

# Dynamic analysis of ball-screw with rotating nut driven

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## Abstract

There is a certain degree difference between the static and operation condition for the high-speed Ball-screw with Rotating Nut. Therefore, this paper establishes a dynamic model of a preload-adjustable ball-screw with rotating nut by means of lumped-parameter and analyses the effects of changeable table position and work piece mass on the first three axial modes of the free vibration. A high-speed feeding system is modelled and its nature characteristics when the feeding system is in static, low and high rotate state. The results show that, at low speed state, the dynamics of the feeding system is the same as stationary state, and in high-speed conditions, the dynamics is quite different with the static state. The natural frequencies are notably changed with the position change of the table movement. The research lays an important theoretical foundation for developing this novel feed drive system.

*Keywords:* Ball screw, Dynamic analysis, Modal analysis, Frequency response

## 1 Introduction

The increasing demands that precision and engineering applications place on positioning systems has prompted an investigation into the ball screws. The reciprocating ball screw mechanism is a force and motion transfer device [1-3]. Chin Chung Wei [4, 5] developed theoretical analyses of the kinematics of a single-nut double-cycle ball screw. Huang and Ravani [6] used the concept of medial axis transform (MAT) to analysis the contact stresses between ball-screw and ball-nut, in their analysis we can get normal forces, contact angles and contact stress in contact areas of ball-screw and ball-nut. These scholars study the dynamics of the traditional ball screw pair. The traditional screw transmission of ball screw pair, particularly long screw for large and heavy high-end NC machine, is quite heavier than nuts, the much more inertia force of ball screw rotation can lead to heat, deformation and serious energy consumption. The new-type nut-driven ball screw pair is the green product with advanced structure and promotion value which integrates rolling bearing functions and routine ball screw pair functions, during transmission, H. Weule [7] describes the advantages and characteristics of a Dynamic feed axis with ball screw drive and driven nut in comparison with the conventional electromechanical drive.

Currently, most literature analysis the feed system dynamics based on static state of, and ignored the influence of the speed of the feed system dynamics [8-11]. In fact, drive nut speed have great influence to feed the system dynamic characteristics, especially high speed feed condition. If you use the stationary state to the analysis of dynamic characteristics of high-speed feed cutting stability, will make a big error. This paper take a high-speed double nut drive type feed system as the

research object. On the basis of actual working condition the lumped mass method is used to comparative analysis of the feed system quiescent state, low-speed operation dynamics differences in the state, the state of the high-speed operation. Related research conclusion can lay the foundation to a scientific analysis of the feed system dynamics characteristic and cutting stability.

## 2 Drive nut component axial stiffness calculation

### 2.1 NUT COMPONENT AXIAL STIFFNESS CALCULATION

Axial stiffness  $k_{nut}$  of nut components can be obtained through axial load exerted on the nut divided by amount of axial deformation:

$$k_{nut} = \frac{F_{axis}}{\delta_{axis}} \quad (1)$$

In order to eliminate the axial clearance of ball screw and to improve the axial contact stiffness of ball screw pair, preloaded spacer is usually used to pre-tighten the double nut mechanism pretension, as shown in Figure 1. Given that normal force applied by each ball on nut A in ball screw pair to screw normal force of preload applied by preloaded spacer through nuts A and B to screw is  $P_p$ .

When the axial working load is  $F_{axis} \neq 0$ , nuts A and B are elastically deformed at the point of contact under joint action of axial load  $F_{axis}$  and normal force of preload  $P_p$ , and axial elastic deformation amount of nut A is equal to axial elastic deformation amount of nut B:

$$F_{axis} = (P_A - P_B) z \sin \alpha \cos \beta \quad (2)$$

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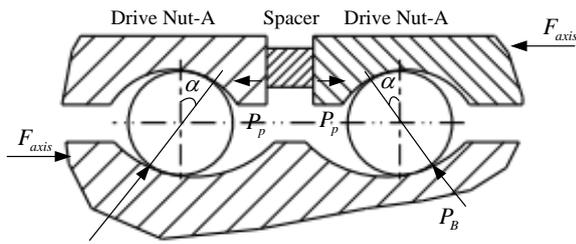


FIGURE 1 The ball screw vice work principle diagram of gasket type double nut prestressing structure

According to Hertz contact theory, elastic approach of two elastomers is proportional to power of their normal pressure  $2/3$ th power, and there

$$P_A^{2/3} - P_p^{2/3} = P_p^{2/3} - P_B^{2/3} \tag{3}$$

When the known axial working load is  $F_{axis}$  and preload is  $P_p$ , values of  $P_A$  and  $P_B$  are obtained through equations (3).

### 2.2 INFLUENCING FACTORS OF DYNAMIC CHARACTERISTICS INDUCED BY ANGULAR VELOCITY

Nut-driven ball screw pair will, during high-speed operation, produce centrifugal force, gyroscopic moment, axial stiffness softening and other phenomena. As shown in Figure 2, centrifugal force of steel ball  $j$  increases with rotating speed of screw  $\omega$  under action of axial load  $F_a$ , and steel ball moves to the sides of nut. Under joint action of preload, centrifugal force and normal force of internal and external channels, steel ball  $j$  revolves around screw axis, with its radius of  $d_0/2$  and angular velocity of  $\omega_{mj}$ :

$$F_{cj} = \frac{\pi}{12} \rho d^3 d_0 \omega^2 \frac{\omega_{mj}}{\omega} \tag{4}$$

where  $d$  is the diameter of steel ball and  $\rho$  is mass density of steel ball.

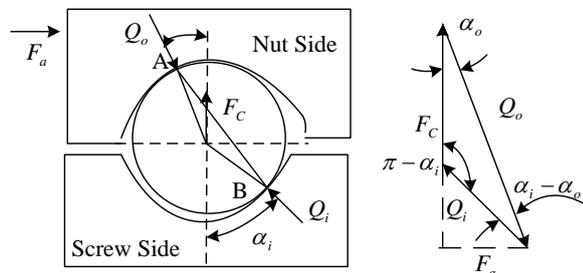


FIGURE 2 Loads acting on a ball at low-speed

At high-speed state, centrifugal force  $F_c$  is divided into  $F_N$  and  $F_p$  along the normal direction of lateral point of contact of nut and tangential direction of race. Under action of force  $F_N$ , steel ball extrudes the nut,

resulting in the increase of lateral contact force of nut, decrease of lateral contact force of screw; under action of force  $F_p$ , steel ball moves upward, as shown in Figure 3.

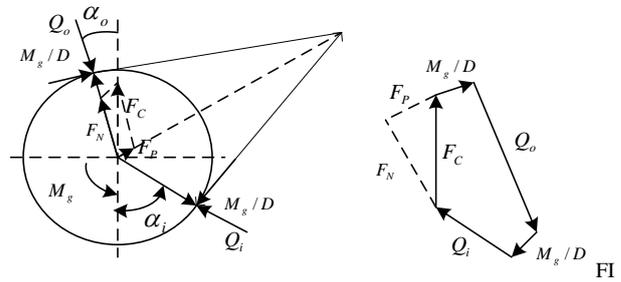


FIGURE 3 Loads acting on a ball at high-speed

At high-speed state, gyroscopic moment of steel ball is:

$$M_{gj} = J \omega^2 \frac{\omega_{bj}}{\omega} \frac{\omega_{mj}}{\omega} \sin \beta_j \tag{5}$$

$$\beta_j = \arctan \frac{\sin \alpha_{oj}}{\gamma + \cos \alpha_{oj}} \tag{6}$$

Expression for the relationship between angular velocity of driving nut  $\omega$  and axial stiffness of driving nut components  $k_x$  is:

$$k_{nut} = \frac{k_o k_i \cos \alpha_o \cos \alpha_i}{k_o \cos \alpha_o + k_i \cos \alpha_i} \tag{7}$$

When the feeding system is in high-speed rotation, axial stiffness of driving nut components will decrease gradually with increase of angular velocity, i.e. stiffness softening phenomenon. The reason for this phenomenon is that the centrifugal force leads to change in deformation of lateral raceway of screw of driving nut and contact area of lateral raceway of nut, resulting in the change of axial contact stiffness of driving nut.

### 3 The establishment of the dynamic model

#### 3.1 ELASTIC MODEL STRUCTURE

In order to analysis the impact on dynamic characteristics of the system from elastic properties of ball screw, stiffness model is used for modeling of ball screw. The model of elastic structure system of nut-driven ball screw pair consisting of concentrated mass (inertia) and spring is shown in Figure 4.

In Figure 4, stiffness parameters of main transmission parts, synthetic axial stiffness  $k_a$  of ball screw and driving nut components, torsional stiffness of ball screw  $k_g$ , and axial stiffness between driving nut and working platform  $k_n$ . Main moments of inertia and quality parameters: moment of inertia of driving nut  $J_b$ , mass of driving nut

$m_n$ , and mass of working platform  $m_t$ . In consideration of synthetic torsional stiffness conditions, there will be certain angle difference between angular misalignment of driving nut  $\theta_m$  input by the motor and angular misalignment of nut  $\theta_n$  output by the motor. Under the action of axial force, screw and drive nut assembly will generate a certain amount of axial elongation; in consideration of synthetic axial stiffness of screw and driving nut components, axial displacement of nut of  $(x_n - p\theta_n)$  is resulted, where  $p$  is the lead of screw.

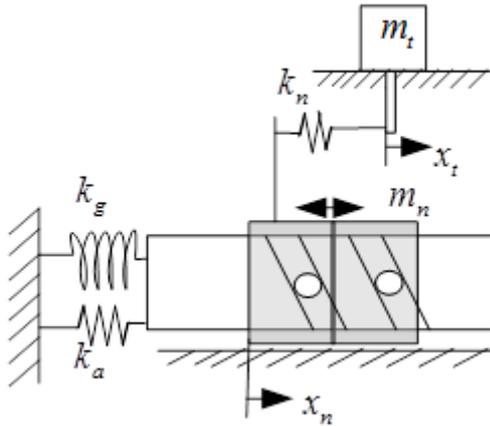


FIGURE 4 Modelling of the preload adjustable feed drive system with a lumped parameter system

Spring stiffness synthesis method can be used to work out axial stiffness of screw and synthetic axial stiffness of driving nut components:

$$k_a = \left( \frac{1}{k_{screw}} + \frac{1}{k_{nut}} \right)^{-1} \quad (8)$$

When determining the freedom degree of motion of moving parts, lagrangian energy method can be used for the construction of model for freedom outputs. First, based on speeds of transmission parts, the total kinetic energy of all moving parts can be obtained, which is:

$$T = \frac{1}{2} m_t \dot{x}_t^2 + \frac{1}{2} m_n \dot{x}_n^2 + \frac{1}{2} J_b \left( \frac{\dot{\theta}_m + \dot{\theta}_n}{2} \right)^2 \quad (9)$$

The total potential energy of the system can be obtained simultaneously according to elastic deformation of the system, which is

$$V = \frac{1}{2} k_n (x_t - x_n)^2 + \frac{1}{2} k_a (x_n - p\theta_n)^2 + \frac{1}{2} k_g (\theta_m - \theta_n)^2 \quad (10)$$

Taking  $L = T - V$ , lagrangian function about generalized coordinate and generalized force can be obtained, which is:

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = Q_i \quad (11)$$

Taking freedoms of motion  $\theta_m$ ,  $\theta_n$ ,  $x_t$  and  $x_n$  as generalized coordinates  $q_i$ ,  $i=1, \dots, 4$ , torque moment input  $T_m$  as generalized force, namely generalized coordinate matrix  $q = (x_t \ x_n \ \theta_m \ \theta_n)$ :

$$Q = (F_x \ 0 \ T_m \ 0)^T \quad (12)$$

Make the lagrange equation represented in equation (11) further into the matrix form:

$$M\ddot{q} + Kq = Q \quad (13)$$

As preload, working platform position and workpiece quality vary with the time during the processing, stiffness matrix and overall mass matrix of the system also vary with the time, which will lead to the change in natural frequency and modality in the dynamic system with the time.

### 3.2 BASIC PARAMETERS

This text cites double nut-driven ball screw pair as an example to construct the dynamic model for double nut-driven ball screw pair with adjustable preload. Table 1 lists geometric and physical parameters of the feeding system.

TABLE 1 List of the parameters used in the present analyses

Parameter	Unit	Value
nominal diameter	d <sub>0</sub> /mm	41.4
screw length	L/mm	3000
helix angle	β/(°)	8.74
contact angle	α/(°)	45
ball's diameter	d <sub>0</sub> /mm	6.35
worktable mass	m <sub>t</sub> /kg	20.0
axial load	F <sub>axi</sub> /N	500.0

## 4 Results and discussion

Immediate integration is one method to calculate the structural dynamic equation, and the commonly used method is Newmark method. Natural frequency and frequency response function of the system can be obtained via equation (14) by the use of Newmark integration method based on known conditions.

### 4.1 THE COMPARATIVE ANALYSIS OF NATURE FREQUENCY IN THE STATIC AND OPERATING STATE

Take the feeding system shown in Table 1 for an example, select stationary state ( $\omega = 0$  rad/s), low-speed state ( $\omega = 200$  rad/s and refer to Figure 4, no stiffness softening significantly occurs to the feed system) and high-speed state ( $\omega = 3000$  rad/s, and at this time, stiffness softening occurs to the feeding system), to conduct dynamic analysis of the feeding system, and conduct comparative analysis on change in dynamic characteristics of the feeding system at such three states.

TABLE 2 The three kinds of operating state natural frequency contrast

Order number	Natural frequency /Hz		
	Static state	Low speeds (200rad/s)	High speeds (3000rad/s)
1 Order	208.7	206.4	157.2
2 Order	429.8	425.7	388.5
3 Order	823.9	815.6	764.3

It can be seen from Table 2 that natural frequency of spindle of the system at the low-speed state is basically the same as that at the stationary state, while the natural frequency at the high-speed state is very much different from that at the stationary state. It can be seen that for the spindle system, high speed induced bearing stiffness softening is an important factor that affects dynamic characteristics.

#### 4.2 PRELOAD ON THE INFLUENCE OF THE DRIVE SYSTEM

Figure 5 shows the frequency response function curve of working platform and axial acceleration of driving nut when the system is at  $F_{axis} = 500.0N$  and preload is 40%.

Figure 6 shows frequency response function curves of working platform and axial acceleration of driving nut when preload is respectively 40% and 35%. It can be observed that a serious resonance phenomenon occurs at  $\omega_a = 157.2Hz$ ; this resonance characteristic is axial vibration of working platform due to synthesis of axial action of the parts, and the frequency value at this point is lower, therefore having a great impact on the system's processing accuracy.

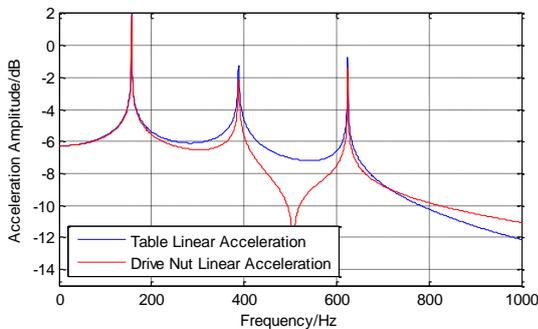


FIGURE 5 Frequency responses of the ball screw acceleration and working table acceleration

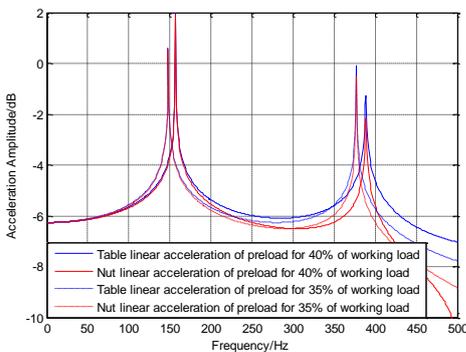


FIGURE 6 The resonant frequency shifts as the preload of the ball screw varies

#### 4.3 STRUCTURE PARAMETERS ON THE INFLUENCE OF THE DRIVE SYSTEM

Figure 7 shows the impact of screw length, screw diameter, operating position of nut and other factors on the first-order natural frequency of the system.

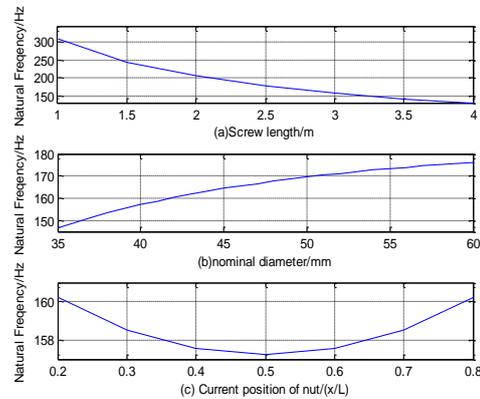


FIGURE 7 The ball screw design parameters of the system's first natural frequency sensitivity analysis

This figure shows that with the increase in ball screw length, the first-order natural frequency of transmission system reduces; the first-order natural frequency of transmission system decreases; the bigger the screw diameter, the larger the first-order natural frequency, so increase of screw diameter is an effective way to improve the feed system's dynamic performance.

#### 4.4 MOVING PARTS QUALITY CHANGE ON THE INFLUENCE OF THE DRIVE SYSTEM

Figure 8 shows the impact of frequency response function curve of mass variation of working platform vs. axial acceleration output when there is no workpiece on the working platform and the working platform is in the middle of screw. It shows that the mass of working platform has a great impact on the system's third-order natural frequency, but has little impact on the first-order and second-order natural frequencies.

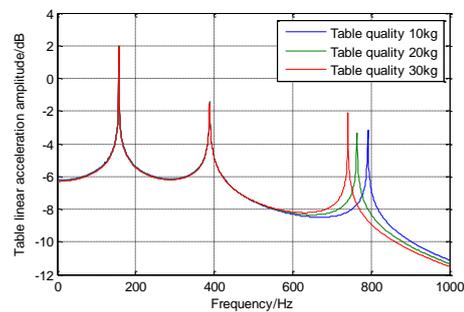


FIGURE 8 Effect of considering working table mass changes

Figure 9 and Figure 10 show the change in natural frequency of the system at the axial movement during the processing. Given that the quality of workpiece blank is 80kg, the mass of part after processing is 60kg. The natural frequency of each order varies linearly with change in mass of workpiece, but in general, although

change in mass of workpiece has an impact on natural frequency of each order, which is less compared to that brought by the position of working platform.

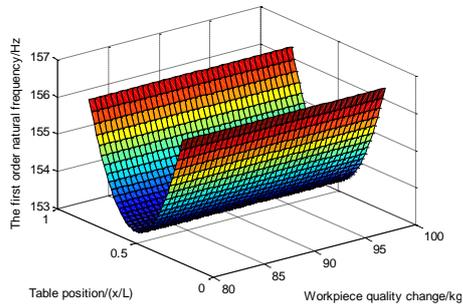


FIGURE 9 The quality and position to the first natural frequency influence

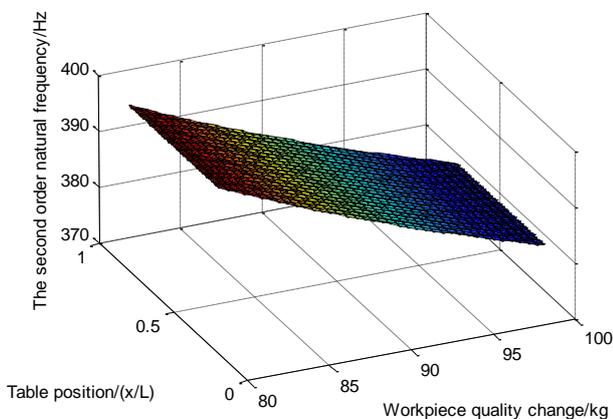


FIGURE 10 The quality and position to the second natural frequency influence

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## 5 Conclusions

This text uses concentrated mass method and lagrange equation to construct the dynamic model for the transmission system with double nut driven ball screw pair under the action of preload. Take the high-speed nut-driven ball screw pair for an example, obtain natural frequency of the system and the corresponding vibration mode. The analysis results show that natural frequency of the system at the low-speed state is basically the same as that at the stationary state, while the natural frequency at the high-speed state is very much different from that at the stationary state. Natural frequency of the system is time-varying, and change in preload, design parameters and position of moving part during the processing, etc. has a remarkable impact on its natural frequency. The research results provide the important theoretical basis for development and manufacturing of such new feeding system as transmission system of nut-driven ball screw pair.