Analysis on road traffic accidents spatial distribution based on the multi-fractal theory

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Abstract

After analysing the characteristics on spatial distribution of road traffic accidents in some areas in China, this paper took road traffic accidents of some provinces/cities in China as an example and thought those provinces/cities as cells. Then the fractal spectrum of road traffic accidents spatial distribution in two-dimensional space and $\ln\varepsilon - \ln\chi_q(\varepsilon)$ curve was obtained by MATLAB programming based on multi-fractal theory. Because of the preferable linear relation $\ln\chi_q(\varepsilon)$ between and $\ln\varepsilon$, the conclusion of which road traffic accidents spatial distribution satisfies power-law form and accord with multi-fractal distribution was obtained. By calculating the relation of related parameters, this paper analysed the characteristics of road traffic accidents spatial distribution further.

Key words: Road traffic accidents, spatial distribution, multi-fractal spectrum, deaths, injuries

1 Introduction

The fractal theory that combine traditional determinate theory with Stochastic theory is one of subjects established and developed in the late 19th century. It prompts people to know the complex natural phenomena more profoundly and has become one of important branches on scientific research about nonlinear (CHENG Q et al., 1995; Harte D., 2001; SUN Hongjun et al., 2005).

The fractal theory applies to establish model on natural phenomena and to forecast, but it is difficult to use fractal geometry in real world. Because genuine fractal is not nonexistent in nature, accurate fractal in mathematics is changed as approximative fractal in nature for applying (CHENG Q., 1999; EVERTSZ C J G et al., 1992).For example, in physical phenomenon, Brownian Movement is persuasive on fractal model (EVERTSZ C J G et al., 1992). So far, the fractal theory has been applied to many fields, including chemistry, life science, astronomy, geography, geology, economics and biology etc. other than physics (Kantelhardt J W et al., 2006; Godano C et al., 1997; MOLCHAN G et al., 2007; Kantelhardt J W et al., 2002). Especially, the fractal science has significant help in many researches of the practical problem, such as earthquake analysis, analysis of the characteristics of rivers and lakes and stock trend analysis etc (LIU Yan et al., 2007; FU Qiang et al., 2010; NIU Zhi-guang et al., 2010; TANG Lizhong et al., 2010; LI Min et al., 2011).

The fractal theory is applied not only in natural science, but also in social science. It have been proved by practical studies that the stock market and changes in profits are related to the fractal, so economists researched the complex economic phenomenon by the fractal theory (YU Jian-ling et al.,2006; YANG Ni et al., 2008). Studies have shown that the road net in traffic system has fractal features also.

In recent years, many researchers applied the fractal theory to study traffic system in China. LIU-Miaolong in Tongji University taken the urban traffic network in Shanghai as a case, the fractal dimension of the traffic network in Shanghai and its districts have been measured and calculated so as to research on the spatial changes of the fractal features (LIU-Miaolong et al., 2003). XU Zhi-hai in Information Engineering University studied several fractal dimensions of characterizing the structure of spatial networks base on the factual geometry theory, and stressed on both the definition and the geographical meaning of similitude dimension, and illustrated its application on the research of traffic networks distribution character with an example. According to the result analysis, some conclusions are given (XU Zhi-hai et al., 2006). JIANG Kejin Southwest in Jiaotong University proposed the consistent evolution between urban morphology dimension and road network dimension and put forward the coordinate between gene conception and model for evaluating the matching relationship between them, finally taken Chengdu city as an example to analyse it's the evolution process of urban morphology and road network (JIANG Kejin et al.,

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2008). ZHANG Xiao-hong, YU Ren-de in Shandong University of Technology studied the multi-fractal spectrum of traffic accident time series in Shandong province from 2008 to 2009 by the partition function and obtained the monthly multi-fractal spectrum of time series based on the multi-fractal theory (ZHANG Xiao-hong, YU Ren-de et al., 2013). CHEN Peng in Southeast University demonstrated the feasibility to analyse traffic accident using fractal theory by power spectrum analysis, and extended outwards the fractal property of inner interval by self similarity and scale invariance of fractal, and on this basis proposed a fractal extrapolation algorithm to realize the prediction of traffic accidents (CHEN Peng et al., 2008). CHEN Zhiyu in Shanghai Maritime University used the fractal interpolation to handle discrete data aggregation of temporal series and evaluate the IFS iteration function system and its attractors, and predicted the quantity trend of marine traffic accidents in the coming years (CHEN Zhiyu et al., 2009).

By studying and analysing, road traffic accident time series has multi-fractal distribution characteristics. However, the law of road traffic accident reflects not only on time series distribution but also on spatial distribution. This paper applies the fractal theory to study the characteristics on spatial distribution of road traffic accidents.

2 Multi-fractal spectrum on two-dimensional space

2.1 DEFINITION OF BINARY FRACTAL SPECTRUM

X is assumed to represent multi-measure fractal limit set, then definition of binary fractal spectrum is given by

$$\begin{cases} d_S^{(u,v)}(X) = h_S^{(u,v)}(X) \times d(\mu), \\ d_R^{(u,v)}(X) = h_R^{(u,v)}(X) \times d(\mu), \end{cases} \tag{1}$$

where, $d(\mu)$ as the dimension of radical measure μ, often as n.

Especially, if μ, m is same measure on R^n, and $\mu = q$, $v = 1$, binary fractal spectrum is converted into one dimensional fractal spectrum as formula (2). It is Renyi dimension, information dimension as $q = 1$, capacity dimension as $q = 0$.

$$\begin{cases} h_{S/R}^{(q,1)}(X) = h_{S/R}^{(q)}(X), \\ d_{S/R}^{(q,1)}(X) = d_{S/R}^{(q)}(X). \end{cases} \tag{2}$$

Conventional fractal dimension include box counting dimension, Renyi dimension, information dimension, capacity dimension.

2.2 THE CALCULATION OF MULTI-FRACTAL SPECTRUM ON BINARY DIMENSION

$(x_i, y_i, z_i), i = 1, 2, ...n_t$ is denoted as distribution of a set of points on binary dimension, (x_i, y_i) as plane coordinates of points, z_i as measuring data, n_t as number of points.

Rectangular area including all points $S = \{(x, y) \mid \min(x_i) \le x \le \max(x_i), \min(y_i) \le y \le \max(y_i)\}$ is divided into a lot of small squares in size ε. Small squares are called unit. Sum of measuring data on i[th] unit is denoted as $\mu_i(\varepsilon)$. Formula (3) is defined.

$$\chi_q(\varepsilon) = \sum_i \mu_i^q(\varepsilon). \tag{3}$$

If $\mu_i(\varepsilon)$ meets multi-fractal condition, and relationship between formula (3) and size ε. Then formula (4) can be obtained for any q meeting $-\infty \le q \le \infty$.

$$\chi_q(\varepsilon) = k \times \varepsilon^{\tau(q)}, \tag{4}$$

where $\tau(q)$ as function of q, and q as integer only.

Formula (4) is converted into formula (5).

$$\ln \chi_q(\varepsilon) = \ln k \times \varepsilon^{\tau(q)}. \tag{5}$$

The linear relation with slope $\tau(q)$ between $\ln \varepsilon$ and $\ln \chi$ is indicated on two logarithm planes. In calculation, part of q is obtained to carry linear fitting by least square method. Then $\tau(q)$ is obtained. Finally multi-fractal spectrum $f(\alpha)$ is obtained by $\tau(q)$.

3 Analysis on road traffic accident spatial distribution based on the multi-fractal theory

As a whole, road traffic accident declines slowly by statistic data in China. For traditional analysis on road traffic accident, object of study is time series. As a result, the time distribution can be obtained.

3.1 SITUATION ANALYSIS ON ROAD TRAFFIC ACCIDENTS SPATIAL DISTRIBUTION

This paper selects road traffic accidents statistic data in some provinces/cities of China from 2009 to 2011 for analysing. Road traffic accidents, deaths, injuries spatial distribution are showed as Figure 1 – Figure 3 in some provinces/cities of China from 2009 to 2011.

The following conclusions are obtained according Figure1-Figure3.

(1) In China, the curve shape of road traffic accidents spatial distribution is almost similar every year.

(2) The difference is clear on road traffic accidents, deaths, injuries in every provinces/cities.

(3) It is subject to the level of economic development and location (such as eastern coast or western interior). For example, road traffic accidents, deaths, injuries in less developed district and western interior are less than in developed district and eastern coast.

Thus, road traffic accidents spatial distribution takes on the inhomogeneity of the characteristics in China. So, it is necessary to study further for grasping the characteristics of road traffic accidents spatial distribution.

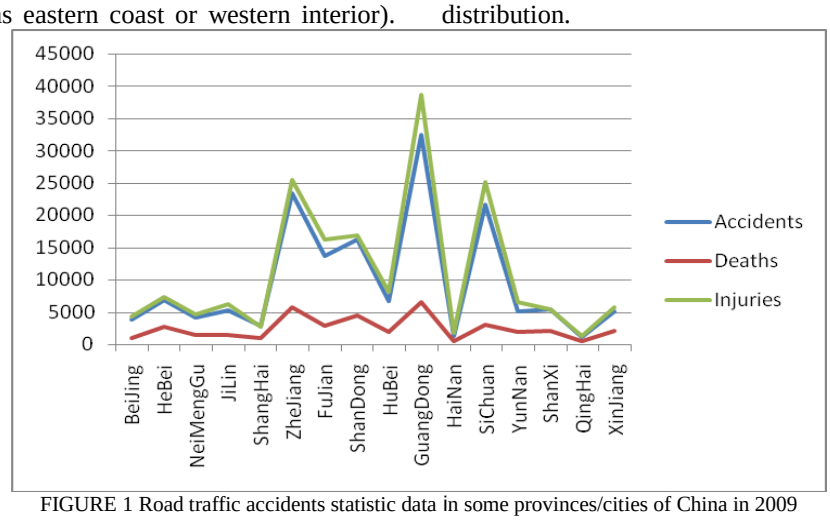

FIGURE 1 Road traffic accidents statistic data in some provinces/cities of China in 2009

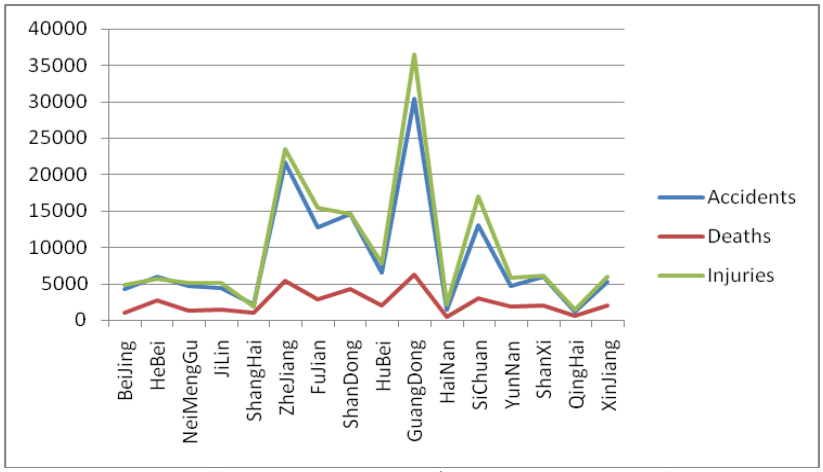

FIGURE 2 Road traffic accidents statistic data in some provinces/cities of China in2010

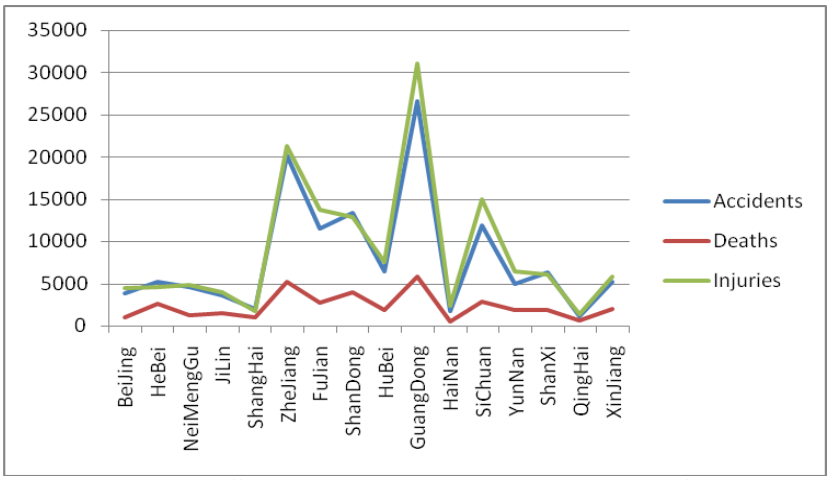

FIGURE 3 Road traffic accidents statistic data in some provinces/cities of China in 2011

3.2 ANALYSIS ON ROAD TRAFFIC ACCIDENTS SPATIAL DISTRIBUTION BASED ON MULTI-FRACTAL SPECTRUM

1) Choosing and analysing data

Road traffic accidents spatial distribution is almost the same in some provinces/cities of China from 2009 to 2011, so we choose road traffic accidents in some provinces/cities of China in 2011 and look provinces/cities are as cells for studying.

2) Multi-fractal spectrum on road traffic accidents spatial distribution

Road traffic accidents in some provinces/cities of China in 2011 is analysed as Table 1

Program is written as Figure 4, then $\lg\varepsilon-\lg\chi$ curve is obtained as Figure 5.

TABLE 1 Road traffic accidents in some provinces/cities of China in 2011

provinces/cities	Road traffic accidents	provinces/cities	Road traffic accidents	provinces/cities	Road traffic accidents
Beijing	3934	Anhui	14005	Sichuan	11860
Tianjin	2600	Fujian	11517	Guizhou	1566
Hebei	5197	Jiangxi	3354	Yunnan	5022
Shanxi	6239	Shangdong	13375	Xizang	943
Neimenggu	4591	Henan	6877	Shanxi	6362
Liaoning	6446	Hubei	6490	Gansu	3027
Jilin	3639	Hunan	8118	Qinghai	1163
Heilongjiang	3435	Guangdong	26586	Ningxia	1829
Shanghai	2085	Guangxi	4290	Xinjiang	5182
Jiangsu	13436	Hainan	1737		
Zhejiang	20178	Chongqing	5729		

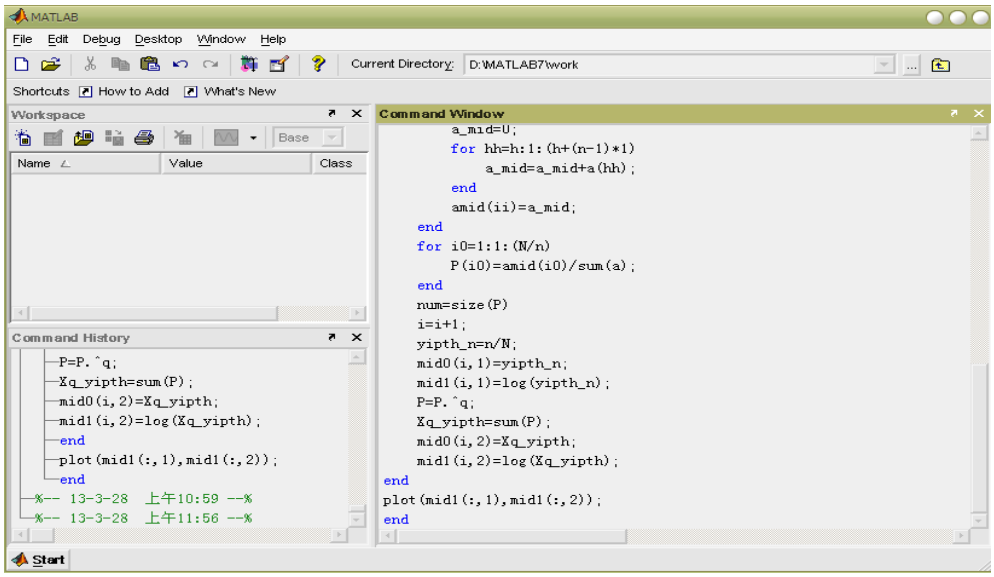

FIGURE 4 Matlab program analysis

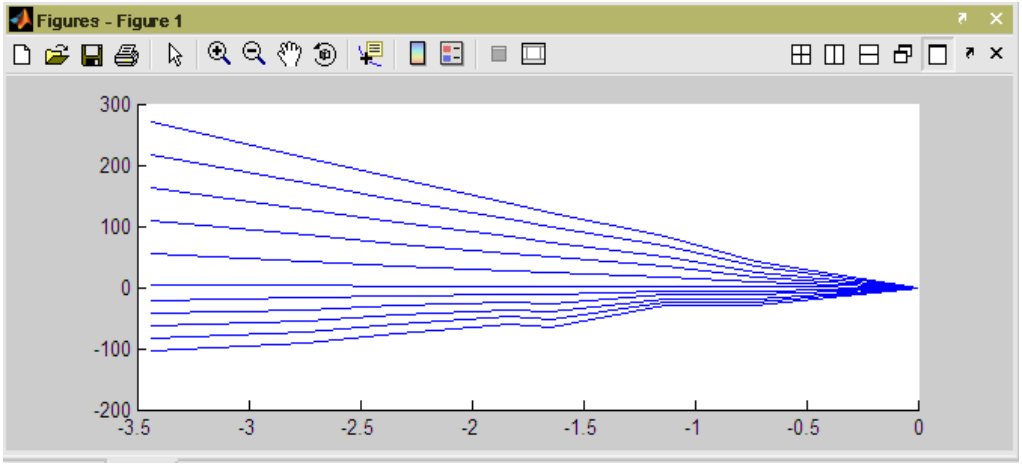

FIGURE 5 $\ln\varepsilon-\ln\chi_q(\varepsilon)$ curve

It is known that there is better linear relation between $\ln \chi_q(\varepsilon)$ and $\ln \varepsilon$ by $\ln \varepsilon - \ln \chi_q(\varepsilon)$ curve in Figure 5.

4 Conclusion

4.1 RELATIVE PARAMETER RELATION

1) $q-\tau(q)$ relation

Road traffic accidents spatial distribution has characteristic of multi-fractal distribution, so first $\tau(q)$, slope of $\ln \varepsilon - \ln \chi_q(\varepsilon)$ curve, must be obtained, then $q-\tau(q)$ relation is confirmed as Figure6.

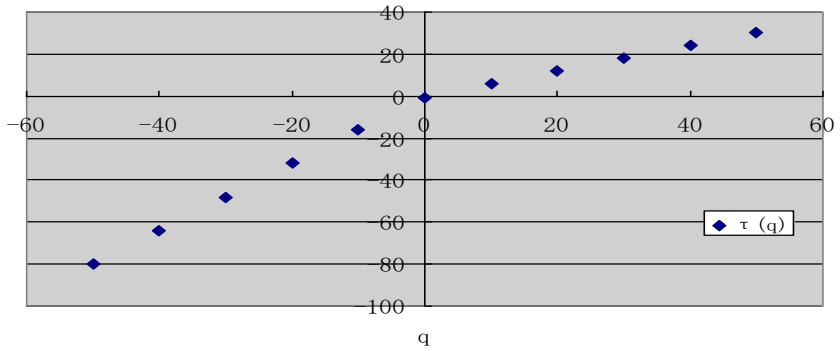

FIGURE 6 $q-\tau(q)$ relation

2) $q-\alpha(q)$ relation

$\alpha(q)$ is obtained by the differential of the $\tau(q)$, and then $q-\alpha(q)$ relation is obtained as Figure7.

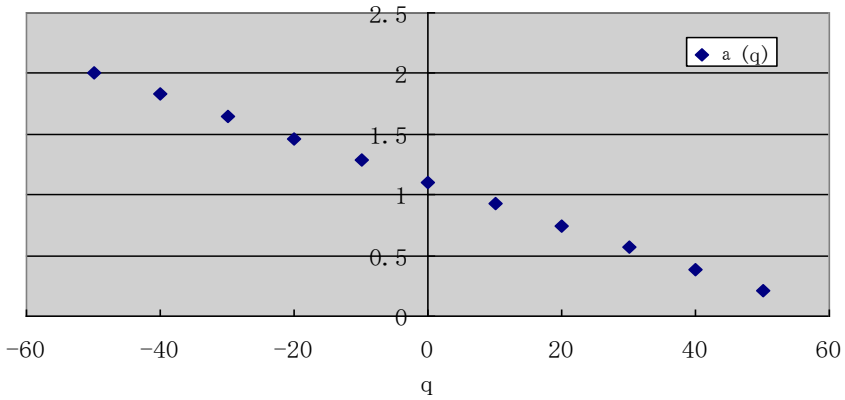

FIGURE 7 $q-\alpha(q)$ relation

3) $f(\alpha)-\alpha(q)$ relation

$f(\alpha)$ is obtained by $\alpha(q)$, and then is obtained $f(\alpha)-\alpha(q)$ relation is confirmed as Figure 8.

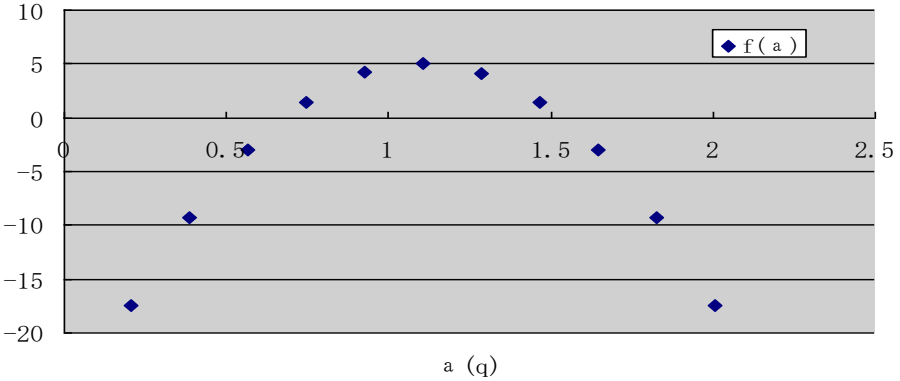

FIGURE 8 $f(\alpha)-\alpha(q)$ relation

4.2 CONCLUSION

By analysing Figure 6 – Figure 8, conclusion follows: $\tau(q)$ is an increasing function of q, $\alpha(q)$ as derivative of $\tau(q)$ decreases as q increases, $f(\alpha)$ is convex function of $\alpha(q)$.

It is known $f(\alpha)-\alpha(q)$ curve has broad and relaxed characteristic by Figure 8. The size of $\Delta\alpha=\alpha_{\max}-\alpha_{\min}$ indicates the size of strangeness on spatial distribution. Because of $\Delta\alpha=\alpha_{\max}-\alpha_{\min}=2.007-0.207=1.8$, the strangeness of road traffic accidents spatial distribution is greater in example in this paper.

In fact, economy development level and geographical location are different, there is obvious difference on road traffic accidents among different princes/cities in China. In general, road traffic accidents in undeveloped and inland provinces/cities are relatively less than in developed and coastal provinces/cities. So studying result based on multi-fractal theory accords with fact in China. It follows that road traffic accidents special distribution has characteristic of continuous and multi-fractal distribution.

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