

Power purchase plan using minimal cost flow

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Abstract

The minimal cost flow method presented in this paper is used to calculate the minimum power purchase cost, because the user's goal is the minimal power purchasing cost under open electricity market. By establishing the minimal power, purchasing cost model and using the minimum cost flow algorithm to solve the objective function, the method is used to calculate a network in a single time. A continuous period of power purchase plan is obtained by accumulating each period network flow together. Example analysis proves that it is feasible to solve the problem with the minimal cost flow algorithm.

Keywords: Power Purchase Cost, Minimal Cost Flow, Network, Electric Market, Power Purchase Plan

1 Introduction

Before 2002, China's electric power system was a vertically integrated monopoly industry, with power plants and grids integrated within one enterprise. With the reform of the power system in China, the electricity market gradually opens to the users. The users can choose their own power supply companies. Direct-purchasing electricity was first performed on large customers [1-3], and then gradually extended to all users. The user's goal is to take minimum power purchase cost as a power purchase plan. Therefore, the minimal purchase cost research has very practical significance.

Different countries have different power systems hence electricity-purchasing researches are different [4-6]. The purpose of China's electric power system reform is to break the monopoly and form competition. This is helpful to power plants, which need to improve efficiency and to power grid in terms of the reduction of line losses, which is beneficial to the development of the power system.

Current electricity purchasing cost model generally does not consider the structure of the grid. As the network flow algorithm can reserve system network topology structure and has the advantage of easiness in considering security constraints, it is suitable for solving constraints of high dimension, linear and nonlinear optimization problems. Therefore, the network flow theory has been widely applied to many fields such as power system dispatching, power system restoration plans [7], observability analysis [8, 9], network reliability [10-13], etc.

In this paper, the minimum cost flow algorithm is used to solve the minimal purchasing cost model and calculate a network in a single time. A continuous period of power purchase plan is obtained by accumulating each

period network flow together. Case studies have proven that the minimal cost flow method was successful in solving the minimal purchase cost problem and therefore has a certain applicational value.

2 The mathematical model

2.1 THE OBJECTIVE FUNCTION

The user's goal is the minimal electricity purchasing cost under open electricity market. The objective function can be expressed as:

$$\min \sum_{t=1}^T \sum_{i=1}^N W_{i,t} \rho_{i,t}, \quad (1)$$

$$W_{i,t} = P_{i,t} \Delta t, \quad (2)$$

where $W_{i,t}$ is the electricity of the i th branch at t time; $\rho_{i,t}$ is the cost of unit electricity corresponding to $W_{i,t}$; $P_{i,t}$ is the power of the i th branch at t time; N represents the number of branches.

2.2 CONSTRAINTS

The objective function need to satisfy the following constraints:

(1) Power constraints

The power constraints include the generator output and line transmission limit.

$$P_{i\min} \leq P_{i,t} \leq P_{i\max} \quad i = 1, 2, \dots, N, \quad (3)$$

where $P_{i\min}$ is the lower and $P_{i\max}$ is the upper output limit of unit i , respectively.

(2) Node voltage constraints

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$$U_{i\min} \leq U_{i,t} \leq U_{i\max} \quad i = 1, 2, \dots, N, \quad (4)$$

where $U_{i,t}$ is the voltage of node i at t time; $U_{i\min}$ is the lower and $U_{i\max}$ is the upper limit of node i , respectively.

(3) Power balance constraints

$$\sum_{i=1}^N P_{i,t} = L_t \quad i = 1, 2, \dots, N, \quad (5)$$

where L_t is the system load at t time.

3 Network flow theory introduction

3.1 NETWORK FLOW INTRODUCTION

Network flow problems belong to the category of graph theory. Graph is also referred to as a network, consisting of a number of nodes and its connecting arcs. If any two nodes in graph G are connected, then G is a connected graph. It is called a connected digraph if each arc has a direction. For digraph $G = (V, E)$, where V is the set of all nodes; E is the set of all arcs. The node which only has outflow arcs is called source point(s), while the one only has inflow arcs is called receive point(r) in V . The rest nodes which there are both outflow and inflow arcs are called intermediate points.

Arc(i,j) is the edge connecting node i and j , which has a predetermined direction, flow f_{ij} , the cost of the unit flow c_{ij} , the flow rate limit (as shown in Figure1 and Figure 2).

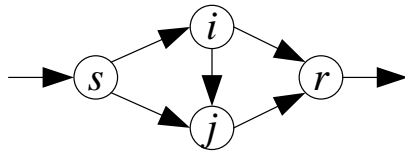


FIGURE 1 A Network

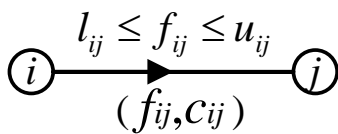


FIGURE 2 Flow Limit and Cost in An Arc

A set of f that meets the following two basic conditions in the graph can be viewed as a feasible flow.

(1) For each arc to meet capacity limits.

$$l_{ij} \leq f_{ij} \leq u_{ij} \quad (i, j) \in E. \quad (6)$$

where l_{ij} is lower limit, u_{ij} is upper limit.

(2) For the intermediate node, inflow is equal to outflow.

$$\sum_{(i,j) \in E} f_{ij} = \sum_{(k,i) \in E} f_{ki} \quad (7)$$

It can be proved that the net flows are equal from s to r . The flows are all set to v , and v is called the flow of the feasible flow f [14].

3.2 THE MINIMAL COST FLOW METHOD

The minimum cost flow problem is to make the minimum total cost under the specified flow taking into account the arc cost. The objective function can be expressed as:

$$\min \sum_{(i,j) \in E} c_{ij} f_{ij} \quad (8)$$

The minimal cost flow algorithm is mainly Klein algorithm, scaling algorithm, out-of-kilter and minimum cost path algorithm, etc. We take Busacker-Gowan iteration method here [15], which is relatively mature, the main steps are as follows:

(1) Find out minimum cost path μ from s to r ;

(2) Distribute the maximum possible flow f to μ :

$$f = \min \sum_{(i,j) \in \mu} \{u_{ij}\} \quad (9)$$

And let all edges capacity reduce f . In this case, the unit flow cost should be changed to infinity for the saturated edges;

(3) Make the reverse side of all edge (i,j) on the pathway μ , then $u_{ji} = f, c_{ji} = -c_{ij}$;

(4) In such a new network, repeat the above steps (1), (2), (3) until the complete flow is equal to v .

3.3 MULTI-PERIOD NETWORK FLOW MODEL

A continuous time network flows is constituted by combining each network flow chart of a single time in space. Multi-period model is a three-dimensional model with each layer corresponding to a single period model (as shown in Figure 3). Each s_i is united together to form new s , in the same way to form new r . Arc(s, s_i) represents the output, and arc(r_i, r) represents the load. Considering each period together, the power purchase plan of continuous time can be obtained.

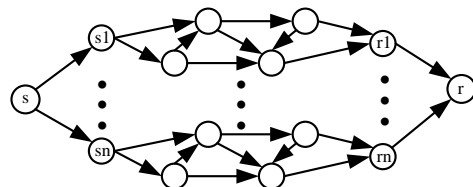


FIGURE 3 Multi-Period Network Flow Model

The above structure model is a typical generalized network flow programming problem. It can be expressed as:

$$\min C^T F, \quad (10)$$

$$\text{satisfied with } AF = b, \quad (11)$$

where C represents the m -dimensional arc cost vector; F represents the m -dimensional arc flow; b represents an n -dimensional vector injected nodes; A represents $n \times m$ dimensional node-arc incidence matrix [16].

4 Example analysis

A system diagram and its corresponding cost flow network diagram are shown in Figure 4 and Figure 5 respectively. When the purchasing electricity changes from minimum to maximum, we calculate the corresponding changes in the minimal power purchase costs. The unit of arc flow and its unit cost are MWh and Yuan/MWh respectively. For example, (15,330) means that the upper flow is 15 MWh and the unit cost is 330 Yuan/MWh.

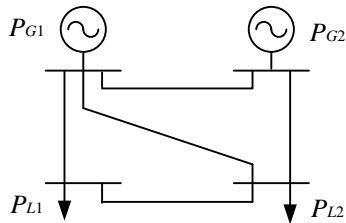


FIGURE 4 Power System Diagram

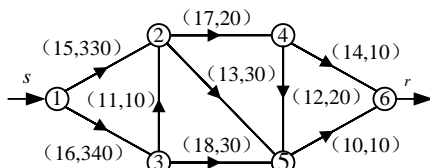


FIGURE 5 Network Diagram

Arc capacity matrix F and arc cost matrix C are

$$F = \begin{bmatrix} 0 & 15 & 16 & 0 & 0 & 0 \\ 0 & 0 & 0 & 17 & 13 & 0 \\ 0 & 11 & 0 & 0 & 18 & 0 \\ 0 & 0 & 0 & 0 & 12 & 14 \\ 0 & 0 & 0 & 0 & 0 & 10 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 330 & 340 & 0 & 0 & 0 \\ 0 & 0 & 0 & 20 & 30 & 0 \\ 0 & 10 & 0 & 0 & 20 & 0 \\ 0 & 0 & 0 & 0 & 20 & 10 \\ 0 & 0 & 0 & 0 & 0 & 10 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Put F and C into the program, the largest flow is 24MWh, the corresponding minimum cost is 8740Yuan, and the minimal cost matrix F_{min} is

$$F_{min} = \begin{bmatrix} 0 & 15 & 9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 14 & 1 & 0 \\ 0 & 0 & 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 14 \\ 0 & 0 & 0 & 0 & 0 & 10 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Minimum cost flow calculation is shown in Table 1 by the maximum flow[17], where C is the unit purchase cost, F is the purchase electricity, and Q is the electricity purchase cost.

TABLE 1 The minimum cost flow calculation

Step	Path	C(Yuan/MWh)	F(MWh)	Q(Yuan)
1	1→2→4→6	360	14	5040
2	1→2→5→6	370	1	370
3	1→3→5→6	370	9	3330

The distribution of the minimum cost flow can be divided into several steps to solve when the maximum flow passed through the network. Combining the steps together, the final result of the minimum cost flow distribution can be obtained (as shown in Figure 6 and Figure 7).

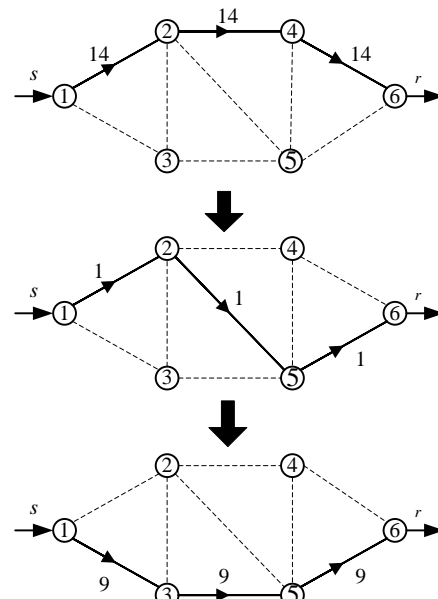


FIGURE 6 The Minimal Cost Flow Solving Process

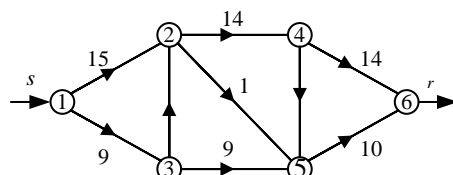


FIGURE 7 The Minimal Cost Flow Distribution

Assuming that the system is running in the same way, when the flow changes from minimum to maximum, the corresponding minimum costs will also increase (as shown in Table 2).

TABLE 2 The minimum cost under different flow

F	1	2	3	4	5	6	7	8
Q	360	720	1080	1440	1800	2160	2520	2880
F	9	10	11	12	13	14	15	16
Q	3240	3600	3960	4320	4680	5040	5410	5780
F	17	18	19	20	21	22	23	24
Q	6150	6520	6890	7260	7630	8000	8370	8740

Assuming that the system is running in period T , the purchasing power and the corresponding minimal cost change with the same trend(as shown in Table 3 and Figure 8). Accumulated purchasing power will gradually increase with the increase of time, the same trend with the accumulated minimal power purchase cost (as shown in Figure 9). Then the optimal power purchase scheme can be obtained from the above curves, which can clearly reflect the power-purchasing path.

TABLE 3 Each period flow distribution

T	1	2	3	4	5	6	7	8
F	18	17	16	17	18	19	20	21
Q	6520	6150	5780	6150	6520	6890	7260	7630
T	9	10	11	12	13	14	15	16
F	22	23	22	20	18	19	20	21
Q	8000	8370	8000	7260	6520	6890	7260	7630
T	17	18	19	20	21	22	23	24
F	22	23	24	23	22	21	20	19
Q	8000	8370	8740	8370	8000	7630	7260	6890

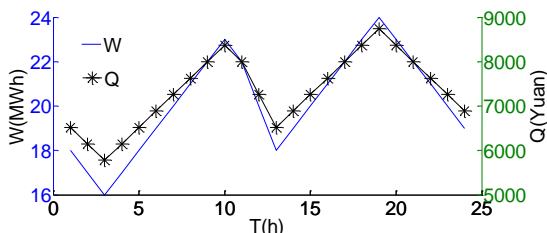


FIGURE 8 Purchasing Power and Minimal Power Purchase Cost Curve

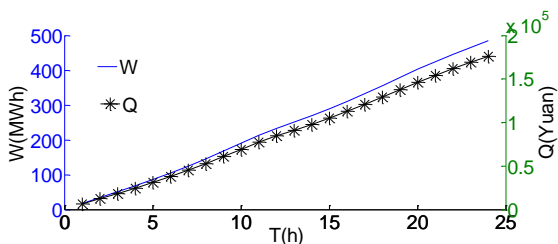


FIGURE 9 Purchasing Power and Minimal Power Purchase Cost Accumulation Curve

When the system operation mode changes, by changing the arc cost matrix and the arc capacity matrix, we can easily adapt to change in operating mode.

5 Conclusions

The minimal cost flow algorithm is used to determine the power purchase plan, which can reflect the system network topology making it easy to consider system constraints. The method is simple, rapid and clear in physical concept. When the operation mode changes, by



changing the network matrix we can quickly adapt to the new operation mode. Example analysis proves that the method is feasible and practical.

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