# Physical parameter identification and wave force inversion research of bridge pier structure

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**Abstract**

According to total compensation composite inversion algorithm, physical parameter identification and inversion of wave time interval are done under the condition of input information of isolated pier in wave force part being unknown. Based on the characteristics of large-diameter piers in shallow water which are affected by wave force and combining with "statistical average method" in probability theory, structural physical parameter identification and wave inversion calculation are done under the condition of unknown wave force. Analysis of numerical values shows that results of structural physical parameter identification cater for accuracy requirements which is feasible for inversion method of parameter identification of large-diameter piers in shallow water with unknown input information thus providing a new method for structural physical parameter identification research of river-spanning and sea-crossing piers affected by wave force and providing references for engineering application.

*Keywords:* bridge pier structure, parameter identification, unknown input, load inversion, time domain correlation

## 1 Introduction

Performance of river-spanning and sea-crossing bridges in service gradually degenerates under the effect of load and environment. Although degeneration process is slow, structural damage is sudden and fragile which keeps high randomness and unpredictability giving considerable difficulties to safety forecasting. Therefore bridges' health monitoring and preservation including detecting the current condition of bridges in service and judging their damage condition are essential. Correctness of bridge structural health monitoring results mainly depends on whether recognition algorithm could accurately and effectively identify the real form of structure from the actual recorded signal.

At present most recognition algorithms are established on the basis of recorded information being complete which is not suitable for damage detection of river-spanning and sea-crossing bridge structures. The reason is that environmental excitation is generally taken as vibrating load in physical testing. However, interaction positions of these vibrating loads are uncertain which brings difficulties for measurement and is difficult to guarantee the completeness of recorded information. Moreover limited by testing cost, not enough number of sensor could be used to record the response messages at all positions with independent freedom degree in the structure, which is also difficult to guarantee the completeness of response. Therefore, how to identify the structural dynamic properties under the condition of testing information being incomplete or unknown is a difficult problem in the research of bridge dynamic detection technology theory at present. According to this, this paper takes irregular wave excitation simulated by linear amplitude superposition method as example and utilizes total compensation composite inversion algorithm to do identification on isolated piers under the following two input conditions:

1) When wind power is low, only underwater part of isolated bridge pier is affected by wave. While force on the part above the water is approximate to 0. Identification is done on it under the condition of input information being unknown.

2) Large-diameter piers in shallow. water are totally submerged. They are done with unknown input information system identification affected by wave force. Identification results show that total compensation composite inversion algorithm, in which wave excitation is manually simulated could better invert wave effect, do large-diameter unknown input system identification and wave force inversion.

## 2 Numerical simulation of random wave force of isolated piers

### 2.1 NUMERICAL SIMULATION OF IRREGULAR WAVE OF LINEAR SUPERPOSITION METHOD

Suppose that the known input wave force $f(t)$ is Gaussian random process whose average value $i$=0 and wave spectrum density function $S(\omega)$ is known. Then numerical simulation could be done on manual wave through three methods of constant amplitude superposition method, various amplitude superposition method and autoregressive method. This paper utilizes various amplitude superposition method, in which Fourier rapidly transforms helping to greatly save

***Corresponding author** e-mail: 121687678@qq.com computation time. Comparing with constant amplitude superposition method, it presents certain improvement and keeps unconditionally stable. Specific derivation and simulation process is as follows [1]:

One random wave process could be described as:

$$\eta(t) = \sum_{n=1}^{\infty} a_n \cos(k_n x - \omega_n t - \varepsilon_n) . \quad (1)$$

In terms of a fixed point, it is advisable that x=0, then:

$$\eta(t) = \sum_{n=1}^{\infty} a_n \cos(\omega_n t + \varepsilon_n) . \quad (2)$$

In order to determine the of each $a_n, \omega_n$ component wave, wave spectrum $S_{\eta\eta}(\omega)$ may be used to make it in which $\varepsilon_n$ is the evenly random distribution number in the range of $0 \sim 2\pi$ . Take wave spectrum $s_{\eta\eta}(\omega)$ as target spectrum whose energy covers the vast majority in the range $\omega_1 \sim \omega_{m+1}$ and other parts are rejected. It is defined that $\Delta\omega_i = \omega_{i+1} - \omega_i$ and $\varpi_i = \frac{1}{2}(\omega_i + \omega_{i+1})$ . Superpose m cosine waves of wave energies in m sections whose wavefront equation is:

$$\eta(t) = \sum_{t=1}^{m} \sqrt{2S_{\eta\eta}\varpi_i \Delta\omega_i} \cos(\omega_i t + \varepsilon_i) . \quad (3)$$

According to [2], fluctuate horizontal velocity and acelerated velocity of water particle to acquire the following equations:

$$\mu(t) = \sum_{t=1}^{m} \sqrt{2S_{\eta\eta}(\omega_i)\Delta\omega_i}\,\omega_i \frac{\cosh(k_i z)}{\sinh(k_i d)} \cos(\omega_i t + \varepsilon_i), \quad (4)$$

$$\mu(t) = -\sum_{t=1}^{m} \sqrt{2S_{\eta\eta}(\omega_i)\Delta\omega_i}\,\omega_i^2 \frac{\cosh(k_i z)}{\sinh(k_i d)} \cos(\omega_i t + \varepsilon_i) . \quad (5)$$

Wave force Equation (6) is affected on piles of per unit length deduced according to Morison.

$$f(t) = f_D(t) + f_I(t) = \frac{\pi}{4}\rho C_m D^2 \dot{u}(t) + \frac{1}{2}\rho C_m D^2 u(t)\left|u(t)\right| . \quad (6)$$

Suppose that the total height of this pier is *l*, height of the below-surface part is *d*, weight is concentrated on the single-degree-of - freedom system of $z_0$ height centre. Then time-varying Equations (7)-(9) of the simplified total inertia force of single degree of freedom, total drag force item and total wave force are shown as Equation (9):

$$P_I(t) = \frac{\pi}{4}\rho C_m D^2 \dot{\mu}(t) d , \quad (7)$$

$$P_D(t) = \frac{1}{2}\rho C_D D \mu(t)\left|\mu(t)\right| d , \quad (8)$$

$$F_l(t) = \frac{1}{2}\rho C_D D f(z_0) \sum_{i=1}^{m} 2S_{\eta\eta}(\omega_i)\Delta\omega_i\omega_i^2 \frac{\cosh(k_i z)}{\sinh(k_i d)} \cos(\omega_i t + \varepsilon_i) \left| \frac{\cosh(k_i z)}{\sinh(k_i d)} \cos(\omega_i t + \varepsilon_i) \right| - \frac{\pi}{4}\rho C_m D^2 d f(z_0) \sum_{t=1}^{m} \sqrt{2S_{\eta\eta}(\omega_i)\Delta\omega_i}\,\omega_i^2 \frac{\cosh(k_i z)}{\sinh(k_i d)} \sin(\omega_i t + \varepsilon_i), \quad (9)$$

$$F_j(t) = \frac{\pi}{4}\rho C_m D^2 \frac{\cosh(k_i z)}{\sinh(k_i d)} \sum_{i=1}^{m} \sqrt{2S_{\eta\eta}(\omega_i)\Delta\omega_i}\,\omega_i^2 \sin(\omega_i t + \varepsilon_i) dz,\ j = 1, 2, \ldots . \quad (10)$$

Total wave force $F_l(t)$ in Equation (9) is the wave force vibration equation, which is acquired after wave is transformed to the top of pier. Substitute this wave force vibration equation into dynamic equation, in which response spectrums of displacement, velocity and acceleration of this pier structure could be acquired through calculating this differential equation.

## 2.2 SIMPLIFIED CALCULATION OF WAVE FORCE OF LARGE-DIAMETER PILE IN SHALLOW WATER

### *2.2.1 Water Depth Keeping Great Influence on Wave Motion*

Wave in deep water (*d≥0.5L*), *d* means water depth, *L* is wave length, the same below.) is called deep-water wave. Wave in shallow water (*0.05L≤d<0.5L*) is called shallow-water wave. Wave in extra-shallow water (*d<0.5L*) is called extra-shallow wave. According to [3], suppose that wave period *T* or wave number *K* in shallow water do not change with water depth namely $K = \frac{2\pi}{L}$.

### *2.2.2 Large-Diameter Components*

Aiming at large-diameter components, the influences of diameter of pier on wave force usually utilize diffraction theory proposed by MacCamy and Fuchs from USACE in 1954. In terms of relatively big-size structures (*D*≥0.2*L*, *D* is the diameter of the structure), viscosity effect could be neglected. Wave force is main the inertia force [3].Thereby formula of simplified wave force of large-diameter piers in shallow water taking *dz* height as unit is as Equation (10). Here *dz* is the height of pier which is divided into single degree of freedom.

$j$ is the dividing segments taking $dz$ as height and $j = d / dz$. $z_j$ is the height from centre of $j$ to the bottom of pier. Other parameters are the same as the above.

It is deduced from the simplified Equation (10) that wave force ratio affected on any adjacent unit is determined only by cosh($kz_j$):

$$\alpha_j = \frac{F_{j+1}(t)}{F_j(t)} = \frac{\cosh(kz_{j+1})}{\cosh(kz_j)}. \quad (11)$$

## 3 Algorithm theory and identification steps

### 3.1 PART OF UNKNOWN EXTERNAL EXCITATION MATRIX

Electrical Dynamic equation of multiple-degree-of-freedom system is:

$$M\ddot{u} + C\dot{u} + Ku = F(t), \quad (12)$$

where $M$, $C$, $K$ are respectively weight, damping and stiffness matrix. $F(t)$ is the external excitation vector. $u$, $\dot{u}$ and $\ddot{u}$ are respectively structural displacement, speed and aceleration response vector.

Suppose that $M$ is the known diagonal moment. Transpose the Equation (12) to acquire that:

$$C\dot{u} + Ku = F(t) - M\ddot{u} = P. \quad (13)$$

Lead $P$ to become system input vector.

1) Part of unknown input external excitation matrix could be shown as:

$$F(t) = [F_k(t) \;\; F_u(t)]'. \quad (14)$$

where $F_k(t)$ is external excitation of known time interval information. $F_u(t)$ is external excitation of unknown time interval information. According to Reference [4], Equation (13) could be transformed as:

$$H\theta = P, \quad (15)$$

where $H$ is system response matrix namely the response of system under excitation of external system $F(t)$. $\theta$ is the system physical parameter vector waiting for being identified. According to Equations (13) and (14), system input vector of $P$ is acquired:

$$P = [P_k \;\; P_u]'. \quad (16)$$

Total compensation inversion algorithm identifies system parameter $\theta$ according to Equation (15) and inverts unknown input vector $P_u$ under the condition of system response matrix $H$ and part of input $P_k$ being known. Inversion algorithm is done on the part of unknown input according to [5] whose main steps are:

Artificially define initial value of any structural parameter vector to be $\hat{\theta}_0$;

Substitute the given initial value $\hat{\theta}_0$ into Equation (15) to calculate out that:

$$\tilde{P}_0 = H\hat{\theta}_0, \quad (17)$$

where $\tilde{P}_0$ could be expressed as:

$$\tilde{P}_0 = [\tilde{P}_k \;\; \tilde{P}_u]'. \quad (18)$$

Replace the estimated $\tilde{P}_k$ in Equation (21) with known excitation time interval $P_k$ to acquire the revised system input vector:

$$\hat{P}_0 = [P_k \;\; \tilde{P}_u]'. \quad (19)$$

According to Equation (18) perform the calculations on the revised $\hat{P}_0$ vector based on least squares criterion to acquire the new estimated value of structural parameter:

$$\hat{\theta}_1 = (H'H)^{-1} H'\hat{P}_0. \quad (20)$$

Determine whether $\hat{\theta}_1$ caters for all convergence conditions. All parameters cater for the following formula:

$$\left| \frac{\hat{\theta}_n^i - \hat{\theta}_{n-1}^i}{\hat{\theta}_{i-1}} \right| \le \varepsilon, \quad (21)$$

where $\hat{\theta}_n^i$ is the i system parameter estimated value of the n iteration. $\varepsilon$ is the artificially given precision value. If all conditions cater for Equation (21), calculation finishes, in which the current system parameter identification result is the final calculation result. If not, new $\hat{\theta}_1$ is taken as initial value, in which Equations (16)-(21) are repeated.

### 3.2 UNKNOWN EXTERNAL EXCITATION MATRIX

$$F(t) = [F_u(t) \quad \alpha F_u(t)]', \quad (22)$$

where $F_u(t)$ is wave force of the unknown input unit. $\alpha F_u(t)$ is the wave force of adjacent unknown input unit. $\alpha$ is the ratio of wave forces between two adjacent units. They are calculated through Equation (11).

Similarly it could be transformed as follows according to Equation (15):

$$H\theta = P. \quad (23)$$

According to Equation (18), system input vector is acquired:

$$\tilde{P}_0 = [\tilde{P}_{ui} \quad \tilde{P}_{u(i+1)}]', \quad (24)$$

where $\tilde{P}_{ui}$ and $\tilde{P}_{u(i+1)}$ are respectively the unknown input vectors of $i$ and $i$+1 degrees of freedom or units.

Main steps of inversion algorithm under similar part of unknown input are:

Initial value of artificially given any structural parameter vector is $\hat{\theta}_0$.

Substitute the given initial value $\hat{\theta}_0$ into Equation (16) to calculate out that:

$$\tilde{P}_0 = H\hat{\theta}_0 , \tag{25}$$

where $\tilde{P}_0$ can be expressed as:

$$\tilde{P}_0 = \left[ \tilde{P}_{ui} \;\; \tilde{P}_{u(i+1)} \right]' . \tag{26}$$

After $\tilde{P}_0$ is calculated, $F(t)$ in Equation (22) becomes known, through which estimated values of $\tilde{F}_u(t)$ and $\alpha\tilde{F}_u(t)$ could be acquired according to Equation (22). Generally values of $\tilde{F}_u(t)$ in estimated $\tilde{F}_u(t)$ and $\alpha\tilde{F}_u(t)$ are unequal which does not cater for the equal result deduced in Equation (11). At this moment statistical average method in probability is done on $\tilde{F}_u(t)$, in which the average $\bar{F}_u(t)$ is utilized to acquire the new estimated input force $\hat{P}$ process [6] of each unit.

$$\bar{F}_u(t_i) = \frac{1}{N}\sum_{l}^{N} \tilde{F}_{u.l}(t_i), i = 1,2,...,M; l = 1,2,...,N , \tag{27}$$

where $N$ is degree of freedom or unit number. $M$ is sampling points.

Utilize the new $\bar{F}_u(t)$ to acquire the revised vector $\hat{P}_0$. Then new parameter estimated value could be acquired under least squares criterion according to Equation (20).

Determine whether the new calculated $\hat{\theta}_1$ and the original $\hat{\theta}_0$ cater for the convergence condition of the given precision $\varepsilon$. Suppose that $\hat{\theta}_1^i$ is the $i$ identification value of structural parameter. If all parameters cater for the following equation:

$$\left| \frac{\hat{\theta}_1^i - \hat{\theta}_0^i}{\hat{\theta}_0^i} \right| \le \varepsilon . \tag{28}$$

Then parameter estimated value of this step $\hat{\theta}_1$ is taken as the final calculation result. Otherwise it would be taken as new parameter initial value to repeat steps 2~5 up to convergence.

Utilize the finally identified parameter $\hat{\theta}_1$ to invert the practical wave force excitation according to Equation (16).

Calculation steps under the above-mentioned two input conditions could be expressed more directly by Figure 1.

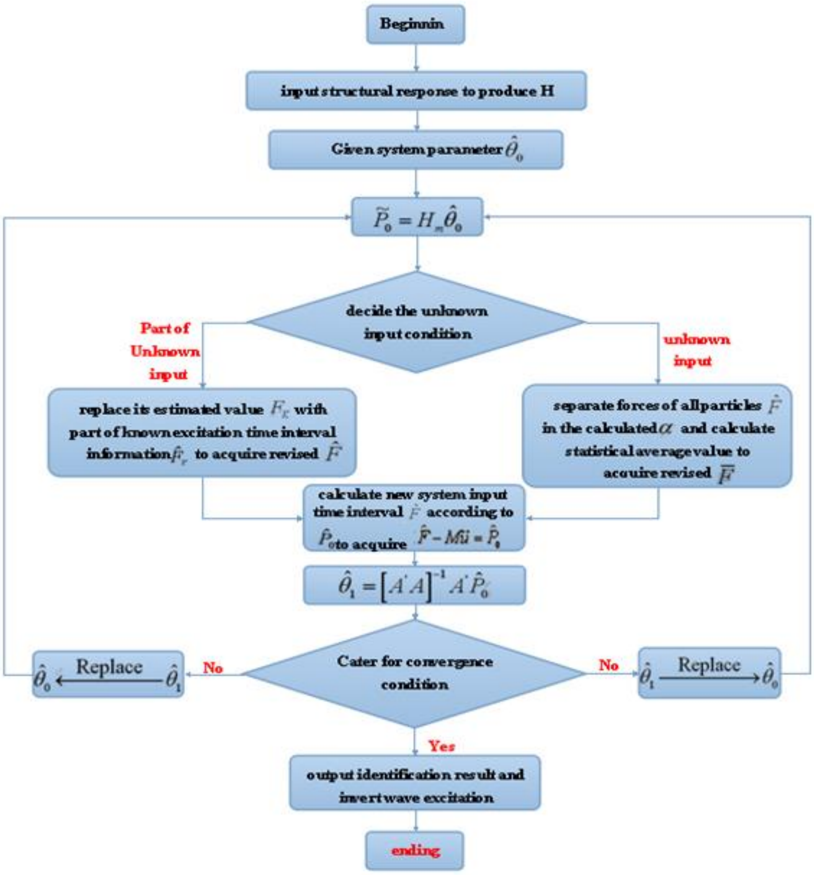


FIGURE 1 Flow diagram of inversion calculation

It is seen from a series of algorithm steps above that physical parameter identifications of pier structure under condition of part of wave input being unknown and of large-diameter piers in shallow water under the condition of wave input being unknown according to total compensation composite algorithm are done whose concept is clear. Thought is of clarity and identification steps are easy for programming realization.

## 4 Example analysis

### 4.1 EXAMPLE ENGINEERING PROFILE

Establish the simplified model of isolated pierstructure Figure 2.

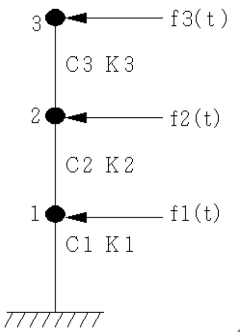


FIGURE 2 Simplified model of isolated pier structure

Suppose that concentrated masses M1 = 18840 kg, M2 = 18000 kg, M3 = 15840 kg, stiffnesses between units K1 = $1.623\times10^7$ N/m, K2 = $1.082\times10^7$ N/m, K3 = $0.952\times10^7$N/m and proportional dampings among units are respectively C1 = 138693 Ns/m, C2 = 108462 Ns/m and C3 = 75786 NS/m. Diameter of the bridge pier is Dm whose heighth is 24m. Here the designed wave height $H = H_{1\%} \approx 1.51 H_{1/3} = 9.0m$ and designed wave length $L = \bar{L}(\text{average wavelength}) = 135.0\text{m}$. According to characteristics of wave method, it is acquired that $P_D \le 0.5 P_I$. Neglect drag force and only take effects of inertial wave force into consideration. Wave spectrum uses P-M Spectrum. Suppose that displacement, speed and acceleration response of each node are all known. It is required to identify all dampings and stiffness parameters and invert the input wave force. Utilizing $Newark-\beta$ Method to calculate dynamic response, in which sampling point $M$ is 1000 and sampling time interval $dt = 1.0s$. Suppose that all mass parameters in calculation process are known and real parameters of structure are shown in Table 1. Physical parameter identification and effect load inversion are done under the following conditions.

TABLE 1 Parameter list of each node of bridge pier

| Node | Weight(kg) | Damping(kg/ms) | Stiffness(N/m) |
|---|---|---|---|
| **1** | 0.1298e+07 | 0.7788e+06 | 0.2384e+06 |
| **2** | 0.1400e+07 | 0.8400e+06 | 0.2540e+06 |
| **3** | 0.1584e+07 | 0.9504e+06 | 0.3010e+06 |

### 4.2 PHYSICAL IDENTIFICATION AND WAVE FORCE INVERSION UNDER PART OF INPUT UNKNOWN AND UNKNOWN INPUT

When diameter of pier d is 9.0m and water depth is 20m, wave time intervals of node1 and node2 affected by irregular wave force which is simulated by variable amplitude superposition method are shown in Figure 3 in which correlation coefficient $\alpha_1 = 1.14$ of $f1(t)$ and $f2(t)$. It is known through numerical calculation that after response, the assumed input wave forces $f1(t)$ and $f2(t)$ are unknown. At this moment known force of pile segment above water surface $\alpha_1 = 1.14$. Thereby a problem of parameter identification under part of input being unknown and input load inversion comes into being.

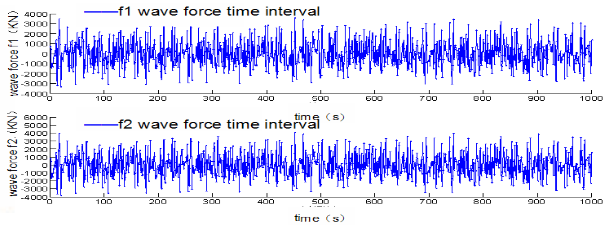


FIGURE 3 Time interval curve of wave force exerted on nodes

Stiffness and damping matrix of bridge pier structure in this case are shown as (29):

$$K = \begin{bmatrix} k_1+k_2 & -k_2 & 0 \\ -k_2 & k_2+k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix}, \quad C = \begin{bmatrix} c_1+c_2 & -c_2 & 0 \\ -c_2 & c_2+c_3 & -c_3 \\ 0 & -c_3 & c_3 \end{bmatrix}. \tag{29}$$

Concrete matrix expressions of $H$, $\theta$ and $P$ deduced by Equation (16) are shown as Equation (30):

$$H(t_i) = \begin{bmatrix} x_1(t_i) & x_1(t_i)-x_2(t_i) & 0 & \dot{x}_1(t_i) & \dot{x}_1(t_i)-\dot{x}_2(t_i) & 0 \\ 0 & x_2(t_i)-x_1(t_i) & x_2(t_i)-x_3(t_i) & 0 & \dot{x}_2(t_i)-\dot{x}_1(t_i) & \dot{x}_2(t_i)-\dot{x}_3(t_i) \\ 0 & 0 & x_3(t_i)-x_2(t_i) & 0 & 0 & \dot{x}_3(t_i)-\dot{x}_2(t_i) \end{bmatrix}, \tag{30}$$

$$P(t) = [P(t_1), P(t_2), P(t_3), ..., P(t_M)]', \tag{31}$$

$$P(t_i) = \begin{bmatrix} f_1(t_i) - m_1\ddot{x}_1(t_i) \\ f_2(t_i) - m_2\ddot{x}_2(t_i) \\ 0 \end{bmatrix}, \tag{32}$$

$$\theta = [k_1, k_2, k_3, c_1, c_2, c_3]', \quad (33)$$

According to the above-mentioned conditions, responses of displacement $u$, speed $\dot{u}$ and acceleration $\ddot{u}$ of structure affected by $f2(t)$ and $f3(t)$ are acquired utilizing Nemark Method. Meanwhile Node 3 not affected by wave load is taken as known part of input information of input load $f3(t) = 0$. Without regard to noise, parameter results identified by the method of this paper are shown in Table 2. Taking the influence of noise on the actually acquired data into consideration, influence of white noise is added into the response acquired by structure to simulate the actually measured results, in which added noise level is determined by percentage between response peak value and noise peak value [7] in order to test the sensitive degree of this algorithm on the influence of noise. Here parameter identification and load inversion are considered under two conditions of 1% and 6%. Parameter identification results under the condition of noise are shown in Table 3 and Table 4.

It is seen from Table 3 and 4, and Figures 4-7 that under condition of part of unknown input wave force being time domain related, method of this paper could not only identify physical parameter of pier but also accurately invert input load of each node. In terms of 6% of noise level, identified parameters are utilized to do inversion on the input wave forces on Nodes 1 and 2. Comparing with actually input wave force, the maximum error is 6.6149% whose results are shown in Figures 4-7. Obviously, its inversion results are ideal.

TABLE 2 Parameter identification list without noise (iteration 169 times, $\varepsilon$=0.000002 )

| Node | Damping identification value (kg/ms) | Damping error (%) | Stiffness identification value (N/m) | Stiffness error (%) |
|---|---|---|---|---|
| 1 | 0.77879e+06 | 0.012 | 0.23839e+06 | 0.004 |
| 2 | 0.83998e+06 | 0.002 | 0.25399e+06 | 0.004 |
| 3 | 0.95040e+06 | 0.000 | 0.30100e+06 | 0.000 |

TABLE 3 Parameter identification list with 1% noise (iteration 172 times, $\varepsilon$=0.000002 )

| Node | Damping identification value (kg/ms) | Damping error (%) | Stiffness identification value (N/m) | Stiffness error (%) |
|---|---|---|---|---|
| 1 | 0.77820e+06 | 0.077 | 0.23876e+06 | 0.151 |
| 2 | 0.8527e+06 | 1.512 | 0.25063e+06 | 1.327 |
| 3 | 0.95407e+06 | 0.386 | 0.29753e+06 | 1.153 |

TABLE 4 Parameter identification list with 6% noise (iteration 171 times, $\varepsilon$=0.000002 )

| Node | Damping identification value (kg/ms) | Damping error (%) | Stiffness identification value (N/m) | Stiffness error (%) |
|---|---|---|---|---|
| 1 | 0.80296e+06 | 3.1022 | 0.25417e+06 | 6.6149 |
| 2 | 0.87081e+06 | 3.6678 | 0.24319e+06 | 4.2559 |
| 3 | 0.98396e+06 | 3.5311 | 0.32047e+06 | 6.4684 |

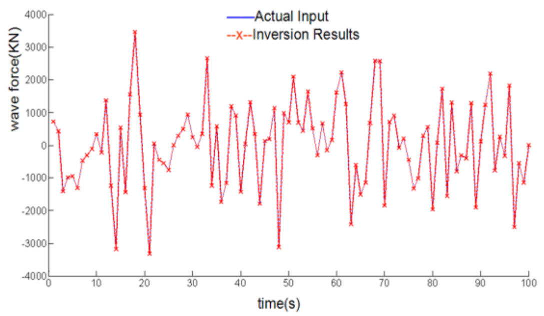

FIGURE 4 Comparison between wave load input and inversion time interval on node 1

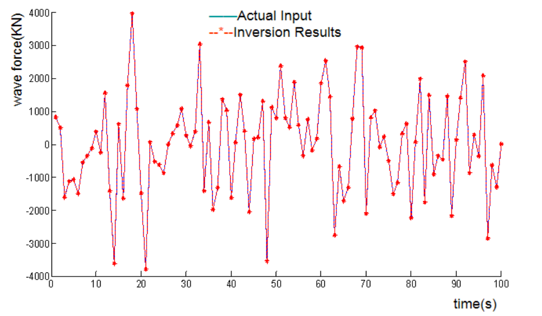

FIGURE 5 Comparison between wave load input and inversion time interval on node 2

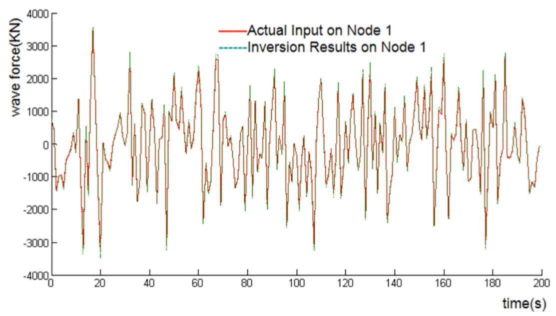

FIGURE 6 Comparison of actual input inversion results on node 1 under 6% noise

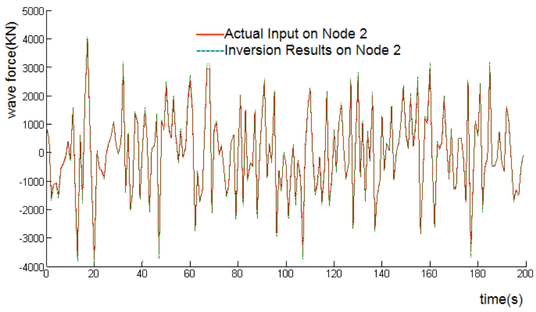

FIGURE 7 Comparison of actual input inversion results on node 2 under 6% noise

When diameter of pier $D$ is 9.0m and water depth is 24m, wave time intervals of Nodes 1, 2 and Node 3, affected by irregular wave force, which is simulated by variable amplitude superposition method are shown in Figure 4, among which correlation coefficient of $f1(t)$ and $f2(t)$ is $\alpha_1 = 1.14$ and the one of $f1(t)$ and $f3(t)$ is $\alpha_2 = 1.44$. Through numerical calculation it is acquired that input wave forces $f1(t)$, $f2(t)$ and $f3(t)$ are unknown after response. Thereby a typical problem of parameter identifycation and load inversion under unknown load input comes into being. Input wave time intervals on pier nodes are shown in Figure 8.

Similar with the condition above, parameter result identified by the method in this paper are shown in Table 5 without taking noise into consideration. Similarly parameter identification and load inversion under 1% and 6% percenttage of noise are considered. Parameter identification results with noise are shown in Table 6 and Table 7.

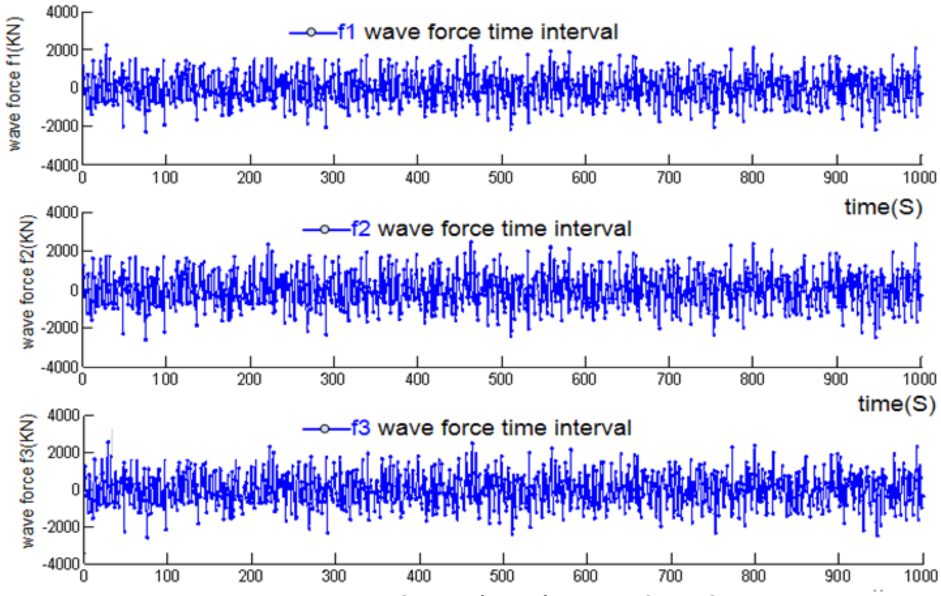


FIGURE 8 Time interval curve of wave force exerted on nodes

TABLE 5 Parameter identification list without noise (iteration 816 times, $\varepsilon$=0.000002 )

| Node | Damping identification value (kg/ms) | Damping error (%) | Stiffness identification value (N/m) | Stiffness error (%) |
|---|---|---|---|---|
| **1** | 0.77876e+06 | 0.0039 | 0.23837e+06 | 0.0084 |
| **2** | 0.83995e+06 | 0.0036 | 0.25397e+06 | 0.0079 |
| **3** | 0.95034e+06 | 0.0063 | 0.30096e+06 | 0.0010 |

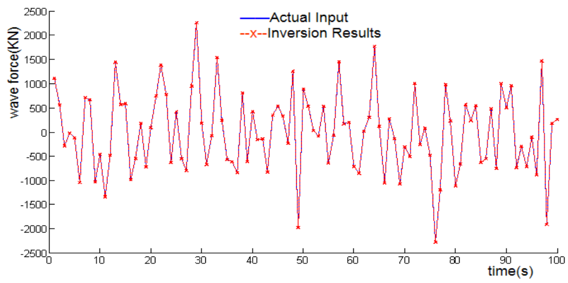


FIGURE 9 Comparison between wave load input and inversion time interval on node 1

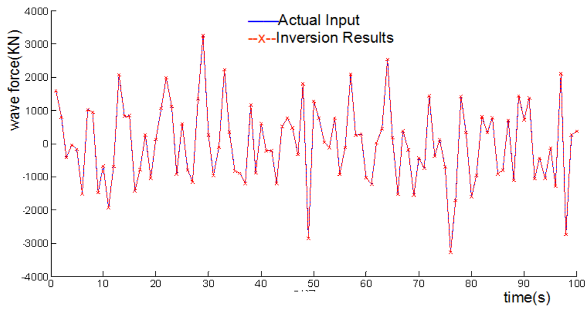


FIGURE 10 Comparison between wave load input and inversion time interval on node 2

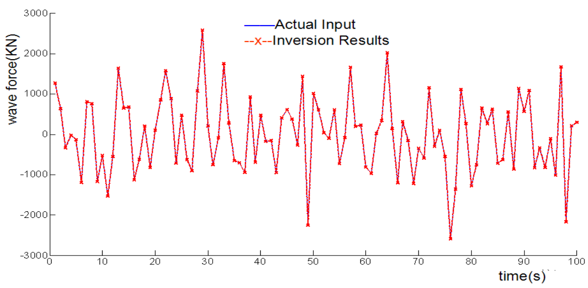


FIGURE 11 Comparison between wave load input and inversion time interval on node 3

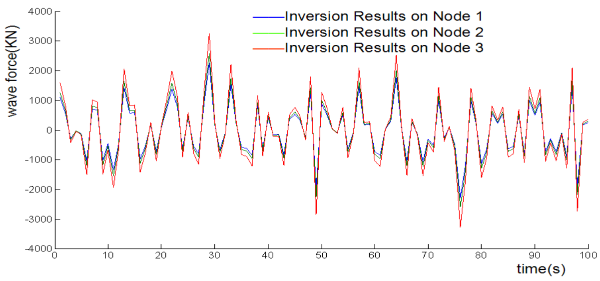


FIGURE 12 Comparison between wave load input and inversion time interval among three nodes

TABLE 6 Parameter identification list with 1% noise (iteration 823 times, $\varepsilon$=0.000002)

| Node | Damping identification value (kg/ms) | Damping error (%) | Stiffness identification value (N/m) | Stiffness error (%) |
|---|---|---|---|---|
| **1** | 0.77645e+06 | 0.3018 | 0.23867e+06 | 0.1133 |
| **2** | 0.84012e+06 | 0.0143 | 0.25128e+06 | 1.0709 |
| **3** | 0.98147e+06 | 3.2691 | 0.30019e+06 | 0.2691 |

TABLE 7 Parameter identification list with 6% noise (iteration 775 times, $\varepsilon$ = 0.000002)

| Node | Damping identification value (kg/ms) | Damping error (%) | Stiffness identification value (N/m) | Stiffness error (%) |
|---|---|---|---|---|
| **1** | 0.7980e+06 | 2.4653 | 0.2370e+06 | 0.5872 |
| **2** | 0.7820e+06 | 6.9048 | 0.2685e+06 | 5.7087 |
| **3** | 1.0292e+06 | 8.2912 | 0.3083e+06 | 2.4252 |

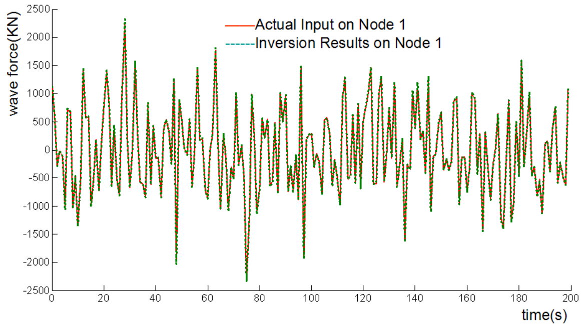


FIGURE 13 Comparison of actual input inversion results on Node 1 under 6% Noise

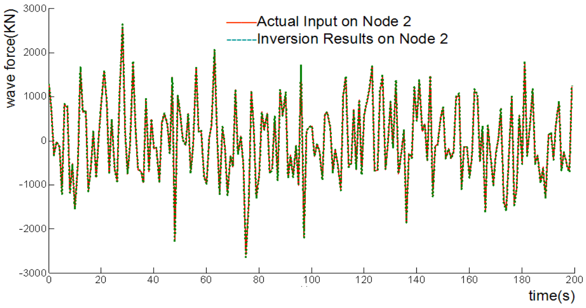


FIGURE 14 Comparison of actual input inversion results on Node 2 under 6% Noise

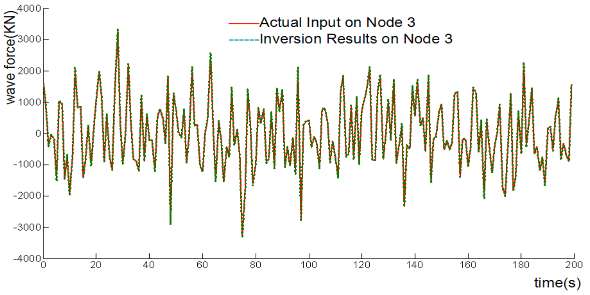


FIGURE 15 Comparison of actual input inversion results on node 3 under 6% noise

It is seen from Tables 2-7 and Figures 13-15 that under the condition of unknown input wave force being time domain related, method of this paper could not only identify physical parameter of pier but also accurately invert input load of each node. In terms of 6% of noise level, identified parameters are utilized to do inversion on the input wave forces on Node 1 and Node 2. Comparing with actually input wave force, the maximum error is 5.7087%. Obviously its inversion results are ideal.

## 5 Conclusions

Under condition of input wave load being known and unknown input wave load being time domain totally related, this paper does researches on physical parameter identification and load inversion of cantilever bridge pier structure. Suitable node units on piers are chosen to establish simplified computation model, in which characteristic parameters of damping and stiffness chosen from each node unit are taken as targets waiting for being identified. Numerical analysis is done utilizing numerical analysis software MALTAB through specific examples whose analysis conclusions are as follows:

Utilize total compensation method to do physical parameter identification and unknown wave force inversion on bridge piers under condition of part of unknown wave input. Its identification and inversion results present good stability.

In terms of circular piers without considering wave drag force, statistical average method of probability and improved total compensation method are combined together to do physical parameter identification and load inversion under unknown wave force input when wave force time intervals affected on each node are almost totally time domain related and correlation coefficient is $\alpha_i$. For cantilever bridge pier structure, this algorithm keeps good applicability and accuracy on parameter identifications of damping and stiffness of each node under the influences of different noise signals when shear is out of shape.

Different initial values of physical parameters waiting for being identified keep little influence on identification result. Whether chosen initial values are near to the true values is just related to convergence time and convergence steps.

Without influences of noise, identification precision of physical parameter on each node of bridge pier structure is higher than that with noise existing whose reason is that there exists big difference between order of magnitude of stiffness parameter and the one of damping parameter. This leads identification result of stiffness parameter to be obviously lower than that of damping parameter. Small scope of noise influence keeps both being within the range of precision requirements.

This paper only takes researches of physical parameter identification and load inversion of cantilever bridge piers affected by wave load into consideration. It does not consider influences of bridge superstructure, cross-section shape of pier, water velocity, wind load, etc. Necessary experiments and actual monitoring are still needed to test or correct the inversion calculation method in order to understand applicability and universality of the proposed method in this paper more clearly.

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