Blind multi-image super resolution reconstruction with Gaussian blur and Gaussian noise

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Abstract

A framework of blind multi-image super resolution reconstruction method is proposed to improve the resolution of low resolution images with Gaussian blur and noise. In the low resolution imaging model, the shift motion, Gaussian blur, down-sampling, as well as Gaussian noise are all considered. Firstly, the Gaussian noise in the low resolution image is reduced through Wiener filtering method. Secondly, the Gaussian blur of the de-noised image is estimated through error-parameter analysis method. Thirdly, the motion parameters are estimated. Finally, super resolution reconstruction is performed through iterative back projection algorithm. Experimental results show that the Gaussian blur and motion parameters are estimated with high precision, and that the Gaussian noise is restrained effectively. The visual effect and peak signal to noise ratio (PSNR) of the super resolution reconstructed image are enhanced. The importance of Gaussian blur estimation and effect of Gaussian de-noising in multi-image super resolution reconstruction are tested in an experimental way.

Keywords: blind, multi-image super resolution, Gaussian blur, Gaussian noise, iterative back projection

1 Introduction

High resolution (HR) images are often required in many imaging applications. HR means that the pixel density within and image is high, and can offer more details. To improve the spatial resolution of image, the most direct way is to improve the precision and stability of the imaging system with expensive cost and some technical difficulties. Super resolution (SR) method is an efficient way with lower cost than hardware method.

In general, SR includes video SR [1] and image SR. Video SR refers to reconstructing a higher resolution video form a low resolution (LR) video by utilizing the redundancy information between the adjacent frames and the prior information of the imaging system. Image SR refers to reconstructing a higher resolution from one image or a set of images acquired from the same scene. On the basis of the number of the LR images, image SR image SR mainly includes multi-image SR [2-4] and single-image SR [5-7]. Multi-image SR is commonly researched, in which the movement with sub-pixel precision is estimated and utilized to reconstruct a HR image. Thus, image registration is very important in multi-image SR.

In many practical applications, the image restoration problem is always blind, which means that the PSF is most likely unknown or is known only to within a set of parameters [8]. In iterative back projection (IBP) SR reconstruction algorithm, the more accurate the imaging model is estimated, the better quality of the reconstructed image will be achieved. However, in most of the current algorithms, the blur is assumed to be a known Gaussian point spread function (PSF) with given parameters, or the blur is not considered at all in some algorithms, which does not meet the real imaging model of optical devices and limits the SR reconstruction quality. Thus, the blind image SR reconstruction [9-10] is one advanced issue and challenge in image restoration, which is expressed as estimating a HR image and the PSF simultaneously. The foremost difficulty of blind de-blurring is rooted in the fact that the observed image is an incomplete convolution. The convolution relationship around the boundary is destroyed by the cut-off frequency, which makes it much more difficult to identify the blurring function.

In addition, the process of noise is seldom considered in the LR imaging model, which restrained the quality of the SR reconstructed image. The noise can worsen the quality of images and bring some difficulty to image analyzing. Thus, noise should be considered in the framework of multi-image SR reconstruction.

In this paper, a framework of blind multi-image SR reconstruction method with Gaussian blurs and noise is proposed. In the LR imaging model, the processes of Gaussian blur, down-sampling, as well as noise are all considered. The Gaussian noise is reduced through Wiener filtering method. The Gaussian blur of the de-noised LR image is estimated through error parameter analysis method. The SR image is reconstructed through iterative back projection (IBP) algorithm.

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2 Framework of blind multi-image SR reconstruction with Gaussian blur and noise

The framework of multi-image SR reconstruction with Gaussian blur and noise is shown in Fig.1. Firstly, the Gaussian noise in the low resolution image is reduced through Wiener filtering algorithm. Secondly, the movement parameters between the de-noised LR images are estimated. Thirdly, the Gaussian blur of the de-noised image is estimated through error-parameter analysis method. Finally, super resolution reconstruction is carried out through iterative back projection algorithm.

2.1 THE LR IMAGING MODEL

In the LR imaging model, the shift movement, Gaussian blur, down-sample, and Gaussian noise are all considered, as shown in Fig.1. The mathematical description of LR imaging model of multi-image SR reconstruction may be expressed as follows:

$$Y=EBDF+N, \tag{1}$$

where, Y is the LR image; F is the HR image; E is the movement; B is the blur function; D is the down-sample process; N is the noise.

The real scene may be expressed by a high resolution (HR) image. Firstly, the HR is moved vertically and horizontally. The moved image is blurred by convolving with a point spread function (PSF). The blur mainly includes the Gaussian blur induced by the optical devices of the imaging system, the motion blur caused by the movement of the scene or the camera, as well as the defocus blur bringing by the false focus while imaging, etc. As Gaussian blur is the most common and is considered here. Secondly, the blurred image is down-sampled by a given integer factor. Here, the down-sampled image is gained by taking the neighborhood average gray value of the blurred image. Thirdly, the down-sampled is noised to generate the LR image. Here, the Gaussian noise is considered.

2.2 ITERATIVE BACK PROJECTION METHOD

Among the current SR reconstruction methods, the iterative back projection (IBP) method has the virtues of small computational amount, fast convergent rate, good reconstruction effect, and so on. In addition, the estimated information about the LR imaging model can be well utilized in the IBP algorithm.

In IBP algorithm, If the LR imaging model is estimated more accurately, the SR reconstructed image will achieve better quality. By back projecting the estimation error between the estimated LR image and the original image onto the HR image grid, the estimation error is gained to modify to estimated HR image. Repeating the iterative process until the iteration time is greater than a given number or the estimation error is less than a threshold, the SR image will be gained.

According to this idea, the IBP algorithm may be expressed as follows:

$$\hat{f}_i^{k+1} = \hat{f}_i^k - \lambda H_i^{BP}(\hat{g}_i^k - g_i^{'}) . \tag{2}$$

Here, the initial value of the estimated HR image is taken as the interpolated image of the LR image by Bilinear interpolation algorithm. According to the LR imaging model proposed in this paper and the idea of IBP algorithm, the framework of the multi-image SR reconstruction method with noise is shown in Fig.1. Here, P is the number of LR images, i=1,…,P; k is the iteration time; $\hat{f}$ is the estimated SR image; g is the observed LR image; y' is the de-noised LR image; $\hat{g}$ is the simulated LR images of $\hat{f}$; E, B and D are the matrix forms of the motion blur and down-sampling respectively; n is the system noise; E^{-1}, B^{-1}, D^{-1} and n^{-1} denote the inverse operation of E, B, D and n; H^{BP} is the back projection operation; $\hat{g} - g'$ is the difference of simulated LR image and the de-noised LR image; λ is the gradient step.

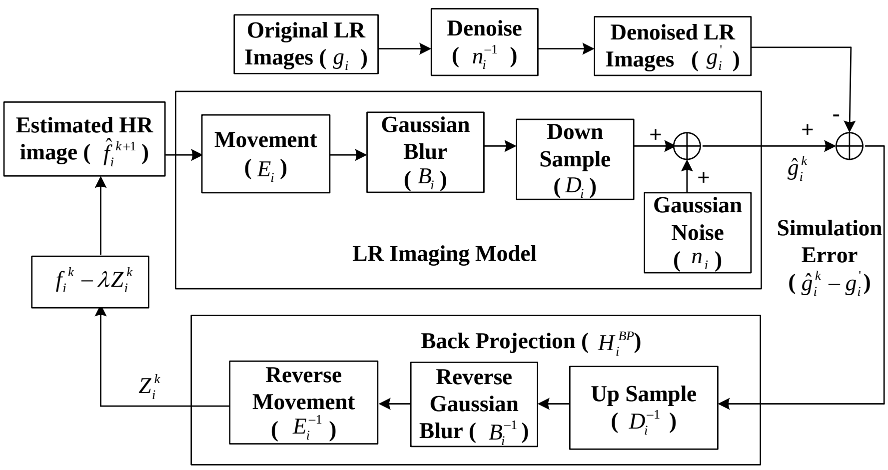


FIGURE 1 Framework of blind multi-image SR reconstruction with Gaussian blur and noise

2.3 WIENER FILTERING DE-NOISING

In Wiener filtering algorithm, both the blur function and the statistical character of system noise are considered. The noise is assumed to be a random process. The aim is to make the mean square error between the original image and the estimated image to be the least. According to this idea, the sketch map of Wiener filtering may be expressed in Fig.2.

The observed low resolution image may be expressed as follows:

$$y(n)=\sum_{k=-\infty}^{\infty} x(n-k)h(k)+\xi(n)=x(n)*h(n)+\xi(n), \tag{3}$$

Where, * is the convolution operation; $x(n)$ is the original high resolution image; $\xi(n)$ is Gaussian white noise with zero mean; $h(n)$ is the blur function, which can be presented by point spread function (PSF).

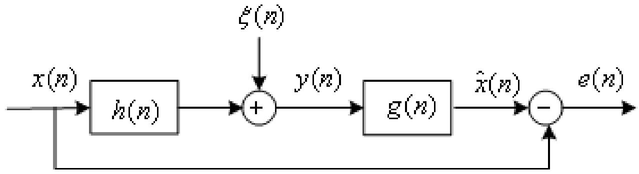


FIGURE 2 The sketch map of Wiener filtering.

When the discrete Fourier transform (DFT) method is used to estimate the restored image, the Wiener filter may be expressed as follows:

$$X=\frac{H^{*}Y}{|H|^{2}+S_{nn}/S_{xx}}, \tag{4}$$

where, X, Y and H are the DFT of the real image (x), the blurred image (y) and the blur function (h) respectively; S_{nn} and S_{xx} denote the power spectrum of the noise and the real image. As it is usually very difficult to estimate S_{nn} and S_{xx}, the Wiener filter is usually approximated by the following formula:

$$X=\frac{H^{*}Y}{|H|^{2}+\Gamma}, \tag{5}$$

where, Γ is a positive constant, which is often taken as an experience value.

2.4 MOVEMENT ESTIMATION

Here, the globle movement with vertical shift and horizontal shift are considered. If the reference image is $r(x',y')$, and the other image is $s(x,y)$, a and b are the horizontal shift and vertical shift respectively, the rigid transformation model between the coordinates of these two images may be denoted as:

$$\begin{bmatrix} x' \\ y' \end{bmatrix}=\begin{bmatrix} x \\ y \end{bmatrix}+\begin{bmatrix} a \\ b \end{bmatrix}. \tag{6}$$

The mathematical relationship of these two images can be expressed as follows:

$$s(x,y)=r(x',y')=r(x+a,y+b). \tag{7}$$

Two-dimensional series expansion at (x,y) is made to the right part of the preceding equation. Ignoring the high order terms, the following approximate expression will be get:

$$s(x,y)\approx r(x,y)+a\frac{\partial r}{\partial x}+b\frac{\partial r}{\partial y}. \tag{8}$$

Thus, the object function can be written as:

$$E(a,b)=\sum\left[r(x,y)+a\frac{\partial r}{\partial x}+b\frac{\partial r}{\partial y}-s(x,y)\right], \tag{9}$$

where, $\sum$represents the summation to the overlapped part of r and s. Monimizing the object fuction. Performing partial derivatives about a and b respectively and letting them equal to zero, the optical estimated parameters will be obtained:

$$X=A^{-1}B, \tag{10}$$

where,

$$X=\begin{bmatrix}\hat{a}\\ \hat{b}\end{bmatrix},\ A=\begin{bmatrix}\sum\frac{\partial r}{\partial x} & \sum\frac{\partial r}{\partial y}\end{bmatrix},\ B=\sum(s-r), \tag{11}$$

2.5 GAUSSIAN BLUR ESTIMATION

Gaussian PSF is the most common blurring function of many optical measurements and imaging systems. Generally, the Gaussian blurring function may be expressed as follows:

$$h(m,n)=\begin{cases}\frac{1}{\sqrt{2\pi}\sigma}\exp\{-\frac{1}{2\sigma^{2}}(m^{2}+n^{2})\} & (m,n)\in C \\ 0 & \text{others}\end{cases}, \tag{12}$$

where, σ is the standard deviation; C is a supporting region. Commonly, C is denoted by a matrix with size of $K\times K$, and K is often an odd number.

Thus, the size and the standard deviation need to be estimated for the Gaussian blurring function, which may be estimated according to the error-parameter curves based on Wiener filtering method [8-9].

Firstly, reflection symmetric extension is performed on the observed image (y) with size of $M\times N$, and the size of the extended image becomes $2M\times 2N$. Then, calculate the Fourier transformation of the extended image (Y). Given a size (K) of the PSF, the error-parameter curves are generated at different standard deviations (σ).

According to the error-parameter curves, the approximate size and standard deviation of the blurring function can be estimated. The size where the distance between the curves decreases greatly is assumed to be the estimated size, and the standard deviation where the corresponding curve increases obviously is assumed to be the estimated standard deviation.

In order to estimate the parameters of Gaussian PSF automatically, two thresholds T_1 and T_2 are set. Firstly, given an estimation error e, the curve where once the distance between curves is smaller than T_1 gives out the estimated size ($\hat{K}$) of the Gaussian PSF. The distance is defined as the absolute difference of the cycle number (j) of standard deviation at e. Then, by calculating the slop of the estimation error at different standard deviations on the estimated curve, the deviation value can be estimated. The deviation once the slop is greater than the threshold T_2 is the estimated deviation ($\hat{\sigma}$).

3 Experiments

3.1 SIMULATE LR IMAGES

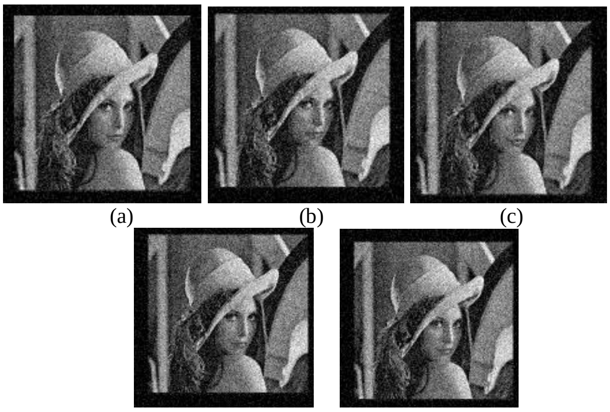
Experiments are performed on multiple simulated LR images to test the algorithm objectively and subjectively. To avoid the boundary effect, a zero window with width of 16 pixels is added to the image ‘lena.bmp’ of size 256×256, and the gained simulated HR image of size 288×288 as shown in Fig.3.

The HR image is passed through the LR imaging model as shown in Fig.1. Firstly, the HR image is horizontally and vertically shifted. Here, the horizontal shift (a_i) and the vertical shift (b_i) are taken as shown in Tab.1. Secondly, the five moved images are convolved by a Gaussian PSF with size of 7 and standard deviation of 0.8 respectively. Secondly, the blurred image is down-sampled by a factor of 2 in horizontal and vertical direction. Finally, Gaussian noise with a density of 0.05 is added. The generated 5 LR images with size of 144×144 are shown in Fig.4. Take the first LR image as reference image.

FIGURE 3 The simulated HR image

TABLE 1 The movement parameters

Sequence number (i)	Horizontal shift (a_i) (in pixel)	Vertical shift (b_i) (in pixel)
1	0	0
2	2.5467	2.5778
3	-2.8796	3.4356
4	2.5478	-3.4566
5	-2.5656	-3.5436



(a) (b) (c) (d) (e)

FIGURE 4 The simulated LR images

3.2 IMAGE DE-NOISE

The simulated LR image is de-noised by Wiener filtering algorithm. The de-noised LR images are generated. The bilinear interpolated reference LR image (Fig.4(a)) by 2 times is shown in Fig.5, and the PSNR is 31.6242dB. The bilinear interpolated de-noised reference LR image by 2 times is shown in Fig.6, and the PSNR is 32.8454dB. We can see that the Gaussian noise is reduced and the PSNR is improved.

FIGURE 5 The Bilinear interpolated image of reference LR image

FIGURE 6 The Bilinear interpolated image of de-noised reference LR image

3.3 MOVEMENT ESTIMATION

Relative to the reference de-noised LR image, the estimated movement parameters of the *i*th LR image are denoted as $\hat{a}_i$ and $\hat{b}_i$. The absolute estimation errors are defined Eq.(13), The estimated absolute estimation errors are shown in Tab.2.

$$\Delta a_i = \left|\hat{a}_i - a_i\right|,\ \Delta b_i = \left|\hat{b}_i - b_i\right|, \tag{13}$$

TABLE 2 The estimated absolute estimation errors

Sequence number (*i*)	Horizontal estimation error (Δa_i) (in pixel)	Vertical estimation error (Δb_i) in pixel)
1	0	0
2	0.0094	0.0112
3	0.0024	0.0083
4	0.0015	0.0067
5	0.0024	0.0069

3.4 GAUSSIAN BLUR ESTIMATION

The Gaussian PSF of the de-noised reference LR image is estimated. The sizes (K) of the Gaussian PSF are taken as 3, 5, 7, 9 and 11 respectively. The range of the standard deviation (σ) is taken as [0.5, 2]. The searching time is taken as 100. The threshold T1 and T2 are taken as 2 and 0.5 respectively. The generated error-parameter (E-σ) curves of the LR image at different sizes are shown in Fig.7.

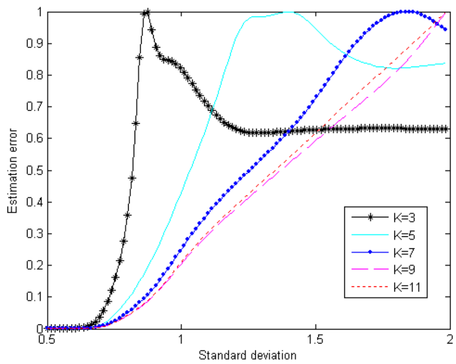


FIGURE 7 The error-parameter curves of the de-noised reference LR image

By analyzing the relationship of the multiple curves, the estimated size ($\hat{K}$) is 7, and the estimated standard deviation ($\hat{\sigma}$) is 0.755. The absolute estimation error of the size and standard deviation are as follows respectively:

$$\left|K_0 - \hat{K}\right| = \left|7-7\right| = 0,\ \left|\sigma_0 - \hat{\sigma}\right| = \left|0.8 - 0.755\right| = 0.005$$

3.5 SUPER RESOLUTION RECONSTRUCTION

Utilizing the estimated movement parameters and Gaussian PSF, super resolution reconstruction is performed on the de-noised LR images through IBP algorithm. When the estimated size and standard deviation of the Gaussian PSF are 7 and 0.755, the SR reconstructed image is shown in Fig.8 (a).

In addition, in order to justify the importance of Gaussian blur estimation in multi-image SR reconstruction, in the case of the estimated size of Gaussian PSF is 7, the estimated standard deviation is taken from 0.1 to 3 with an increment of 0.1, the PSNRs of the estimated SR images are shown in Fig.9. The SR reconstructed images when the standard deviations are 0.1and 3 are shown in Fig.8 (b) and (c) respectively.

Relative to the simulated HR image, the peak signal to noise ratio (PSNR) of the reconstructed image gained by different methods are shown in Tab.3.

The experimental results show the effectiveness of the proposed method. The Gaussian noise is reduced in the SR reconstructed image. The movement parameters and Gaussian PSF are estimated with high accuracy. The SR reconstructed image has better visual effect and higher PSNR than other methods. When the Gaussian PSF is near the real value, the SR reconstructed image has better visual effect and higher PSNR. When the estimated is smaller than the real value, the SR reconstructed image is ambiguous. When the estimated is larger than the real value, the SR reconstructed image has obvious ring effect.

(a) $\hat{\sigma}$ =0.755

(b) $\hat{\sigma}$ =0.1

(c) $\hat{\sigma}$ =3

FIGURE 8 The SR reconstructed images at different estimated standard deviations

TABLE 3 The PSNRs of the images gained by different methods (in dB)

Figure	Fig.5	Fig.6	Fig.8(a)	Fig.8b)	Fig.8(c)
PSNR	31.6242	32.8454	33.7324	33.5085	32.8253

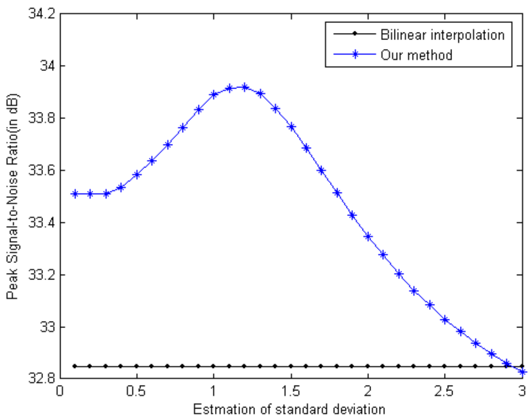


FIGURE 9 The PSNR of the SR reconstructed image at different estimated $\hat{\sigma}$

4 Conclusion

A framework of blind multi-image SR reconstruction with Gaussian blur and Gaussian noise is proposed. The degrading processes of movement, Gaussian blur, down sample and Gaussian noise are all considered in the LR imaging model. The simulated LR images are de-noised by Wiener filtering algorithm. The horizontal shift and the vertical shift between the de-noised LR images are estimated. The Gaussian blur of the de-noised LR image is estimated through error-parameter analysis method. The SR image is reconstructed through IBP algorithm. The experimental results justified the effectiveness of the proposed method. The Gaussian noise is well restrained in the SR reconstructed image. The visual effect and PSNR of the SR reconstructed image are improved. The proposed framework may be widely applied in other SR image reconstruction cases, such as different movement, different types of blur, different noise, and so on.

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