Frequency analysis of the area error in the triangular partition of a discrete global grid

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Abstract

The discrete global grid is a method of hierarchical space expression with global scale, multi-resolution, and multi-scale transform features. It is a basic theory related to global geographic information systems and spatial grid computation. The main challenge in the practical application of discrete global grids is the generation of errors. Thus, four typical kinds of global discrete grid models were investigated in this study of partition errors. Area was considered an indicator of partition errors. Furthermore, this study computed the area errors in levels 3 to 10 partitions of each kind of global discrete grid model. The frequency of each indicator was also analysed. Analysis results show that the Synder models have the smallest area deformation.

Keywords: discrete global grid, triangular partition, frequency analysis

1 Introduction

The discrete global grid is a method of hierarchical space expression with global scale, multi-resolution and multi-scale transform features [1]. In this research field, however, errors are a significant problem, especially the deformation of discrete global grid model partitions. This deformation influences the geometrical homogenization of the discrete global grid model [2]. The basic partition deformation problems occur in terms of area, distance, and direction. In this study, the frequency analysis method is employed to investigate the area error in triangular partitions. The partition deformation change rule is analysed by computing the area error and the corresponding frequency characteristics.

2 Study objects

Four kinds of discrete global grid models were selected as the study objects. They represented typical discrete global grid categories.

The QTM model has an octahedral inscribed geometry [3]. Each of the chords between the adjacent nodes of the octahedral is divided into two equal segments. The midpoint of each chord is projected onto the surface of the sphere and becomes a new node. This new node forms a new geometry along with the original nodes in a process called partition, which can be repeated. Each partition is a project process from plane to sphere.

SQT has an icosahedral inscribed geometry [4]. The surface of the sphere is divided into 20 partitions of equal area. Each partition is recursively divided into four spherical triangles of similar area.

The Synder model also has an icosahedral inscribed geometry [5]. The projection transformation equation of this model was developed by Synder in 1992, and its partition method is similar to that of the Fuller model.

STQIE model [6] uses a projection transformation equation based on ERLPR projection. The projection plane is divided into triangles of equal area. The vertices of each triangle are then reversely projected onto the surface of the sphere to form many spherical triangles. These spherical triangles in turn comprise the STQIE grid.

3 Computation method of partition area error

The ideal discrete global grid model is an evenly partitioned unit sphere. Moreover, nodes are uniformly distributed on its surface. The partitions of the ideal model are equal in terms of area and are not afflicted with geometric deformation. Therefore, the ideal model does not display systematic errors. The area values of the ideal model can be regarded as true values. Once determined, the difference between true and calculated values of each discrete global grid model can then be used to analyse the errors.

The area calculation formula of ideal partition is as follows:

$$S_n = 4\times\frac{\pi}{2^n}, \quad (1)$$

where S_n is the area of a partition in level n and n is the depth of the partition level.

The actual partition area is calculated using the following spherical triangle area formula:

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$$S = E \times \pi \times R^2 \times 180^0 , \quad (2)$$

where E is spherical excess, and R is spherical radius.

A, B, and C are set as the three angles of a spherical triangle. Thus, the spherical excess Equation is as follows:

$$E = A + B + C - 180^0 . \quad (3)$$

The area error Equation is as follows:

$$S_{\triangle} = E - S_n . \quad (4)$$

In discrete global grids with inscribed geometries, the vertices of the inscribed geometry distribute evenly on the surface of the sphere prior to partition. Thus, the partitions formed by these vertices are equal in terms of area and do not exhibit any errors. These partitions are called level 1 partitions. Level 2 partitions display errors because of the partition algorithms and the systematic error in the model. According to the results of previous research [2], deformation changes in the partitions stabilized after level 10. Thus, the aforementioned grid models are partitioned from levels 3 to 10 in this study to analyse the error change rule.

The area errors in the partitions from levels 3 to 10 are analysed through frequency analysis. This analysis evenly divides the interval between the maximum and the minimum statistical values. Furthermore, the frequency analysis of partition error can reflect whether it is influenced by model systematic error. It can also reflect the influence of subdivision methods on partition deformation.

4 Frequency analysis of partition area error

Figures 1 to 32 present the frequency analyses of the STQIE, SQT, Synder, and QTM models. Each model is divided into partitions ranging from levels 3 to 10.

Figures 1 to 8 depict a single peak shape in the representation of STQIE partition error frequency. The error intervals are both positive and negative. Furthermore, the entire interval is 9.775E-8. The error rate of the negative interval is 53.26%, whereas that of the positive interval is 46.74%.

Figures 9 to 16 also display a single peak shape, but the interval is positive. The entire interval is 3.595E-7, and the error interval of SQT is 3.67 times that of STQIE. Moreover, the distribution of area error in the latter is more concentrated than that in the former. In addition, STQIE has both positive and negative intervals. By contrast, SQT has only a negative interval. Therefore, the area deformation in the STQIE partition is smaller than that in the SQT partition.

The interval of area error frequency in the Synder model is symmetric, and the centre is located at point 0 as shown in Figures 17 to 24. Error distribution is increasingly concentrated in line with the increase in level depth. Furthermore, the partition area is little deformed by systematic error.

Figures 25 to 32 indicate that the frequency representation of the QTM model is irregular in shape. Partition area deformation is greater in this model than in the three other models under the influence of systematic error. Thus, the QTM partition method generates severe area error.

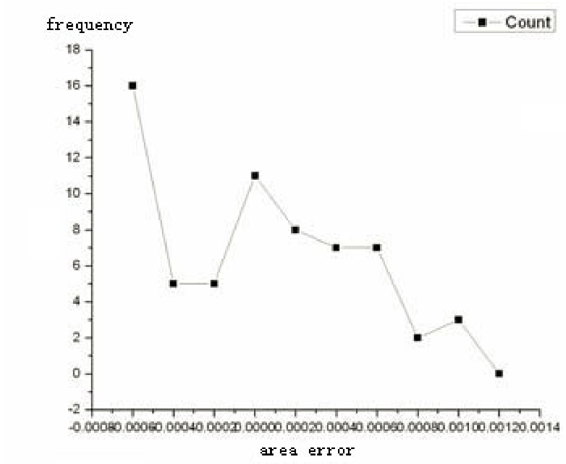

FIGURE 1 Frequency of the STQIE partition error in level 3

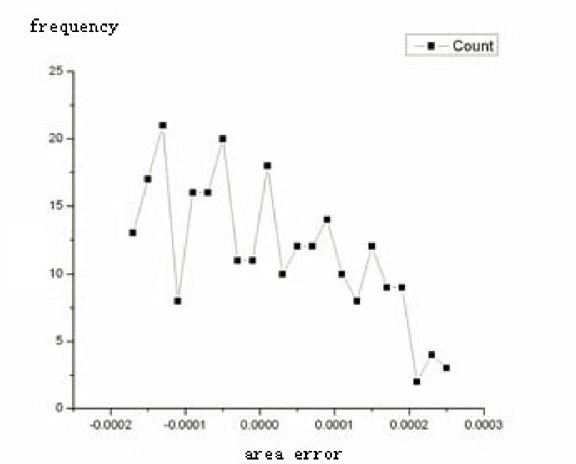

FIGURE 2 Frequency of the STQIE partition error in level 4

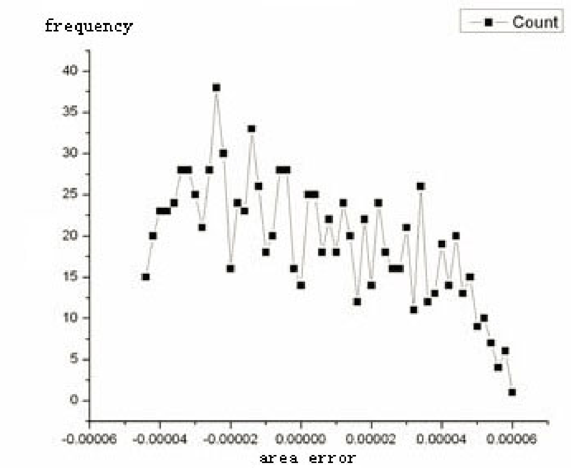

FIGURE 3 Frequency of the STQIE partition error in level 5

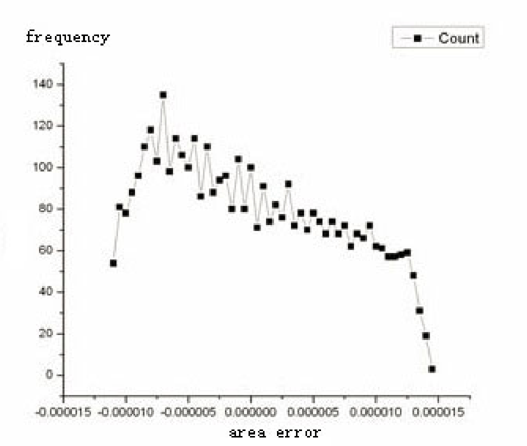

FIGURE 4 Frequency of the STQIE partition error in level 6

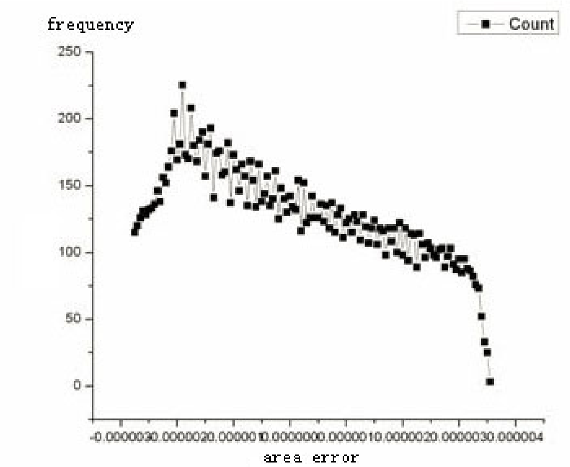

FIGURE 5 Frequency of the STQIE partition error in level 7

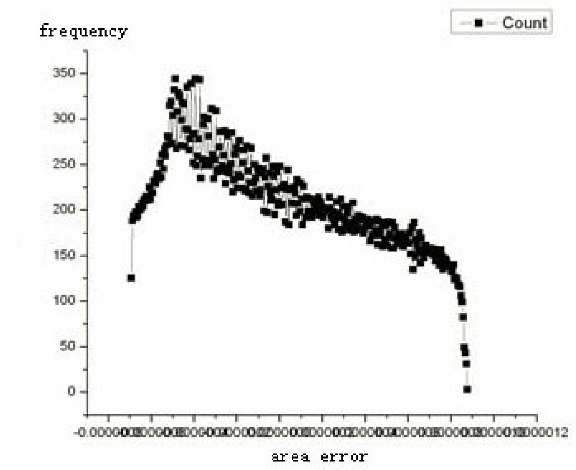

FIGURE 6 Frequency of the STQIE partition error in level 8

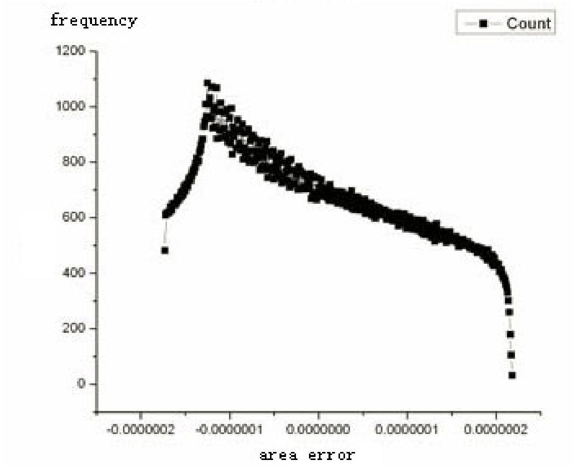

FIGURE 7 Frequency of the STQIE partition error in level 9

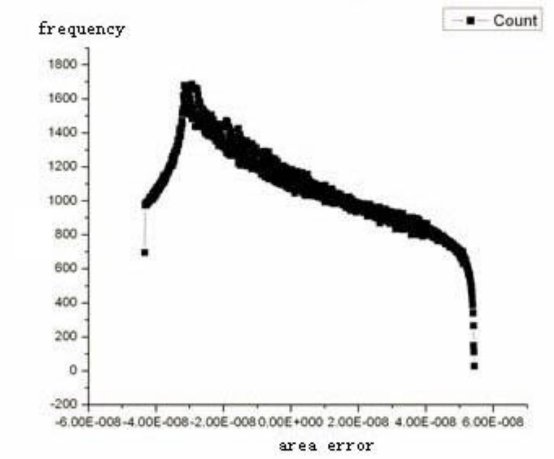

FIGURE 8 Frequency of the STQIE partition error in level 10

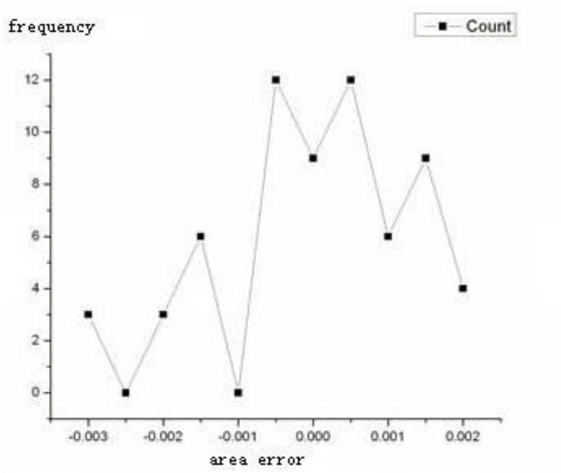

FIGURE 9 Frequency of the SQT partition error in level 3

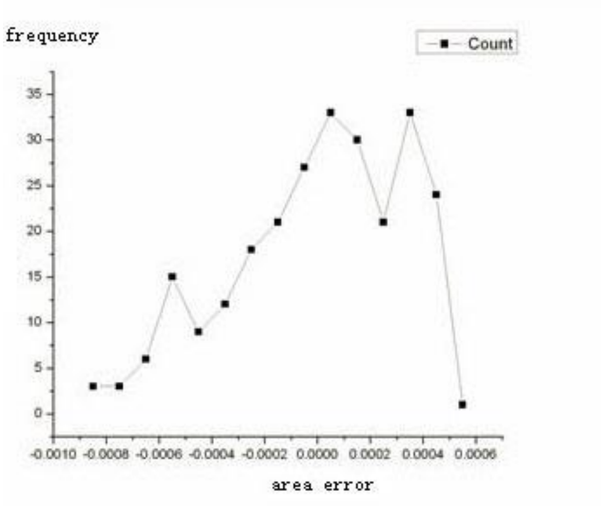

FIGURE 10 Frequency of the SQT partition error in level 4

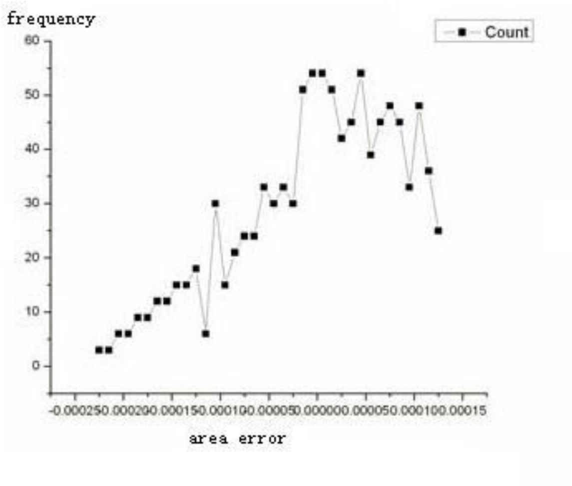

FIGURE 11 Frequency of the SQT partition error in level 5

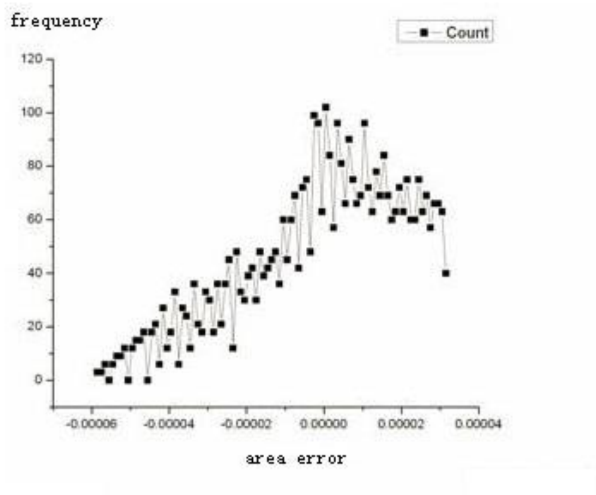

FIGURE 12 Frequency of the SQT partition error in level 6

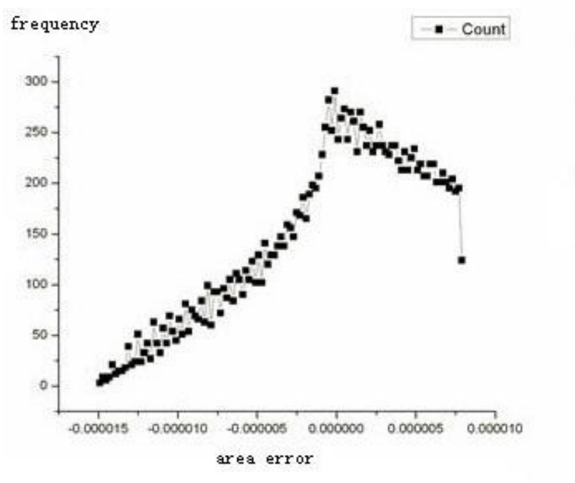

FIGURE 13 Frequency of the SQT partition error in level 7

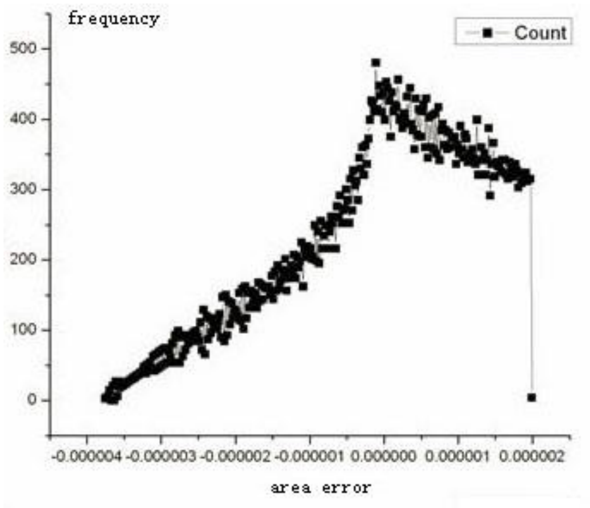

FIGURE 14 Frequency of the SQT partition error in level 8

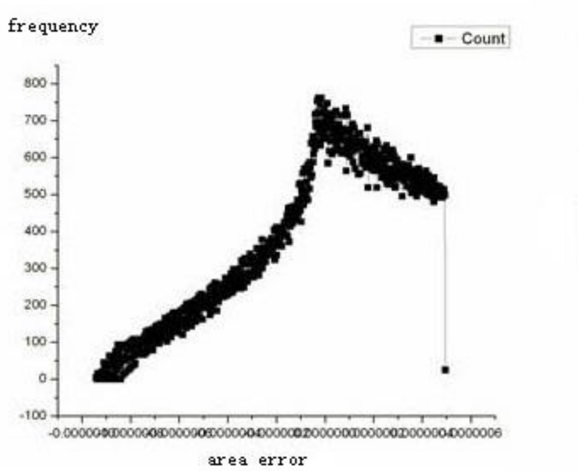

FIGURE 15 Frequency of the SQT partition error in level 9

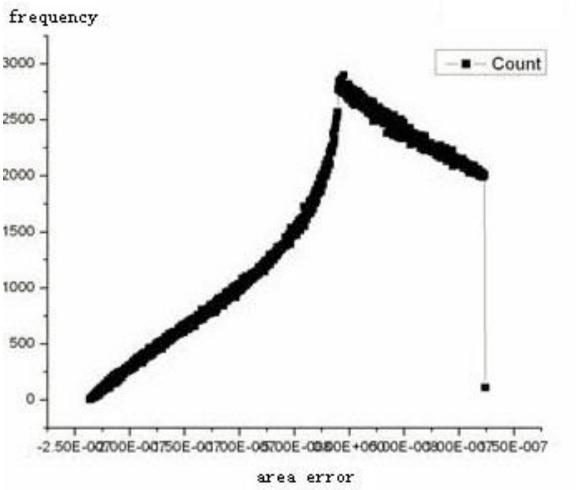

FIGURE 16 Frequency of the SQT partition error in level 10

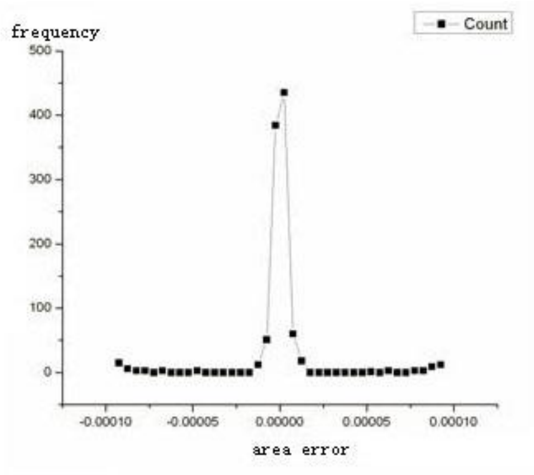

FIGURE 19 Frequency of the Synder model partition error in level 5

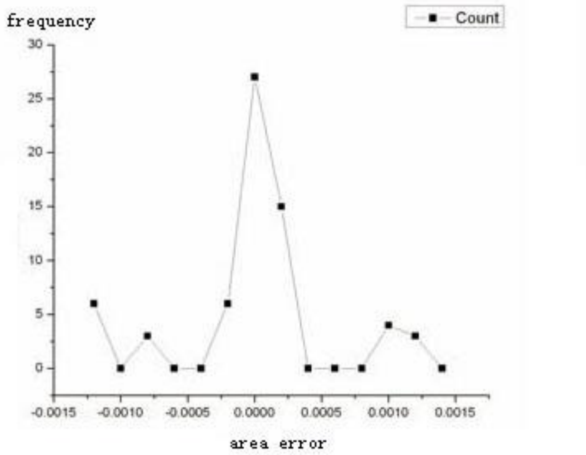

FIGURE 17 Frequency of the Synder model partition error in level 3

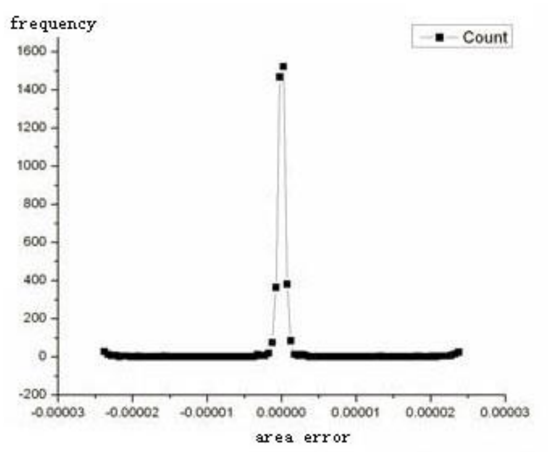

FIGURE 20 Frequency of the Synder model partition error in level 6

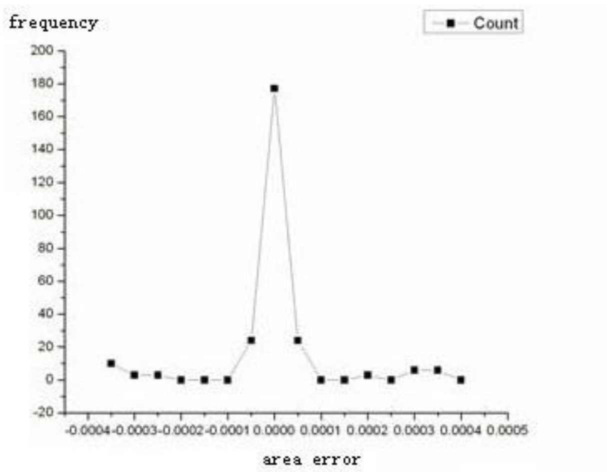

FIGURE 18 Frequency of the Synder model partition error in level 4

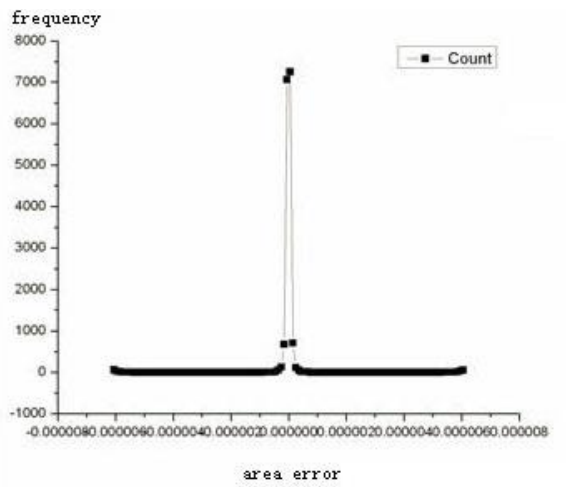

FIGURE 21 Frequency of the Synder model partition error in level 7

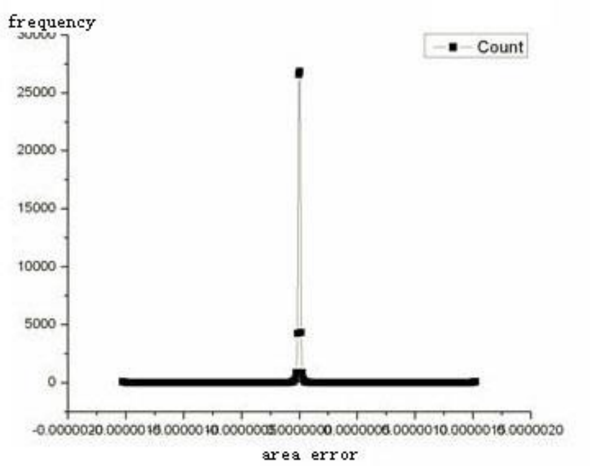

FIGURE 22 Frequency of the Synder model partition error in level 8

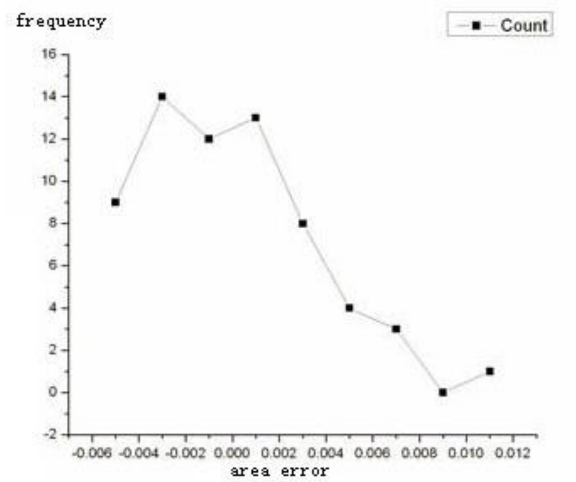

FIGURE 25 Frequency of the QTM partition error in level 3

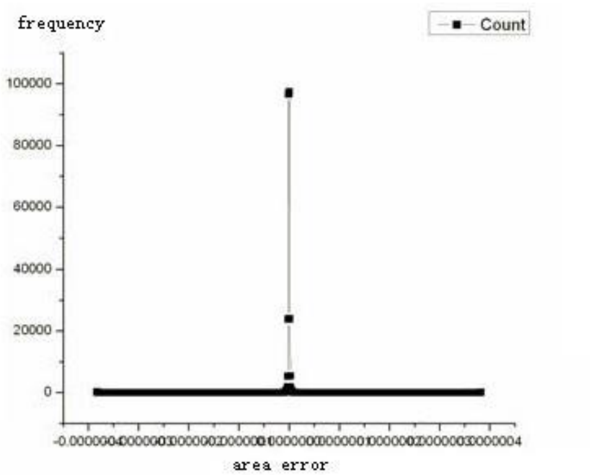

FIGURE 23 Frequency of the Synder model partition error in level 9

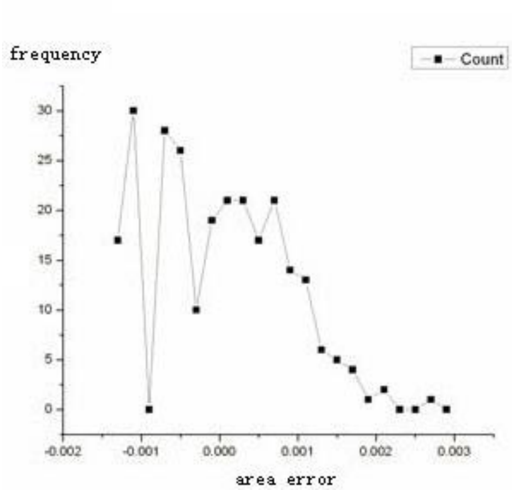

FIGURE 26 Frequency of the QTM partition error in level 4

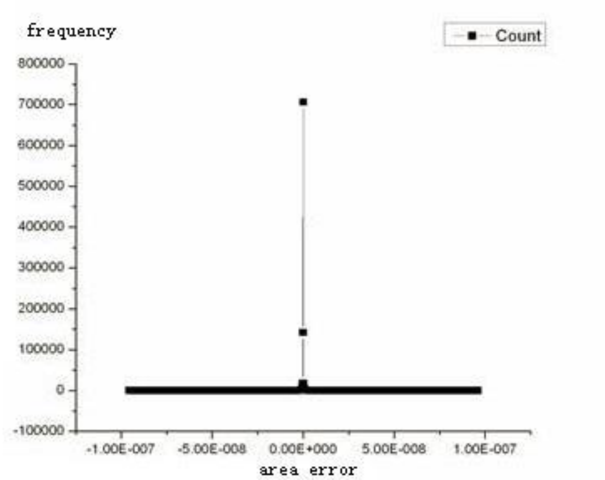

FIGURE 24 Frequency of the Synder model partition error in level 10

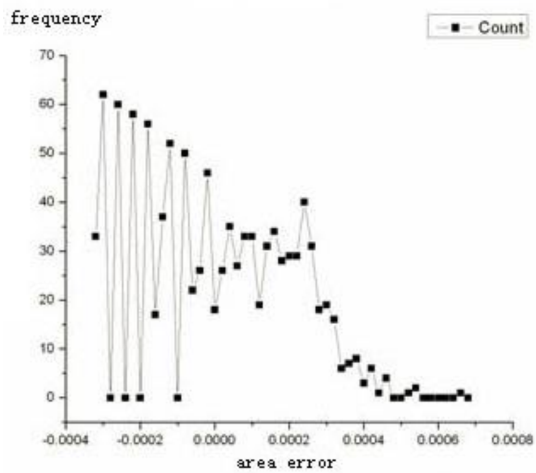

FIGURE 27 Frequency of the QTM partition error in level 5

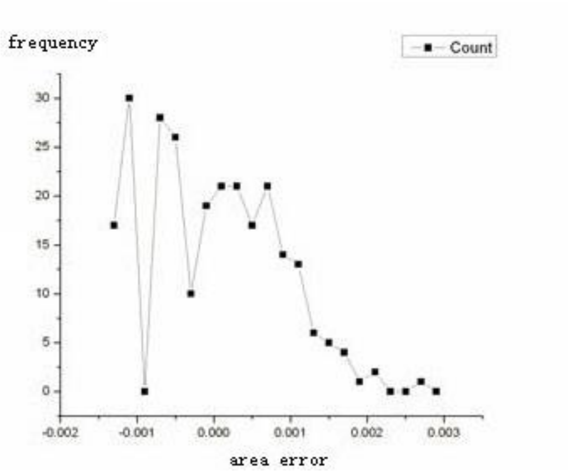

FIGURE 28 Frequency of the QTM partition error in level 6

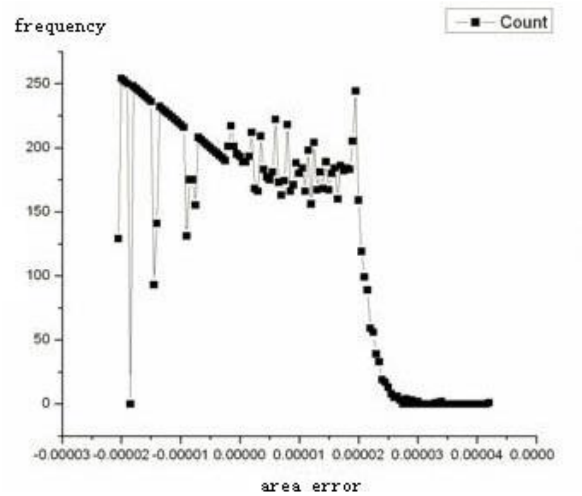

FIGURE 29 Frequency of the QTM partition error in level 7

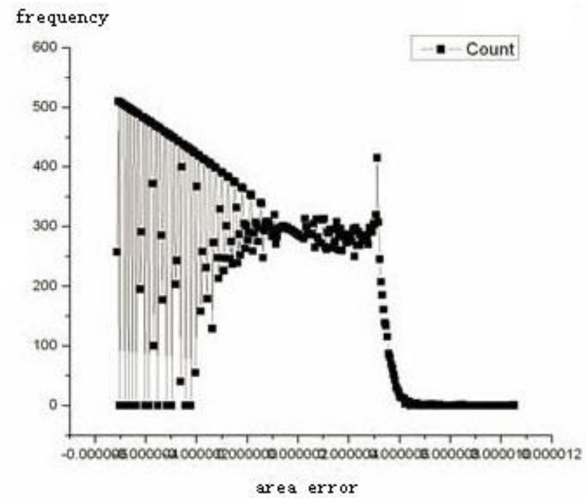

FIGURE 30 Frequency of the QTM partition error in level 8

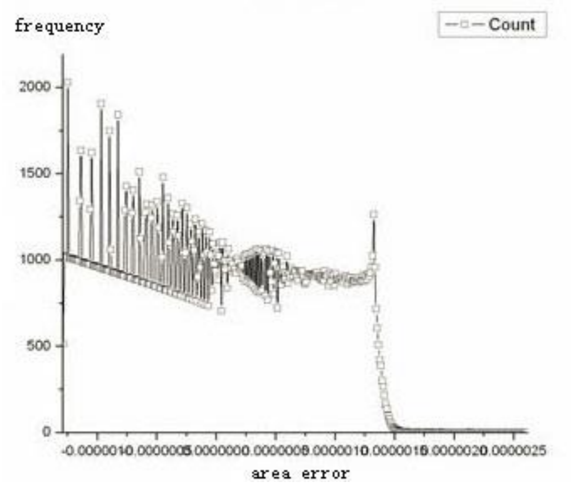

FIGURE 31 Frequency of the QTM partition error in level 9

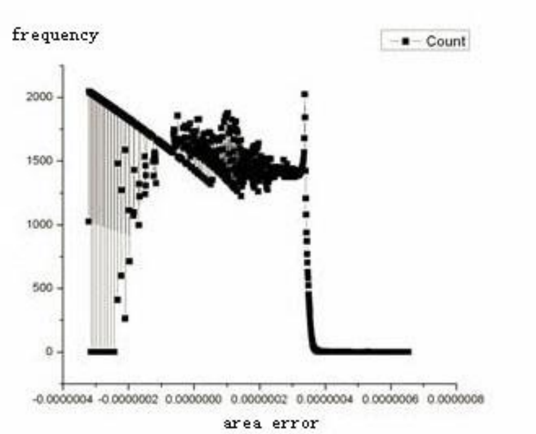

FIGURE 32 Frequency of the QTM partition error in level 10

5 Conclusions

A frequency analysis can clearly reflect the characteristics of area error in different grid model types. In particular, the distribution of these errors in the Synder model is symmetrical, and they are mainly induced by the accidental error generated during the computational process. This phenomenon is consistent with the characteristics of equal area projection in this model.

As a result of this projection, the Synder model displays the smallest area error compared with the other three models as per frequency analysis. By contrast, the QTM model exhibits the largest area error among the four models because of its partition method and octahedral inscribed geometry. With respect to area error, the four models are thus ranked in ascending order as follows: Synder, STQIE, SQT, and QTM.

Acknowledgments

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