Research on growth opportunity and liquidity monitoring by mathematical optimization

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Abstract

Based on the assumption of variable-investment, this paper introduces growth opportunity into the model of liquidity needs (Tirole, 2006). Through the establishment of mathematical optimization model, we analyse the influence of growth opportunities on liquidity needs and liquidity investment decisions. Both of mathematical derivation and numerical simulation show that, the entrepreneur tends to overinvest in illiquid assets if the growth opportunity is small; otherwise, he will overhoard of liquid assets. In addition, the agency costs due to information asymmetry may also affect the entrepreneur's decisions of liquid assets investment.

Keywords: Growth opportunity, liquidity monitoring, variable-investment model, mathematical optimization

1 Introduction

As the vitality of enterprises, liquidity is necessary to ensure the normal operation of enterprises. As ongoing entities, firms are concerned that they may in the future be deprived of the funds that would enable them to take advantage of exciting growth prospects, strengthen existing investments, or simply stay alive. Enterprises in the process of actual operation often face a liquidity shocks, and then the liquidity demand will happen. Liquidity is the most important part of the business, whether purchasing, production, sales and other business sectors, or investment in new projects, distribution of profits to shareholders, as well as the repayment of debt principal and interest, all of these require a lot of liquidity.

As the first one explained the meaning of mobility, Keynes believes that liquidity refers to how easy is it to convert asset into payment [1]. In the definition of enterprises' liquidity, however, the academia has yet formed a clear definition. The study of [2] points out that the liquidity of enterprises not only includes the solvency, but also contains the whole cash needs. However, the author of [3] holds that corporate liquidity is the total liquidity of each single asset. The research for liquidity assets describes enterprise liquidity as frequency of fund flows [4]. Liquidity risk is divided into market liquidity risk and financing liquidity risk [5]. The market liquidity risk can be further divided into two categories, exogenous and endogenous [6].

As the vitality of enterprises, liquidity is necessary to ensure the normal operation of enterprises. The occurrence of liquidity risk accidents tends to endanger the survival of the enterprise, and even spread to the entire community. Unsurprisingly, liquidity planning is central to the practice of corporate finance and consumes a large fraction of chief financial officers' time. Studies for liquidity monitoring have never stopped.

There are lots of factors that affect liquidity demand and liquidity risk, and growth prospect is also one of them. The writer of [7] firstly puts forward the concept of growth opportunity, and he thinks growth opportunity refers to the part of enterprise's value depending on discretionary expenditures in the future. Growth opportunities, in other words, not only including traditional investment opportunities, also including discretionary spending, which can bring greater success for the enterprise [8]. Growth opportunities depend not only on the external environment, but also depend on the enterprise itself [9]. And different enterprises may own different growth opportunities [10].

The essence of liquidity supervision is liquidity risk management, and lots of scholars study the factors that influence the liquidity and risk management. The purpose of liquidity risk management is to seek anticipated cost trade-off in the ample liquidity and abundant liquidity [11]. The major factors that affect the liquidity are company size, growth opportunities, physical assets investment yields, the company's ability to create operating cash flow, working capital management efficiency as well as the debt ratio and so on [12]. Otherwise, maintaining high liquidity can improve the value of the inter-temporal investment options [13].

At present, research on liquidity monitoring is concentrated on definition, classification and measurement through qualitative analysis. Some scholars have discussed the impact and causes of liquidity monitoring by empirical analysis. However, few scholars

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have made theoretical study for liquidity monitoring, from the perspective of demand for liquidity.

On the basis of liquidity needs model [14-17], this paper analyses the impact of growth opportunities on liquidity needs. Illiquidity may affect the normal operation, make it impossible to repay maturing debt, and even endanger the survival of the enterprise; conversely, excess liquidity will reduce the profits. Therefore, we try to study the influence of growth opportunities on the firm's liquidity investment decisions by establishing and solving the mathematical optimization model. Ultimately, we want to prove the need for monitoring liquidity assets.

The result shows that the entrepreneur will overinvest in illiquid assets when the growth opportunity is relatively small. Conversely, the entrepreneurs will overboard liquid assets when the growth opportunity is relatively big. In addition, agency costs will also affect the entrepreneur's investment decisions on liquid assets. Therefore, in order to prevent entrepreneurs make some investment decision not conducive to the development of enterprises, the company's liquidity ratio must be formulated in loan agreement.

2 Assumption

This paper introduces growth prospects on the basis of the liquidity risk management model, in the contest of the variable-investment framework. At date 0, the entrepreneur has a project requiring fixed investment I, he initially has "assets" A and needs to borrow $I-A$ from investors. At date 1, the firm meets a new investment chance requiring an amount ρI, where ρ is ex ante unknown and has cumulative distribution function $F(\rho)$ with density $f(\rho)$ on $\rho\in[0,\infty)$. The realization of ρ is learned at date 1.

The probability of success p is affected by the effort degree of the entrepreneur, which is unobservable. Behaving yields probability $p=p_H$ of success, and misbehaving results in probability $p=p_L<p_H$ of success and private benefit $BI>0$.

Let $\Delta p=p_H-p_L>0$. If the firm does not reinvest ρI, then it yields, at date 2, RI with probability p and 0 with probability $1-p$. If the firm reinvests ρI, then it yields, at date 2, RI with probability $p+\tau$ and 0 with probability $1-(p+\tau)$, where $\tau>0$.

The investment has positive NPV. Both the entrepreneur and investors are risk neutral. The entrepreneur is protected by limited liability. Investors behave competitively in the sense that the loan, if any, makes zero profit. We summarize the timing in Figure 1:

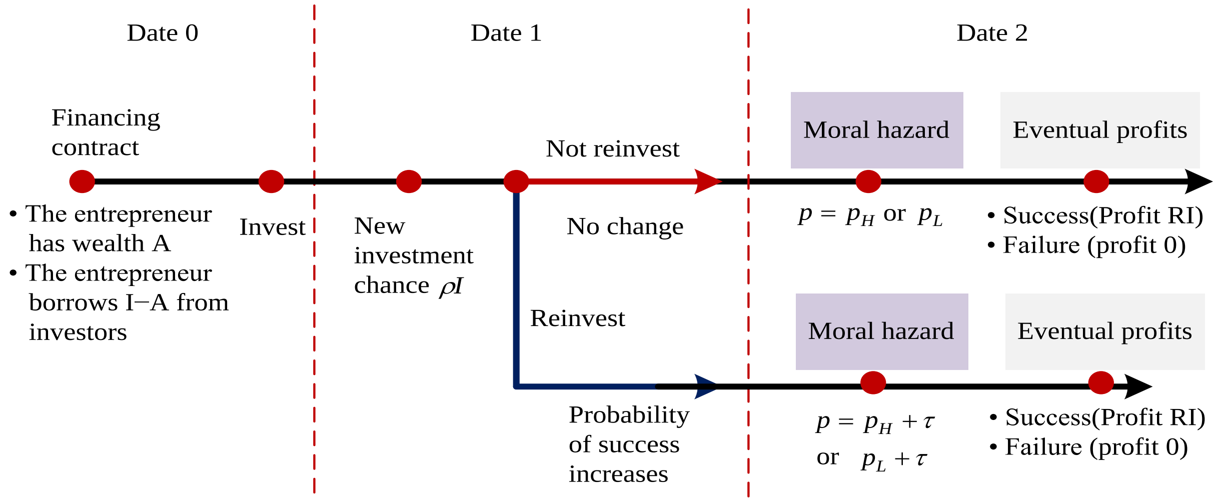

FIGURE 1 Figure of the timing

3 Optimal models

Suppose that the financing contract takes the following state-contingent form: $\{I;\rho^c;(R_b,0);(RI-R_b,0)\}$.

The contract specifies that ρ^c is a cut-off of reinvestment: only if $\rho\le\rho^c$, the firm reinvests and the investment level is I. If the project success, the entrepreneur and investors get R_b and $RI-R_b$ respectively; if the project fail, both of them get 0. For any cut-off of reinvestment ρ^c:

$$0<\frac{[\tau F(\rho^c)+p_H]R-[1+\int_0^{\rho^c}\rho f(\rho)]}{\tau F(\rho^c)+p_H}<\frac{B}{\Delta p}.$$

The probability of reinvestment is $\Pr ob\{\rho\le\rho^c\}=F(\rho^c)$.

So the optimization problem becomes

$$\begin{cases} \max\limits_{R_b,\rho^c,I} & F(\rho^c)(p_H+\tau)R_b + [1-F(\rho^c)]p_H R_b - A \\ s.t.\ (a1) & F(\rho^c)(p_H+\tau)R_b \geq F(\rho^c)[(p_L+\tau)R_b+BI], \\ (b1) & [1-F(\rho^c)]p_H R_b \geq [1-F(\rho^c)](p_L R_b + BI), \\ (c1) & F(\rho^c)(p_H+\tau)(RI-R_b)+[1-F(\rho^c)] \times p_H(RI-R_b) \geq I + \int_0^{\rho^c} \rho I f(\rho)d\rho - A \end{cases} . \quad (1)$$

(a1) is the objective function is the entrepreneur's utility, when the entrepreneur's incentive-compatibility constraint if the firm can come up with enough cash to reinvest. (b1) is the incentive-compatibility constraint if not, and both of them could be simplified as: $R_b \geq BI/\Delta p$. (c1) is the investors' individual-rationality constraint and it holds with equality. So the optimal model (1) will be simplified as:

$$\begin{cases} \max\limits_{R_b,\rho^c,I} & \{F(\rho^c)(p_H+\tau)+[1-F(\rho^c)]p_H\}RI - [I+\int_0^{\rho^c}\rho I f(\rho)d\rho], \\ s.t.\ (a2) & R_b \geq BI/\Delta p, \\ (b2) & \{F(\rho^c)(p_H+\tau)+[1-F(\rho^c)]p_H\} \times (RI-R_b) = I + \int_0^{\rho^c}\rho I f(\rho)d\rho - A \end{cases} . \quad (2)$$

The optimal solution of the model (2) can be got through the following three steps.

Firstly, get R_b for a given ρ^c and I. We may illustrate the "feasible contract set" of (2) by Figure 2, and it may be constituted by the shaded area OEF.

First of all, it is easy to get that the intercept of the line (b2) is $A/[\tau F(\rho^c)+p_H]$ and the slope is: $k_1 = \dfrac{[\tau F(\rho^c)+p_H]R - [1+\int_0^{\rho^c}\rho f(\rho)d\rho]}{\tau F(\rho^c)+p_H}$.

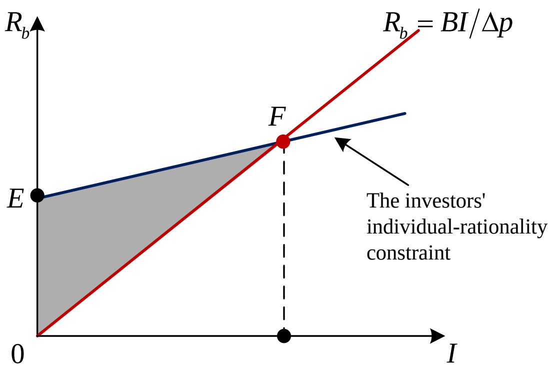

FIGURE 2 The solution for the model

The intercept of the line (b1) is 0 and the slope is $B/\Delta p$. Since $0 < k_1 < B/\Delta p$, the "feasible contract set" is not an empty set if $A \geq 0$. And then, it is the bigger the better for R_b. So the point F constitutes the optimal contract: $R_b^* = BI/\Delta p$.

Secondly, consider the optimal I for a given ρ^c. Actually, take R_b^* into the problem (2) and it can be further simplified as that

$$\begin{cases} \max\limits_{\rho^c,I} & m(\rho^c)I \\ s.t. & \{F(\rho^c)(p_H+\tau)+[1-F(\rho^c)]p_H\} \times (RI-R_b) = I+\int_0^{\rho^c}\rho I f(\rho)d\rho - A \end{cases} , \quad (3)$$

where

$$m(\rho^c) = [p_H + \tau F(\rho^c)]R - [1+\int_0^{\rho^c}\rho f(\rho)d\rho],$$

$$I^*(\rho^c) = k(\rho^c)A = \frac{A}{[1+\int_0^{\rho^c}\rho f(\rho)d\rho] - [\tau F(\rho^c)+p_H](R-B/\Delta p)},$$

$$U_b(\rho^c) = m(\rho^c)k(\rho^c)A.$$

Finally, we could solve ρ^c.

Proposition 1: If ρ^{c*} is equal to the expected unit cost of effective investment, $U_b(\rho^c)$ reaches its maximum, that is: $\rho^{c*} = c(\rho^{c*}) = \dfrac{1+\int_0^{\rho^c}\rho f(\rho)d\rho}{p_H/\tau + F(\rho^c)}$.

Proof: In fact

$$U_b(\rho^c) = \frac{\{[p_H+\tau F(\rho^c)]R - [1+\int_0^{\rho^c}\rho f(\rho)d\rho]\}A}{[1+\int_0^{\rho^c}\rho f(\rho)d\rho] - [\tau F(\rho^c)+p_H](R-B/\Delta p)} = \frac{\tau R - c(\rho^c)}{c(\rho^c)-\tau(R-B/\Delta p)}A .$$

So $U_b(\rho^c)$ reaches its maximum means that $\min\limits_{\rho^c} c(\rho^c) = [1+\int_0^{\rho^c}\rho f(\rho)d\rho] / [p_H/\tau + F(\rho^c)]$.

The first-order condition is:

$$\rho^c f(\rho^c)[p_H/\tau + F(\rho^c)] - f(\rho^c) - f(\rho^c)\int_0^{\rho^c}\rho f(\rho)d\rho = 0 .$$

That is

$$\rho^{c*}\frac{p_H}{\tau} + \int_0^{\rho^{c*}} F(\rho)d\rho = 1. \quad (4)$$

Assume that $Z = 1 - \rho^{c*}(p_H/\tau) - \int_0^{\rho^{c*}} F(\rho)d\rho = 0$, then from (4) we can get that $c(\rho^{c*}) = \dfrac{Z + \rho^{c*}[p_H/\tau + F(\rho^{c*})]}{p_H/\tau + F(\rho^{c*})} = \rho^{c*}$.

Proposition 2: ρ^{c*} is increasing with the increase of τ/p_H.

Proof: Because $\rho^{c*} = c(\rho^{c*})$

$\Leftrightarrow \rho^{c*}F(\rho^{c*}) + \rho^{c*}(p_H/\tau) = 1 + \int_0^{\rho^{c*}} \rho f(\rho)d\rho$.

Solving the partial derivative of ρ^{c*} with respect to τ/p_H on both sides of the above formula:

$$[\rho^{c*} f(\rho^{c*}) + F(\rho^{c*})]\frac{\partial\rho^{c*}}{\partial(p_H/\tau)} + \rho^{c*} + \frac{p_H}{\tau}\frac{\partial\rho^{c*}}{\partial(p_H/\tau)} = \rho^{c*} f(\rho^{c*})\frac{\partial\rho^{c*}}{\partial(p_H/\tau)}.$$

That means $\rho^{c*} = -[F(\rho^{c*}) + \frac{p_H}{\tau}]\frac{\partial\rho^{c*}}{\partial(p_H/\tau)}$.

It is easy to get that $\rho^{c*} > 0$, $F(\rho^{c*}) + p_H/\tau > 0$, and then $\frac{\partial\rho^{c*}}{\partial(p_H/\tau)} < 0 \Leftrightarrow \frac{\partial\rho^{c*}}{\partial(\tau/p_H)} > 0$.

4 Monitoring 0verinvestment in illiquid assets

In most instances, loan agreements do not focus solely on the borrower's solvency, but also strictly constrain the borrower's liquidity. For example, many loan agreements require that the borrower maintain a minimum level of working capital. It is not a priori clear why this is so. Let us bring one answer to this puzzle, and show that it may be optimal for lenders to simultaneously impose gearing and liquidity ratios.

In the absence of a liquidity requirement, suppose that the borrower invests the full $I^*(1+\rho^{c*}) = I'$ in illiquid assets; despite the lack of cash left for reinvestment, the project will often be continued. An interesting issue relates to whether the investors should renegotiate the borrower's compensation scheme so as to account for the unexpectedly high scale of operations. The answer to this question depends on the way the managerial compensation contract was initially drawn.

- ✓ If the borrower owns a share in the firm's final profit, then managerial compensation scales up with investment, and the initial incentive scheme remains incentive compatible as investment increases and is not renegotiated by lenders to account for the altered firm size.
- ✓ If the entrepreneur gets a fixed reward for "success"; because the private benefit scales up with investment, the initial incentive scheme is then no longer incentive compatible. Lenders then offer to increase the borrower's reward in the case of "success" and so they raise the borrower's payoff in the case of success to $BI'/\Delta p$ in order to make sure the borrower behaves.

It should be noted that the high payoff can be achieved only if investors agree that. Now suppose that investor may get more when the entrepreneur behaving than misbehaving, the entrepreneur's high payoff will not be cancelled. Then the following proposition can be drawn.

Proposition 3: The entrepreneur's high payoff will not be cancelled if τ, p_L and B are relatively small, p_H and R are relatively big.

Proof: In fact, as long as

$$\{F(\rho^{c*})(p_H+\tau) + [1-F(\rho^{c*})]p_H\}(R - B/\Delta P)I' \geq \{F(\rho^{c*})(p_H+\tau) + [1-F(\rho^{c*})]p_L\}(RI' - B/\Delta PI^*)$$

or

$$(\Delta P)(R - B/\Delta P) \geq [p_L + \tau F(\rho^{c*})](B/\Delta P)[\rho^{c*}/(1+\rho^{c*})]. \quad (5)$$

This means $Y_1 \equiv (\Delta P)(R - B/\Delta P) - [p_L + \tau F(\rho^{c*})](B/\Delta P)[\rho^{c*}/(1+\rho^{c*})] \geq 0.$

Then the high payoff may not be cancelled. ρ^{c*} will reduce as τ decreases and p_H increases, and $\rho^{c*}/(1+\rho^{c*})$ will also reduce. Furthermore, with the decreasing of p_L and B and the increasing of R, the left side of (5) will increase and the right side will decrease, that means (5) will be set up.

Proposition 4: The entrepreneur will overinvestment in illiquid assets if τ, p_L and B are relatively small, p_H and R are relatively big.

Proof: Indeed, the borrower, who, regardless of the design of her initial compensation contract, receives expected rent $p_H B/\Delta p$ per unit of illiquid assets, prefers investing I' rather than I^* if:

$$\{F(\rho^{c*})(p_H+\tau) + [1-F(\rho^{c*})]p_H\}(BI^*/\Delta P) < \{F(\hat{\rho}_0)(p_H+\tau) + [1-F(\hat{\rho}_0)]p_L\}(BI'/\Delta P),$$

or

$$F(\rho^{c*}) - F(\hat{\rho}_0) < [(p_H/\tau)F(\hat{\rho}_0)]\rho^{c*}. \quad (6)$$

That means $Y_2 \equiv F(\rho^{c*}) - F(\hat{\rho}_0) - [(p_H/\tau)F(\hat{\rho}_0)]\rho^{c*} < 0$ and if τ/p_H is relatively small, p_H/τ will be relatively big, then (6) will be set up. Furthermore, with the decreasing of p_L and B and the increasing of R, $\hat{\rho}_0$ will increase and (6) will be set up more easily.

Corollary 1: The overinvestment in illiquid assets will be achieved if τ, p_L and B are relatively small, p_H and R are relatively big.

From proposition 3 and proposition 4, we may get that the entrepreneur will overinvestment in illiquid assets and the investors will not cancel his high payoff if τ, p_L and B are relatively small, p_H and R are relatively big.

Alternatively, the entrepreneur may have been granted in the initial agreement a fixed reward for "success". Because the private benefit scales up with investment, the initial incentive scheme is then no longer incentive compatible. Because the borrower is then strictly better off overinvesting, the lender should rationally anticipate to lose money overall (That the lender loses money results from the facts that the borrower deviates from investment I^* to obtain more than U_b, and that U_b is the maximum utility for the borrower consistent with a nonnegative profit for the lender.). Hence, the optimal investment lever is not I^* and the rationale for a liquidity requirement.

5 Monitoring overhoarding of liquid assets

As mentioned earlier, lenders may also need to verify that the borrower does not underinvest in illiquid assets in order to overinsure against liquidity shocks. We select a specific set of assumptions for the sole purpose of illustrating a possible incentive to underinvest in illiquid assets.

- ✓ The borrower can use the excess liquidity in order to withstand the liquidity shock;
- ✓ The borrower and investors receive shares of the date 2 profit with share $(B/\Delta p)R$ held by the borrower and share $(R-B/\Delta p)R$ held by the investors;
- ✓ Unused liquidity is returned to investors and the firm only issue stock.

Proposition 5: The entrepreneur will overhoard of liquid assets if τ/p_H is relatively big.

Proof: Suppose further that the borrower invests $I" < I^*$ in illiquid assets and thus hoards liquidity equal to $\rho^{c^*}I^* + (I^* - I")$. She can then withstand liquidity shocks ρ such that:

$$\rho I" \le \rho^{c^*}I^* + (I^* - I") \Leftrightarrow \rho \le [\rho^{c^*}I^* + (I^* - I")]/I" \equiv \bar{\rho}.$$

Letting $\varepsilon = (I^* - I")/I"$, and using the all-equity-firm assumption, the borrower prefers to underinvest only if:

$$\{F(\bar{\rho})(p_H+\tau)+[1-F(\bar{\rho})]p_H\}(BI"/\Delta P) \ge \{F(\rho^{c^*})(p_H+\tau)+[1-F(\rho^{c^*})]p_H\}(BI^*/\Delta P). \quad (7)$$

For small underinvestments, that is $\varepsilon \to 0$

$$\{F[\rho^{c^*}+(1+\rho^{c^*})\varepsilon]-F(\rho^{c^*})\} = f(\rho^{c^*})[\rho^{c^*}+(1+\rho^{c^*})\varepsilon-\rho^{c^*}] = f(\rho^{c^*})(1+\rho^{c^*})\varepsilon$$

and (7) will be like that

$$\varepsilon f(\rho^{c^*})(1+\rho^{c^*}) \ge [F(\rho^{c^*})+p_H/\tau]\varepsilon \Leftrightarrow f(\rho^{c^*})(1+\rho^{c^*}) \ge [F(\rho^{c^*})+p_H/\tau]. \quad (8)$$

That means $Y_3 \equiv f(\rho^{c^*})(1+\rho^{c^*}) \ge [F(\rho^{c^*})+p_H/\tau] \ge 0$ and if τ/p_H is relatively small, p_H/τ will be relatively big, then (8) will be set up.

Roughly, if liquidity shocks around the threshold ρ^{c^*} are quite likely, hoarding a bit more liquidity than allowed is privately profitable for the borrower. The borrower would always prefer underinvesting to investing I^* if she had a fixed claim (namely, $BI^*/\Delta p$ in the case of success).

6 Numerical simulation and conclusions

As mentioned earlier, lenders may also need to verify that the borrower does not underinvest in illiquid assets in order to overinsure against liquidity shocks. We select a specific set of assumptions for the sole purpose of illustrating a possible incentive to underinvest in illiquid assets.

Now, we make some numerical calculations on the theoretical results. Table 1 shows the influence of τ on financing. Where, basic parameters are $R=3$, $B=0.8$, $p_L=0.07$, $p_H=0.36$, $\rho \in U[0,1]$. From table 1, it can be get that ρ^{c^*} increases with the increasing of τ and the entrepreneur will overinvestment in illiquid assets if τ is relatively small. (In all of the following tables, "Ent." means the entrepreneur and "Inv." Means investors).

Table 2 shows the influence of τ/p_H on financing. Where, basic parameters are $R=3$, $B=0.8$, $p_L=0.07$, $\rho \in U[0,1]$. From table 2, we may get that ρ^{c^*} is increasing with the increase of τ/p_H and the entrepreneur's incentive to overboard of liquid assets increases with the increasing of τ/p_H.

Table 3 shows the influence of p_L on financing. Where, basic parameters are $R=3$, $B=0.8$,

$p_H = 0.38$, $\tau = 0.55$, $\rho \in U[0,1]$. From Table 3 some endings may be got: (i) ρ^{c*} has nothing to do with p_L; (ii) The entrepreneur's incentive to overinvest in illiquid assets decreases with the increasing of p_L; (iii) Investors will gradually become opposed to their high salaries with the increasing of p_L. Therefore, overinvestment in illiquid assets would be achieved only if p_L is comparatively small.

Table 4 shows the influence of B on financing. Where, basic parameters are $\tau = 0.5$, $p_L = 0.05$, $R = 3$, $p_H = 0.4$, $\rho \in U[0,1]$. From table 3 we may get the following conclusions:

(i) ρ^{c*} has nothing to do with B;

(ii) The entrepreneur's incentive to overinvest in illiquid assets decreases with the increasing of B;

(iii) Investors will gradually become opposed to their high payoff with the increasing of B. Therefore, overinvestment in illiquid assets would be achieved only if B is relatively small.

Generally speaking, this paper analysed the influence of growth opportunities on liquidity needs and liquidity investment decisions. Firstly, the "first-best cutoff" of reinvestment increases with the increasing of growth opportunity. Secondly, with the increase of growth opportunities, the entrepreneur's incentive to overinvest of illiquid assets decreases and investors will gradually become against the high payoff. As long as one participant does not agree, overinvesting will not happen. In other words, overinvesting of illiquid assets becomes possible only if the growth opportunity is small.

TABLE 1 The growth opportunity τ and overinvesting of illiquid assets

τ	ρ^{c*}	I^*	Y_1	**Ent.**	Y_2	**Inv.**	**Overinvest**
0.01	0.03	1.09	-0.97	√	0.06	√	√
0.06	0.16	1.08	-0.84	√	0.04	√	√
0.11	0.29	1.05	-0.70	√	0.01	√	√
0.16	0.41	1.02	-0.56	√	-0.04	×	×
0.21	0.51	0.98	-0.44	√	-0.09	×	×
0.26	0.59	0.95	-0.33	√	-0.16	×	×
0.31	0.67	0.92	-0.23	√	-0.24	×	×
0.36	0.73	0.89	-0.15	√	-0.32	×	×
0.41	0.79	0.87	-0.08	√	-0.41	×	×
0.46	0.83	0.86	-0.02	√	-0.50	×	×
0.51	0.87	0.84	0.03	×	-0.59	×	×
0.56	0.91	0.83	0.07	×	-0.69	×	×

TABLE 2 The ratio of growth opportunity τ / p_H and overhoarding of liquid assets

p_H	τ	ρ^{c*}	I^*	Y_3	**Overhoard**
0.36	0.21	0.51	0.98	-0.71	×
0.36	0.26	0.59	0.95	-0.38	×
0.36	0.31	0.67	0.92	-0.16	×
0.36	0.36	0.73	0.89	0.00	√
0.36	0.41	0.79	0.87	0.12	√
0.36	0.46	0.83	0.86	0.22	√
0.44	0.40	0.69	1.57	-0.10	×
0.42	0.40	0.71	1.33	-0.05	×
0.40	0.40	0.73	1.15	0.00	√
0.38	0.40	0.75	1.00	0.05	√
0.36	0.40	0.78	0.88	0.10	√
0.34	0.40	0.80	0.77	0.15	√

TABLE 3 The success probability of misbehaving p_L and overinvesting of illiquid assets

p_L	ρ^{c*}	I^*	I'	Y_2	**Ent.**	Y_1	**Inv.**	**Overinvest**
0.01	0.883	5.085	9.575	-1.155	√	0.133	√	√
0.03	0.883	3.612	6.801	-1.059	√	0.035	√	√
0.05	0.883	2.726	5.134	-0.951	√	-0.067	×	×
0.07	0.883	2.135	4.021	-0.830	√	-0.174	×	×
0.09	0.883	1.713	3.225	-0.691	√	-0.289	×	×
0.11	0.883	1.396	2.628	-0.533	√	-0.411	×	×
0.13	0.883	1.149	2.163	-0.348	√	-0.543	×	×
0.15	0.883	0.951	1.791	-0.132	√	-0.688	×	×
0.17	0.883	0.790	1.487	0.125	×	-0.849	×	×
0.19	0.883	0.655	1.234	0.436	×	-1.031	×	×

TABLE 4 The private benefit B and overinvesting of illiquid assets

B	ρ^{c*}	I^*	I'	Y_2	**Ent.**	Y_1	**Inv.**	**Overinvest**
0.60	0.825	3.383	6.173	-1.008	√	0.092	√	√
0.65	0.825	2.429	4.432	-0.878	√	0.012	√	√
0.70	0.825	1.895	3.458	-0.747	√	-0.068	×	×
0.75	0.825	1.553	2.834	-0.617	√	-0.148	×	×
0.80	0.825	1.316	2.401	-0.487	√	-0.228	×	×
0.85	0.825	1.142	2.083	-0.356	√	-0.308	×	×
0.90	0.825	1.008	1.840	-0.226	√	-0.387	×	×
0.95	0.825	0.902	1.647	-0.096	√	-0.467	×	×
1.00	0.825	0.817	1.491	0.035	×	-0.547	×	×
1.04	0.825	0.759	1.386	0.139	×	-0.611	×	×

Finally, if the growth opportunity is big, the entrepreneur tries to overhoard of liquid assets. In addition, agency costs, which may be measured by $B/\Delta p$ will affect the entrepreneur's investment decisions on liquid assets. In order to prevent the entrepreneur to make wrong investment decisions, it is optimal for lenders to simultaneously impose gearing (leverage) and liquidity ratios.

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