

Research on the robustness of supply chain network with uncertainty

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Abstract

A new method is proposed to enhance robustness of complex supply chain network structure in uncertain conditions of market, costs and others. The researcher constructs the topology of a supply chain network firstly, and then explains the concept of central nodes of network and robust supply chain network based on scale-free network model from the perspective of quality control and places research emphasis on the method of enhancing the robustness of complex supply chain network structure. Finally, the supply chain network of auto parts enterprises in Zhejiang Province is cited as an example to verify the validity of the method.

Keywords: supply chain network, network quality control, scale-free network, robustness

1 Introduction

With the increasingly fierce market competition and complex economic environment changes such as variation of the users' needs, the uncertainty of the supply chain is more and more prominent; however, the uncertainty may arise from within the supply chain such as demand, production, sales, management, and operation [1]. Robustness of the supply chain is a universal basic attribute of it. Robustness is the ability to maintain the functions of its system when faced with changes in the internal structure or external environment. At present, a group of scientists led by Dirk Helbing found in the study of complex supply chain network that bullwhip effect in supply chain management is relevant with the topological properties of network [2,3]. This paper focuses on construction of a robust supply chain network under uncertainty, modeling analysis of complex supply chain network based on network static geometric quantities and analytical methods in graph theories and social network analysis, and exploration of the geometric properties of the network topology, robustness of the structure, dynamic robustness in the process of network evolution, etc. from the perspective of the quality control based on statistical principles and analytical method.

However, there are no existing studies on complex supply chain network topology and its robustness in the process of evolution under uncertainty. Most studies only point out the robustness and vulnerability according to the nature of the complex network [4-6]. Research on the complex supply chain network at home and abroad is mainly about supply chain adaptability analysis based on the

complexity theory and knowledge sharing between the nodes of the supply chain companies [7-9], without taking the properties of the complex supply chain network topology under uncertainty into consideration. Complex supply chain network modeling based on network graph theory clearly shows the relationship between the enterprise nodes within the supply chain network [10], but can't express explicitly the robustness and vulnerability of the supply chain network. As a matter of fact, the robustness of the supply chain network is not only related to and the stability of the node enterprises in the supply chain but also to the structure of the supply chain network.

2 Construct scale-free supply chain network

Uncertainty is in most of the supply chain system. Robustness, a universal basic attribute of the uncertainty of a supply chain network, is the ability to maintain the functions of its system when faced with changes in the internal structure or external environment. The system of the supply chain develops from scratch gradually. The supply chain network, which is spontaneous and market-driven, has strong uncertainty. Currently, some scholars at abroad have found in the supply chain bullwhip effect – the amplification effects are all related to the supply network topology. Therefore, the effective supply network topology can weaken the bullwhip effect while enhance the stability and resistance to attack of the complex supply chain network.

First, we build a complex supply chain network modeling; the modeling process is shown in Figure 1:

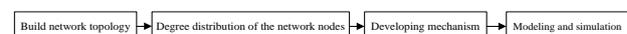


FIGURE 1 Complex supply chain network modeling process

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The formation, development and changes of the complex supply chain network can be seen as the result of interactions between the elements of the supply chain system and the external environment. Supply chain network is not just supply chain management of a single enterprise, but a complicated Multi-layer industry supply chain network. The construction method is as follows: the different business entities in the supply chain are taken as the network nodes, and the connections between the enterprises as the edges of the network. The edge direction is from the higher-level enterprise to the lower-level one; the edge weight is the number of the child nodes contained in the parent node, which is shown in Figure 2.

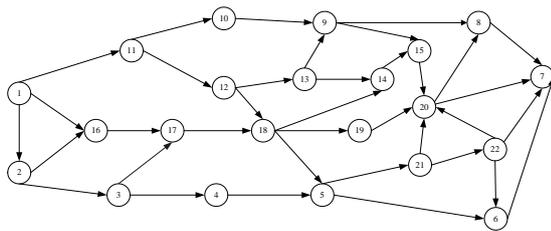


FIGURE 2 Supply chain network topology

In the supply chain network topology diagram shown in Figure 2, indegree refers to the number of edges pointing to the node, and outdegree refers to the number of edges starting from that point. We assume that x represents the node of complex enterprise cooperation network, and r represents the edge between the nodes in the network. i and j stand for any two nodes in the network. If there is an edge between i and j , the enterprise i and enterprise j are in cooperative relations, which is represented as $r(i, j)=1$. If N is the total number of the nodes in the network static statistics, the corresponding static statistics of the enterprise cooperation are defined as follows:

1) Degree of node: it is the number of edges the node connects with:

$$k_i = \sum_{i,j} r(i, j). \tag{1}$$

2) Average path length of node: the distance between i and j in the network is d_{ij} , which is the number of the edges on the shortest path connecting the two nodes, namely:

$$L = \frac{1}{\frac{1}{2} N(N+1)} \sum_{i \geq j} d_{ij}, \tag{2}$$

where N is the number of the nodes in the network.

3) Clustering coefficient: it is the parameter used to measure clustering situation of a node. We assume that i has k_i edges in the network, then the number of the edges between these k_i nodes may be up to $k_i(k_i-1)/2$. The ratio of E_i (the actual number of edges) between these k_i nodes and $k_i(k_i-1)/2$ (the total possible number of edges) is defined as the clustering coefficient of the node:

$$C_i = \frac{2E_i}{k_i(k_i-1)}. \tag{3}$$

Compile network processor based on the constructed supply chain network topology, and the aforementioned statistic eigenvalues of each node in the network can be achieved by applying the formulas mentioned above. Assuming that k_i (the degree of each node) is as shown in Table 1, the clustering coefficients of these nodes obtained by using Equation (3) are as follows.

TABLE 1 Node degree and clustering coefficient

No.	k_i	C_i	No.	k_i	C_i
1	3	0.06	12	1	0.00
2	22	0.01	13	1	0.00
3	1	0.03	14	23	0.00
4	1	0.00	15	45	0.07
5	39	0.09	16	8	0.13
6	56	0.12	17	4	0.02
7	86	0.15	18	5	0.01
8	1	0.13	19	12	0.01
9	12	0.00	20	23	0.02
10	112	0.04	21	20	0.04
11	1	0.04	22	1	0.03

By computing the clustering coefficient of each node in the supply chain network, it is found that the node with high node degree also has a high clustering coefficient; by performing statistical analysis on the distribution probability of each node degree, it is found that most nodes have a small number of connections in the network while a few nodes have a large number of connections, and there is no peak in the probability distribution of these node degrees, indicating that the node degree distribution follows the power-law distribution, in line with the characteristics of scale-free network.

3 Robustness analysis of the supply chain network under uncertainty

3.1 MODE AND FEATURES OF THE SUPPLY CHAIN UNDER UNCERTAINTY

The uncertainty of the supply chain contains two types. One is the uncertainty of the outer join. In scale-free supply chain network, partners often opt to quit due to reasons of their own. Such event, though occurs randomly, is considered to be attack on the network. For instance, the partners drop out of the supply chain network because they are in shortage of funds or stop producing some models of products. The other is uncertainty of emergency. Due to fierce market competition and inner motivation of achieving maximum interests, some important suppliers (i.e., core node) in the supply chain network are tempted by high profit from some other enterprises and some other enterprises or units are interested in inviting some important experienced intermediary retailers to enter into their network. Once these important suppliers and retailers are off the network, they will inevitably give a crushing blow to the core enterprises in raw material supply and sales of products. This is what vandalism means.

When the supply chain network is in uncertain conditions, the robustness of the scale-free supply chain network can be interpreted as: when a node in the network is des-

troyed, it continues to maintain its operational capacity, without causing significant impact to the entire network. A node which brings a catastrophic failure to the network when subjected to random external attack is judged to be unqualified. In accordance with the principles of quality control, only when the failure rates of the network nodes are all less than some expected number, this network is considered to be robust. Although Barabasi and some other scholars have pointed out that scale-free network shows unusual robustness when the node is under random attack [11,12], it is vulnerable to attack and leads to damage of the entire network. The robustness of scale-free supply chain network under conditions of uncertainty is to be analyzed from the perspective of network quality control in the following.

3.2 DEFINE THE CENTRAL NODE IN THE SUPPLY CHAIN NETWORK

First $(V(t), E(t))$ represents a random complex supply chain network. $V(t)$ is a node set of the supply chain network at time t , that is, all enterprises in the supply chain; $E(t)$ is an edge set of the enterprises' nodes at time t ; $N(t)$ is the number of enterprises in the network at time t ; $E(N(t))$ is the average of the enterprises in the network at time t ; $k_i(t)$ is the degree of the enterprises at time t ; $P\{k_i(t)=k\}(k=1,2,...)$ is the degree distribution of enterprises at time t .

According to the robustness of the scale-free supply chain network, the central node of the scale supply chain network is defined firstly:

The central node of the scale-free supply chain network is defined as: if A and B are two proper subsets of $V(t)$, $A \neq \Phi$, $B \neq \Phi$, $A, B \subset V(t)$, $A \cup B = V(t)$, $A \cap B = \Phi$, $K > 0$, when $k \geq K$, $\forall t > 0$, $\forall i \in A$, $j \in B$, so that $P\{k_i(t) \geq k\} > P\{k_j(t) \geq k\}$, then A is named the central node of the scale-free the supply network. These nodes are the core enterprises in the supply chain.

3.3 DEFINITION OF ROBUST SUPPLY CHAIN NETWORK

This paper draws on SPC (Statistical Process Control) theory in quality control and mathematical statistical analysis theory to monitor and control the degree distribution of each node in the supply chain network, in order to realize the real-time regulation on node degree distribution and reduce variation of the robustness of the supply chain network. In the event that abnormal situation occurs to the central node of the network, i.e., the degree of the node exceeds the safety threshold σ , measures must be taken immediately to adjust it so as to ensure steady operation of the entire supply chain network [13].

The robust supply chain network refers to the network which is still able to maintain its functions when some nodes or edges in the network are under random attack. $p(t)=|A|/E(N(t))$ and $|A|$ is the potential of the set A . If $p(t) < \varepsilon$ and $\varepsilon \in (0,1)$, the failure rate of the central

nodes in the network is less than the expected number \square . Such network $(V(t), E(t))$ has certain robustness. The degree of each node in the supply chain network is often used in the study to facilitate the statistical analysis of robustness. In scale-free supply chain network, the degree of the central node is always very high, however, if it goes beyond a certain range and comes under attack, the entire network may malfunction. In accordance with quality control principles, the degrees of all nodes in the network are desired to be within the safety threshold. This kind of network has definite robustness [14,15].

The safety threshold is σ is divided into upper control limit σ_{UCL} and lower control limit σ_{LCL} , average is T , then:

Upper control limit: $\sigma_{UCL} = T + A_2R$,

Lower control limit: $\sigma_{LCL} = T - A_2R$,

where R is the average of the range of each node; A_2 is a constant. They vary with n (sample size). Please refer to Xbar-R coefficient table for the parameter values. In Table 1, $n=22, A_2=0.634$.

Therefore, for degree distribution of each node in the supply chain network, MSA (measurement system analysis) in SPC software is used for experimental statistics of the 24 subgroups. The following results can be obtained: control coefficient is 0; controls parameter: both upper control limit and the lower control limit are 22.27; stability judgment: the average number beyond the control limit is 8, which is judged to be unacceptable. Figure 3 is a diagram for stability analysis, from which the nodes beyond the control limits can be checked at any time.

4 Enhance the robustness of the supply chain network

Although the supply chain network is robust for random failure in the process of continuous preferential growth, it can't be ignored that the network is also vulnerable to attack. As long as the external attack impacts on the central high degree node, namely, the core enterprise in the supply chain network, it will lead to a destructive failure of the entire supply chain network, resulting in paralysis of the network. Therefore, how to enhance the stability of the network is essential. Two methods of enhancing the robustness in the process of supply chain network growth will be introduced in the following.

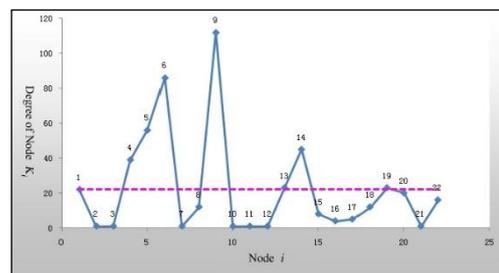


FIGURE 3 Diagram for stability analysis

4.1 CHANGE THE PREFERENTIAL MECHANISM

The first method: in the formation process of the entire supply chain network, the traditional way is to change the growth mechanism of the network so as to form a small-world network in the future development, in order to improve the stability of the entire network. In this paper, the referential mechanism is changed on the basis of two mechanisms growth mechanism and preferential attachment mechanism that the scale-free network follows so as to form a new network topology of exponential degree distribution. Network production model is as follows:

1) Growth: start from a network with m_0 nodes. Each time a new node is introduced, it is connected to m existing nodes, where $m \leq m_0$.

2) Priority connection: the probability that a new node is connected to an existing node in the network i at time t is:

$$\prod_i(t) = 1 / (m_0 + t - 1) \tag{4}$$

It is clear that in the network generated by the above-mentioned growth mechanism, the connection probability of its nodes has nothing to do with node degree. According to probability theory, the average degree distribution is

$p(k) = \frac{e^{-k/m}}{m}$, showing that its network nodes are exponentially distributed, so the role of core node in the network is weakened. Because of that, there is no major attack

point in the entire network and the stability of the network is enhanced.

4.2 THE DYNAMIC ADJUSTMENT MODEL

The second method is: after an old node drops out, a new comparable point must be quickly found out to replace it. This requires real-time monitoring of the each node in the supply chain network to make different judgments in different situations.

- 1) In the operation process of supply chain network, a node may apply to quit the supply chain network due to some reason;
- 2) A node can't offer enough products because of its limited production capacity;
- 3) After a period of operation of the supply chain network, the clients' demand increases suddenly and the original supply chain network cannot satisfy the customers in time, quality and quantity;
- 4) After a period of operation of the supply chain network, the clients' demand decreases suddenly and the original supply chain network has overcapitalization, resulting in increase of operating costs in the supply chain network.

According to the aforementioned four situations, a dynamic adjustment model of scale-free supply chain network is constructed as follows, shown in Figure 4.

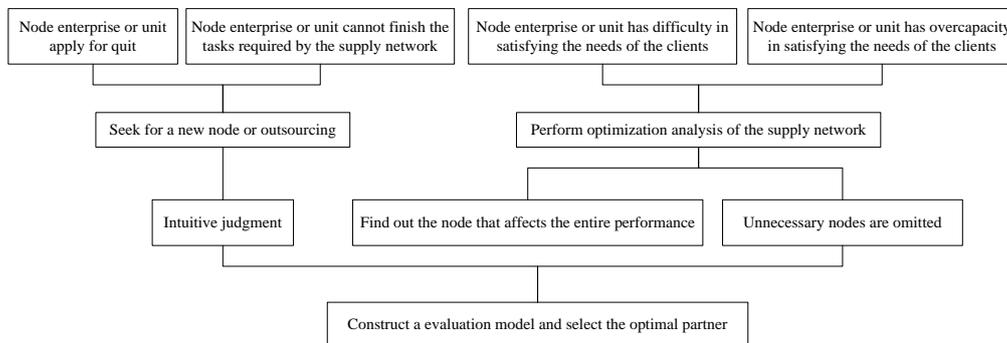


FIGURE 4 Scale-free supply chain network dynamic regulation model

The entire network needs to be adjusted continuously in the operation process. Only when the adjustment is finished and the new tasks of the adjusted supply chain network have been coordinated, can the new network realize effective conversion and start operation of new supply chain network. In this way, anti-risk capacity of the supply chain network is enhanced.

5 Application examples

Considering that components and parts of automotive industry have a wide range of distribution and are used in large quantity currently, auto parts enterprise clusters in Zhejiang province are taken as the object in this example, in order that automobile manufacturers can better manage

the auto parts suppliers. By taking into account of various factors, with Matlab as the development tool, the researcher intuitively simulates the network distribution of the suppliers of automobile parts and motorcycle parts industry by studying the node degree distribution and random generation of point and edge, displays the central node of the supply chain network through the degree distribution of each node in the network, and checks if there are abnormalities in node distribution through network robustness test.

Here the information of 102 auto parts suppliers obtained from purchase department of China Youngman Automobile Group is taken as examples. We assume that the growth characteristics of the supply chain network are as follows: the initial number of nodes of the supply chain $m_0=20$; each time a new node is introduced, the number of

newly generated edges $m=5$; network size after growth $N=102$; network, m_0 nodes in the network are all isolated when the network is initialized.

Figure 5 is a diagram for supplier network topology generated by Matlab simulation software:

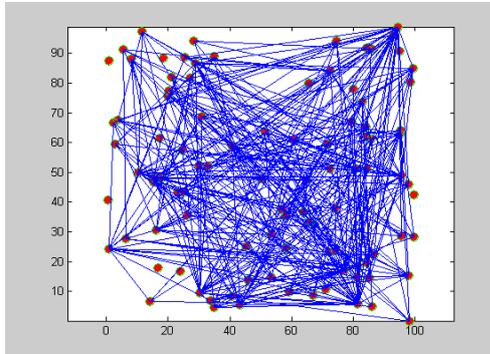
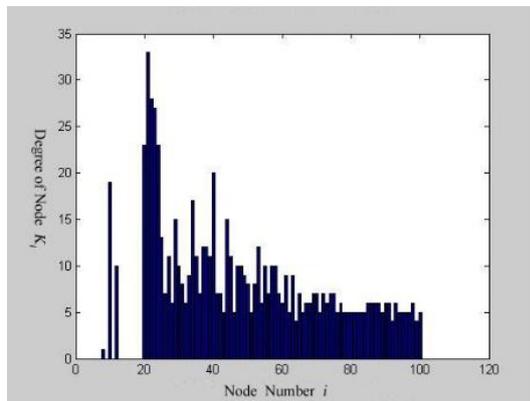


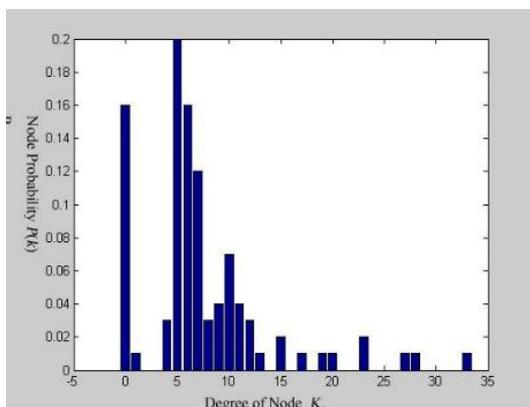
FIGURE 5 Diagram for network distribution

5.1 NETWORK DEGREE ANALYSIS

After growth in the above pattern, MATLAB program is used for statistics and analysis of the degree and distribution of each node. The final output of degree distribution and probability distribution of each node is shown in Figure 6.



a) Node degree distribution



b) Node probability distribution

FIGURE 6 Node degree distribution and node probability distribution

In Figure 6, there are 17 nodes with degree 5, 14 with degree 6, 10 with degree 7, 1 with degree 28, 1 with degree 29, and 10 with degree 0. It shows that only few nodes have a high degree, for example, node 21, 22, and 23 meet the conditions in $P\{k_i(t) \geq k\} > P\{k_j(t) \geq k\}$ and become the central nodes of the supply chain network.

5.2 NETWORK ROBUSTNESS ANALYSIS

Previous data proves that the vast majority of nodes in the supply chain network have only a few connections, in line with the characteristics of scale-free network. By degree analysis of each node in Figure 3 and MSA analysis of degree distribution of the entire supply chain network, a few nodes have significantly exceeded the upper control limit. Therefore, if these enterprises are destroyed, the entire auto parts supply chain network will suffer a devastating injury. Attack on node 21, 22, or 23 will have an impact on the whole auto parts supply, while attack on node 1 or 0 will not.

It suggests that the supply chain network has a strong immunity to general random errors in the continuous growth process, but the central node in the supply chain network must be protected in order to maintain the stability of the network consistently, and as a result, the random network is shown to be in a steady state.

6 Conclusion

When the supply chain network is faced with many uncertain conditions, it is simple and intuitive to describe the supply chain network with the complex network theory. The theory of complex networks is a feasible method to complex supply chain network topology construction. The scale-free supply chain network is analyzed with emphasis through statistical analysis on each enterprise node in the supply chain network. The scale-free supply chain is observed from the perspective of quality control, and thus to propose the definition and quantitative analysis method of the robust supply chain network, providing a more scientific basis for scale-free analysis of the supply chain network. On that basis, two network optimization methods of enhancing the stability of the supply chain are proposed. Finally, practical examples show that the combination of network of quality control principles and complex network theory is a more practical and effective research method.

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References

- [9] Yeh C T, Lin Y K, Huang C F 2014 *IIE Transactions* **46**(10) 1066-1078.
- [10] Dominguez R, Cannella S, Framinan J M 2014 *Computers & Industrial Engineering* **73** 85-95
- [11] Liu G M, Che J G and Lei C C 2009 *Journal of Sichuan Ordnance* **30**(3) 44-5 (in Chinese)
- [12] Santoso T, Ahmed S, Goetschalckx M, Shapiro A 2005 *European Journal of Operational Research* **167**(1) 96-115.
- [13] Xu X, Li R W and Wu X L 2011 *Modern manufacturing engineering* **8** 62-9(in Chinese)
- [14] Khalili-Dambghani K, Tavana M, Amirkhan M 2014 *International Journal of Advanced Manufacturing Technology* **73**(9-12) 1567-95
- [15] Verma A, Seth N 2011 *International journal of Human and Social Sciences* **6**(1) 5-10
- [16] Zhang X, Zhao Q H and Xia G P 2012 *Journal of Convergence Information Technology* **7**(1) 45-53.
- [17] Longinidis P and Georgiadis M C 2014 *Omega-International Journal of Management Science* **47** 73-89
- [18] Guo J L 2006 *Acta Physica Sinica* **55**(8), 3916-3920 (in Chinese)
- [19] Li X L 2012 *Advances in Information Sciences and Service Sciences* **4**(8) 389-96
- [20] Albert R and Barabási A L 2002 *Reviews of Modern Physics* **74** 47-97
- [21] Lin J, Wang G, Hu Z J, Long Q Q 2012 *International Journal of digital content technology and its application* **6**(5) 225-34
- [22] Xu X, Li R W, Wu X L and Liu H X 2012 *China Mechanical Engineering* **23**(8) 941-6 (in Chinese)
- [23] Huang E and Goetschalckx M 2014 *European Journal of Operational Research* **237**(2) 508-18

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