Source enumeration algorithm based on eigenvector: revisit from the perspective of information theory

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Received 11 February 2014, www.tsi.lv

Abstract

In case of low signal to noise ratio (SNR) and small snapshot condition, it is difficult to separate sources and noises, and the performance of classical eigenvector source estimation algorithm drops quickly. To solve the problem, further research is carried out around the characters of eigenvalue and eigenvector, and a novel eigenvalue algorithm is presented based on the theory of source enumeration. In detail, the eigenvectors of sample covariance matrix are employed as the decision factor, which is insensitive to SNR. And an improved Predictive Description Length (PDL) criterion is adopted to enumerate source number. Theoretical analysis and simulation results demonstrate that the proposed algorithm is available and efficient in case of low SNR and small snapshot condition compared with those of Minimum Description Length (MDL) and PDL.

Keywords: source enumeration, eigenvector, signal-to-noise ratio, information theory, predictive description length

1 Introduction

The estimation signal number in noise background is one of the key problems in array signal processing, and widely applied in radar, communication, biomedical, seismic signals and the field of electronic countermeasures and so on. The accuracy number of signals is the prerequisite for many super-resolution array processing algorithm. If the estimated signal number was different with the accurate number of signals, many super-resolutions estimation algorithms would suffer from severe performance degradation. While in the condition of low Signal to Noise Ratio (SNR) and small snapshot, the process of source enumeration becomes much more complex and difficult. Among classic subspace decomposition estimation algorithms for Difference of Arrival (DOA) [1], source number is critical in dividing signal eigenvector from noise eigenvector, which is applied in spanning the independent noise subspace, and signal subspace as well [2]. When estimated source number is distorted or even deviated, the orthogonality between the two subspaces will be damaged [3, 4], which will lead to DOA estimation performance degradation directly. For source number estimation, the algorithm based on information theory criterion is now declared the most common and effective, such as the sequence hypothesis criterion, Akaike information criterion (AIC) [5,6] and minimum description length (MDL) criterion [7]. However, for information theory criterion and algorithms of source number estimation and their improved algorithms, the estimation performance reduction is more serious under the condition of low *SNR* or missing snapshot. The algorithm based on *MDL* criterion is used to multiple coherent source number's estimation and positioning [3], and an improved algorithm applying random geometry-array and covariance matrix noise is proposed [7], but a large number of arrays are required and difficult to realize effective estimation at low SNR. As for the performance and calculation speed, the recursion method of PDL criterion [8-10] provided lately have great improvement than MDL and G&T criterion [11].

All the above algorithms are based on covariance matrix's eigenvalue, and the performance is decided by the eigenvalue capability in dividing signal eigenvalue and noise eigenvalue. However, in case of limited snapshot data, the division process is quite difficult, especially at low SNR. The solution is to change eigenvalue, in which a reconstructed eigenvalue cluster at low SNR is used to extract coherent and non-coherent signal in BEM criterion [7, 8]. The performance of eigenvalue is depressed seriously by various noises, and the eigenvector is also used to replace eigenvalue to estimate source number [12].

In the paper, a source number estimation algorithm is presented based on sampling covariance matrix eigenvector. Eigenvector of sampling covariance matrix is used to construct judgment variable in the algorithm, and the number of source is estimated according to improved predictive description length *(PDL)* criterion. The paper is organized as follows. The second part gives the preliminary of signal model, and the eigenvector algorithm and source enumeration is provided in third part. The fourth part demonstrates the simulation and the results, and the last gives the conclusion.

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2 Signal model

Denote *N*-Array spaced array has *M* (*M* < *N*) signals with narrow-band signal and fixed centre frequency from far-field source shooting with angles $\theta_k \quad (k = 1, 2, 3, ..., M)$. Denote $X(t)$ the complex envelope receiving vector of antenna arrays [10], and then: $X(t) = [x_1(t), x_2(t), ..., x_N(t)]^T$ can be expressed as follows at snapshot time *t*:

$$X(t) = A(\theta)S(t) + N(t) = \sum_{i=1}^{M} \alpha(\theta_i) \cdot s_i(t) + N(t), \quad t = 1, 2, 3, ... \qquad (1)$$

In the Equation (1), $S(t) = [s_1(t), s_2(t), \cdots, s_N(t)]^T$ is the complex envelope vector of *M* receiving source signals, $N(t) = [n_1(t), s_2(t), \cdots, n_N(t)]^T$ complex Gauss noise vector of antenna arrays, and $A(\theta) = [a_1(\theta_1), a_2(\theta_2), ..., a_N(\theta_M)]$ manifold array matrix with $N \times M$ dimensions. Element $a(\theta_k)(k = 1, 2, ..., M)$ is a steering vector with $M \times 1$ dimensions aiming at wave direction and a_i can be expressed as follows:

$$a_i = [e^{-j\mu_{1i}}, e^{-j\mu_{2i}}, ..., e^{-j\mu_{Mi}}]^T, \qquad (2)$$

where $\mu_{mi} = \frac{2\pi}{\lambda}[x_m \cos\theta_i \cos\varphi_i + y_m \sin\theta_i \cos\varphi_i]$, (x_m, y_m) is the *m* array signal position, and (θ_i, φ_i) is the parameter of incident angle.

Covariance matrix of original receiving data vector $X(t)$ can be expressed as follow:

$$R = E[X(t)X^H(t)]. \qquad (3)$$

In the Equation (3), *E*[ ] denotes the mathematical expectation, and "*H*" denotes Hermitian transformation. And then the covariance matrix [13] can be decomposed as follow:

$$R = U\sum^2 U^H = U_S \sum_N^2 U_S^H + U_N \sum_N^2 U_N^H, \qquad (4)$$

where U_S is the matrix consisting of eigenvectors corresponding with top *M* large eigenvalues, and $\sum^2 = diag(\lambda_1, \lambda_2, ..., \lambda_N) = diag(\sigma_1^2, \sigma_2^2, ..., \sigma_N^2)$, the diagonal matrix consisting of eigenvalues, U_N the matrix consisting of eigenvectors corresponding with last $N - M$ small eigenvalues. Elements of diagonal matrix $\sum_S^2 = diag(\lambda_1, \lambda_2, \cdots, \lambda_M)$ consist of the top *M* large eigenvalues corresponds with U_S eigenvalues. Elements of diagonal matrices $\sum_S^2 = diag(\lambda_{M+1}, \lambda_{M+2}, \cdots, \lambda_N)$ denote the last $N - M$ small eigenvalues corresponding to U_N eigenvalues. Eigenvalues meet the condition $\lambda_1 > \lambda_2 > ... > \lambda_M > \lambda_{M+1} = \lambda_{M+2} = ... = \lambda_N = \sigma^2$. Source number can be determined by the number of the last $N - M$ eigenvalues.

To determine the number of *M* eigenvalues, principle component analysis (PCA) and similar theory provide some methods [13]. The popular power method is based on the ratio of eigenvalues, in which the sum of *M* large eigenvalues is around 85% to sum of all eigenvalues. The ratio can be expressed as follow:

$$\frac{\lambda_1 + \lambda_2 + ... + \lambda_M}{\lambda_1 + \lambda_2 + ... + \lambda_M + ... + \lambda_N} \approx 85\% \qquad (5)$$

and signal-to-noise model provided another method in the case of low noise. In detail, relationship between eigenvalues can be expressed as follow:

$$\lambda_1 > \lambda_2 > ... > \lambda_M >> \lambda_{M+1} \approx \lambda_{M+2} \approx ... \approx \lambda_N \approx \sigma^2 \qquad (6)$$

in which the variances of noises are much smaller than those of signals, and around the value of σ^2.

In practical consideration, for the reason that matrix *R* is decided by limited snapshot points, and the eigenvalues are different and difficult to be evaluated at low SNR and small snapshot, it is difficult to accurately estimate the source number.

3 Source enumeration via eigenvectors

3.1 TRANSFORM OF EIGENVECTOR

Define signal subspace as the subspace spanned by U_S, and noise subspace spanned by U_N, where the two subspaces are orthogonal. For the subspace spanned by U_S and subspace spanned by *A* belong to the same signal subspace, and U_S's column vectors are basic vectors for signal subspace, the steering vector $a(\theta_i)(i = 1, 2, ..., M)$ can be expressed as follow:

$$a(\theta_i) = \sum_{j=1}^{M} c_{ij} \cdot s_j \qquad (7)$$

in which, S_j is the basic vector for signal subspace and column vectors of U_S, $c_{ij}(i, j = 1, 2, ..., M)$ the linear combination index for U_S.

Then it can be obtained that the noise subspace and steering vector subspace spanned from *A* are mutually orthogonal. That is:

$$n_K^H \alpha(\theta_i) = 0, (i = 1, 2, ..., N - M). \qquad (8)$$

Define vector $y_k(k = 1, 2, ..., N - M)$ the function of column vectors n_K^H and $X(t)$, and then y_k can be expressed as follow:

$$y_k(t) = n_K^H X(t) = n_K^H \cdot \left(\sum_{i=1}^{M} \alpha(\theta_i) s_i(t) + N(t) \right) = . \quad (9)$$
$$n_K^H N(t) = \omega_{NK}(t).$$

Similarly, eigenvectors of U_S column vectors can be defined as standard orthogonal basis, and a new vector $z_i(t)(i=1,2,...,M)$ can be obtained.

$$z_i(t) = S_i^H X(t) = S_i^H \left(\sum_{j=1}^{M} \alpha(\theta_j) s_j(t) \right) + S_i^H N(t) =$$
$$\sum_{j=1}^{M} \left[\sum_{\rho=1}^{M} c_{j\rho} s_i^H s_\rho s_j(t) \right] + S_i^H N(t) = \quad , \quad (10)$$
$$\sum_{j=1}^{M} c_{jt} s_j(t) + \omega_{Ni}(t),$$

where $\omega_{Nk}(t)$ and $\omega_{Ni}(t)$ are independent and identically distributed complex Gauss random vector, with zero mean and σ^2 variance.

From the Equations (9) and (10), it can be concluded that if the steering vector is weight disposed by noise subspace eigenvectors, the output variable will not comprise any signal component. Otherwise, if the steering vector is weighted by signal subspace eigenvectors, the output variable will comprise both signal component and noise component. Therefore, column vector of snapshot time *t* can be expressed as follow:

$$d(t) = [z_1(t),..., z_M(t), y_1(t),..., y_{N-M}(t)]^T \quad (11)$$

and observation matrix $D = [d_1, d_2, \cdots, d_T]$ is constructed by *T* snapshots. Each column's power spectrum and peak value P_n of matrix *D* can be obtained via Fast Fourier Transform (*FFT*). And then the judgment variable $q_n(n = 1, 2, \cdots, N)$ are defined as follow:

$$q_n = N \cdot \frac{p_n}{\sum_{i=1}^{N} p_i}. \quad (12)$$

3.2 ALGORITHMS BASED ON INFORMATION THEORY

Information theory criterion is to solve the question of pattern recognition. For given observe signal $X = [X_1, X_2, ..., X_n]$, the purpose of information theory criterion is to choose the most matching model in the serial of parameter probability model $f(x|\Theta)$ [14,15]. When observed signals are narrow band and satisfy independent and identically distributed (*i.i.d*), and noise is Additive White Gauss Noise (*AWGN*), the Akaike information criterion (*AIC*) and minimum description length (*MDL*) criterion are expressed as follows:

$$MDL(M) = L(N-M)\ln\Lambda(M) + \frac{1}{2}M(2N-M)\ln L \quad , (13)$$
$$AIC(M) = 2L(N-M)\ln\Lambda(M) + 2n(2M-N)$$

where $\Lambda(M)$ can be denoted by the Equation (14).

$$\Lambda(M) = \frac{\frac{1}{N-M}\sum_{i=M+1}^{N}\lambda_i}{\left(\prod_{i=M+1}^{N}\lambda_i\right)^{\frac{1}{N-M}}} \quad (14)$$

and when *AIC* value (or *MDL value)* is minimized, the corresponding value *M* is just the number of estimated sources.

3.3 IMPROVED JUDGMENT CRITERION

According to the Equation (2), the covariance matrix of original data $X(t)$ can also be expressed as follow:

$$R = AR_s A^H + \sigma^2 I \, , \quad (15)$$

where R_s is the covariance matrix, σ^2 noise variance, and matrix *A* the function of incident azimuth vectors $\theta_k = [\theta_1, \theta_2, \cdots, \theta_M]^T$.

Covariance matrix *R* can also be viewed as the function of parameter set $\mathbf{\Phi} = \left[M, \theta_k, \sigma^2, R_s\right]$, and the conditional probability density function of receiving data vector is as [5]:

$$f(X \mid \mathbf{\Phi}) = \frac{1}{\pi^n |R|} \exp\left\{-X^H (R)^{-1} X\right\}, \quad (16)$$

where symbol $|R|$ stands for matrix's determinant. The aim of source number estimation is to estimate source number *M* and then the incidence angle vector $\theta_k = [\theta_1, \theta_2, \cdots, \theta_M]^T$.

Define *PDL* of *L* data column vector as follows:

$$P(L) = -\sum_{i=1}^{L} \log f(X(t) \mid \Phi_{t-1}) \, , \quad (17)$$

where Φ_{t-1} is Maximum Likelihood (*ML*) estimator of all receiving data vector before time *t*. For any given moment, parameter vectors' estimators are based on previous observation, thus it is called prediction of description length estimation.

To estimate source number, the following cost function is designed [5]:

$$PDL(m) = \arg\min_{m \in K} P(L) =$$
$$\arg\min_{m \in K}\left[-\sum_{i=1}^{L} \log f(X(t) \mid \Phi_{t-1})\right], \quad (18)$$

where $K = \{0,1,\cdots,N-1\}$. And source number m is estimated only when $PDL(m)$ is minimized.

From the classic PDL judgment criterion provided in [8], $P(L)$ can be redefined as follows:

$$P(L) = -\sum_{i=1}^{L} \log f(x_i(t) \mid \lambda_1) . \quad (19)$$

Among which, λ_i is the eigenvalue of Φ_{t-1}, $x_i(t)$ the i-th receiving data of antenna arrays.

Replace λ_i with judgment variable q_n, and the source number estimation criterion can be expressed as follows:

$$P(L) = -\sum_{i=1}^{L} \log f(x_i(t) \mid q_i) \quad (20)$$

Therefore, the process of source number estimation algorithm can be concluded in following steps.

Step1: Construct covariance matrix R;

Step2: Calculate corresponding eigenvectors by the eigenvalues of R;

Step3: Weight $X(t)$ with eigenvectors and obtain observation matrix D;

Step4: Calculate each column's power spectrum of D;

Step5: Calculate the peak value and q_n in the Equation (12);

Step6: Confirm source number according to the Equation (20).

4 Simulation

In the simulations, different noise conditions are considered, such as non-stationary, non-Gaussian or even colored, and the proposed algorithm is compared with those of MDL and PDL. Simulations demonstrate similar results that the proposed algorithm has better performance in the case of low SNR and slow snapshot, and for simplicity only part of simulation process and results are demonstrated here.

4.1 SIMULATION CONDITION

Monte-Carlo numerical simulation method is adopted to estimate source number estimation algorithm's performance. Simulation parameters are set as follows:

- Antenna arrays is an eight uniform array, and array element spacing is 0.5λ.
- At every Monte-Carlo simulation, snapshot number is set as 8192.
- Input signals are three incoherent narrow-band signal (BPSK signal with 32Kbps), and the incidence angles are 45°, 80° and 89°.
- The channel is AWGN channel.

4.2 SIMULATION RESULTS

For given SNR, average value of 200 times Monte-Carlo simulation results is employed to obtain the testing success probability. When $Eb/N0 = 0db$, the distribution result and comparison of judgment variable q_n and eigenvalue in proposed algorithm are shown in Figure 1. From Figure 1, it can be seen that after the arrays output vector weighted, the difference between q is quite obvious. For the value of q array, the top three ones are larger than the last five ones, and the last five ones are almost equal. All the eigenvalue are almost equal expect the first one, which provides guarantee for next source number estimation.

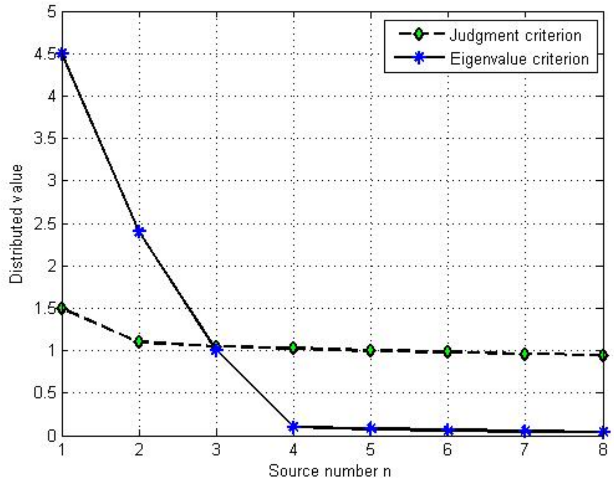

FIGURE 1 Distribution of judgment variance and eigenvalue

Simulation provides the change between provided algorithm's, MDL's and PDL's estimation success probability in Figure 2. There the mean values are used to balance the probability at each $Eb/N0$.

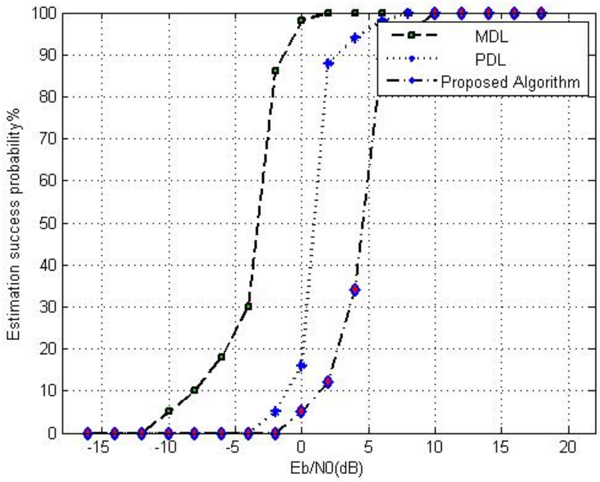

FIGURE 2 Estimation success probability when the value of *Eb/N0* changes

In detail, Figure 2 shows that when $Eb/N0 \geq 6db$ (signal input SNR is high), the proposed algorithm has similar estimation success probability with that of MDL and PDL. But in the case that the input SNR is low, estimation success probability of proposed algorithm is higher than those of *MDL* and *PDL*. Under the condition that estimation success probability is 90%, proposed algorithm has almost 4dB improvement than that of *PDL* and 6dB improvement than that of *MDL*.

5 Conclusion

A new source number estimation algorithm is provided to solve the problem in case of low SNR. In the proposed algorithm, eigenvectors of covariance matrix is adopted to replace eigenvalue to obtain a new judgment variable, and then the characteristics of source number are more effectively displayed. Therefore, the improved PDL judgment criterion is used to implement the estimation. The simulation results shows that the algorithm provided has obvious improvement in SNR compared with classic MDL and PDL at the same estimation success probability.

Acknowledgement

The results discussed in this paper are part of the theses works of UAV data link and satellite communication anti-jamming system. This work is supported by National Natural Science Foundation of China (61174162), and Shaanxi Provincial Science & Technology Department (Program No. 2011K06-36), and Shaanxi Provincial Education Department (Program No. 2013JK1145) and the second batch of College Scientific Research Foundation in 2013 (Program No. XJ130225).

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