Dynamic pricing model of monopolistic manufacture based on the after-sale service

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Received 6 October 2013, www.tsi.lv

Abstract

Dynamic pricing is concerned by business and academia as a pricing method, and has also made extensive research in this field. But the dynamic pricing theory with multi period is not mature considering monopolistic environment and after-sale service of manufacture. Because consumer not only pays attention to the product itself, but more emphasis on after-sale service of product with the changes of consumption concept and increasingly fierce market competition. Therefore good after-sale service is an important reason for consumers to purchase repetitively, which has become the key to the success of manufacture. This paper puts forward to the demand function with learning character, and constructs the multi-period dynamic pricing model on account of monopolistic manufacture and after-sale service level. Then it has important theoretical and practical significance when the conclusions are applied to the monopolistic manufacture. The research findings show the product price of manufacture fluctuates with oscillation both in the short and long term. But it is gradually reduced to a constant value for the magnitude or extent of the price oscillation with certain rate convergence in the long term. Finally, the price may tend to consumer's reservation price or unit operating cost of manufacture.

Keywords: Dynamic Pricing Model, After-sale Service, Monopolistic Manufacture

1 Introduction

The study of pricing theory is nothing more than from macro and micro perspective according to the existing literatures. The first kind of product pricing is about economic analysis based on macro perspective, which summarized the price formation mechanism by using the basic economics principles, such as the marginal utility theory of Adam Smith (1776), the labour value theory of Marx (1845), the equilibrium price theory of Marshall (1920) and the incomplete competition theory putted forward by Chamberlain (1938) and Robinson (1933), etc.. It has derived many products pricing methods based on the above pricing theory, such as elastic analysis method, correlation product pricing method and complete sets of products pricing method, etc. [1]. The second is the specific product pricing method because the cost profit for traditional pricing mode was broke out by many economists with the development of market and economy, and a series of pricing models were developed. These pricing models are developed to the practical application and the direction of diversification, which in addition to considering the internal variety and other factors, but also considering the external market volatility and the competitive environment. Dynamic pricing is that the manufacture adjusts product prices for different customer groups in real time in order to obtain the maximum benefit. As an important pricing technology of revenue management, dynamic pricing has been integrated into the various optimization software of management, which provides an important support for manufactures to develop a reasonable price decision, and has been widely used in the tourism industry, such as aviation, automobile, railway, etc. [2-3].

There is great development about dynamic pricing model of the monopolistic manufacture. Smith and Achabal (1998) construct the dynamic optimal control model according to the retail commodity with clearance at the end of season, which considers the change of sales volume is influenced by lower prices and the seasonal change [4]. Zhao and Zheng (2000) study the demand is non homogeneous Poisson process based on Gallego and van Ryzin (1994). Their results show the optimal price increases with the decrease of inventory quantity in a certain period, and it is monotone under certain conditions [5]. Constantinos Maglaras, Joern Meissner maximize firms' total expected revenues over a finite horizon and put forward to dynamic pricing strategies for multiproduct revenue management problems [6]. Xuanming Su (2010) studies a monopolist firm selling a fixed capacity, and his findings show that the firm's expected profits will increases when the presence of speculators increase, and the speculative behaviour may generate incentives which lead to lower capacity investments [7]. Josef Broder and Paat Rusmevi chientong (2012) build a stylized dynamic pricing model, in which the purchasing decisions for a sequence customers make a monopolistic prices [8]. Bhalla and Manaswini (2012) solve a monopolist's optimal price strategy, and his results show the prices and per period profits will increase over the period of time [9]. But many researches seldom consider the dynamic pricing from the manufacturer's own point of view. The

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consumer's behaviour is not only related to consumption preferences, consumption habits, income and other factors of consumers, but also more closely linked with the product quality, reputation and after-sale service etc. Therefore, the after-sale service is the key to win for manufacture, which has already become the new focus of market competition [10].

In summary, the dynamic pricing theory with multi period is not mature considering monopolistic environment and after-sale service of manufacture. The literature is still relatively small, which is need to be researched further in-depth and systematically. This paper discusses the dynamic pricing strategy based on after-sale service under the monopoly environment. Firstly, it establishes the demand model with learning nature on account of the early and current period purchase price, the after-sale service. Secondly, it also solves the optimal dynamic pricing strategy of manufactures, and the result show that the price of product tends to a constant value with oscillation according to the dynamic pricing rules of manufacture. Finally, the optimal price and after-sale service of every period are solved, and it proves that the sales price tends to customer's reservation price or unit cost under certain conditions.

2 Model assumptions and model construction

2.1 MODEL ASSUMPTIONS

2.1.1 The market structure

Assume that there is only a monopolistic manufacturer selling a product in a particular industry. For example, it is inevitable to form a monopoly in a certain period when new products promote to the market because of the novelty of product, the adoption of new technology, enhance of function and the change of appearance, which is mainly reflected in the monopoly of proprietary technology.

2.1.2 The price and production cost

Assume that p_t is the sales price of product in period t, and c is the unit production cost of product, which is equal in each period. Assume $p_t \geq c$, where t=1, 2….

2.1.3 The objective function

The goals are not the same for different types of manufacture in different development period, such as the maximum of profits, value, and market share or customer satisfaction and so on. This paper assumes that the manufacture's goal is to maximize profit.

2.1.4 The cost function of after-sale service

Assume it is independent for production cost and after-sale service provided by manufacture and the cost function of after-sale service is

$$C(a_t) = \frac{ka_t^2}{2}, \quad (1)$$

where k is the cost coefficient of after-sale service, this coefficient is closely related to the product type. It is higher when the complexity of product becomes greater. It means that the cost paid more when the manufacturer raises a unit of after-sale service. And a_t is the level of after-sale service for product in period t. The larger a_t indicates the cost of after-sale service provided by manufacture to consumers is greater, and the increase is in multiples square level.

2.1.5 The homogeneity assumption

In addition to the same product price and after-sale service level of manufacture in each period, the other aspects in different periods are also homogeneous.

2.1.6 The assumption of discount coefficient

The products of manufacture generally have a certain life cycle, so it is worth considering the time value of money. Assume the discount coefficient between the two cycles of the products is δ, where $\delta \in (0,1)$.

2.2 DEMAND MODEL

When manufacturer provides after-sale service to consumers, their demand function in period t can be represent as

$$D_t = M - \beta p_t + \lambda a_t. \quad (2)$$

Here, D_t is the demand function of manufacturer in period t. M represents the market capacity of product. And β is the price-sensitive parameter, which indicates the reverse relationship between the demand and price. Then λ is the sensitive coefficient of after-sales service to the demand, which shows the positive changes relation between the demand and after-sales service. The larger coefficient indicates the current demand of manufacturer, D_t, will be increased when manufacturer increases the after-sale service, a_t.

Eq. (2) portrays only the current relationship between the demand function of manufacturer and the prices, after-sale service level. In fact, the current price and after-sale service have not only certain effect on the current demand, but also influence the sales of product for the future periods. Therefore, the two assumptions are made as follows. The first is learning hypothesis. Assume there has learning process for consumer to the deal, but this learning process of consumers can be traced back to the previous transaction only. The second is the diffusion effect hypothesis of price or after-sale service. The lower price in period t-1 will lead to the increase of the demand in period t, and the higher price will reduce the future demand. But there is positive relationship between the product demand and after-sale service. This assumption implies that the current price and after-sale service will be affected by the next consumer surplus, which is determined by the difference between them. So Eq. (2) can be rewritten as follows according to the two assumptions.

$$D_t = M - \beta p_t + \lambda a_t + m a_{t-1} - n p_{t-1}, m, n \in [0,1], \quad (3)$$

where m and n are the learning coefficients of after-sale service and price in period t-1. And $ma_{t-1} - np_{t-1}$ is learning effect of consumer, which is determined by the transaction price and after-sale service. Here, assume the influence degree is much bigger than the last for the current market price and after-sales service levels to the demand, which is $\beta > n$ and $\lambda > m$.

2.3 MODEL CONSTRUCTION

According to Eq. (1), Eq. (2) and the unit cost hypothesis, the profit function in period t can be written as follows.

$$\pi_t = (p_t - c)D_t - {ka_t^2}/{2} = (p_t - c)(M - \beta p_t + \lambda a_t + m a_{t-1} - n p_{t-1}) - {ka_t^2}/{2}. \quad (4)$$

Then the total profits of product in infinite period can be converted into the following decision problems.

$$\pi = \max \sum_{t=1}^{\infty} \delta^{t-1} [(p_t - c)(M - \beta p_t + \lambda a_t + m a_{t-1} - n p_{t-1}) - {ka_t^2}/{2}, \quad (5)$$

where the discount coefficient for the first stage is equal to 1. And p_0 and a_0 are respectively the sale price and after-sale service level in the beginning period.

Assume the optimal total profit of manufacturer is $\Pi_t(p_t, a_t)$ from the period t to the end period. So Eq. (5) can be rewritten as the Bellman equation.

$$\Pi_t(p_t, a_t) = \max\left([(p_t - c)(M - \beta p_t + \lambda a_t + m a_{t-1} - n p_{t-1}) - {ka_t^2}/{2} + \delta \Pi_{t+1}(p_{t+1}, a_{t+1}) \right\}. \quad (6)$$

Eq. (6) shows that the dynamic programming problem is closely related to the sales price and after-sale service level, but there is not much relationship with the time. Therefore, the dynamic pricing problem with monopoly is converted into the maximization problem of profit.

3 Model analyses

It reveals the dynamic pricing rule of the monopolistic manufacture on account of after-sales service level according to the optimal solution and its property of model by partial differential analysis.

3.1 PRICING RULES GIVEN AFTER-SALE SERVICE LEVEL

In order to solve the after-sale service level, the first derivative of π_t for p_t can be solved, and assume it is equal to zero according to Eq. (4) when the after sale service level is given. That is

$$\frac{\partial \pi_t}{\partial p_t} = p_t'(p_{t-1}) = \frac{M + \lambda a_t + m a_{t-1} - n p_{t-1} + c\beta}{2\beta} = 0. \quad (7)$$

The second order derivative of Eq. (7) can be expressed as follows:

$$\frac{\partial p_t'(p_{t-1})}{\partial p_{t-1}} = \frac{-n}{2\beta} < 0. \quad (8)$$

The profit function can reach the maximum value at a certain point because the second order derivative is less than 0. By using the mathematical induction, the optimal selling price of monopolistic manufacture is

$$p_t' = \frac{\sum_{i=1}^{t} (2\beta)^{t-i} (-n)^{i-1} (M + \lambda a_{t-i+1} + m a_{t-i} + c\beta) + (-n)^t p_0}{(2\beta)^t}. \quad (9)$$

Relaxing the assumption of after-sales service level, assume that $a_1 = a_2 = ... = a_t = a$, Eq. (9) can be simplified as follows:

$$p_t' = \frac{[1 - ({-n}/{2\beta})^t][M + (\lambda + m)a + c\beta]}{2\beta + n} + ({-n}/{2\beta})^t p_0. \quad (10)$$

There is β>n according to the hypothesis of demand function. While $t \to \infty$, so the limit of Eq. (10) can be written as follows.

$$p_t^* = \lim_{t\to\infty} p_t' = \lim_{t\to\infty} \left\{ \frac{[1 - ({-n}/{2\beta})^t][M + (\lambda + m)a + c\beta]}{2\beta + n} + ({-n}/{2\beta})^t p_0 \right\} = \frac{M + (\lambda + m)a + c\beta}{2\beta + n}. \quad (11)$$

The following conclusion can be drawn from the above analysis.

Conclusion 1 The price of monopolistic manufacturer is $p_t^{'} = \frac{\sum_{i=1}^{t}(2\beta)^{t-i}(-n)^{i-1}(M+\lambda a_{t-i+1}+ma_{t-i}+c\beta)+(-n)^t p_0}{(2\beta)^t}$ when the after-sale service level is fixed. And the product price is $p_t^{''} = \lim_{t\to\infty} p_t^{'} = \frac{M+(\lambda+m)a+c\beta}{2\beta+n}$ while $a_1 = a_2 = ... = a_t = a$ and $t\to\infty$.

The sale price of product tends to a fixed value when the cycle of product sales is long enough and there is the same after-sale service according to the conclusion 1. This price has positive correlation with the market capacity, learning coefficient of after-sale service and unit operation cost, but there is reverse connection with the coefficient of price learning and price sensitivity. And this price does not reflect the trajectory before the price tends to be fixed, so it is worth discussing the change trajectory. Because of $\beta > \text{n}$, there is $-n/2\beta \in (-1/2, 0)$ according to Eq. (10) while $a_1 = a_2 = ... = a_t = a$. So the first half of $p_t^{'}$ is always greater than 0 whatever the number of product cycle change, and the second part, $(-n/2\beta)^t p_0$, ill appear the motion rules of vibration with the increase of cycle number. When t is even cycle, then $-n/2\beta > 0$, but when t is odd cycle, there has $-n/2\beta < 0$. The changes of odd cycle and even cycle will lead to the regular changes of $p_t^{'}$.

Taking $p_t^{''} = \frac{M+(\lambda+m)a+c\beta}{2\beta+n}$ of Eq. (11) into $p_t^{'}$ of Eq. (10), $p_t^{'}$ can be represented as follows.

$$p_t^{'} = [1-(-n/2\beta)^t]p_t^{''} + (-n/2\beta)^t p_0 = p_t^{''} - (p_t^{''} - p_0)(-n/2\beta)^t = \frac{M+(\lambda+m)a+c\beta}{2\beta+n} - \frac{M+(\lambda+m)a+c\beta-(2\beta+n)p_0}{2\beta+n}(-n/2\beta)^t. \quad (12)$$

There are two kinds of situations for the change rule of $p_t^{'}$, such as $p_t^{''} > p_0$ and $p_t^{''} < p_0$.

(1) The changes rule for $p_t^{''} > p_0$

Firstly, due to $-n/2\beta \in (-1/2, 0)$ and $-(p_t^{''} - p_0) < 0$, $(-n/2\beta)^t$ will reduce with the increase of t when t is even cycle, such as $t = 2n, n = 1, 2...$. So $p_t^{'}$ will increase with the increase of t. Secondly, $(-n/2\beta)^t$ will increase with the increase of t when t is odd cycle, such as $t = 2n-1, n = 1, 2...$, but $p_t^{'}$ will decrease.

(2) The changes rule for $p_t^{''} < p_0$

Firstly, due to $-(p_t^{''} - p_0) > 0$, $(-n/2\beta)^t$ will reduce with the increase of t when t is even cycle, such as $t = 2n, n = 1, 2...$, but $p_t^{'}$ will also decrease with the increase of t. Secondly, $(-n/2\beta)^t$ will increase with the increase of t when t is odd cycle, such as $t = 2n-1, n = 1, 2...$, but $p_t^{'}$ will also increase with the increase of t due to $-(p_t^{''} - p_0) > 0$.

Conclusion 2 The sales price of product, $p_t^{'}$, will increase(reduce) with the increase(reduce) of t due to $a_1 = a_2 = ... = a_t = a$ and $p_t^{''} > p_0 (p_t^{''} < p_0)$ when t is even cycle, such as $t = 2n, n = 1, 2...$, and there always is $p_t^{'} < p_t^{''} (p_t^{'} > p_t^{''})$. And $p_t^{'}$ will increase (reduce) with the increase (reduce) of t when t is odd cycle, namely, $t = 2n-1, n = 1, 2...$, and there is $p_t^{'} > p_t^{''} (p_t^{'} < p_t^{''})$.

This conclusion shows the sale price of product has oscillation law with the increase of period number in a certain extent. The sales price of product is fluctuating in the short term, but tends to a fixed value in the long run. Because consumer is not familiar with all aspects of the product information when new product promotes to market, the price will fluctuate more frequently with the change of demand. And the price of product stabilizes when consumer is familiar with the product when the product is used by a certain cycle.

But this conclusion does not consider the after-sale service level, which is derived in harsh conditions. Then the following will discuss the pricing rules when the after-sale service level is a constant value, and there have only two cases analyzed, such as strictly increasing and strictly decreasing of the after-sale service

(1) The change of $p_t^{'}$ for $a_t < a_{t+1}$

Firstly, there have $(-n)^{i-1} > 0$ and $a_t < a_{t+1}$ when t is even cycle. So

$$p_t^{'} = \frac{\sum_{i=1}^{t}(2\beta)^{t-i}(-n)^{i-1}(M+\lambda a_{t-i+1}+ma_{t-i}+c\beta)+(-n)^t p_0}{(2\beta)^t} > \frac{\sum_{i=1}^{t}(2\beta)^{t-i}(-n)^{i-1}[M+(\lambda+m)a_0+c\beta]+(-n)^t p_0}{(2\beta)^t}. \quad (13)$$

Then solving the limit of Eq. (13) according to $-n/2\beta \in (-1/2, 0)$, so the limit of $\overline{p_t^{''}}$ is

$$\overline{p_t^{''}} = \lim_{t\to\infty} p_t^{'} = \lim_{t\to\infty}\frac{\sum_{i=1}^{t}(2\beta)^{t-i}(-n)^{i-1}(M+\lambda a_{t-i+1}+ma_{t-i}+c\beta)+(-n)^t p_0}{(2\beta)^t} > \lim_{t\to\infty}\frac{\sum_{i=1}^{t}(2\beta)^{t-i}(-n)^{i-1}[M+(\lambda+m)a_0+c\beta]+(-n)^t p_0}{(2\beta)^t} = \overline{p_0^{''}}, \quad (14)$$

where $\overline{p_t^{''}}$ is the limit of sale price in the period t, and $\overline{p_0^{''}}$ represents the limit of sale price for $a_t = a_0$.

Secondly, when t is even cycle, there is $(-n)^{i-1} < 0$, so

$$p_t^{'} < \frac{\sum_{i=1}^{t}(2\beta)^{t-i}(-n)^{i-1}[M+(\lambda+m)a_0+c\beta]+(-n)^t p_0}{(2\beta)^t}. \quad (15)$$

Similarly, the limit of $\overline{p_t^{''}}$ is

$$\overline{p_t^{''}} = \lim_{t\to\infty} p_t^{'} < \frac{M+(\lambda+m)a_0+c\beta}{2\beta+n} = \overline{p_0^{''}}. \quad (16)$$

(2) The change of $p_t^{'}$ for $a_t > a_{t+1}$

Firstly, there have $(-n)^{i-1} > 0$ and $a_t < a_{t+1}$ when t is odd cycle, so

$$p_t^{'} < \frac{\sum_{i=1}^{t}(2\beta)^{t-i}(-n)^{i-1}[M+(\lambda+m)a_0+c\beta]+(-n)^t p_0}{(2\beta)^t}. \quad (17)$$

The limit of Eq. (17) is

$$\overline{\overline{p_t^{''}}} = \lim_{t\to\infty} p_t^{'} < \frac{M+(\lambda+m)a_0+c\beta}{2\beta+n} = \overline{\overline{p_0^{''}}}, \quad (18)$$

where $\overline{\overline{p_t^{''}}}$ is the limit of the sale price in period t, and $\overline{\overline{p_0^{''}}}$ is the limit of the sale price for $a_t = a_0$.

Secondly, there has $(-n)^{i-1} < 0$ when t is even cycle, so

$$p_t^{'} > \frac{\sum_{i=1}^{t}(2\beta)^{t-i}(-n)^{i-1}[M+(\lambda+m)a_0+c\beta]+(-n)^t p_0}{(2\beta)^t}. \quad (19)$$

The limit of Eq. (19) is

$$\overline{\overline{p_t^{''}}} = \lim_{t\to\infty} p_t^{'} > \frac{M+(\lambda+m)a_0+c\beta}{2\beta+n} = \overline{\overline{p_0^{''}}}. \quad (20)$$

Conclusion 3 The limit of product sales price in period t is greater than the price limit for $a_t = a_0$ when t tends to infinite with odd cycle and $a_t < a_{t+1}$, that is $\overline{p_t^{''}} > \overline{p_0^{''}}$, but there is $\overline{p_t^{''}} < \overline{p_0^{''}}$ while t is odd cycle. The limit of product sales price is less than the price limit for $a_t = a_0$ when t tends to infinite with odd cycle and $a_t > a_{t+1}$, that is $\overline{\overline{p_t^{''}}} < \overline{\overline{p_0^{''}}}$, but there is $\overline{\overline{p_t^{''}}} > \overline{\overline{p_0^{''}}}$ while t is even cycle.

Conclusion 3 shows there has a lower bound for the sale price of product with the increase of cycle number and after-sale service level when t is odd cycle, in other words, the sale price is increasing. The sales price of product has an upper bound when t is an even cycles, that is to say the price is decreasing. In the same way, there have similar change rules for the decline of after-sale service level. It implies the sales price is dynamic with oscillation from a certain perspective. Therefore, the sales price has rules to follow in certain conditions.

3.2 THE PRICING RULES FOR THE CHANGE OF PRICE AND AFTER-SALES SERVICE LEVELS SIMULTANEOUSLY

The part regards after-sales service level and sales price as the decision variables simultaneously. In order to obtain the maximum profits, manufacture should determine the optimal sale price and after-sales service level according to the actual situation of manufacture. Solving the first-order partial derivatives of Eq. (4) for p_t and a_t, and make them equal to zero, so there have

$$\frac{\partial \pi_t}{\partial p_t} = M - 2\beta p_t + \lambda a_t + m a_{t-1} - n p_{t-1} + c\beta = 0, \quad (21)$$

$$\frac{\partial \pi_t}{\partial a_t} = \lambda(p_t - c) - k a_t = 0. \quad (22)$$

After finishing Eq. (21) and Eq. (22), p_t and a_t are

$$p_t = \frac{M + \lambda a_t + m a_{t-1} - n p_{t-1} + c\beta}{2\beta}, \quad (23)$$

$$a_t = \frac{\lambda(p_t - c)}{k}. \quad (24)$$

Taking $a_{t-1} = \frac{\lambda(p_{t-1} - c)}{k}$ of Eq. (24) into Eq. (23), there has

$$p_t(p_{t-1}) = \frac{kM - c\lambda^2 + c\beta k + (m\lambda - kn)p_{t-1} - m\lambda c}{2\beta k - \lambda^2}. \quad (25)$$

Similarly, the optimal price and after-sales service level can be derived through the mathematical induction, namely the conclusion 4.

Conclusion 4 The optimal price and after-sales service level are equal to the initial sales price and after-sales service respectively when the profit of manufacture achieves maximization. That is

$$\begin{aligned} p_1^{'} &= \frac{A_1 + k(ma_0 - np_0)}{A_2}, a_1^{'} = \frac{\lambda(p_1^{'} - c)}{k} \\ p_t^{'} &= \frac{A_3(1-\eta^{t-1})}{A_2(1-\eta)} + \eta^{t-1} p_1^{'}, a_t^{'} = \frac{\lambda(p_t^{'} - c)}{k}, t = 2,3.... \end{aligned}, \quad (26)$$

where $A_1 = km - c\lambda^2 + c\beta k$, $A_2 = 2\beta k - \lambda^2$, $A_3 = T_1 - m\lambda c$, $A_4 = m\lambda - kn$ and $\eta = A_4 / A_2$.

Conclusion 4 means the manufacture's optimal sale price and after-sale service is affected by the initial sale prices of product and after-sale service level. The sale price and after-sale service will be gradually reduced along with the increase of the sales cycle number when the initial price is higher, but the reduction extent will be smaller. In another word, it is smaller for the initial price and after-sale service level influenced by the late price and after-sale service level. But there is no further explanation for the convergence of price, which is reduced to what extent in conclusion 4. Therefore, the following discusses the degree of convergence effect.

According to previous consumption experience, relevant alternative product prices and other product information, consumer as a whole has a subjective expectation value for product, which is the reserve price. This price is the most acceptable price for consumers. Assume the reserve price as a constant value is p^*, and it does not change with the change of cycle number. It is different for the reservation price and sales price. Assume the reserve price of consumer is higher than the sales price of product, once the product price is higher than the reserve price, so the demand will be reduced to 0. Therefore, there is $p^* \geq p_t$. It also assumes that the sale price is generally greater than the unit operation cost, namely $p_t \geq c$. From the above analysis, there has $p_t \geq p_t^{''} = \lim_{t\to\infty} p_t^{'} \geq c$.

So the Eq. (26) can be rewritten as follows.

$$p_t^{'} = \frac{A_3}{A_2(1-\eta)} - \frac{A_3\eta^{t-1}}{A_2(1-\eta)} + \eta^{t-1}p_1^{'} = \frac{A_3}{A_2(1-\eta)} + \eta^{t-1}[p_1^{'} - \frac{A_3}{A_2(1-\eta)}]. \quad (27)$$

Assume $A_5 = p_1 - \frac{A_3}{A_2(1-\eta)}$, $p_t^{'}$ is

$$p_t^{'} = \frac{A_3}{A_2(1-\eta)} + \eta^{t-1}A_5. \quad (28)$$

The value of $p_t^{'}$ and $p_t^{''}$ are also different due to the different value of η, therefore there are five kinds of cases.

(1) If t is odd cycle, there has $\eta^{t-1} > 0$, so $p_t^{''} = \lim_{t\to\infty} p_t^{'} = p^*$ when t tends to infinity for $\eta < -1$ and $A_5 > 0$. But when t is even cycle, there has $\eta^{t-1} < 0$, then $p_t^{''} = c$. If t is even cycle, there has $\eta^{t-1} > 0$, so $p_t^{''} = c$ when t tends to infinity for $\eta < -1$ and $A_5 < 0$. But when t is even cycle, there has $\eta^{t-1} < 0$, then $p_t^{''} = p^*$.

(2) When t tends to infinity, there has $p_t^{''} = \frac{A_3}{2A_2} + (-1)^{t-1}A_5 = \frac{A_3}{2A_2} + (-1)^{t-1}[p_1 - \frac{A_3}{2A_2}]$ for $\eta = -1$.

(3) When t tends to infinity, there has $p_t^{''} = \lim_{t\to\infty}[\frac{A_3}{A_2(1-\eta)} + \eta^{t-1}A_5] = \frac{A_3}{A_2(1-\eta)}$ for $-1 < \eta < 1$.

(4) There has $p_t^{''} = c$ for $\eta = 1$ when t tends to infinity.

(5) There has $p_t^{''} = p^*$ for $\eta > 1$ and $A_5 > 0$ when t tends to infinity. And there has also $p_t^{''} = c$ for $\eta > 1$ and $A_5 < 0$ when t tends to infinity.

According to the analysis of the above five cases the conclusion 5 can be obtained as follows

Conclusion 5 The sale price of product has four kinds of choices when the cycle number tends to infinity. Firstly, the sale price of product is equal to the unit operating cost, that is $p_t^{''} = c$. Secondly, the sale price of product is equal to the unit operating cost and the consumer's reservation price, which is $p_t^{''} = \lim_{t\to\infty} p_t^{'} = p^*$. Thirdly, the sale price is $p_t^{''} = \frac{A_3}{2A_2} + (-1)^{t-1}[p_1 - \frac{A_3}{2A_2}]$ for $\eta = -1$. Finally, the sale price is $p_t^{''} = \frac{A_3}{A_2(1-\eta)}$ for $-1 < \eta < 1$.

The sale price of product tends to the consumer's reservation price or the unit operating cost of manufacture with the increase of sale cycle while it does not consider the case for $-1 \leq \eta < 1$ from the conclusion 5. This confirms the truth of much realistic economic life. The sale price of product is reduced gradually when the sale is sluggish for the product promoting to market or it is the end of product life cycle. And the product will exit the market while the price tends to the unit operating costs of manufacture. The sale price is high, which is generally lower than the reserve price of consumer when the product is sold well or is promote to the market for the first.

4 Example analyses

The change trend of the optimal sale price and after-sale service level is studied by the example of an agricultural machinery manufacturer in the pursuit of profit maximization. In order to study the change trend, assume that the initial conditions of manufacture are $k = 0.5, m = 20, c = 80, \lambda = 0.1, \beta = 0.3, a_0 = 20, n = 3, p_0 = 120$ and $T_1 = 175$ according to Eq. (26). The optimal sale price and after-sale service level for the first ten periods can be derived, as shown in table 1.

TABLE 1 the calculation results for the first ten periods

periods(t)	**The optimal price ($p_t^{'}$)**	**The optimal after-sales service ($a_t^{'}$)**
1	142.07	12.41379
2	134.1241379	10.82483
3	126.8434979	9.3687
4	123.8728265	8.774565
5	123.2342014	8.64684
6	123.1580581	8.631612
7	123.152913	8.630583
8	123.1527139	8.630543
9	123.1527094	8.630542
10	123.1527094	8.630542

It is seen from table 1, the optimal price of manufacture tends to 123.152 in the seventh period, and the optimal after-sales service tends to 8.63 in the sixth period.

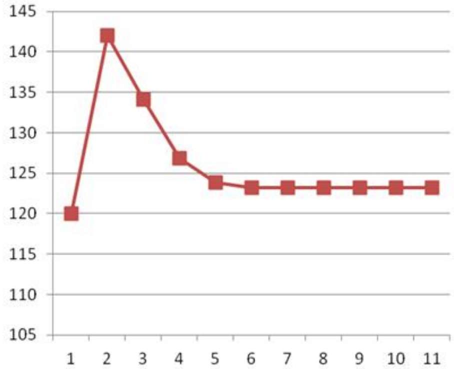

FIGURE 1 the optimal price trend

Figure 1 shows that the optimal price of manufacture is at a higher level during initial period. But it gradually reduced and tends to be a stable value, which is consumer's reservation price or unit operating costs. Because the manufacture selects the higher initial price generally in order to recover the upfront cost of as soon as possible.

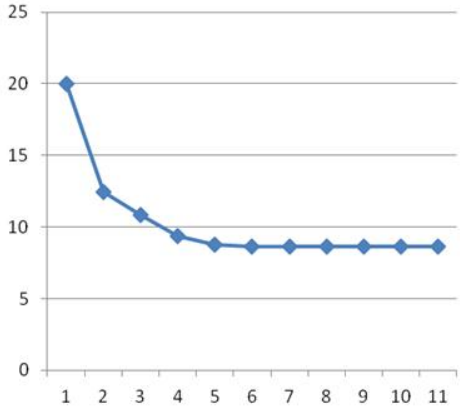

FIGURE 2 the optimal after-sales service trend

Figure 2 shows the optimal after-sale service level of will gradually reduce and tends to a constant value with the increase of cycles. Higher initial after-sale service level is also to recover the initial investment as soon as possible. Because the cost of after-sale service provide by manufacture is decreased, so the optimal after-sale service level will also reduce gradually.

5 Conclusions

This paper proposes the demand function of consumer with learning character, and then constructed the multi period dynamic pricing model for monopolistic manufacture according to the purchase price and after-sale service level in the prior and current period. In order to try to reveal the dynamic pricing rules of manufacture, the optimal sale price and after-sales service level is solved by dynamic programming method, then the limit value of product sale price is derived considering after-sales service. The findings show as follows. Firstly, the product pricing of manufacture fluctuates with oscillation both in the short and long term, but in the long term it tends to a constant value. Secondly, the oscillatory amplitude or degree of product price is gradually reduced with a certain convergence when the period number tends to infinity. The sale price may tend to consumer's reservation price and unit operation cost of manufacture.

There are two possible research directions to study the pricing problem. Firstly, the demand function with learning can be developed from linear to nonlinear model. Secondly, the uncertain demand function will be considered to study the effect to pricing.

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