Anti-synchronization of a class of fractional-order chaotic system with uncertain parameters

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Abstract

In order to pull the fractional-order theory to better application, the detailed of computer numerical simulation of the Adams-Bashforth-Moulton Algorithm is proposed in this paper. Anti-synchronization of a class of fractional-order chaotic system with uncertain parameters is realized on this basis and the stability theorem of the system is presented at the same time. And thus it indicates that this method can be adapted to chaotic system with certain parameters and a class of chaotic system with not equal fractional-order. And corresponding implementation conditions is given as well. Besides, it is pointed out that the method, which unites the synchronization and anti-synchronization is also suitable for synchronization issues of the system. Finally, take classic Lorenz system for instance, track time domain and error map about drive system and response system of anti-synchronization are given. The results prove the effectiveness of the control method in the realization of anti-synchronization of a class of fractional-order chaotic system with uncertain parameters and the feasibility of fractional order computer numerical simulation

Keywords: fractional-order, chaotic system, anti-synchronization, chaos control

1 Introduction

Fractional calculus is known as a more common form of integer order, and it also has its own unique features - memory function, meanwhile, a growing number of scientific workers have been attracted to fight for it gradually [1-6]. As for fractional definitions and mathematical description, it has gained initial achievements, but how to make a more accurate numerical simulation analysis through computer is still in its infancy. It is can be predicted that computer numerical simulation of fractional order system will greatly promote the engineering applications of fractional order [7-9], meanwhile, it will certainly open up another important research area of computer applications, which will help us describe the objective world better.

Chaos is a movement pattern of nonlinear system, whose feature is a special, unstable and class of random. The system in a chaotic movement is usually called chaotic system, which was first found by the U.S. meteorologist Lorzenz in his proposed meteorological equations. With further research, the discovery and understanding of chaotic feature began to shift to the control and use of it. In 1990, physicist Ott. Grebogi and Yorke at the university of Maryland successfully controlled chaos by parameter perturbation method (i.e. OGY method), which is an iconic achievement. Closely followed, Pecora and Carroll first raised a synchronization scheme of two similar chaotic systems with different initial conditions, which is called PC method. Since then, Control of Chaotic Synchronization has aroused widespread concern. So far, various control methods of Chaotic Synchronization have been proposed by domestic and foreign scholars, such as adaptive control method [10, 11], Backstopping control method [12], fuzzy control [13, 14], sliding mode control [15, 16], etc. With the deepening of the research, the concept of synchronization has also been expanded, which includes anti-synchronization, generalized synchronization, projective synchronization, phase synchronization, trailing synchronization, etc. [17-19]. Among them, the anti-synchronization is a very interesting concept, which means a state that the drive system and response system achieve equal magnitude but opposite in sign.

In real industrial practice, the noise is ubiquitous, and many system parameters of the system cannot be accurately measured, and even unknown. In order to better realize the engineering practical value of computer numerical simulation, it is very meaningful to consider the uncertainty of noise and parameter, and it has preliminary obtained some related research results [20-22].

In summary, anti-synchronization of a class of fractional-order chaotic system with uncertain parameters is studied in this paper, and a stability theorem of the system is presented. Besides, this method is also suitable to chaotic system with certain parameters、drive and response systems with unequal Fractional Order and synchronization of a class of fractional-order chaotic

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system. Finally, take classic Lorenz system as an example, the detailed iterative formula of computer numerical simulation of the Adams-Bashforth-Moulton Algorithm is proposed, and the results of computer numerical simulation are given.

2 Description of fractional system

Theorem 1 [23] a general fractional order linear system can be described as

$$D^q x = Ax \,, \tag{1}$$

where $q=[q_1,q_2,\ldots,q_i,\ldots,q_n]$ and $(0<q_i\le 1)$. If and only if all eigenvalues λ_i of matrix A is satisfied with $|\arg(\lambda_i)|>q\pi/2$, the system (1) is absolutely stable.

In this paper, a class of fractional order chaotic systems with uncertain parameters is presented as:

$$D^{\alpha} x = (A+\Delta A)x + f(x) \,. \tag{2}$$

Its response system is

$$D^{\beta} y = (A+\Delta A)y + f(y) + U(x,y) \,, \tag{3}$$

where $x, y\in R^n$ are all n-dimensional state vectors; *f* is a continuous vector function; $U(x,y)$ is the designed cont roller; α and β are the fractional orders; A is the coeffic ient matrix of the fractional order system; ΔA indicates u ncertain parameters, which satisfies $\|\Delta A\|\le\delta<M$. By th e way, *M* is a constant.

Our goal is to design a controller $U(x,y)$, which could make the system (2) and system (3) to achieve an equal and opposite anti-synchronization.

3 Design of adaptive sliding mode controller

In order to obtain the designed form of the controller, we first suppose is the error vector. Thus, its error system is

$$\begin{aligned} D^{\beta} e &= (A+\Delta A)e + f(y) - f(x) + U(x,y) \\ &= (A+\Delta A)e + F(x,y) + U(x,y) \end{aligned} , \tag{4}$$

where $F(x,y)=f(y)-f(x)$.

Here, we chose a switch sliding surface as $S(t)=D^{\beta-1}e(t)-\int_0^t (A+K)e(\tau)d\tau$, where K is a designed parameter matrices. Usually, the two conditions $S(t)=0$ and $\dot S(t)=0$ are must satisfied simultaneously.

Because $\dot S(t)=0$, we can get

$$\dot S(t) = D^{\beta}e(t) - (A+K)e(t) = 0 \,. \tag{5}$$

Thus, taking Eq. (4) into (5), we can get $\dot S(t)=(\Delta A-K)e(t)+F(x,y)+U(x,y)=0$.

To meet the conditions of sliding mode, we set $DS(t)=-p*sign(S)-r*abs(S)^{m/n}*sign(S)$, in which $sign(S)=\begin{cases}+1, S>0\\ 0, S=0\\ -1, S<0\end{cases}$, *p*>0, *r*>0 are both Controller gain.

On the sliding surface, because $Ds(t)=\dot s(t)=0$ is satisfied, the controller is

$$U(x,y) = (K-\Delta A)e(t) - F(x,y) - (rS + psignS) \,. \tag{6}$$

Theorem 2: When the controller (6) is applied to the system (3), the system (2) and system (3)can achieve the anti-synchronization. In other words, the error system is zero, and it reaches absolute stability.

Proof: When the system is running on the sliding surface, namely $S(t)=0$, the error system can be simplified as $D^{\beta}e=(A+K)e$.

According to Theorem 1, as long as $(A+K)$ meets the condition $|\arg(\lambda_i)|>q\pi/2$, the error system is stable absolutely.

Therefore, the theorem 2 has been proved.

Lemma 1: If the parameters of the fractional order system is determined, i.e $\Delta A=0$, the controller is still valid, and also can control the drive and response system to achieve anti-synchronization.

Lemma 2: If the order α of drive system and β of response system is not equal, then it needs to introduce a compensation controller $U_I(x,y)=D^{\beta}(\chi x)-f(x)-(A+\Delta A)\chi x$. So, the controller (6) is still valid.

Lemma 3: If the error system is configured as $e=y-x$, this means synchronization of the drive system and the response, and the controller is still valid.

4 Numerical simulations

We consider the Lorenz system as the drive system $\begin{cases} D^{\alpha_1}x_1 = a(y_1-x_1)\\ D^{\alpha_2}y_1 = bx_1 - x_1z_1 - y_1\\ D^{\alpha_3}z_1 = x_1y_1 + cz_1\end{cases}$ and response system $\begin{cases} D^{\beta_1}x_2 = a(y_2-x_2)\\ D^{\beta_2}y_2 = bx_2 - x_2z_2 - y_2\\ D^{\beta_3}z_2 = x_2y_2 + cz_2\end{cases}$, where the parameters (a, b, c) = (10, 28, 8/3), $\alpha=\beta=0.99$, and the initial value (x_1, y_1, z_1) = (1, 0, 9), (x_2, y_2, z_2) = (1, 1, 1).

We set the parameters are p=0.2, r=6, m=3and n=2;

Coefficient matrix of the system is $A=\begin{pmatrix}-10 & 10 & 0\\ 28 & -1 & 0\\ 0 & 0 & -8/3\end{pmatrix}$;

parameter matrix $K=\begin{pmatrix}-20 & 20 & 0\\ -28 & 0 & 0\\ 0 & 0 & -1/3\end{pmatrix}$. Thus, A + K are $A+K=\begin{pmatrix}-40 & 40 & 0\\ -10 & -1 & 0\\ 0 & 0 & -3\end{pmatrix}$, and its eigenvalues are $(\lambda_1,\lambda_2,\lambda_3)=(-20.5+4.44i,-20.5-4.44i,-3)$.

According to Theorem 1, it satisfies $\left|\arg(\lambda_i)\right| > \beta_i\pi/2 = 0.99\pi/2$, so the error system is absolutely stable, which means the drive system and response system achieve to synchronization. Therefore, the expression of the controller is

$$\begin{cases} u = (-30e_1+30e_2)-(5s_1+0.2sign(s_1)) \\ u = -38e_1-(5s_2+0.2sign(s_2)) \\ u = -\frac{1}{3}e_3-(5s_3+0.2sign(s_3)) \end{cases}.$$

The fractional order nonlinear system equation can be solved by using Adams-Bashforth-Moulton algorithm. So, the iterative formula of solving fractional Lorenz system is

$$\begin{cases} x_{n+1}=x_0+\dfrac{h^\alpha}{\Gamma(\alpha+2)}\left\{10[y_{n+1}^p-x_{n+1}^p]+\sum_{j=0}^{n}a_{1,j,n+1}10(y_j-x_j)\right\} \\ y_{n+1}=y_0+\dfrac{h^\beta}{\Gamma(\beta+2)}\left\{-x_{n+1}^p z_{n+1}^p+(24-4c)x_{n+1}^p+cy_{n+1}^p\right\} \\ \qquad +\sum_{j=0}^{n}a_{2,j,n+1}\left(-x_j z_j+(24-4c)x_j+cy_j\right) \\ z_{n+1}=z_0+\dfrac{h^\gamma}{\Gamma(\gamma+2)}\left\{[x_{n+1}^p y_{n+1}^p-\dfrac{8z_{n+1}^p}{3}]+\sum_{j=0}^{n}a_{3,j,n+1}\left(x_j y_j-\dfrac{8z_j}{3}\right)\right\} \end{cases},$$

in which

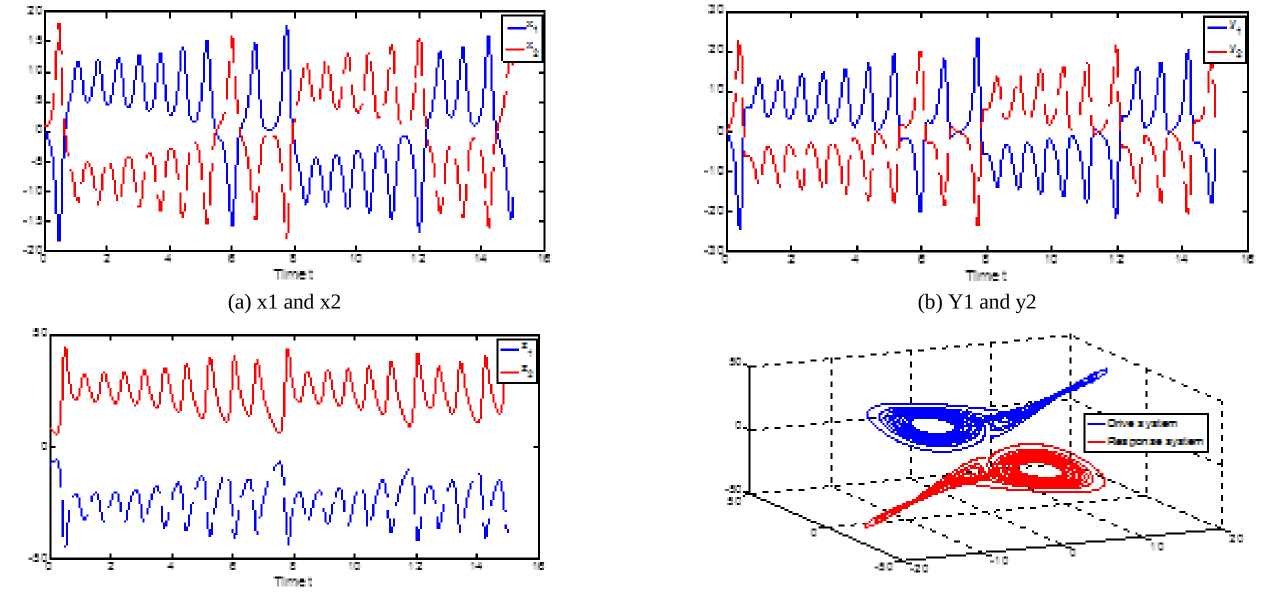
$$\begin{cases} x_{n+1}^p=x_0+\dfrac{1}{\Gamma(\alpha)}\sum_{j=0}^{n}b_{1,j,n+1}10(y_j-x_j) \\ y_{n+1}^p=y_0+\dfrac{1}{\Gamma(\beta)}\sum_{j=0}^{n}b_{2,j,n+1}(-x_j z_j+(24-4c)x_j+cy_j) \\ z_{n+1}^p=z_0+\dfrac{1}{\Gamma(\gamma)}\sum_{j=0}^{n}b_{3,j,n+1}\left(x_j y_j-\dfrac{8z_j}{3}\right) \end{cases}$$

and

$$\begin{cases} b_{1,j,n+1}=\dfrac{h^\alpha}{\alpha}((n-j+1)^\alpha-(n-j)^\alpha, 0\le j\le n \\ b_{2,j,n+1}=\dfrac{h^\beta}{\beta}((n-j+1)^\beta-(n-j)^\beta, 0\le j\le n \\ b_{3,j,n+1}=\dfrac{h^\gamma}{\gamma}((n-j+1)^\gamma-(n-j)^\gamma, 0\le j\le n \end{cases},$$

$$\begin{cases} a_{1,j,n+1}=\begin{cases} n^\alpha-(n-\alpha)(n+1)^\alpha & j=0 \\ (n-j+2)^{\alpha+1}+(n-j)^{\alpha+1}-2(n-j+1) & 0\le j\le n \end{cases} \\ a_{2,j,n+1}=\begin{cases} n^\beta-(n-\beta)(n+1)^\beta & j=0 \\ (n-j+2)^{\beta+1}+(n-j)^{\beta+1}-2(n-j+1) & 0\le j\le n \end{cases} \\ a_{3,j,n+1}=\begin{cases} n^\gamma-(n-\gamma)(n+1)^\gamma & j=0 \\ (n-j+2)^{\gamma+1}+(n-j)^{\gamma+1}-2(n-j+1) & 0\le j\le n \end{cases} \end{cases}.$$

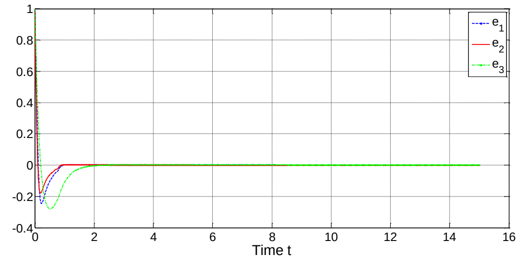
Programming numerical analysis with Matlab, we can get time-domain diagram as shown in Fig. 1 and error map as shown in Fig. 2, which demonstrates drive system and response system of Lorenz system achieve anti-synchronization. From the Fig. 1 and Fig. 2, we know the system quickly achieve to anti-synchronization, and the corresponding anti-synchronization error is 0.



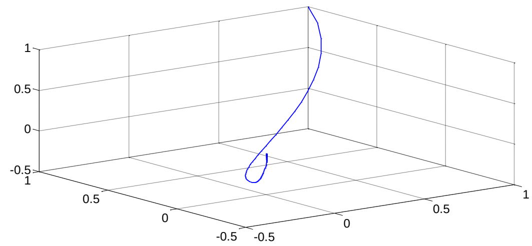
(a) x1 and x2 (b) Y1 and y2

(c) Z1 and z (d)Three -dimensional plot

FIGURE 1 Time-domain of Lorenz drive system and response system achieving to anti-synchronization



(a) e1, e2 and e3



(b)Three -dimensional plot

FIGURE 2 Error map of Lorenz drive system and response system achieving to anti-synchronization

5 Conclusions

In this paper, anti-synchronization of a class of fractional-order chaotic system with uncertain parameters is realized based on Sliding Mode Control Theory. Moreover, taken Lorenz system as an example, a detailed iterative formula of computer numerical simulation based on Adams-Bashforth-Moulton Algorithm is proposed, and the simulation results are presented. We can draw the following conclusions:

(1) This method has good robustness, and can well control a class of chaotic system with uncertain parameter to achieve anti-synchronization;

(2) From the three Lemmas, we can also get that this method is also suitable to chaotic system with certain parameters, unequal fractional orders;

(3) The feasibility of fractional order numerical simulation is also been proved.

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