

# Study on T-S fuzzy sliding mode control based on a new reaching law

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## Abstract

Fuzzy Sliding Mode controller based on fuzzy T-S Model is designed for the nonlinear, uncertainties and fast variable time characterizes question of bank-to-turn aircraft control and guidance system model. BTT control model was obtained by using T-S modelling method, asymptotically stable sliding surface was designed, and a new sliding mode reaching law is proposed. Based on the new reaching law, sliding mode stable tracking controller is designed. At last, the rationality and the effectiveness of the designed T-S fuzzy sliding mode stable tracking controller with the new reaching law are verified by the theoretical proof and the simulation experiments.

*Keywords:* Bank-to-turn Aircraft, Fuzzy Control, Sliding Mode Control, Reaching Law

## 1 Introduction

BTT(bank-to-turn is called BTT for short.) aircraft control technology is a kind of advanced control technology, It is great theoretical significance and the value of engineering applications for the development of high-speed, high precision and high mobile aircraft system[1]. However, BTT aircraft is a multivariate controlled object of kinematic coupling, inertia coupling, pneumatic coupling and control coupling [2].

Fuzzy control has not depended on accurate mathematical model of the characteristics and robustness and adaptability, it is an effective method to solve complex control problems [3-4]. T-S fuzzy model is an important tool for the modelling of nonlinear uncertain systems; Literature [5] has proved the T-S fuzzy model is a better approximation performance than the Mamdani fuzzy model. Sliding mode control method has strong robustness for parameter perturbation and the external interference, Therefore it is subject to the attention of scholars from various countries [6]. this program of Fuzzy control and sliding mode control combining is maintained the advantages of sliding mode control and overcome the sliding mode control shaking, has strong robustness for model uncertainties and external disturbances, improved reaching the dynamic quality of the system segment. The Lin and Boulkroune [7-9, 13] design controller the application of fuzzy system, many scholars have adopted the fuzzy basis function system. Combining the characteristics of BTT control, reaching law T-S model fuzzy sliding mode control will be applied to the control of the BTT aircraft in order to meet the needs of the BTT Control. Simulation results show

that the system has good dynamic qualities and strong robustness.

## 2 T-S Fuzzy modelling of nonlinear systems

### 2.1 PROBLEM DESCRIPTION

Considering a class of second-order multi-input and multi-output nonlinear systems:

$$\ddot{\mathbf{y}} = \mathbf{F}(\mathbf{x}) + \mathbf{G}(\mathbf{x})\mathbf{u}. \quad (1)$$

In the formula,  $\mathbf{y} = [x_1 \ x_2 \ \dots \ x_n]^T \in \mathbb{R}^n$  is the output vector of system;  $\mathbf{x} = [x_1 \ x_2 \ \dots \ x_n]^T$  is the state vector of system, and State can be measured;  $\mathbf{F}(\mathbf{x}) \in \mathbb{R}^n$  and  $\mathbf{G}(\mathbf{x}) \in \mathbb{R}^n$  are nonlinear function of system state matrix, and  $\text{rank}(\mathbf{G}) = n$ ;  $\mathbf{u} \in \mathbb{R}^n$  is the input vector of system.

System (1) is a nonlinear system, this nonlinear model is difficult to design a global control law, because an essentially nonlinear system cannot carry out its global linear model through a global linearization, but it can be represented as a series of local linear system. Fuzzy sliding mode control method combines the advantages of both closely, while maintaining the fuzzy control system does not rely on the characteristics of the model, but it also has the advantage of easy sliding mode control system design and stability analysis theory.

### 2.2 FUZZY T-S MODEL

T-S fuzzy model is a nonlinear model and easy to express dynamic characteristics of complex systems.

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Consider nonlinear uncertain systems described by the fuzzy T-S model. The system is described as the following  $m$  fuzzy rules, the  $i$ th fuzzy inference rule: if  $z_1$  is  $F_1^i$  and  $z_2$  is  $F_2^i \cdots z_n$  is  $F_n^i$ , then

$$\begin{aligned} \dot{x}(t) &= A_i x(t) + B_i u(t) \\ y(t) &= x(t) \end{aligned} \tag{2}$$

In the formula:  $z(t) = [z_1(t), z_2(t), \dots, z_n(t)]^T$  is Fuzzy antecedent variables.  $F_j^i$  is Fuzzy sets.  $x(t) \in R^n$  is State variables.  $u(t) \in R^m$  is Input of fuzzy system.  $i = 1, 2, \dots, n$  is the number of system input.  $y(t) \in R^n$  is Fuzzy output of system.  $A_i \in R^{n \times n}$  and  $B_i \in R^{n \times m}$  are system matrix and output matrix.

The state equation of fuzzy system from a single point of fuzzification, product inference and centre weighted average and defuzzification is [10-11].

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^n \bar{\alpha}_i(z(t)) [A_i x(t) + B_i u(t)] \\ y(t) &= \sum_{i=1}^n \bar{\alpha}_i(z(t)) x(t) \end{aligned} \tag{3}$$

where  $\bar{\alpha}_i(z(t)) = \prod_{j=1}^n \mu_{F_j^i}(z_j(t)) / \sum_{i=1}^m \prod_{j=1}^n \mu_{F_j^i}(z_j(t))$ ,  $\mu_{F_j^i}$  is the membership function of  $z(t)$  related to the fuzzy sets.

Assume that the fuzzy system is controllable, the fuzzy control rules  $i$  is  $R^i$ : if  $z_1$  is  $F_1^i$  and  $z_2$  is  $F_2^i \cdots z_n$  is  $F_n^i$ , then  $u(t) = -K_i x(t)$

The entire state feedback control law is:

$$u(t) = \sum_{i=1}^n \bar{\alpha}_i(z(t)) K_i x(t) \tag{4}$$

The essentially of T-S fuzzy modelling method is that nonlinear dynamical system model can be regarded as fuzzy multiple local linear approximation model, so in order to strike a T-S model for nonlinear systems, you must obtain each local subsystem linear model.

### 2.3 CONSTRUCTED THE T-S FUZZY MODEL

For a class of nonlinear dynamic process model:

$$\dot{x} = F(x) + G(x)u \tag{5}$$

In the formula,  $x = [x_1, x_2, \dots, x_n]^T$  is the output vector of the system;  $F(x) \in R^n$  and  $G(x) \in R^{n \times m}$  are nonlinear

smooth vector function,  $u \in R^m$  is the input vector of the system.

If  $T(x, u) = F(x) + G(x)u$ , the formula (5) can be expressed as:

$$\dot{x} = T(x, u) \tag{6}$$

If  $T(x_0, u_0) = F(x_0) + G(x_0)u_0$ , and at the operating point  $(x_0, u_0)$ , the  $T(x, u)$  launched by Taylor methods available:

$$\begin{aligned} \dot{x} &= T(x_0, u_0) + \left. \frac{\partial T}{\partial x} \right|_{\substack{x=x_0 \\ u=u_0}} (x - x_0) \\ &+ \left. \frac{\partial T}{\partial u} \right|_{\substack{x=x_0 \\ u=u_0}} (u - u_0) + \dots \end{aligned} \tag{7}$$

Assuming  $g_{ij}$  is the  $(i, j)$ -th element of the matrix  $G(x)$ , then:

$$\left. \frac{\partial T}{\partial x} \right|_{\substack{x=x_0 \\ u=u_0}} (x - x_0) = \left. \frac{\partial T}{\partial x} \right|_{x=x_0} + H(x_0, u_0) \tag{8}$$

In the formula,  $\sum_{k=1}^m u_k \left. \frac{\partial g_{ik}(x)}{\partial x_j} \right|_{\substack{x=x_0 \\ u=u_0}}$  is the  $(i, j)$ -th element of the matrix  $H(x_0, u_0)$ , the formula (8) finishing as:

$$\left. \frac{\partial T}{\partial x} \right|_{\substack{x=x_0 \\ u=u_0}} (x - x_0) = G(x_0) \tag{9}$$

If  $T(x_0, u_0) = 0$ ,  $(x_0^T, u_0^T)^T$  is the equilibrium point of the formula (6), scilicet the point at  $(x_0, u_0)$  has  $\dot{x} = 0$ .

Assuming  $\delta x = x - x_0$ , and ignoring the Taylor expansion of the higher order terms, linear model can be obtained at the equilibrium point  $(x_0, u_0)$ :

$$\frac{d}{dt} x = Ax + B\delta u \tag{10}$$

In the formula,  $A = \left. \frac{\partial T}{\partial x} \right|_{\substack{x=x_0 \\ u=u_0}}$ ;  $B = \left. \frac{\partial T}{\partial u} \right|_{\substack{x=x_0 \\ u=u_0}}$ .

In the equilibrium point  $x = 0$ , it can be constructed locally linear model. In the non-equilibrium point at work, Taylor linearization of the nonlinear model is a result of the radiation model, rather than a linear model.

Assuming for a given operating point  $x_0$ , but it is not necessarily the equilibrium point of the formula (6), construct matrices  $A$  and  $B$ , so that in the neighbourhood  $x_0$  there are:

$$\begin{aligned} F(x) + G(x)u &\approx Ax + Bu, \quad \forall u \\ F(x_0) + G(x_0)u &= Ax_0 + Bu, \quad \forall u \end{aligned} \tag{11}$$

Because  $u$  is arbitrary, so there must be  $G(x_0) = B$ . Then, in the  $x_0$ 's neighbourhood to find a matrix  $A$  such that there is:

$$\begin{aligned} F(x) &\approx Ax \\ F(x_0) &= Ax_0 \end{aligned} \tag{12}$$

$a_i^T$  is assumed that the  $i$ -th row matrix,  $F_i$  is the  $i$ -th component of  $F$ , the formula (12) may be expressed as:

$$\begin{aligned} F_i(x) &\approx a_i^T x, \quad i=1,2,\dots,n \\ F_i(x_0) &= a_i^T x_0, \quad i=1,2,\dots,n \end{aligned} \tag{13}$$

In points  $x_0$ , expand the left side of the above equation and ignore the second and higher order terms, finishing can be:

$$\nabla^T F_i(x_0)(x - x_0) \approx a_i^T (x - x_0). \tag{14}$$

For vector  $a_i$ , it should be kept as close to  $\nabla F_i(x_0)$ , and to satisfy the constraints  $F_i(x_0) = a_i^T x_0$ .

Defines  $E = \frac{1}{2} \|\nabla F_i(x_0) - a_i\|_2^2$ , then  $a_i$  can be attributed to the following optimization problem:

$$\min E_{a_i}, \quad a_i^T x_0 = F_i(x_0). \tag{15}$$

Formula (15) is a convex constrained optimization problem. Then, the first condition of the optimization problem is:

$$\begin{aligned} \nabla_{a_i} E + \lambda \nabla_{a_i} (a_i^T x_0 - F_i(x_0)) &= 0 \\ a_i^T x_0 &= F_i(x_0) \end{aligned} \tag{16}$$

In the formula,  $\lambda$  is Lagrange multiplier; The  $a_i$  of  $\nabla_{a_i}$  subscript gradient computation of  $\nabla$  is associated with  $a_i$ . The formula (16) derived:

$$\begin{aligned} a_i - \nabla F_i(x_0) + \lambda x_0 &= 0 \\ a_i^T x_0 &= F_i(x_0) \end{aligned} \tag{17}$$

Finishing the above formula can be obtained:

$$a_i = \nabla F_i(x_0) + \frac{F_i(x_0) - x_0^T \nabla F_i(x_0)}{\|x_0\|^2} x_0.$$

The key to the stability problem of nonlinear systems is to construct the controller, make it in a neighbourhood point within an arbitrary starting point; the trajectory converges to the point of the closed loop system. If the starting point is just the working point, then the trajectories of the closed loop system can stop at this point; expect the controller at after any time. Therefore, the work point must be asymptotically stable equilibrium point of the closed-loop system.

### 3 Reaching Law Designed

The dynamic quality of sliding movement depends on the switching function and its parameters. In many cases, the reaching law design is features with a simple structure, high quality, it cannot output  $u$  limit structure, and only rely on the calculation of its structure and form. When the other variable structure cannot be applied, the reaching law method can still give the results [6, 10].

Reaching law is the following Common.

Equal to the speed of approaching law

$$\dot{s} = -\varepsilon \operatorname{sgn}(s), \tag{18}$$

$\varepsilon > 0$  is the rate of system motion point approach the switching surface.  $\varepsilon$  is the smaller, slower approach, the adjustment process is slow,  $\varepsilon$  is the greater, the closer to the fast adjustment process is fast jitter.

Exponential reaching law

$$\dot{s} = -\varepsilon \operatorname{sgn}(s) - ks \quad \varepsilon > 0, k > 0. \tag{19}$$

The reaching law approach speed from the large value gradually reduced to zero, to shorten the approach time, the motion point to the speed of the switching surface is very small. Good by adjusting the parameters can improve the dynamic quality, and reduce the chattering.

Power reaching law:

$$\dot{s} = -k |s|^a \operatorname{sgn}(s) \quad 0 < a < 1, k > 0. \tag{20}$$

The appropriate choice the value of  $a$  and  $k$  obtained with the exponential reaching law similar results.

Generally reaching law:

$$\dot{s} = -\varepsilon \operatorname{sgn}(s) - f(s) \quad \varepsilon > 0, \tag{21}$$

where  $f(0) = 0$ , when  $s \neq 0$ ,  $sf(s) > 0$ .

Exponential reaching law to reach a relatively short time, while the reaching law will be relatively large speed when the motor points to reach the switching surface, causing relatively large buffeting. Power reaching law and the exponential reaching law closer. The general results of reaching law and selection of  $f(s)$  is closely related, the selection  $f(s)$  of appropriate can effectively eliminate chattering.

In the sliding mode control system, the control structure exists switching characteristics, so that the sliding mode was buffeting form. Thus, scholars proposed saturation function instead of using the sign function. The difference is shown in Figure 1.

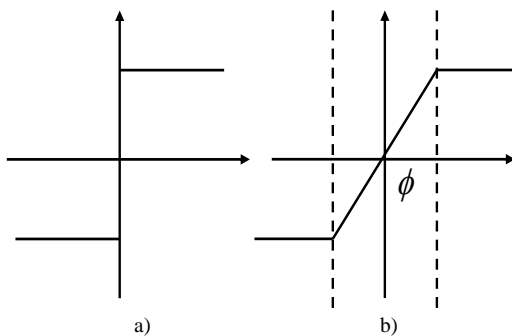


FIGURE 1 The difference between symbolic function and saturation function: a)  $\text{sgn}(s)$ , b)  $\text{sat}(s)$

Saturation function can be expressed as:

$$\text{sat}(s) = \begin{cases} \text{sgn}(s), & s \geq \phi \\ \frac{s}{\phi}, & s < \phi \end{cases}$$

To enable smooth sliding mode presents the movement, to eliminate chattering of sliding movement, Saturation function can be deformed to obtain saturation function modified form, shown in Figure 2.

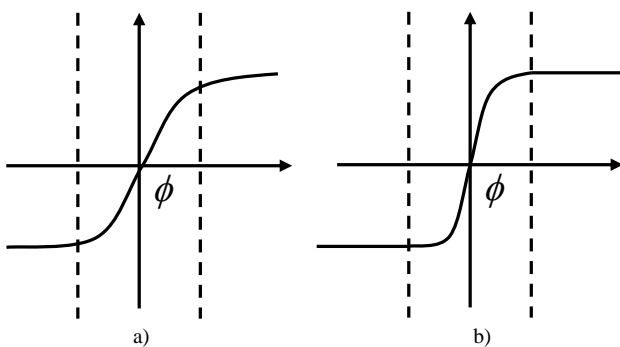


FIGURE 2 Deformation saturation function: a)  $\text{sat}_1(s)$ , b)  $\text{sat}_2(s)$

Deformed saturation function  $\text{sat}_1(s)$  is expressed as:

$$\text{sat}_1(s) = \begin{cases} \text{sgn}(s), & s \geq \phi \\ \tanh\left(\frac{2\pi s}{\phi}\right), & s < \phi \end{cases} \quad (22)$$

Deformed saturation function  $\text{sat}_2(s)$  is expressed as:

$$\text{sat}_2(s) = \begin{cases} \text{sgn}(s), & s \geq \phi \\ \left(\frac{s}{\phi}\right)^p, & s < \phi \end{cases} \quad (23)$$

For saturation function  $\text{sat}_2(s)$ :

- 1) When  $s < \phi$ , that is  $s(t)$  when the value is very small,  $s(t)$  instead of using  $s/\phi$ , and extended the form of powers reaching law :  
 $\dot{s} = -k_2 |s|^\beta \text{sgn}(s)$   $k_2 > 0, 0 < \beta < 1$
- 2) When  $s \geq \phi$ , that is  $s(t)$  when the value is very big, we want a faster rate close to equilibrium, the index reaching law can be at a faster rate approaching equilibrium, so select the form of index reaching law:  
 $\dot{s} = -k_1 s |s|^\alpha \text{sgn}(s)$   $k_1 > 0, 0 < \alpha < 1$

Through the above analysis, design new fast reaching law is

$$\dot{s} = -k_1 s |s|^\alpha \text{sgn}(s) - k_2 |s|^\beta \text{sgn}(s), \quad (24)$$

where  $k_1 > 0, k_2 > 0, 0 < \alpha < 1, 0 < \beta < 1$ .

When the system state away from the sliding surface, the value of a great  $s(t)$ , the approach speed depends primarily on the first item of the equation (24) in the right; when the system state is close to the sliding surface, that is when the  $s(t)$  value is very small, approaching the speed depends on the type (24) in the right.

#### 4 T-S fuzzy sliding mode control

Dynamic quality of sliding movement depends on switching function and its parameters. In many cases, reaching law cannot restrict the output design structure, relying only calculated its structure and form, has a simple structure, high quality features. When other variable structure cannot be applied, reaching law still gives the result.

Consider the nonlinear controlled object, and it can be described by fuzzy TS model, fuzzy TS model to describe the  $j$ -th fuzzy inference rules of system are:

$$\begin{aligned} R^j : & \text{IF } x_1 \text{ is } F_1^j \text{ and } x_2 \text{ is } F_2^j \text{ and } \dots \text{ and } x_n \text{ is } F_n^j, \\ & \text{THEN } \dot{x}(t) = A_j x(t) + B_j u(t). \quad (25) \\ & y(t) = C_j x(t). \end{aligned}$$

In the formula,  $F_j^i$  is the fuzzy set;  $x_j$  the  $j$ -th variable system;  $\mathbf{x} = (x_1, x_2, \dots, x_n) \in \mathbf{R}^n$  is the input of fuzzy system,  $i = 1, 2, 3, \dots, r$  is the number of rules fuzzy inference system;  $\mathbf{y} \in \mathbf{R}^m$  is the output of fuzzy system;  $\mathbf{u}$  is the control input vector;  $\mathbf{A}_i \in \mathbf{R}^{n \times n}$  and  $\mathbf{B}_i \in \mathbf{R}^{n \times m}$  are the system matrix and the input matrix;  $\mathbf{C}_i$  is the output matrix.

Assuming  $\alpha_i(z(t))$  is the membership function, which  $x(t)$  relevant in fuzzy set  $F_j^i$ , through a single point of blurring, product inference and centre weighted average anti-blur constitute the global fuzzy model can be expressed as:

$$\begin{cases} \dot{\mathbf{x}}(t) = \frac{\sum_{i=1}^r \alpha_i(\mathbf{x}(t))[\mathbf{A}_i \mathbf{x}(t) + \mathbf{B}_i \mathbf{u}(t)]}{\sum_{i=1}^r \alpha_i(\mathbf{x}(t))} \\ \mathbf{y}(t) = \sum_{i=1}^r \bar{\alpha}_i(\mathbf{x}(t)) \mathbf{C}_i \mathbf{x}(t) \end{cases}, \quad (26)$$

Among:  $\alpha_i(z(t)) = \prod_{i=1}^r \mu_{F_i^i}(z(t))$  ;

$$\bar{\alpha}_i(z(t)) = \alpha_i(z(t)) / \sum_{i=1}^r \alpha_i(z(t)), \quad i = 1, 2, \dots, r.$$

#### 4.1 FUZZY SLIDING MODE CONTROL BASED ON REACHING LAW

According to the characteristics of reaching law method, reaching law sliding mode controller is designed based on T-S fuzzy model.

Control law is derived as follows:

$$\mathbf{s} = \mathbf{C}\mathbf{x}. \quad (27)$$

Among:  $\mathbf{C} = \text{diag}(c_1, c_2, \dots, c_{13}), c_i (i = 1, 2, \dots, n)$  is constant.

Assuming  $\text{slaw}$  is reaching law, by derivation and organizes the formula can get:

$$\dot{\mathbf{s}} = \mathbf{C}\dot{\mathbf{x}} = \text{slaw}. \quad (28)$$

Through state equation (25) into equation (28), we obtain T-S fuzzy sliding mode controller is:

$$\begin{aligned} \mathbf{R}^i : \text{IF } x_1 \text{ is } F_1^i \text{ and } x_2 \text{ is } F_2^i \text{ and } \dots \text{ and } x_n \text{ is } F_n^i, \\ \mathbf{u}_i = (\mathbf{C}\mathbf{B}_i)^{-1}(-\mathbf{C}\mathbf{A}_i \mathbf{x}(t) + \text{slaw}). \end{aligned}$$

Through new fast reaching law (24) into the above formula, T-S fuzzy sliding mode controller based on the new reaching law is:

$$\begin{aligned} \mathbf{R}^i : \text{IF } x_1 \text{ is } F_1^i \text{ and } x_2 \text{ is } F_2^i \text{ and } \dots \text{ and } x_n \text{ is } F_n^i, \\ \mathbf{u}_i = (\mathbf{C}\mathbf{B}_i)^{-1}(-\mathbf{C}\mathbf{A}_i \mathbf{x}(t) - K_1 \mathbf{S} |\mathbf{S}|^\alpha \text{sgn}(\mathbf{S}) - K_2 |\mathbf{S}|^\beta \text{sgn}(\mathbf{S})), \quad (29) \end{aligned}$$

Among:  $0 < \alpha < 1, 0 < \beta < 1, K_1 = \text{diag}[k_{11} \ k_{12} \ k_{13}], K_2 = \text{diag}[k_{21} \ k_{22} \ k_{23}], k_{ji} > 0, (i = 1, 2, 3; j = 1, 2).$

#### Stability proof:

Take Lyapunov function is  $\mathbf{V} = \frac{1}{2} \mathbf{s}^T \mathbf{s}$ . Derivation of this formula and equation (24) into it, it can be obtained:  $\dot{\mathbf{V}} = \mathbf{s}^T \dot{\mathbf{s}} = -K_1 \mathbf{s}^T \mathbf{s} |\mathbf{s}|^\alpha \text{sgn}(\mathbf{s}) - K_2 \mathbf{s}^T \mathbf{s} |\mathbf{s}|^\beta \text{sgn}(\mathbf{s}) < 0$ .

Each fuzzy sliding mode subsystem is asymptotically stable; each local subsystem global controller is a weighted sum of the control law, based on the above model analysis can be obtained:  $\mathbf{u} = \sum_{i=1}^r \alpha_i \mathbf{u}_i$ .

Take the following Lyapunov function is  $\mathbf{V} = \mathbf{s}^2/2$ . Derivation of this formula, it can get:

$$\begin{aligned} \dot{\mathbf{V}} &= \mathbf{s}\dot{\mathbf{s}} \\ &= -\sum_{i=1}^r \alpha_i [K_1 \mathbf{S} |\mathbf{S}|^\alpha \text{sgn}(\mathbf{S}) + K_2 |\mathbf{S}|^\beta \text{sgn}(\mathbf{S})] \cdot \\ &< 0 \end{aligned} \quad (30)$$

Therefore, global T-S fuzzy model is asymptotically stable.

#### 4.2 FUZZY SLIDING MODE TRACKING CONTROL

According to the characteristics of reaching law approach. Reaching law sliding mode controller is designed based on T-S fuzzy model. Assuming a given input signal is  $\mathbf{x}_d$ , error vector is defined as:

Control law is derived as follows:

$$\mathbf{s} = \mathbf{C}\mathbf{E}. \quad (31)$$

Among:  $\mathbf{C} = \text{diag}(c_1, c_2, \dots, c_n), c_i (i = 1, 2, \dots, n)$  is constant.

Assuming  $\text{slaw}$  is reaching law, By derivation and organize the formula (31) can get:  $\dot{\mathbf{s}} = \mathbf{C}\dot{\mathbf{E}} = \text{slaw}$ .

Through state equation (25) into the above formula, we obtain T-S fuzzy sliding mode controller is  $\mathbf{R}^i : \text{IF } x_1 \text{ is } F_1^i \text{ and } x_2 \text{ is } F_2^i \text{ and } \dots \text{ and } x_n \text{ is } F_n^i,$

$$\mathbf{u}_i = (\mathbf{C}\mathbf{B}_i)^{-1}(\mathbf{C}\dot{\mathbf{x}}_d - \mathbf{C}\mathbf{A}_i \mathbf{x}(t) - \text{slaw})$$

Through new fast reaching law (24) into the above formula, T-S fuzzy sliding mode controller based on the new reaching law is:

$$R^i : \text{IF } x_1 \text{ is } F_1^i \text{ and } x_2 \text{ is } F_2^i \text{ and } \dots \text{ and } x_n \text{ is } F_n^i, \tag{32}$$

$$u_i = (CB_i)^{-1} (C\dot{x}_d - CA_i x(t) + K_1 S |S|^{\alpha} \text{sgn}(S) + K_2 |S|^{\beta} \text{sgn}(S))$$

Among:  $0 < \alpha < 1, 0 < \beta < 1, K_1 = \text{diag}[k_{11} \ k_{12} \ k_{13}], K_2 = \text{diag}[k_{21} \ k_{22} \ k_{23}], k_{ji} > 0, (i = 1, 2, 3; j = 1, 2).$

**Stability proof:**

Take Lyapunov function is  $V = \frac{1}{2} s^T s$ .

Derivation of this formula and equation (24) into above formula, it can be obtained:

$$\dot{V} = s^T \dot{s} = -K_1 s^T s |s|^{\alpha} \text{sgn}(s) - K_2 s^T |s|^{\beta} \text{sgn}(s) < 0 \tag{33}$$

Each fuzzy sliding mode subsystem is asymptotically stable; each local subsystem global controller is a weighted sum of the control law, based on the above model analysis can be obtained:  $u = \sum_{i=1}^r \alpha_i u_i$ .

Take the following Lyapunov function is  $V = s^2/2$ . Derivation of this formula, it can get:

$$\dot{V} = s\dot{s} = -\sum_{i=1}^r \alpha_i [K_1 S |S|^{\alpha} \text{sgn}(S) + K_2 |S|^{\beta} \text{sgn}(S)] < 0 \tag{34}$$

Therefore, global TS fuzzy model is asymptotically stable.

**5 Controller Designed and Stability Analysis**

**5.1 CONTROL MODEL OF SYSTEM**

Body coordinates, the BTT cruise aircraft control system simplified mathematical model as follows [4, 12]:

Roll channel mathematical model

$$\begin{cases} \dot{\gamma} = \omega_x \\ \dot{\omega}_x = -c_1 \omega_x - c_2 \beta - c_3 \delta_x - c_4 \omega_y - c_5 \delta_y \end{cases} \tag{35}$$

Yaw channel mathematical model:

$$\begin{cases} \dot{\beta} = -b_4 \beta + \omega_y + \omega_x \alpha - b_3 \delta_y - b_7 \delta_x \\ \dot{\omega}_y = -b_2 \beta - b_1 \omega_y + b_6 \omega_x - b_3 \delta_y - b_8 \delta_x + b_7 \omega_x \omega_z \end{cases} \tag{36}$$

Roll channel Mathematical model:

$$\begin{cases} \dot{\gamma} = \omega_x \\ \dot{\omega}_x = -c_1 \omega_x - c_2 \beta - c_3 \delta_x - c_4 \omega_y - c_5 \delta_y \end{cases} \tag{37}$$

Consider the control of the BTT aircraft model (35) ~ (37) If the aircraft longitudinal and lateral loading as the output of the system, the system dynamics do not expect the non-minimum phase phenomenon, at this time feedback linearization theory cannot be directly used for non-minimum phase systems, it will easily lead to system instability.

Therefore select the output vector is  $y = [\gamma, \beta, \alpha]^T$ . To simplify the problem, ignoring control the lift and side force generated by the rudder, and assuming that the BTT aircraft control model in dynamic coefficient changes with time for a small amount of time to change the system state variables, for each output variable and on derivative of the time variable, Until at least one control volume is  $u = [\delta_x, \delta_y, \delta_z]^T$ .  $x = [w_x, w_y, w_z, \gamma, \beta, \alpha]^T$  is State variables. Using formula (35) - (37) to get:

$$\ddot{y} = \begin{bmatrix} \ddot{\gamma} \\ \ddot{\beta} \\ \ddot{\alpha} \end{bmatrix} = F(x) + G(x)u, \tag{38}$$

where  $F(x) = [f_1(x) \ f_2(x) \ f_3(x)]^T$ ,

$$f_1(x) = -c_1 \omega_x - c_2 \beta - c_4 \omega_y,$$

$$f_2(x) = (b_4^2 - b_2 - c_2 \alpha) \beta - (b_4 \alpha - b_6 + c_1 \alpha - a_4 \alpha + \omega_z) \omega_x - (b_1 + b_4 + c_4 \alpha) \omega_y,$$

$$f_3(x) = (a_4^2 - a_2) \alpha - (a_4 + a_1) \omega_z,$$

$$G(x) = \begin{bmatrix} -c_3 & 0 & 0 \\ 0 & b_4 b_3 - b_3 & -c_3 \alpha \\ 0 & 0 & a_4 a_5 - a_3 \end{bmatrix}.$$

Other relevant parameters can refer to the literature [1].

**5.2 CONTROLER DESIGNED**

Consider the BTT aircraft control system model (38), the application of section one section of T-S fuzzy modelling method of BTT aircraft control system, T-S fuzzy model is established.

Select the feature points in the entire flight airspace  $t_i (i = 1, 2, \dots, r)$ , Feature points on the local subsystem is:

$$\ddot{y} = F_i(X) + G_i(X)u. \tag{39}$$

Following  $r$  rules to describe the dynamic behaviour of the system,  $i$  fuzzy inference rules are:

$$R^i : \text{IF } t_i \text{ is } F^i, \text{ THEN} \\ \ddot{\mathbf{y}} = \mathbf{F}_i(\mathbf{X}) + \mathbf{G}_i(\mathbf{X})\mathbf{u} \quad (40)$$

where  $\mathbf{X} = [\gamma, \beta, \alpha, \dot{\gamma}, \dot{\beta}, \dot{\alpha}]^T$ .

Set  $\alpha_i(t_i)$  is  $t_i$  related to the fuzzy sets membership function of  $F^i$  global T-S fuzzy model of BTT aircraft control system is:

$$\ddot{\mathbf{y}} = \sum_{i=1}^r \alpha_i(t_i)(\mathbf{F}_i(\mathbf{X}) + \mathbf{G}_i(\mathbf{X})\mathbf{u}) \quad (41)$$

Conversion aircraft control system model (38). Make  $\mathbf{x}_1 = \mathbf{y}$ ,  $\mathbf{x}_2 = \dot{\mathbf{y}}$ , the system model can be expressed as:

$$\begin{cases} \dot{\mathbf{x}}_1 = \mathbf{x}_2 \\ \dot{\mathbf{x}}_2 = \mathbf{F}(\mathbf{X}) + \mathbf{G}(\mathbf{X})\mathbf{u} \end{cases} \quad (42)$$

where  $\mathbf{x}_1 = \mathbf{y} = [\gamma, \beta, \alpha]^T$ ,  $\mathbf{X} = [\mathbf{x}_1^T, \mathbf{x}_2^T]^T$ .

Given input signal is  $\mathbf{X}_d = [\mathbf{x}_{1d}^T, \mathbf{x}_{2d}^T]^T$ , through the design of control laws, the state  $\mathbf{X}$  track the desired state  $\mathbf{X}_d$ . Define the error vector as follows:

$$\mathbf{E} = \mathbf{X} - \mathbf{X}_d = [\mathbf{e}^T \ \dot{\mathbf{e}}^T]^T, \quad (43)$$

where  $\mathbf{e} = \mathbf{x}_{1d} - \mathbf{x}_1 = [e_1 \ e_2]^T$ .

Using reaching law control model, the control law is pushed to the following:

$$\mathbf{S} = \mathbf{C}\mathbf{E}, \quad (44)$$

where  $\mathbf{C} = [\mathbf{C}_1 \ \text{diag}(1,1,1)]$  is the matrix,  $\mathbf{C}_1 = \text{diag}(c_{11}, c_{12}, c_{13})$ ,  $c_{1i} (i=1,2,3)$  is the normal number.

On the type derivation and finishing available:

$$\begin{aligned} \dot{\mathbf{S}} &= \mathbf{C} \begin{bmatrix} \dot{\mathbf{X}}_d - \dot{\mathbf{x}}_1 \\ \ddot{\mathbf{X}}_d - \ddot{\mathbf{x}}_2 \end{bmatrix} \\ &= \mathbf{C} \begin{bmatrix} \dot{\mathbf{X}}_d \\ \ddot{\mathbf{X}}_d \end{bmatrix} - \mathbf{C}_1\mathbf{x}_2 - \ddot{\mathbf{x}}_2 = \text{slaw} \end{aligned} \quad (45)$$

Finishing (42) and (45) and substituted into the T-S fuzzy inference rules (40), the T-S-type fuzzy sliding mode controller is:

$$R^i : \text{IF } t_i \text{ is } F^i, \text{ THEN} \\ \mathbf{u}_i = \frac{1}{\mathbf{G}_i} \left[ \mathbf{C} \begin{bmatrix} \dot{\mathbf{X}}_d \\ \ddot{\mathbf{X}}_d \end{bmatrix} - \mathbf{C}_1\mathbf{x}_2 - \mathbf{F}_i(\mathbf{X}) - \text{slaw} \right] \quad (46)$$

The new fast reaching law (24) into the above equation, T-S fuzzy sliding mode controller based on the new reaching law for BTT Aircraft is:

$$R^i : \text{IF } t_i \text{ is } F^i, \text{ THEN} \\ \mathbf{u}_i = \frac{1}{\mathbf{G}_i} \left[ \mathbf{C} \begin{bmatrix} \dot{\mathbf{X}}_d \\ \ddot{\mathbf{X}}_d \end{bmatrix} - \mathbf{C}_1\mathbf{x}_2 - \mathbf{F}_i(\mathbf{X}) + K_1|\mathbf{S}|^\alpha \text{sgn}(\mathbf{S}) + K_2|\mathbf{S}|^\beta \text{sgn}(\mathbf{S}) \right], \quad (47)$$

where  $k_{ji} > 0$ ,  $K_2 = \text{diag}[k_{21} \ k_{22} \ k_{23}]$ ,  $(i=1,2,3; j=1,2)$ ;  $0 < \alpha < 1$ ,  $0 < \beta < 0$ .

Take the following Lyapunov function  $V = s^2/2$ .

This derivation and reaching law formula (24) into the above formula, you can have:

$$\begin{aligned} \dot{V} &= s\dot{s} \\ &= -K_1\mathbf{S}|\mathbf{S}|^\alpha \text{sgn}(\mathbf{S}) - K_2|\mathbf{S}|^\beta \text{sgn}(\mathbf{S}) \\ &< 0 \end{aligned} \quad (48)$$

Each fuzzy sliding mode subsystem is asymptotically stable; the global controller for each local subsystem control law is weighted and sum:

$$\mathbf{u} = \sum_{i=1}^r \alpha_i \mathbf{u}_i \quad (49)$$

Take the following Lyapunov function  $V = \frac{1}{2}s^2$ . This type derivation, we can obtain:

$$\begin{aligned} \dot{V} &= s\dot{s} \\ &= -\sum_{i=1}^r \alpha_i [K_1\mathbf{S}|\mathbf{S}|^\alpha \text{sgn}(\mathbf{S}) + K_2|\mathbf{S}|^\beta \text{sgn}(\mathbf{S})] \\ &< 0 \end{aligned} \quad (50)$$

Therefore, global T-S fuzzy model is asymptotically stable.

### 6 Controller simulation

In order to verify the tracking performance of the system, get parameters and mathematical models a certain type of BTT aircraft aerodynamics during the flight. Set the roll angle tracking instructions is  $\gamma = 120^\circ$ , 0.25Hz square wave signal; angle of attack tracking instructions for the amplitude is 0.25Hz square wave signal,  $\alpha = 10^\circ$ , Sideslip angle  $\beta = 0$ .

Throughout the flight, Select five characteristic points constitute fuzzy T-S model of aircraft system; Design membership function is triangular membership functions, According to the T-S fuzzy model rules, as in Figure 3.

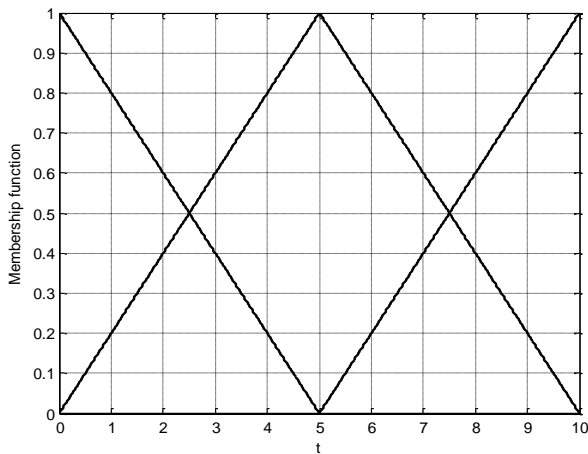


FIGURE 3 Fuzzy membership function in simulation

Using pole placement method, obtain the matrix  $C = [C_1 \ C_2]$ ,  $C_1 = \text{diag}[10 \ 2 \ 5.2]$ ,  $C_2 = \text{diag}[1 \ 1 \ 1]$ . Select  $K_1 = \text{diag}(18, 6, 18)$ ,  $K_2 = \text{diag}(2, 1, 2)$ .

The simulation results shown in Figure 4 and Figure 5. The simulation results show that the roll angle  $\gamma$  and  $\alpha$  angle of attack can quickly track the command signal.

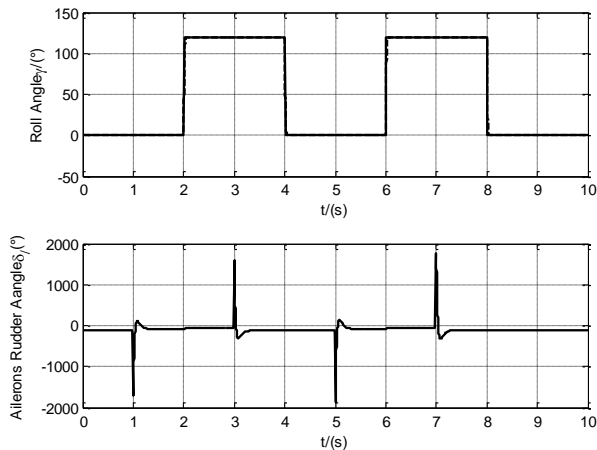


FIGURE 4 Roll angle tracking and aileron rudder angle response curve

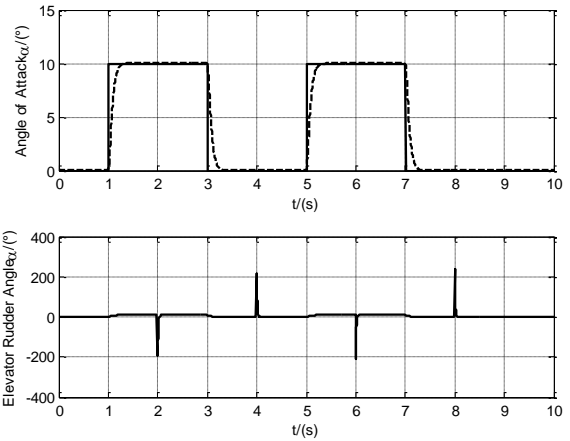


FIGURE 5 Angle of attack tracking and pitch rudder angle response curve

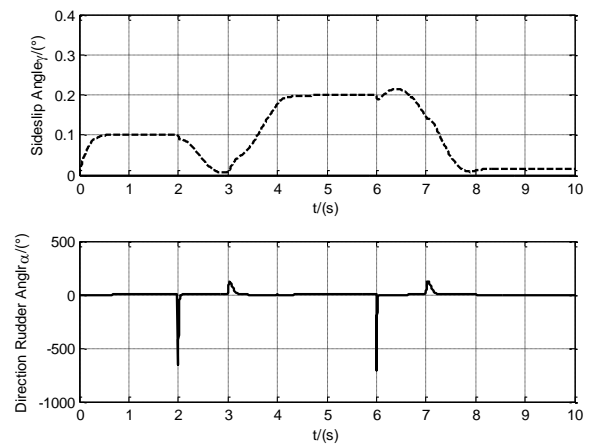


FIGURE 6 Sideslip angle tracking response and yaw rudder angle response curve

It can be seen from Figure 6, the sideslip angle  $\beta$  remain at  $|\beta| \leq 3^\circ$  on to meet the performance requirements of the system, it can effectively control for BTT aircraft.

### 7 Conclusion

BTT aircraft is a multi-variable nonlinear coupling object, to solve this problem, a new BTT aircraft controller is been designed based on Control method of sliding mode control theory and fuzzy T-S. Reaching law proposed by the T-S fuzzy sliding mode controller design method applied to BTT aircraft controller design is feasible, simulation results show that Control effect is better in the aircraft control, and it is able to fast track Guided instruction and Meet the design requirements.

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