can be achieved by the longest path method. Unfortunately, the longest path method leans to produce long arcs such that more dummy nodes were needed, making an influence on the node order phase. So we turned to use the longest path method in conjunction with integer programming to produce the short arcs. The second phase needs to reduce the number of arc crossings. Garey and Johnson proved that arc crossing reducing is a NP-complete problem [5]. Some heuristics [6] and hybridized genetic algorithm (HGA) [7] were proposed to solve that problem. But the crossover operation in HGA was too complex to effectively minimize arc crossings. Due to many arcs and nodes in the actual mine ventilation system, the number of solutions about node order problem is relatively large, which makes genetic algorithm run into a local optimal solution, not global optimal. So an algorithm mixed with genetic algorithm and simulated annealing algorithm was adopted to optimize the arc crossings problem. Finally node coordinates and arc shapes were calculated by the modified longest parallel path method, ensured that the shape of the drawing is oval.

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## 2 Optimizing on node rank

### 2.1 THE LONGEST PATH METHOD

For a network with $n$ nodes, the longest path of any the $j^{th}$ node in the network is the longest path length of the $i^{th}$ source of all the $jth$ node's inflow arcs plus one [3]:

$$\begin{cases} l(1) = 0 \\ l(j) = 1 + \max\{l(i_k) \mid k = 1,...,m\} \\ j = 2,...,n \end{cases}, \qquad (1)$$

where $l(j)$ is the longest path length of the $j^{th}$ node, and $l(i_k)$ is the longest path length of $i_k^{th}$ source of all the $j^{th}$ node's inflow arcs.

Let $\lambda(v)$ be the node rank that equal to the longest length of the node $v$, where $\lambda(v) = l(v)$. Let $l_{\max}$ be the max value of the longest path length in the ventilation network, $s$ and $t$ be ventilation network's source and sink node. The nodes of ventilation network are layered according to the vertical direction from bottom to up. The source node is lay out on the lowest layer $\lambda(s) = 0$, while the sink is lay out on the highest layer $\lambda(t) = l_{\max}$. Other nodes are lay out on the same layer with the same rank $\lambda(v)$.

The span of an arc $(v,w)$ is defined by $\delta(v,w) = \lambda(w) - \lambda(v)$. If the span $\delta$ is larger then 1, $\delta - 1$ dummy nodes are used to split the arc $(v,w)$ into shorter arcs, whose spans are all equal to 1.

### 2.2 THE OPTIMAL NODE RANK

The node rank based on the longest path method is easy to produce a lot of long arcs, whose span are larger than 1. It will make the arcs and nodes of ventilation network focus on the bottom of the graphic such that the drawing is not beautiful. In order to make the node rank produce short arcs and avoid long arcs, we tried to minimize the sum of the arc lengths on the basis of node rank by the longest path method. The optimization problem was defined as follows:

$$\min \sum_{(u,v)\in E} \omega(u,v)(\lambda(v) - \lambda(u))$$
$$subject\ to: \begin{cases} \lambda(v) - \lambda(u) \ge \delta(u,v) \\ \lambda(u) \ge 0 \end{cases}, \qquad (2)$$

where $\omega(u,v)$ is the weight coefficient of an arc $(u,v)$, $\delta(u,v)$ is the minimum length of the arc $(u,v)$. In practical, the default value of $\omega(u,v)$ and $\lambda(u,v)$ is always 1. Obviously, this is a multi-variable integer programming problem, while all the variables must be integers [8].

In fact, the node rank of the longest path method can be used as the initial value of the IP problem to speed up the iterative convergence process.

## 3 Optimization on node order

The aim of the second phase is to find an optimal node order and reduce arc crossings in ventilation network graph.

### 3.1 THE PROBLEM OF ARC CROSSINGS MINIMIZATION

After the node rank phase, the nodes of ventilation network $G = (V,E)$ are divided into several layers, then the ventilation network could be transformed into a hierarchical graph $H_G = \{L_1, L_2, \cdots, L_h\}$. The span of each arc in $H_G$ is 1, and there are no arcs whose spans are more than 2. The nodes on each layer generate a sequence, called node order. The initial node order is defined by $\Pi_0$.

The number of arc crossings between two adjacent layers is not dependent on the node coordinates x, only related to the nodes order in the layer. We consider the layer $L_k$ and its adjacent layer $L_{k+1}$. The source and target node of the arc $(v,w)$ and $(v',w')$ is located in $L_k$ and $L_{k+1}$, the position of the node $v$ in $L_k$ is defined by $\pi_k(v)$. If and only if the following condition is met, there is an arc cross between the arc $(v,w)$ and $(v',w')$ [9]

$$[\pi_k(v) - \pi_k(v')] \times [(\pi_{k+1}(w) - \pi_{k+1}(w')] < 0. \qquad (3)$$

So the problem was transformed into finding an optimal node order, avoiding the complex judgment of geometry intersection.

The optimization process of the drawings was based on an algorithm called GA-SA, which is a mixed algorithm with genetic algorithm and simulated annealing algorithm. We have introduced problem-based chromosome encoding, fitness function and SA algorithm which significantly improve the result of crossings reducing by the enhanced local search ability.

### 3.2 CHROMOSOME ENCODING

After the node rank phase, the nodes are divided into H layers: $L_1, L_2, ..., L_h$. Naturally, the nodes on each layer can be encoded into a permutation. The permutation encoding of the $k$-$th$ layer is defined by $P_k$, and the encoding of a node order is defined by $\Pi$ as follow:

$$\begin{cases} N_k = |L_k| \\ \sum_{k=1}^{h} N_k = |V| \\ P_k = (\sigma_k(1), \cdots, \sigma_k(N_k)) \\ \Pi = (P_1, \cdots, P_h) \end{cases}, \quad (4)$$

where $L_k$ is the $k^{th}$ layer, $\sigma_k(i)$ is the node in the $i^{th}$ position on the $k^{th}$ layer, $|L_k|$ is the number of nodes on the $k^{th}$ layer, and $|V|$ is the number of nodes in ventilation network. Therefore, the sum of the nodes on each layer is equal to $|V|$.

When the number of nodes in ventilation network is too large, the encoded string is also too long, which can slow down the convergence rate of genetic algorithm. In this paper, a small adjustment is applied to the encoding in order to reduce the length of the encoded string. A decimal integer can be used to represent the permutation encoding of each layer. Then we detail the new encoding as follow:

$$\begin{cases} k_i \in [1, (N_i)!] \\ \Pi' = (k_1, k_2, \cdots, k_h) \end{cases}, \quad (5)$$

where $k_i$ is the integer state of the $i^{th}$ layer corresponding to $P_i$. The length of new encoding string $\Pi'$ is h, while the length of $\Pi$ is equal to $|V|$. In general, it is far less than the number of nodes in the ventilation network, $h << |V|$.

Algorithm 1 Pseudo-code of decoding an integer value to the node order of a layer

Where $L_k$ is *k-th* layer, $\Pi_0$ is the original order of nodes on $L_k$, $N_k$ is the number of nodes on $L_k$ and $P$ is an integer value.

```
function decode_to_order( Π0 , Nk , P) {
        Π ← Π0 ;
        Pos, Val ← 0, P;
        K ← Nk -1
        while K > 0 {
        Π ← Π0 ;
        Cf ← ∏_{i=1}^{K} i ;
        Cp ← Val/Cf+Pos;
        Π [Pos] ← Π0 [Cp];
        Π [Pos+1 : Cp+1] ← Π0 [Pos : Cp];
        Π0 ← Π ;
        Pos, Val ← Pos+1, Val%Cf;
          K ← K-1
     }
   return Π
}
```

### 3.3 OBJECTIVE FUNCTION AND FITNESS FUNCTION

The objective function is defined as the number of arc crossings by $c(\Pi)$, then the fitness function is defined by $f(\Pi) = 2^{-c(\Pi)}$. In order to guarantee the efficiency of genetic algorithm, the fitness function is required as simple as possible. Therefore, the efficiency of the algorithm that counts the number of arc crossings between adjacent layers is critical.

Wilhelm Barth and Petra Mutzel proposed a rapid cross counting algorithm based on radix sort and inverse number counting [9]. For a directed graph $G = (V, E)$, the time complexity of the algorithm is $O(|E| \log |V_{small}|)$, where $V_{small}$ is the smaller part of $V$ that is partitioned into two layers.

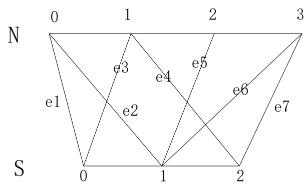


FIGURE 1 Bilayer Graph

### 3.4 SIMULATED ANNEALING LOCAL SEARCH

Simulated annealing algorithm (SA) is a kind of random optimization algorithm based on the mechanism of metal annealing. The sampling technology based on the probability is adopted to update the optimal solution. Under the temperature of $T_k$, a new individual $\Pi_i'$ is got from the disturbed $\Pi_i$, which is the $i^{th}$ individual of current population. Then the acceptance probability of $\Pi_i'$ is determined by Meteopolis sampling criterion:

$$p_i = \begin{cases} 1 & \Delta_i < 0 \\ \exp(-\Delta_i / T_k) & \Delta_i \geq 0 \end{cases}, \quad (6)$$

where $p_i$ is the acceptance probability, $\Delta_i = c(\Pi_i') - c(\Pi_i)$ is the objective function increment. When the new individual $\Pi_i'$ is a more optimal solution $(\Delta_i < 0)$, we completely accept the new individual as the current optimal solution. But when the new individual $\Pi_i'$ is the poor solution $(\Delta_i \geq 0)$, it is accepted by a small probability as the current optimal solution. As the temperature $T_k$ decreased, the acceptance probability of the poor solution would be gradually smaller. Eventually, SA converged to the global optimal solution [10]. The method of geometry cooling is used to control of the temperature as follow:

$$T_k = \lambda T_{k-1} = \lambda^{n-1} T_0, \quad (7)$$

where $T_k$ is the current temperature, $T_0$ is the initial temperature, $\lambda$ is the temperature drop coefficient, the value range is $\lambda \in (0,1)$, and $k$ is the iterations of cooling process.

The initial temperature $T_0$ is calculated as follow:

$$T_0 = c_{\max}(\Pi)/\ln(p_0), \quad (8)$$

where $c_{\max}(\Pi)$ is the number of arc crossings about the worst individual in a population; $p_0$ is initial acceptance probability.

It is considered that the algorithm is convergent, when the optimal solution does not improve by several cooling iterations. Then the search process of SA will stop.

## 4 The longest parallel path method

After the node hierarchy through the longest path method, it analyses the hierarchy chart which adds the secondary node, we can find that: “all paths between any two nodes are the longest path, and the longest path length is the same”. That is to say that the longest path method has failed at this time.

The longest parallel path method was put forward to calculate node coordinates and arc shapes on the improvement of the original method [3], which made the shape of the drawing seem to be oval.

The node $v$ is laid out on the layer of $L_k$, all the targets of the node $v$ out-arcs are laid out on the layer of $L_{k+1}$, the node order of $L_{k+1}$ is $\{w_1, w_2, \cdots, w_n\}$. We iterate nodes on the layer of $L_{k+1}$ by the order $\{(v,w_1),\ldots,(v,w_n)\}$, called forward search, while backward search is defined by the order $\{(v,w_n),\ldots,(v,w_1)\}$.

The longest parallel path method always tries to draw parallel paths. When it searches the longest path between two nodes each time by the depth first search (*DFS*), it firstly determine whether there is a drawn path that contains two nodes. If present, it will make a backward search try to find a symmetric path, which would be drawn in an arc. If not present, the search direction remains the same as the last search, and the path would be drawn in a straight line.

Forward search and backward search can ensure that the long paths are laid out on both sides, while the short paths are laid out in the middle. Eventually, the shape of ventilation network graph leans to be oval.

Algorithm 2 Pseudo-code of the longest parallel path method to draw nodes and arcs

```
S, T ← [], [];
Push the source node s into S;
Push the sink node t into T;
While S not empty and T not empty {
      u ← Pop a node from S;
      v ← Pop a node from T;
      If (u,v) in a drawn path P {
            New_P ← DFS search find a symmetric path of P from u
to v without colored arcs;
      }
      Else {
            New_P ← DFS search find a path from u to v without
colored arcs;
      }
      Caculate node cooridnates and arc shapes in New_P and color
the drawn arcs;
      Analyse the path New_P to find more sources and targets;
      Update S and T;
}
```

## 5 Computational Experiments

We developed a procedure to test two ventilation networks, whose size parameters are listed in Table 1. And control parameters of GA-SA algorithm are listed in Tables 2 and 3. The two ventilation network graphs are shown in Figures 2 and 3.

TABLE 1 Ventilation network size parameters

| | **A simple ventilation network** | **Ventilation network in Zhao Zhuang coal mine** |
|---|---|---|
| The number of nodes | 26 | 155 |
| The number of Arcs | 35 | 213 |

TABLE 2 Control parameters of GA

| **Control parameters** | **A simple ventilation network** | **Ventilation network of Zhaozhuang coal mine** |
|---|---|---|
| Coding scheme | Decimal integer coding | Decimal integer coding |
| Population size | 15 | 30 |
| Termination condition | Maximum evolution algebra 20 | Maximum evolution algebra 50 |
| Crossover rate | 0.75 | 0.75 |
| Mutation rate | 0.05 | 0.05 |
| Selection Operator | Roulette selection | Roulette selection |
| Crossover operator | Single-point crossover | Single-point crossover |
| Mutation Operator | Single-point mutation | Single-point mutation |

TABLE 3 Control parameters of SA

| **Control parameters** | **A simple ventilation network** | **Ventilation network of Zhaozhuang coal mine** |
|---|---|---|
| Termination condition | The best individual has no improvement for 3 generations | The best individual has no improvement for 10 generations |
| Initial acceptance rate | 0.8 | 0.618 |
| Temperature drop coefficient | 0.9 | 0.99 |

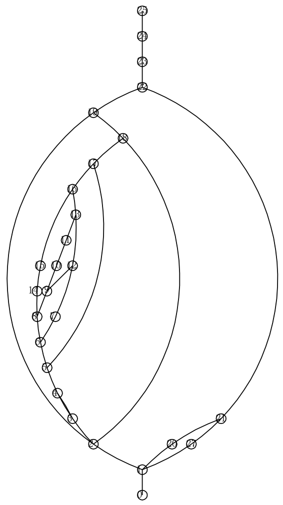

FIGURE 2 A simple ventilation network graph

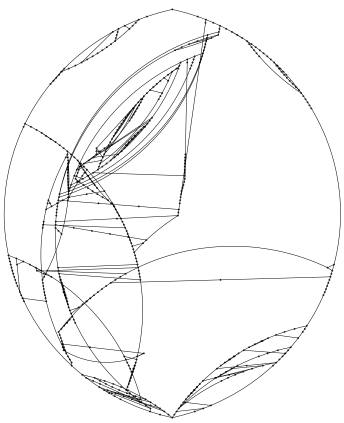

FIGURE 3 Ventilation network graph of Zhao zhuang coal mine

## 6 Conclusions

We developed a procedure for drawing ventilation network graph based on Sugiyama method, ranking nodes on layers by the longest path method, reducing arc crossings by GA-SA algorithm, and calculating node coordinates and arc shapes by the improved longest parallel path method to make the drawing seem to be oval.

Computational experiments were done with two ventilation networks. They showed that our new approach could find a better layout to reducing arc crossings and make the drawing seem to be oval. Actually, it could be concluded that the drawing is better for the small ventilation network (probably less than 100 arcs).

So there is also another interesting point to study further. We need to simplify the ventilation network, removing part of the arc which does not affect the connectivity and the overall structure of ventilation network. So the ventilation network is splitted into some smaller sub-networks, then this new approach is applied to these small networks to produce a better drawing.

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