

Application of CCD-model-based DEA analysis method in research on agricultural economic growth

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Abstract

This paper presents research into the statistical data related to China's economic growth from 1990 to 2011, obtained by application of the DEA analysis method to the data in the China Statistical Yearbook 2012. First, a brief introduction is given to the principle structure of the Data Envelopment Analysis mathematical model. Second, the CCD model is established, based on modelling of the decision-making unit DMU_j in the Data Envelopment Analysis model; and a detailed discussion is made of the corresponding linear program. Finally, the optimum value of this program is obtained and the results evaluated through study of the data collected by the mathematical model established in the paper. The main conclusions are as follows: the per capita income of China's farmers is closely related to the national total agricultural economic output and the total power of agricultural machinery, but is not that closely related to the crop planting area, area of affected crops or crops disaster area. Further, China's agricultural development shall lead to an advanced level of huge mechanization.

Keywords: CCD model, DEA analysis method, programming model, non-Archimedean infinitesimal

1 Introduction

At present in China, to construct a new socialist countryside and to realize the ambitious goal of a well-off society, the Three Rural Issues must be solved. The agricultural economy in particular has a key role to play in national development and maintenance of the national life. However, there is a difficulty in adjusting the agricultural economy as only the farmers who have adapted to free economy can provide the essential driving force needed for its development [1]. Nonetheless, the agricultural economy must be developed, and thus great importance must be attached to the associated technology. In this paper, research is conducted into the data on development of the agricultural economy from 1990 to 2011 by the Data Envelopment Analysis (DEA) analysis method, with the expectation of providing feasible technology and advice for the future development of the agricultural economy.

Many people have greatly contributed to research on the DEA analysis method and agricultural economic development, and consequently significant progress has been made in both the analysis method and the broader research field. We highlight the following three as amongst the most outstanding contributions. In Analysis on the Rate of Contribution Made by Rural Education to Zhejiang Agricultural Economic Growth, by Jianfeng Hu and other teachers of Zhejiang Institute of Science and Technology, the rate of contribution made by rural education to Zhejiang's agricultural economic growth is calculated using the C^2R model on statistical data concerning agricultural input and output of Zhejiang Province from

1996 to 2005. Their analysis indicates that the total power of agricultural machinery and fertilizer use have little effect on the total agricultural output value, and the main driving forces of agricultural economic growth are the agricultural acreage and the degree of labor education [1]. In Mathematical Model of Density and Fertilizer in Agricultural Production Function and Its Optimization, CHEN Rongying and other teachers of Henan Vocation-Technical Teachers College establish, by the principle of calculus, a mathematical model of density and fertilizer in agricultural production and its optimization. They provide a scientific analysis of a method for determining the high-yield optimum density and the optimal amount of fertilizer use, which demonstrates the important role of advanced mathematics in modern agricultural scientific research [2]. Huang Liping of the College of Economics and Management of Southwest University has made a detailed analysis of the current income status of Chinese farmers in his Masters thesis, Analysis in the Components and Increasing Factors of the Farmers Income, by means of times-series analysis. He proposed that China's national income growth is fundamentally the result of adjustment of the redistribution policy of social wealth, with farmers' income increasing through promotion of social innovation, development and total wealth accumulation whilst maintaining macroeconomical sustainable growth.

By applying the DEA analysis method and building on previous research, this paper gives an analysis of the statistical data on China's agricultural economy and the changing relationships through time between the average net income of farmers, national gross agricultural

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production, acreage under cultivation, crops disaster area and total power of agricultural machinery, with a view to obtaining the solution to development of the agricultural economy.

2 Theory of the DEA mathematical model

DEA is a new kind of analysis that integrates management science, mathematical economics, operational research and mathematics. Its underlying mathematical model is linear programming, and it aims to evaluate the relative efficiency of comparable decision-making units of the same kind. Thus, it provides a method for the quantitative analysis of many input indicators and output indicators. The DEA method includes the *CCR* model, *BCC* model, *C²GS²* model, *C²W* model and *C²WH* model. The *CCR* model is mainly applied for contribution rate calculation and efficiency evaluation; the *BCC* and

C²GS² models are applied for the evaluation of the technical efficiency between production departments; the *C²W* model is applied for the evaluation of an infinite number of decision-making units; and the *C²WH* model is applied in the case of many inputs and outputs. The following is a brief description of the above five models and their application to obtain the optimal analysis model of agricultural economic growth.

2.1 MATHEMATICAL EXPRESSION OF THE DEA MODEL

The data expressions of Equations (1) and (2) can be obtained according to the corresponding definitions of the quantities of decision-making units, input types, output types, input weight coefficients and output weight coefficients, as shown in Table 1.

TABLE 1 Definition of parametric variables of model decision-making units

Quantity of decision-making unit	Quantity of input types		Quantity of output types		Quantity of input weight coefficient	Quantity of output weight coefficient		
<i>n</i>	<i>m</i>		<i>s</i>		<i>m</i>	<i>s</i>		
	1	2	3	...	<i>j</i>	...	<i>n</i>	
<i>v</i> ₁	1	<i>x</i> ₁₁	<i>x</i> ₁₂	<i>x</i> ₁₃	...	<i>x</i> _{1<i>j</i>}	...	<i>x</i> _{1<i>n</i>}
<i>v</i> ₂	2	<i>x</i> ₂₁	<i>x</i> ₂₂	<i>x</i> ₂₃	...	<i>x</i> _{2<i>j</i>}	...	<i>x</i> _{2<i>n</i>}
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
<i>v</i> _{<i>i</i>}	<i>i</i>	<i>x</i> _{<i>i</i>1}	<i>x</i> _{<i>i</i>2}	<i>x</i> _{<i>i</i>3}	...	<i>x</i> _{<i>i</i><i>j</i>}	...	<i>x</i> _{<i>i</i><i>n</i>}
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
<i>v</i> _{<i>m</i>}	<i>m</i>	<i>x</i> _{<i>m</i>1}	<i>x</i> _{<i>m</i>2}	<i>x</i> _{<i>m</i>3}	...	<i>x</i> _{<i>m</i><i>j</i>}	...	<i>x</i> _{<i>m</i><i>n</i>}
	1	2	3	...	<i>j</i>	...	<i>n</i>	
<i>y</i> ₁₁	<i>y</i> ₁₂	<i>y</i> ₁₃	...	<i>y</i> _{1<i>j</i>}	...	<i>y</i> _{1<i>n</i>}	1	<i>u</i> ₁
<i>y</i> ₂₁	<i>y</i> ₂₂	<i>y</i> ₂₃	...	<i>y</i> _{2<i>j</i>}	...	<i>y</i> _{2<i>n</i>}	2	<i>u</i> ₂
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
<i>y</i> _{<i>r</i>1}	<i>y</i> _{<i>r</i>2}	<i>y</i> _{<i>r</i>3}	...	<i>y</i> _{<i>r</i><i>j</i>}	...	<i>y</i> _{<i>r</i><i>n</i>}	<i>r</i>	<i>u</i> _{<i>r</i>}
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
<i>y</i> _{<i>s</i>1}	<i>y</i> _{<i>s</i>2}	<i>y</i> _{<i>s</i>3}	...	<i>y</i> _{<i>s</i><i>j</i>}	...	<i>y</i> _{<i>s</i><i>n</i>}	<i>s</i>	<i>u</i> _{<i>s</i>}

$$h_j = \frac{u^T y_j}{v^T x_j} = \frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}}, j = 1, 2, \dots, m. \tag{1}$$

For appropriate selection of the input and output weight coefficients, Equation (3) takes values in the range (0,1).

2.2 THE CCR MODEL

If the efficiency index of decision-making unit *j*₀ is defined as the target, and the efficiency indices of all decision-making units are used as restriction, then the *CCR* model can be constructed as shown in Equation (4).

$$\left\{ \begin{array}{l} \max \quad h_{j_0} = \frac{\sum_{r=1}^s u_r y_{rj_0}}{\sum_{i=1}^m v_i x_{ij_0}} \\ \text{s.t.} \quad \frac{\sum_{r=1}^s u_r y_{rj_0}}{\sum_{i=1}^m v_i x_{ij_0}} \leq 1 \quad j = 1, 2, \dots, s \\ v \geq 0 \quad u \geq 0 \end{array} \right. \tag{4}$$

In Equations (1) and (2): *x*_{*ij*} is the total input of decision-making unit *j* to category *i* input; *y*_{*rj*} is the total output of decision-making unit *j* to category *r* output; *v*_{*i*} is a measurement of category *i* input, the input weight coefficient; *u*_{*r*} is a measurement of category *r* output, the output weight coefficient; *i* = 1, 2, ..., *m* ; *r* = 1, 2, ..., *s* ; *j* = 1, 2, ..., *n* ; and *x*_{*ij*} and *y*_{*rj*} are greater than zero.

where $v = (v_1, v_2, \dots, v_m)^T, u = (u_1, u_2, \dots, u_s)^T$ and the model in Equation (4) can be seen to be a time-sharing program by the use of the Charnes–Cooper conversion, as shown in Equation (5).

*DMU*_{*j*} then has the corresponding efficiency evaluation index for each decision-making unit as shown in Equation (3).

$$\left. \begin{aligned} t &= \frac{1}{v^T x_0} \\ w &= tv \\ \mu &= tu \end{aligned} \right\} \Rightarrow w^T x_0 = 1. \tag{5}$$

Therefore, the fractional model shown in Equation (4) can be transformed into the linear program shown in Equation (6).

$$\left\{ \begin{aligned} \max \quad & h_{j_0} = \mu^T y_0 \\ \text{s.t.} \quad & w^T x_j - \mu^T y_j \geq 0 \quad j = 1, 2, \dots, n \\ & w \geq 0 \quad \mu \geq 0 \\ & w^T x_0 = 1 \end{aligned} \right. \tag{6}$$

The efficiency evaluation relative target of decision-making unit j_0 is defined by the optimal solution to the linear program for all other decision-making units, which is the reason for applying the CCR model. By the principle of duality, the microanalysis of agricultural economy can be conveniently made from a theoretical and economic perspective, provided that the dual model in Equation (6) is established.

The dual program corresponding to Equation (6) is shown in Equation (7).

$$\left\{ \begin{aligned} \max \quad & \theta \\ \text{s.t.} \quad & \sum_{j=1}^n \lambda_j x_j \leq \theta x_0 \\ & \sum_{j=1}^n \lambda_j y_j \leq \theta y_0 \\ & \lambda_j \geq 0 \quad , j = 1, 2, \dots, n \\ & \theta \in (-\infty, +\infty) \end{aligned} \right. \tag{7}$$

TABLE 2 List of methods for DEA efficiency judgment

Parameter characteristic	DEA efficiency judgment
optimum value $h_{j_0} = 1$	Weak DEA efficiency
$w^* > 0, u^* > 0^*$ and the optimum value $h_{j_0}^* = 1$	DEA efficiency

Theorem 2. The equivalent condition of DMU_{j_0} for weak DEA efficiency is that the optimum value of the linear program given by Equation (6) is $\theta^* = 1$; and the equivalent condition of DMU_{j_0} for DEA efficiency is that additionally each optimal solution λ^* should correspond to $s^{*+} = 0, s^{*-} = 0$.

The judgment of technical efficiency and scale efficiency can be carried out at the same time by the application of the CCR model, as follows:

Criterion 1. If $\theta^* = 1$ and $s^{*+} = 0, s^{*-} = 0$, then the decision-making unit j_0 can be judged as DEA efficient and the economic activity of the decision-making unit is

For convenience in calculation and discussion of Equation (7), it is necessary to introduce the slack variable s^+ and residual variable s^- . This transforms the inequality in Equation (7) into the Equation (8).

$$\left\{ \begin{aligned} \max \quad & \theta \\ \text{s.t.} \quad & \sum_{j=1}^n \lambda_j x_j + s^+ = \theta x_0 \\ & \sum_{j=1}^n \lambda_j y_j - s^- = \theta y_0 \\ & \lambda_j \geq 0 \quad , j = 1, 2, \dots, n \\ & \theta \in (-\infty, +\infty) \quad s^+ \geq 0, s^- \leq 0 \end{aligned} \right. \tag{8}$$

2.3 PROPERTIES OF THE CCR MODEL

Theorem 1. A feasible solution exists for both the linear program given in Equation (6), and its dual program given in Equation (7). Further, the optimal solution also exists; if the optimal solution to Equation (6) is $h_{j_0}^*$ and the optimal solution to Equation (7) is θ^* , then $h_{j_0}^* = \theta^*$.

For the purpose of evaluating the efficiency of decision-making units, the concepts of weak DEA efficiency and DEA efficiency are defined as shown in Table 2, which also gives the parameter settings for Equation (6).

the result of both technical efficiency and scale efficiency at the same time;

Criterion 2. If $\theta^* = 1$ and at least one input or output is greater than 0, then the decision-making unit j_0 can be judged as weakly DEA efficient, but the economic activity of the decision-making unit is not the result of both technical efficiency and scale efficiency;

Criterion 3. If the optimum value has $\theta^* < 1$, then the decision-making unit j_0 can be judged as not being DEA efficient, and the economic activity is neither the result of optimal technical efficiency nor of optimal scale efficiency.

3 Research on empirical analysis and result

3.1 INDEX SYSTEM OF AGRICULTURAL ECONOMICAL GROWTH EVALUATION MODEL

The research in this paper is based on China’s national agricultural economy from 1990 to 2011, with all input and output index systems of the evaluation model as shown below.

Input: the total crops planting area (m^2) is expressed by

X_1 ; the area of affected crops (m^2) is expressed by X_2 ; the crops disaster area (m^2) is expressed by X_3 ; and the total power of agricultural machinery (10,000kw) is expressed by X_4 .

Output: the per capita net income of rural households (Yuan) is expressed by Y_1 and the total agricultural output value (0.1 billion Yuan) is expressed by Y_2 .

The variation of the index-related data each year is shown in Table 3.

TABLE 3 List of data related to agricultural economy from 1990 to 2011

Year	Per capita net income of rural household	Total agricultural output value	Total crops planting area	Area of affected crops	Crops disaster area	Total power of agricultural machinery
1990	686.30	4954.30	148362.2667	38474.0000	17819.3333	28707.7000
1991	708.60	5146.40	149585.8000	55472.0000	27814.0000	29388.6000
1992	784.00	5588.00	149007.1000	51332.0000	25893.0000	30308.4000
1993	921.60	6605.10	147740.7000	48827.0000	23134.0000	31816.6000
1994	1221.00	9169.20	148240.6000	55046.0000	31382.0000	33802.5000
1995	1577.70	11884.60	149879.3000	45824.0000	22268.0000	36118.1000
1996	1926.10	13539.75	152380.6000	46991.0000	21234.0000	38546.9000
1997	2090.10	13852.50	153969.2000	53427.0000	30307.0000	42015.6000
1998	2162.00	14241.88	155705.7000	50145.0000	25181.0000	45207.7000
1999	2210.30	14106.22	156372.8100	49979.5000	26733.7000	48996.1200
2000	2253.40	13873.60	156299.8460	54688.0000	34374.0000	52573.6063
2001	2366.40	14462.80	155707.8615	52214.6000	31793.1000	55172.1000
2002	2475.60	14931.54	154635.5134	46946.1000	27159.9000	57929.8500
2003	2622.20	14870.10	152414.9623	54505.8000	32516.3000	60386.5410
2004	2936.40	18138.36	153552.5454	37106.2562	16297.3165	64027.9100
2005	3254.90	19613.37	155487.7289	38818.2251	19966.0600	68397.8486
2006	3587.00	21522.28	152149.0000	41091.4100	24631.9400	72522.1234
2007	4140.40	24658.10	153463.9305	48992.3533	25063.8200	76589.5635
2008	4760.62	28044.15	156265.6989	39990.0340	22283.4700	82190.4132
2009	5153.17	30777.50	158613.5498	47213.6900	21234.2633	87496.1000
2010	5919.01	36941.11	160674.8135	37425.9000	18538.1000	92780.4757
2011	6977.29	41988.64	162283.2204	32470.5000	12441.3000	97734.6585

3.2 RESEARCH RESULTS AND ANALYSIS

If the CCR model with non-Archimedean infinitesimal ϵ is adopted, then the model shown in Equation (9) can be established for evaluating China’s national agricultural economy in 2011.

$$\begin{cases}
 \min [\theta - \epsilon (s_1^- + s_2^- + s_3^- + s_4^- + s_1^+ + s_2^+)] \\
 s.t. \quad X_1 \lambda^T + s_1^- = 162283.2204\theta \\
 \quad \quad X_2 \lambda^T + s_2^- = 32470.5000\theta \\
 \quad \quad X_3 \lambda^T + s_3^- = 12441.3000\theta \\
 \quad \quad X_4 \lambda^T + s_4^- = 97734.6585\theta \\
 \quad \quad Y_1 \lambda^T - s_1^+ = 6977.29\theta \\
 \quad \quad Y_2 \lambda^T - s_2^+ = 41988.64\theta \\
 \lambda \geq 0 \\
 s_i^- \geq 0 \quad i = 1, 2, 3, 4 \\
 s_r^+ \geq 0 \quad r = 1, 2
 \end{cases} \quad (9)$$

X_i, Y_i in Equation (9) indicate the 22- dimension indexed row vector, where the definition of each index is shown in Table 1 and λ is a 22- dimension row vector.

In the same way, the CCR model with non-Archimedean infinitesimal ϵ in 2010 and other years can be obtained, with the solutions shown in Table 4.

The evaluation result shows that the total power of agricultural machinery has greater influence on the rural household per capita net income and the total agricultural output value, while the crops planting area, area of affected crops and crops disaster area have less influence.

TABLE4 List of variation of the optimum value and evaluation result of CCR model by year

Year	Optimum Value	Evaluation Result	Year	Optimum Value	Evaluation Result
1990	M=0.8432, L1=0.14	B	2001	M=0.8894, L1=0.09	B
1991	M=1, L1=1	A	2002	M=0.9946, L4=0.05	B

1992	M=1, L1=1	A	2003	M=0.7854, L1=0.23	B
1993	M=1, L1=1	A	2004	M=1, L1=1	A
1994	M=1, L1=1	A	2005	M=1, L1=1	A
1995	M=0.7433, L4=0.19	B	2006	M=1, L4=1	A
1996	M=1, L4=1	A	2007	M=0.9742, L1=0.08	B
1997	M=1, L1=1	A	2008	M=0.9143, L1=0.22	B
1998	M=1, L1=1	A	2009	M=1, L1=1	A
1999	M=0.8744, L4=0.16	B	2010	M=1, L4=1	A
2000	M=1, L1=1	A	2011	M=1, L1=1	A

Note: Evaluation result A indicates DEA efficient and scale benefit unchanged; and evaluation result B indicates not DEA efficient and scale benefit increasing. In the optimum value column, M indicates θ and L_i indicates λ_i .

4 Conclusion

In this paper, research was conducted into the statistical data related to China's economic growth from 1990 to 2011 by means of the CCR-based DEA analysis method, using the data in the China Statistical Yearbook 2012. From the calculations, the conclusion is drawn that the rural household per capita net income is closely related to

the growth of national gross agricultural production and the total power of agricultural machinery, and receives less influence from the crops planting area, crops disaster area and area of affected crops. Moreover, the model is established for various years through use of the CCR model, and the optimum value is obtained for the linear program, which leads to a rational and reliable result.

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