An efficient method for acquiring and processing signals based on compressed sensing

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Abstract

Compressed sensing (CS) theory provides a novel sensing/sampling and processing paradigm that breaks through the limitation of Nyquist rate to some applications. However, it is usually happened to the instability and redundancy of the acquired CS measurements. In view of this, we propose an efficient method to achieve adaptive minimal measurements with fewer measurements and good reconstruction performance by adding the pre-processing block into CS data acquiring and processing paradigm. In the proposed method, we firstly obtain the measurements to perfectly reconstruct the signal, and then design the optimization method to obtain adaptive minimal measurements by eliminating the redundant measurements. Experimental results show that the proposed method can obtain fewer measurements to perfectly reconstruct the signal than that of classical CS and sequential compressed sensing frameworks.

Keywords: compressed sensing, sequential compressed sensing, signal reconstruction, homotopy method

1 Introduction

In the conventional approach to sampling signal, the sampling rate must satisfy the Shannon/Nyquist sample theorem to not lose information [1-3]. Then the signal must be compressed to transmit or store since the high Nyquist rate results in too more redundant samples. FIGURE 1 illustrates the procedure for acquiring and processing signals in the conventional approach, including sampling and compressing the signal, transmitting/storing the data, and decompressing from the received data. While too high Nyquist rate limits some applications [2, 4], such as medical scanners, radar imaging and high-speed analogy-to-digital converters (AIC). Fortunately, emerging compressed sensing (CS) theory [1-7] provides a novel sensing/sampling paradigm that breaks through the limitation of the traditional approach. The CS theory claims that far fewer samples or measurements than the conventional approach can be used to perfectly recovery the signals when restricted isometry property (RIP) is satisfied and the underlying signal is sparse [1-8]. FIGURE 2 demonstrates data acquirement by CS method and then data processing, including obtaining the measurements at the sender, transmitting/storing, and decoding at the receiver. However, such classical CS framework cannot ensure that the acquired measurements can certainly be used to perfectly reconstruct the signal [9]. In addition, the measurements transmitted/stored are usually redundant [10], which will result in the waste of transmission/storage resources.

FIGURE 1 The conventional approach to sampling and processing signals

FIGURE 2 Compressed sensing to sampling and processing signals

To address the above two problems of classical CS framework, this paper proposes a new efficient method shown in FIGURE 3 to acquire and process the data based on CS. In the proposed method, we adds the pre-processing block B to ensure that the measurement results can be used to perfectly reconstruct the signal and obtain adaptive minimal measurements, whose any proper subset cannot be used to perfectly reconstructed the signal. In the proposed method, we firstly judge whether initial measurements are excess or not via sequential compressed sensing (SCS) [11]. Then for two cases of non-excess and excess initial measurements, we design the optimization method to obtain adaptive minimal measurements. Experimental results show that the measurement of a certain measurement set has indeed different important degree to signal reconstruction, and the proposed method can obtain fewer measurements to perfectly reconstruct the signal than that of classical CS and sequential compressed sensing frameworks.

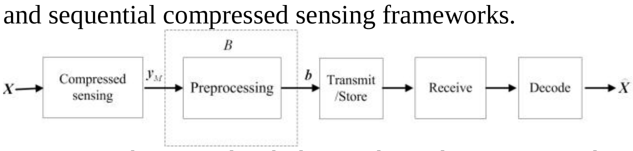

FIGURE 3 The proposed method to sampling and processing signals

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Before proceeding, we define some denotations. Let $\boldsymbol{X}^*$ be the signal with the length N and the scarcity level k. Let $\boldsymbol{y}^M=[y_1,y_2,\cdots,y_M]^T$ represent initial measurements, where M is the number of initial measurements. Then $\boldsymbol{y}^M=\boldsymbol{\Phi}\boldsymbol{X}^*$, where $\boldsymbol{\Phi}=[\boldsymbol{\Phi}_1,\boldsymbol{\Phi}_2,\cdots,\boldsymbol{\Phi}_M]^T$ is $M\times N$ Gaussian measurement matrix. The measurements $\boldsymbol{s}=[s_1,s_2,\cdots,s_L]^T$ are called as SCS or MSCS measurements since it is obtained by SCS or MSCS method, and its corresponding measurement matrix denoted by $\boldsymbol{S}$. Let $\boldsymbol{b}=[b_1,b_2,\cdots,b_K]^T$ $(K\le L)$ be adaptive minimal measurements to transmit/store, whose corresponding measurement matrix denoted by $\boldsymbol{B}$. Let $\widehat{\boldsymbol{X}}_M$ represent the reconstruction signal by using initial measurements $\boldsymbol{y}^M$. Let $\widehat{\boldsymbol{X}}_i$ represent the reconstruction signal by using $\boldsymbol{y}^i=[y_1,y_2,\cdots,y_i]^T$, where i is a positive integer.

2 The proposed method for acquiring and processing the signal

In this section, we propose an efficient method to acquire and process signals based on compressed sensing, recall FIGURE 3. In the proposed method, adaptive minimal measurements with fewer measurements and good reconstruction performance can be achieved by adding the pre-processing block. In the block, we firstly achieve the measurements, which can be used to perfectly reconstruct the signal with probability 1 for the Gaussian measurement ensemble. Then the optimization method is used to reduce redundant measurements to obtain adaptive minimal measurements.

2.1 THE FLOW CHART OF THE PRE-PROCESSING BLOCK

To understand easily the function of the block B, we give the flow-chart of B in FIGURE 4. After acquiring initial measurements $\boldsymbol{y}^M$, we firstly judge that $\boldsymbol{y}^M$ is excess or non-excess. If $\boldsymbol{y}^M$ is non-excess, SCS method is used to achieve SCS measurements $\boldsymbol{s}$. Otherwise, we provide MSCS method to obtain MSCS measurements $\boldsymbol{s}$ by removing some redundant measurements from $\boldsymbol{y}^M$. So SCS or MSCS measurements can be used to perfectly reconstruct the signal. And we find that the signal will not can be perfectly reconstructed if the last measurement of $\boldsymbol{s}$ is removed. It demonstrates that the last measurement of is important.

Inspired by the finding that the last measurement of $\boldsymbol{s}$ is important, we study each measurement of $\boldsymbol{s}$ and conclude that some measurements are important since the signal cannot be perfectly reconstructed if any of them is removed from $\boldsymbol{s}$, and some measurements are unimportant since the signal can still be perfectly reconstructed if any of them is removed from $\boldsymbol{s}$. Therefore, each measurement of a certain measurement set has different important degree to signal reconstruction. Based on this, the optimization method is used to achieve adaptive minimal measurements.

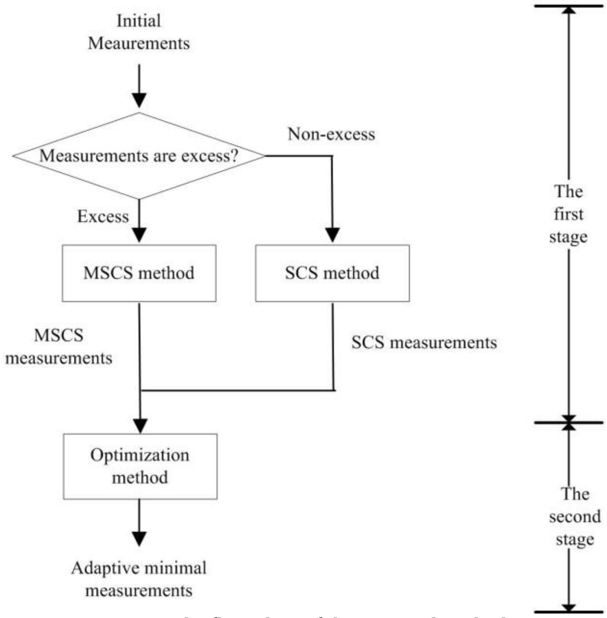

FIGURE 4 The flow-chart of the proposed method

2.2 THE PRE-PROCESSING BLOCK

In the first stage of the processing block B, we need to judge whether initial measurements $\boldsymbol{y}^M$ are excess or not. To this aim, MSCS method is given in FIGURE 5 and Proposition 2 is provided as judgment criterion based on Proposition 1.

Step 1: $\widehat{\boldsymbol{X}}_M$, $\widehat{\boldsymbol{X}}_{M-1}$, and $\widehat{\boldsymbol{X}}_M=\widehat{\boldsymbol{X}}_{M-1}$ are known, and let $i=M-2$.

Step 2: $\boldsymbol{y}^i$ is used to reconstruct $\widehat{\boldsymbol{X}}_i$.

Step 3: If $\widehat{\boldsymbol{X}}_M=\widehat{\boldsymbol{X}}_i$, then let $i=i-1$, and repeat step 2. Otherwise go to step 4.

Step 4: $\boldsymbol{y}^{i+1}$ is MSCS measurements $\boldsymbol{s}$.

FIGURE 5 MSCS method

In FIGURE 5, we know $\widehat{\boldsymbol{X}}_M=\widehat{\boldsymbol{X}}_{M-1}$ since MSCS method deals with the case that initial measurements are excess, and let $i=M-2$, see step 1. The first i measurements $\boldsymbol{y}^i$ of initial measurements $\boldsymbol{y}^M$, whose corresponding measurement matrix is $[\boldsymbol{\Phi}_1,\cdots,\boldsymbol{\Phi}_{i-1},\boldsymbol{\Phi}_i]^T$, is used to reconstruct the signal $\widehat{\boldsymbol{X}}_i$, see step 2. If $\widehat{\boldsymbol{X}}_M\ne\widehat{\boldsymbol{X}}_i$, then $\boldsymbol{y}^{i+1}$ is just MSCS measurements. Otherwise let $i=i-1$ and repeat steps 2-3, see steps 3-4. MSCS measurements $\boldsymbol{y}^{i+1}=[y_1,\cdots,y_i,y_{i+1}]^T$ are rewritten as $\boldsymbol{s}=[s_1,s_2,\cdots,s_L]^T$, where $L=i+1$. And the corresponding measurement matrix is $\boldsymbol{S}=[\boldsymbol{\Phi}_1,\cdots,\boldsymbol{\Phi}_i,\boldsymbol{\Phi}_{i+1}]^T$.

MSCS method is so named because it adopts such a procedure in which the measurement is eliminated one-by-one from initial measurements until the stop condition is satisfied, while SCS method increases the measurement one-by-one until the agreement rule is satisfied. According to the procedure shown in FIGURE 5, we know that the signal will not can be perfectly reconstructed if the last measurement of MSCS measurements is removed. It demonstrates that the last measurement of MSCS measurements is also important, which is the same as that of SCS method.

Proposition 1 [11] In the Gaussian (generic continuous) measurement ensemble, if $\widehat{\boldsymbol{X}}_{M+1}=\widehat{\boldsymbol{X}}_M$ holds, then $\widehat{\boldsymbol{X}}_M=\boldsymbol{X}^*$ with probability 1.

Proposition 2 In the Gaussian (generic continuous) measurement ensemble, if $\widehat{\boldsymbol{X}}_M=\widehat{\boldsymbol{X}}_{M-1}$, then $\boldsymbol{y}^M$ is excess. Otherwise $\boldsymbol{y}^M$ is non-excess.

According to the principles of SCS and MSCS methods, the measurements $\boldsymbol{s}$ can be used to perfectly reconstruct the signal with probability 1 for the Gaussian measurement ensemble. To reduce the number of the measurements to store/transmit, the optimization algorithm shown in FIGURE 6 can be used to achieve adaptive minimal measurements according to the different important degree of the measurement.

In FIGURE 6, we take SCS or MSCS measurements $\boldsymbol{s}$ as the input of the optimization method, see steps 1-2. In steps 3-6, $\boldsymbol{s}$ are divided into two sets: the key set T_1 and the non-key set T_2. In step 4, the important degree of each measurement is illustrated by the reconstruction error denoted by $E_j=\left\|\widehat{\boldsymbol{X}}_p-\widehat{\boldsymbol{X}}_{p-1,j}\right\|/\left\|\widehat{\boldsymbol{X}}_p\right\|$, which can be solved since $\widehat{\boldsymbol{X}}_p$ can be regarded as the original signal. If T_2 is empty or only has a measurement, then T_1 is just adaptive minimal measurements $\boldsymbol{b}$, see step 7. Otherwise, $\boldsymbol{b}$ should be composed of T_1 and some measurements of T_2. Since T_2 is sorted by the descending order of E_j, we may consider that the important degree of the measurement in T_2 is also decreasing. Next we try to remove the measurements as many as possible from the back of T_2 so that the remainder of T_2 together with T_1 can be used to perfectly reconstruct the signal, see step 8. The measurements w are updated, then repeat the above procedure until the condition in step 7 is satisfied, see step 9. So we obtain adaptive minimal measurements $\boldsymbol{b}$, which is a subset of $\boldsymbol{s}$. And the corresponding measurement matrix $\boldsymbol{B}$ is a submatrix of $\boldsymbol{S}$.

Step 1: Let $\boldsymbol{w}=\boldsymbol{s}$, and w contains p components. Then $p=L$.

Step 2: $\widehat{\boldsymbol{X}}_p$ is reconstructed by using $\boldsymbol{w}$.

Step 3: $\widehat{\boldsymbol{X}}_{p-1,j}$ is reconstructed by taking $p-1$ measurements obtained by removing the *j-th* measurement from $\boldsymbol{w}$ ($1\le j\le p$).

Step 4: Compute reconstruction error $E_j=\left\|\widehat{\boldsymbol{X}}_p-\widehat{\boldsymbol{X}}_{p-1,j}\right\|/\left\|\widehat{\boldsymbol{X}}_p\right\|$.

Step 5: Sort $\boldsymbol{w}$ into $\boldsymbol{z}$ according to the descending order of E_j.

Step 6: The measurements of $\boldsymbol{z}$ are divided into two sets T_1 with m_1 components and T_2 with m_2 components ($m_1+m_2=p$ according to *Theorem 1*), where the corresponding $E_j=0$ for each measurement of T_2, and the corresponding $E_j\neq 0$ for each measurement of T_1.

Step 7: If T_2 contains 0 or 1 measurement, then T_1 can be used to perfectly reconstruct the signal. So T_1 is just low-redundancy measurements $\boldsymbol{b}$. Otherwise go to step 8.

Step 8: Let $l=m_2-1$. T_1 together with the first l measurements of T_2 are taken to reconstruct the signal. If the signal can be perfectly reconstructed, then $l=l-1$ and repeat step 8. Otherwise go to step 9.

Step 9: Update $\boldsymbol{w}$ with T_1 and the first $l+1$ measurements of T_2, and $p=m_1+l+1$. Repeat the above steps 2-9.

FIGURE 6 The optimization algorithm to achieve adaptive minimal measurements

3 Experimental results

According to the principles of SCS and MSCS methods, the measurements $\boldsymbol{s}$ can be used to perfectly reconstruct the signal with probability 1 for the Gaussian measurement ensemble. So in this section, we need design some experiments to verify the following issues. (i) The measurement has different important degree to signal reconstruction for the measurements $\boldsymbol{s}$ and $\boldsymbol{b}$. (ii) The proposed method can obtain fewer measurements than that of the classical CS and SCS with good reconstruction performance. In the experiments, sparse signals are used as test signals, and homotopy method [12-14] is selected as the reconstruction algorithm since it is suitable to the recovery of sparse signals.

Experiment 1

In this experiment, a random signal with the length $N=200$ and the sparsity level $k=10$ is generated. And initial measurement numbers $M=30$ are adopted. The proposed method firstly obtains SCS measurements $\boldsymbol{s}$ with $L=36$ measurements. Then the reconstruction error $E_j=\left\|\widehat{\boldsymbol{X}}_L-\widehat{\boldsymbol{X}}_{L-1,j}\right\|/\left\|\widehat{\boldsymbol{X}}_L\right\|$ ($\widehat{\boldsymbol{X}}_L$, $\widehat{\boldsymbol{X}}_{L-1,j}$ refer FIGURE 6, $1\le j\le L$) is used to illustrate the important degree of each measurement in $\boldsymbol{s}$, the results are shown in FIGURE 7.

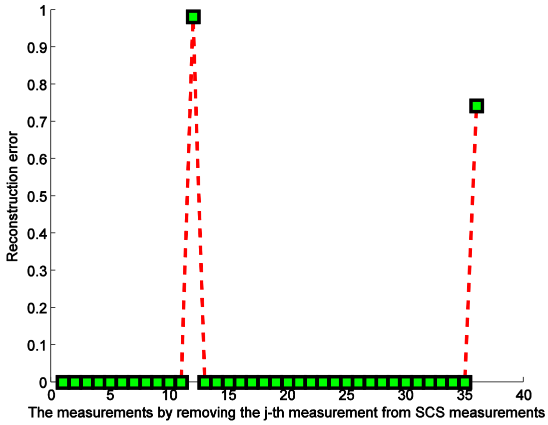

FIGURE 7 Reconstruction error E_j for the measurements obtained by removing the *j-th* measurement from SCS measurements $\boldsymbol{s}$

From FIGURE 7, we see that the signal cannot be perfectly reconstructed when the 12-th or 36-th measurement is removed from $\boldsymbol{s}$, while the signal can still be perfectly reconstructed when any other measurement is removed from $\boldsymbol{s}$. Apparently, the 12-th and 36-th measurements are more important than other measurements in $\boldsymbol{s}$, i.e., they are key measurements. So we can consider that the measurement in $\boldsymbol{s}$ has the different important degree to signal reconstruction. From FIGURE 7, it is easy to see $E_{12} > E_{36}$, which illustrates that the 12-th measurement is more important than the 36-th measurement. This viewpoint can also be verified by the next experiment.

For SCS measurements above, each key measurement is randomly replaced 1000 times to reconstruct the signal, respectively. The results show that, among 1000-time replacements, for the 12-th measurement, the signal can be perfectly reconstructed 19-time and the signal cannot be perfectly reconstructed 981-time. For the 36-th measurement, the signal can be perfectly reconstructed 446-time and the signal cannot be perfectly reconstructed 554-time. So the 12-th measurement is more important than the 36-th measurement from the perspective of probability. It illustrates that the measurement in $\boldsymbol{s}$ has different important degree.

For the above SCS measurements $\boldsymbol{s}$, the optimization method in FIGURE 6 is used to achieve adaptive minimal measurements $\boldsymbol{b}$ with $K = 23$ measurements. Each measurement of $\boldsymbol{b}$ is randomly replaced 1000 times to reconstruct the signal, respectively. The reconstruction probability for each measurement is shown in FIGURE 8. From FIGURE 8, we find that the signal can be perfectly reconstructed with a small probability ($0 \sim 0.257$) when the i-th measurement of $\boldsymbol{b}$ is randomly replaced 1000 times. Different reconstruction probability illustrates that each measurement of $\boldsymbol{b}$ has the different important degree to signal reconstruction. Especially for the 1-st measurement, the reconstruction probability is 0 among 1000-time replacements. It demonstrates that the measurement in $\boldsymbol{b}$ has different important degree.

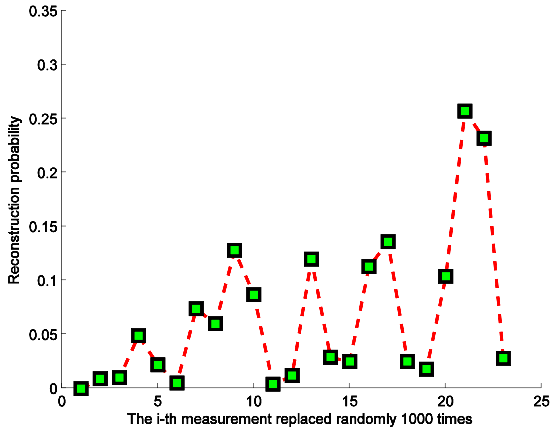

FIGURE 8 Reconstruction probability for the *i-th* measurement of $\boldsymbol{b}$ replaced randomly 1000 times

Experiment 2

Next experiments are used to verify that the proposed method can obtain fewer measurements with good reconstruction performance than that of the classical CS and SCS.

A signal with the length $N = 128$ and the sparsity level $k = 10$ is generated. We choose uniformly initial measurement numbers M from 10 to 60 with the interval 10. The experimental results are shown in TABLE 1.

In TABLE 1, the 1-st column M, the 3-rd column L and the 5-th column K represent the measurement numbers of $\boldsymbol{y}^M$, $\boldsymbol{s}$, $\boldsymbol{b}$, respectively. The 2-nd column E_1, the 4-th column E_2 and the 6-th column E_3 give the reconstruction errors when $\boldsymbol{y}^M$, $\boldsymbol{s}$, $\boldsymbol{b}$ are used to reconstruct the signal, respectively. When the sparsity level k is known, it is well known the fact that $3k \sim 5k$ measurements can be taken to perfectly reconstruct the signal with high probability. The results of the first two columns in TABLE 1 are consistent with the above conclusion. From the 3-rd column, we can see that the number of $\boldsymbol{s}$ lies between $3k \sim 4k$ which is a smaller range than that of the classical CS framework $3k \sim 5k$. The 5-th column shows that measurement numbers of $\boldsymbol{b}$ lie between $1k \sim 3k$, which is fewer than that of $\boldsymbol{s}$. According to CS theory, the minimal measurement numbers of this signal are $k+1=11$ which is obtained by solving the NP hard problem of l_0 model. For $M = 40$, the number of $\boldsymbol{b}$ is $K = 14$, which is very near to the minimal value 11. Compared to initial measurements $\boldsymbol{y}^M$, $\boldsymbol{b}$ decrease 26 measurements, but it can be used to obtain almost the same reconstruction quality as $\boldsymbol{y}^M$.

TABLE 1 Reconstruction errors for different initial measurement numbers

M	E_1	L	E_2	K	E_3
10	1.1580	39	1.6999e-10	30	3.1438e-10
20	0.8543	34	1.9212e-10	21	7.8366e-11
30	0.8125	32	3.7478e-10	27	3.3689e-10
40	3.5374e-11	38	3.6561e-11	14	1.2733e-10
50	3.0281e-11	36	1.3316e-10	26	4.5560e-10
60	2.9799e-11	40	1.3754e-10	28	4.7159e-10

FIGURE 9 is used to intuitively illustrate the reconstruction performance of SCS measurements and adaptive minimal measurements sequences for $M=20$ of TABLE 1. The dashed line shows that the signal can be perfectly reconstructed when the initial measurements $M=20$ is increased adaptively to SCS measurements ($L=34$). The solid line shows that adaptive minimal measurements ($K=21$), which is a subset of SCS measurements, can also be used to perfectly reconstructed the signal with almost the same reconstruction error, see TABLE 1.

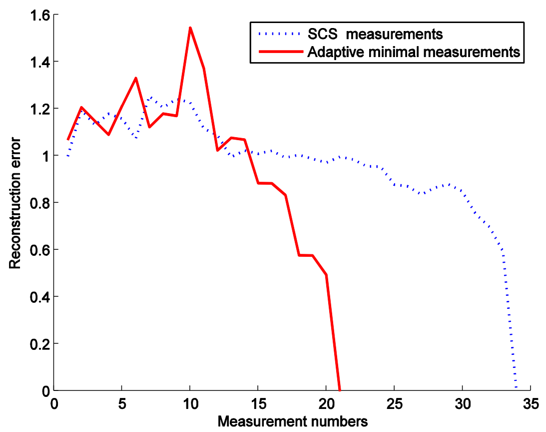

FIGURE 9 Reconstruction errors of SCS measurements and adaptive minimal measurements sequences for $M=20$

To verify the generality of the conclusion reflected by TABLE 1, we do the following statistical experiment. The above signal is still adopted, and 1000 experiments are run for $M=35$. The detailed results are shown in FIGURE 10. In this figure, the horizontal-axis represents the measurement times, the vertical-axis shows the measurement numbers for each measurement task. The solid line represents adaptive minimal measurement numbers, and the dashed line shows SCS or MSCS measurement numbers. From FIGURE 10, all adaptive minimal measurement numbers are fewer than that of SCS or MSCS measurements in these 1000 random measurements, while they can obtain almost the same reconstruction performance. Among 1000 random measurements, 827 SCS or MSCS measurement numbers are not more than $4k$, 743 adaptive minimal measurement numbers are not more than $3k$.

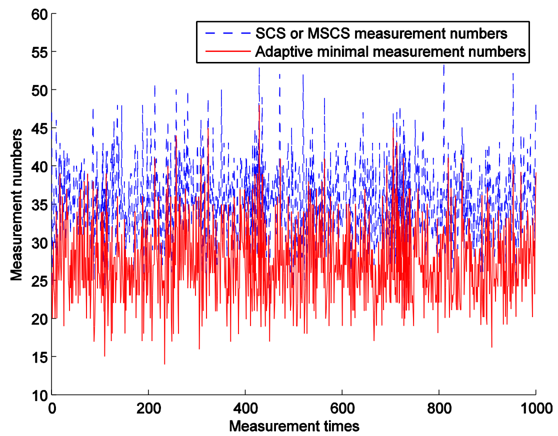

FIGURE 10 Comparisons with SCS or MSCS measurements, and adaptive minimal measurements. ($M=35$)

4 Discussions and conclusions

For the instability and redundancy of the acquired CS measurements, we propose an efficient method to achieve adaptive minimal measurements with fewer measurements and good reconstruction performance by adding the pre-processing block into CS data processing paradigm. In the proposed method, we firstly obtain the measurements to perfectly reconstruct the signal, and then design the optimization method to obtain adaptive minimal measurements by eliminating the redundant measurements. Experimental results show that the proposed method can obtain fewer measurements to perfectly reconstruct the signal than that of classical CS and SCS framework. Therefore, the proposed method provides the whole clue to sampling, pre-processing, storing, transmitting and decompressing, which is helpful to improve CS data processing framework.

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