A method for probabilistic decision making with distance measures under 2-tuple linguistic environment

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Abstract

In this paper we develop a new approach for 2-tuple linguistic multiple attribute decision making with probabilistic information, named the probabilistic 2-tuple linguistic ordered weighted averaging distance (P2LOWAD) operator. The main advantage of this operator is that it uses the probabilistic information in a unified framework between 2-tuple linguistic distance measures and the OWA operator that considers the degree of importance of each concept in the aggregation. Some of its main properties and different families are also studied. Moreover, a practical method based on the P2LOWAD operator for multi-criteria decision making with 2-tuple linguistic information is presented. Finally, an illustrative example demonstrates the practicality and effectiveness of the proposed method.

Keywords: 2-tuple linguistic variables; Distance measure; OWA operator; Probabilistic information; Multi-criteria decision making

1 Introduction

Multi-criteria decision-making (MCDM) problems are the important parts of modern decision theory. In real decision making, because the decision-making problems are fuzzy and uncertain, the attribute values are always assessed precisely in a quantitative form but may be in a qualitative one, for example, when evaluating the "comfort" or "design" of a car, terms like "good", "medium", "bad" are usually used instead of numeric values. In such instances, a better approach might be provided by the use of linguistic assessments rather than numerical values. The use of the fuzzy linguistic approach [1] provides a direct way to manage the uncertainty and model the linguistic assessments by means of linguistic variables. In order to effectively avoid the loss and distortion of information in linguistic information processing process, Herrera and Martínez [2] future developed a fuzzy linguistic representation model called 2-tuple linguistic variables. In the literature, Various approaches have been forwarded for dealing with 2-tuple linguistic information [3-8]. For example, Liu [3] presented an approach based on 2-tuple is to solve the hybrid multiple attribute decision making problem with weight information unknown. Merigó and Gil-Lafuente [4] introduced the induced 2-tuple linguistic generalized ordered weighted averaging (2-TILGOWA) operator and studied its application in multi-person linguistic decision-making problem concerning product management. Wei [5] proposed the GRA-based linear-programming methodology for multiple attribute group decision making with 2-tuple linguistic assessment information. Xu and Wang [6] developed some 2-tuple linguistic power aggregation operators. Zeng et al. [7] developed the 2-tuple linguistic generalized ordered weighted averaging distance (2LGOWAD) operator. Very recently, Li et al. [8] introduced a new method for 2-tuple linguistic decision making called the 2-tuple linguistic induced generalized ordered weighted averaging distance (2LIGOWAD) operator.

Recently, Merigó et al. unified the probabilistic information with distance measures and the OWA operator to result in new aggregation operators called the probabilistic OWAD (POWAD) operator [9]. This operator includes the probability and the OWA operator at the same time in the Hamming distance. Moreover, it also permits to analyze distance measures in a probabilistic way considering the attitudinal character of decision maker. More recently, Zeng et al. [10] develop a new extension of the POWAD operator, called the uncertain POWAD (UPOWAD) operator.

The aim of this paper is to develop a new 2-tuple linguistic decision making method called the probabilistic 2-tuple linguistic ordered weighted distance (P2LOWAD) operator. It is an aggregation operator that normalizes the probabilistic decision making information with the 2LOWAD operator [7]. It is also an extension of the 2-tuple linguistic distance measures that unifies the OWA operator and the probability in the same formulation considering the degree of importance that

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each concept has in the aggregation. Thus, we can use the attitudinal character of the decision maker and the probabilistic information of the specific problem considered. In order to do so, the rest of paper is organized as follows. Section 2 discusses a few preliminaries related to 2-tuple linguistic set theory, OWA operator approach. Section 3 presents the P2LOWAD operator. Some of its desirable properties and special cases are also discussed in this section. Section 4 develops an application of the P2LOWAD in a decision making problem and presents a numerical example. Section 5 summarizes the main conclusions of the paper.

2 Preliminaries

This section briefly reviews the linguistic approach, the OWA operator and the POWAD operator.

2.1 LINGUISTIC APPROACH

Let $S=\{s_0,s_1,...,s_g\}$ be a linguistic term set with odd cardinality. Any label, s_i represents a possible value for a linguistic variable, and it should satisfy the following characteristics:

(1) A negation operator: $Neg(s_i)=s_j$, such that $j=g-i$ (g is the Cardinality);

(2) The set is ordered: $s_i \le s_j$ if and only if $i \le j$.

Therefore, there exists a minimization and a maximization operator. For example, a set of seven terms S could be given as follows [11-12]:

$$S= \{s_1=N, s_2=VL, s_3=L, s_4=M, s_5=H, s_6=VH, s_7=P\} \quad (1)$$

Note that N = None, VL = Very low, L = Low, M = Medium, H = High, VH = Very high, P = Perfect.

Herrera and Martínez [2] developed the 2-tuple fuzzy linguistic representation model based on the concept of symbolic translation. It is used for representing the linguistic assessment information by means of a 2-tuple (s_i,α_i) , where s_i is a linguistic label from predefined linguistic term set S and α_i is the value of symbolic translation, and $\alpha_i \in [-0.5,0.5)$.

Definition 1 ([2]). Let β be the result of an aggregation of the indexes of a set of labels assessed in a linguistic term set S , i.e., the result of a symbolic aggregation operation. $\beta \in [0,g]$, being g the cardinality of S. Let $i=round(\beta)$ and $\alpha=\beta-i$ be two values such that $i\in[0,g]$ and $\alpha\in[-0.5,0.5)$, then α is called a symbolic translation.

Definition 2 ([2]). Let $S=\{s_0,s_1,...,s_g\}$ be a linguistic term set and $\beta\in[0,g]$ be a value representing the result of a symbolic aggregation operation, then the 2-tuple that expresses the equivalent information to β is obtained with the following function:

$$\Delta:[0,g]\to S\times[-0.5,0.5) \quad (2)$$

$$\Delta(\beta)=(s_i,\alpha_i),$$

with

$$\begin{cases} s_i, i=round(\beta) \\ \alpha_i=\beta-i, \alpha_i\in[-0.5,0.5) \end{cases} \quad (3)$$

where $round(\cdot)$ is the usual round operation, s_i has the closest index label to β and α_i is the value of the symbolic translation.

Definition 3 ([2]). Let $a=(s_k,\alpha_k)$ and $b=(s_l,\alpha_l)$ be two 2-tuples, they should have the following properties:

(1) If $k<l$, then $a<b$;

(2) If $k=l$, then

(a) If $\alpha_k=\alpha_l$, then $a=b$;

(b) If $\alpha_k<\alpha_l$, then $a<b$;

(c) If $\alpha_k>\alpha_l$, then $a>b$;

In order to measure the deviation between 2-tuple linguistic variables, Liu [3] defined the concept of distance measures for 2-tuple linguistic variables:

Definition 4 Let (s_k,α_k) and (s_l,α_l) be two 2-tuple linguistic variables, then

$$d\left((s_k,\alpha_k),(s_l,\alpha_l)\right)=\frac{\left|(k+\alpha_k)-(l+\alpha_l)\right|}{g} \quad (4)$$

is called a distance between 2-tuple linguistic (s_k,α_1) and (s_l,α_2) .

2.2 THE OWA OPERATOR

The OWA operator [13] provides a parameterized family of aggregation operators that include the maximum, the minimum and the average criteria as special cases. This operator can be defined as follows:

Definition 5 An OWA operator of dimension n is a mapping OWA: $R^n\to R$ that has an associated weighting W with $w_j\in[0,1]$ and $\sum_{j=1}^{n} w_j=1$ such that:

$$OWA(a_1,a_2,...,a_n)=\sum_{j=1}^{n}w_j b_j \qquad (5)$$

where b_j is the j th largest of the a_i .

The OWA operator is commutative, monotonic, bounded and idempotent. This distance operator provides a parameterized family of aggregation operators ranging from the minimum to the maximum distance.

2.3 THE POWAD OPERATOR

The POWAD operator [9] is a new a new index for decision making method that uses the probability and the OWA operator in the normalization process of the distance measure. Thus, the reordering of the individual distances is developed according to the values of the individual distances formed by comparing two sets. For two sets $A=\;a_1,...,a_n$ and $B=\;b_1,...,b_n$, the POWAD operator can be defined as follows:

Definition 6 A POWAD operator of dimension n is a mapping POWAD: $R^n\times R^n\rightarrow R$ that has an associated weighting vector W with $w_j\in[0,1]$ and $\sum_{j=1}^{n}w_j=1$, according to the following formula:

$$POWAD\;\langle a_1,b_1\rangle,...,\langle a_n,b_n\rangle\;=\sum_{j=1}^{n}\hat{p}_j d_j \qquad (6)$$

where d_j is the j th largest individual distance of the $|a_i-b_i|$, each argument $|a_i-b_i|$ has an associated weight (probability) p_i with $\sum_{i=1}^{n}p_i=1$ and $p_i\in[0,1]$, $\hat{p}_j=\beta w_j+(1-\beta)p_j$ with $\beta\in[0,1]$ and p_j is the probability p_i ordered according to d_j, that is, according to the j th largest of the $|a_i-b_i|$.

3 The P2LOWAD operator

The probabilistic 2-tuple linguistic ordered weighted distance (P2LOWAD) is an extension of the POWAD with 2-tuple linguistic information. Therefore, we can consider the probabilistic information, the attitudinal character of the decision maker and distance measures in the same formulation. Moreover, it provides a model that is able to assess the information in situations with high degree of uncertainty by using 2-tuple linguistic variables. Let $X=\{x_i\,|\,x_i=(s_{x_i},\alpha_{x_i}),i=1,2,...,n\}$ and $Y=\{y_i\,|\,y_i=(s_{y_i},\alpha_{y_i}),i=1,2,...,n\}$ ($s_{x_i},s_{y_i}\in S$, $\alpha_{x_i},\alpha_{y_i}\in[-0.5,0.5)$, $i\in N$) be two sets of linguistic 2-tuples, the P2LOWAD operator can be defined as follows.

Definition 7 A P2LOWAD operator of dimension n is a mapping P2LOWAD: $\Omega^n\times\Omega^n\rightarrow R$ that has an associated weighting vector W with $w_j\in[0,1]$ and $\sum_{j=1}^{n}w_j=1$, according to the following formula:

$$P2LOWAD\;(x_1,y_1),...,(x_n,y_n)\;=\sum_{j=1}^{n}\hat{p}_j d\;\;x_j,y_j \qquad (7)$$

where $d\;\;x_j,y_j$ is the j th largest individual distance of the $d\;\;x_i,y_i$, and $d\;\;x_i,y_i$ is the argument variable represented in the form of distance defined by (6), which has an associated weight (probability) p_i with $\sum_{i=1}^{n}p_i=1$ and $p_i\in[0,1]$, $\hat{p}_j=\beta w_j+(1-\beta)p_j$ with $\beta\in[0,1]$ and p_j is the probability p_i ordered according to d_j , that is, according to the j th largest of the $d\;\;x_i,y_i$.

Note that it is also possible to formulate the P2LOWAD operator separating the part that strictly affects the 2LOWAD operator and the part that affects the probabilistic distance. This representation is useful to see both models in the same formulation but it does not seem to be as a unique equation that unifies both models.

Proposition 1 A P2LOWAD operator is a mapping P2LOWAD: $\Omega^n\times\Omega^n\rightarrow R$ of dimension n, if it has an associated weighting vector W with $w_j\in[0,1]$ and $\sum_{j=1}^{n}w_j=1$ and a probabilistic vector P , with $\sum_{i=1}^{n}p_i=1$ and $p_i\in[0,1]$, such that:

$$\begin{aligned}&P2LOWAD\;(x_1,y_1),...,(x_n,y_n)\\&=\beta\sum_{j=1}^{n}w_j d\;\;x_j,y_j\;+(1-\beta)\sum_{i=1}^{n}p_i d\;\;x_i,y_i\end{aligned} \qquad (8)$$

where $d\;\;x_j,y_j$ is the j th largest of the $d\;\;x_i,y_i$, and $\gamma\in[0,1]$.

In the following example, we present a simple numerical example showing how to use the P2LOWAD operator in an aggregation process.

Example 1 Let S be a linguistic term set defined by (1), $X=\;x_1,x_2,x_3,x_4\;=\;(s_4,\alpha_{0.4}),(s_1,\alpha_{0.2}),(s_5,\alpha_{-0.2}),(s_6,\alpha_0)$ and $Y=\;y_1,y_2,y_3,y_4\;=\;(s_4,\alpha_{-0.4}),(s_4,\alpha_{0.3}),(s_7,\alpha_{-0.3}),(s_2,\alpha_{0.2})$ be two sets of linguistic 2-tuples, then

$$d(x_1, y_1) = d\left((s_4, \alpha_{0.4}), (s_4, \alpha_{-0.4})\right) = \frac{|(4+0.4)-(4-0.4)|}{8} = 0.1 .$$

Similarly, we have

$$d(x_2, y_2) = 0.388,\ d(x_3, y_3) = 0.238,$$

$$d(x_4, y_4) = 0.475 .$$

Assume the following weighting vector $W = (0.2, 0.2, 0.3, 0.3)$ and the following probabilistic weighting vector $P = (0.2, 0.4, 0.1, 0.3)$. Note that the probabilistic information has a degree of importance of 70% while the weighting vector W a degree of 30%. If we want to aggregate this information by using the P2LOWAD operator, we will get the following result. The aggregation can be solved either with Eq. (7) or Eq. (8).

With Eq. (7) we calculate the new weighting vector as:

$$\hat{p}_1 = 0.3 \times 0.2 + 0.7 \times 0.3 = 0.27$$

$$\hat{p}_2 = 0.3 \times 0.2 + 0.7 \times 0.4 = 0.34$$

$$\hat{p}_3 = 0.3 \times 0.3 + 0.7 \times 0.1 = 0.16$$

$$\hat{p}_4 = 0.3 \times 0.3 + 0.7 \times 0.2 = 0.23$$

And then, we calculate the aggregation process as follows:

$$P2LOWAD = 0.27 \times 0.475 + 0.34 \times 0.388 + 0.16 \times 0.238 + 0.23 \times 0.1 = 0.321$$

With Eq. (8), we aggregate as follows:

$$P2LOWAD = 0.3 \times (0.2 \times 0.475 + 0.2 \times 0.388 + 0.3 \times 0.238 + 0.3 \times 0.1) + 0.7 \times (0.2 \times 0.1 + 0.4 \times 0.388 + 0.1 \times 0.238 + 0.3 \times 0.475) = 0.321$$

Obviously, we get the same results with both methods. From a generalized perspective of the reordering step, we can distinguish between the descending P2LOWAD (DP2LOWAD) operator and the ascending P2LOWAD (AP2LOWAD) operator by using wj = w*n–j+1, where wj is the jth weight of the DP2LOWAD and w*n–j+1 the jth weight of the AP2LOWAD operator. Similar to the POWAD operator, the P2LOWAD operator is also monotonic, bounded and idempotent. The P2LOWAD operator provides a parameterized family of aggregation operators. Basically,

(1) If $\beta = 1$, we get the 2LOWAD operator. With the 2LOWAD operator it is also possible to obtain another parameterized family of aggregation operators such as the IFNHD, the IFWHD, the IFWED, the IFOWAD and the IFOWED operator.

(2) If $\beta = 0$, we get the probabilistic 2-tuple linguistic weighted distance distance (P2LWD) approach.

The more β approaches to 1, the more importance we give to the 2LOWAD operator, and vice versa. Note that we could develop a lot of other families of P2LOWAD weights in a similar way as it has been developed in a lot of studies [14-17].

4 Decision making with the P2LOWAD operator

The P2LOWAD operator is applicable in a wide range of situations, such as decision making, statistics, engineering and economics. In this section, we focus on an application in the selection of investments. We analyze a company that operates in Europe and North America that wants to invest some money in a new market (adapted from [10]). They consider five alternatives:

(1) A_1 =Invest in the Asian market;

(2) A_2 =Invest in the South American market;

(3) A_3 =Invest in the African market;

(4) A_4 =Invest in all three markets;

(5) A_5 =Do not invest money in any market;

In order to evaluate these investments, the investor considers that the key factor is the economic situation of the world economy for the next period. He considers the following five possible states of nature that could happen in the future:

(1) S_1 = Very bad economic situation.

(2) S_2 = Bad economic situation.

(3) S_3 = Regular economic situation.

(4) S_4 = Good economic situation.

(5) S_5 = Very good economic situation.

Due to the fact that the general characteristics are very imprecise because they contain a lot of particular aspects, the decision maker cannot use numerical values in the analysis. Instead, he uses the linguistic variables to evaluate the general results obtained for each candidate depending on the characteristic considered.

After careful analysis of these characteristics, the decision maker has given the following information shown in Table 1. Note that the results are linguistic values represented with the 2-tuple linguistic approach.

He also establishes the values of an ideal investment as it is shown in Table 2. This ideal investment represents the optimal results for the company.

TABLE 1. Available investment strategies

	C1	C2	C3	C4	C5
***A*₁**	$(s_4,\alpha_{0.1})$	$(s_3,\alpha_{-0.2})$	$(s_3,\alpha_{-0.2})$	$(s_3,\alpha_{0.2})$	$(s_5,\alpha_{0.2})$
***A*₂**	$(s_4,\alpha_{-0.2})$	(s_4,α_{0})	$(s_3,\alpha_{0.4})$	$(s_2,\alpha_{0.4})$	$(s_5,\alpha_{0.1})$
***A*₃**	$(s_5,\alpha_{0.3})$	$(s_6,\alpha_{0.2})$	$(s_5,\alpha_{-0.3})$	$(s_5,\alpha_{0.1})$	$(s_4,\alpha_{0.1})$
***A*₄**	$(s_5,\alpha_{-0.2})$	$(s_3,\alpha_{0.1})$	$(s_7,\alpha_{0.2})$	$(s_6,\alpha_{0.1})$	(s_6,α_{0})
***A*₅**	$(s_2,\alpha_{0.2})$	$(s_1,\alpha_{0.2})$	$(s_4,\alpha_{0.3})$	$(s_5,\alpha_{0.2})$	$(s_4,\alpha_{0.3})$

TABLE 2 Ideal investment strategy

	C1	C2	C3	C4	C5
I	$(s_6,\alpha_{0.3})$	$(s_7,\alpha_{0.2})$	$(s_7,\alpha_{0.2})$	$(s_6,\alpha_{0.3})$	$(s_7,\alpha_{0.4})$

In this problem, the experts of the company find probabilistic information given as follows: $P=(0.3,0.2,0.3,0.1,0.1)$, $\gamma=50\%$ and the OWA weighting vector $W=(0.1,0.2,0.15,0.25,0.3)$. With this information, it is now possible to aggregate the available information in order to take a decision. Utilize the P2LOWAD operator to calculate the distance between the available robots with the ideal one, we obtain:

$$P2LOWAD(A_1,I)=0.237,$$

$$P2LOWAD(A_2,I)=0.293,\ P2LOWAD(A_3,I)=0.288,$$

$$P2LOWAD(A_4,I)=0.221,\ P2LOWAD(A_5,I)=0.254$$

The optimal choice would be the alternative closest to the ideal. Rank all the alternatives and select the best one(s) according to the $P2LOWAD(A_i,I)(i=1,2,3,4,5)$:

$$A_4 \succ A_1 \succ A_5 \succ A_3 \succ A_2$$

As we can see, the best one is the A_4 , namely, invest in all three markets.

Furthermore, in order to analyze how the different particular cases and the parameter of the P2LOWAD operator have affection for the results, in this example, we consider the 2LOWAD, P2LWD and some different parameter β. The results are laisted in the table 3.

TABLE 3. The Ordering obtained by the particular cases of the P2LOWAD approach

particular cases of the P2LOWAD	Ordering
2LOWAD	$A_4 \succ A_1 \succ A_5 \succ A_3 \succ A_2$
P2LWD	$A_4 \succ A_3 \succ A_5 \succ A_1 \succ A_2$
P2LOWAD($\beta=0.1$)	$A_3 \succ A_4 \succ A_5 \succ A_3 \succ A_2$
P2LOWAD($\beta=0.4$)	$A_4 \succ A_5 \succ A_1 \succ A_3 \succ A_2$
P2LOWAD($\beta=0.8$)	$A_5 \succ A_4 \succ A_1 \succ A_3 \succ A_2$

As we can see, depending on the particular cases of the P2LOWAD operator used, the ordering of the airlines is different. Due to the fact that each particular family of P2LOWAD operator may give different results, the decision maker will select for his decision the one that is closest to his interests.

5 Conclusions

In this paper, we have introduced a new method for probabilistic decision making with distance measures under 2-tuple linguistic environment, named the P2LOWAD operator. This operator uses the main characteristics of the probability, the OWA operator, the distance measure and uncertain information represented in the form of 2-tuple linguistic information. We have also presented an application of the new approach in a decision making problem concerning the selection of investment. We have seen that the P2LOWAD is very useful because it represents very well the uncertain information by using 2-tuple linguistic information. Moreover, it shows a lot of different scenarios that could happen depending on the particular type of P2LOWAD operator used in the problem. Therefore, decision makers can consider many different scenarios depending on his interests or actual needs.

In future research we expect to develop further extensions by adding new characteristics in the problem such as the use of order-inducing variables. We will also consider other decision making applications such as human resource management and product management.

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