UAV trajectory optimization generation based on Pythagorean hodograph curve

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Abstract

The study of autonomous planning of UAVs (Unmanned Aerial Vehicles) flyable on-line path to adapt unstructured environment and improve manoeuvring warfare capability has an important practical significance. A path planning algorithm on-line of UAV based on Pythagorean Hodograph (PH) curve is put forward, which can consider the kinematics and dynamic constraints. The effect of the key parameters on the trajectory generation are analysed, and the appropriate value range that satisfy the constrains are given. To overcome the blindness iteration, the method of improved estimation of distribution algorithm (EDA) is used to trajectory optimization. In the estimation of distribution algorithm, the probability selection mechanism of global elite individual based on interval selection is raised, which improve the speed and precision of the path generation. When the obstacle is detected by the UAV, only if the position and the direction of the interrupt point is given, the trajectory can be replaned online. Simulation results for UAV trajectory optimization generation prove the validity and practicability of the algorithm.

Keywords: unmanned aerial vehicle, PH curve, estimation of distribution algorithm, trajectory generation, backstepping

1 Introduction

Unmanned Aerial vehicles (UAVs) hold good promise for autonomously carrying out various operations in unstructured environment. However, autonomous trajectory planning is still among the most difficult and important problems for the UAVs should allow more flexibility and thus more complex applications, such as avoid collision with moving obstacles. The literature that deals with this problem is rapidly growing.

Some planning algorithms, such as rapidly-exploring random trees [1], genetic algorithm [2] and A* algorithm [3], divide the object region or explore large areas, and find the collision avoidance path points, then the path points are smoothed to flyable path, where vehicles on a team may be regarded as obstacles, making cooperation almost an inevitable approach for the problem.

Some scholars put forward the method of applying curve to plan the whole path directly, such as Shanmugavel [4] put forward that using Dubins curve to plan multiple UAVs' paths. Dubin path is composed by circular arcs and straight line, and from the perspective of the theory of differential geometry, in order to get a smooth flight trajectory, the first two derivative of the path is existed at least, that is, the curvature of the curve is continuous, but Dubin path is C_1 type curve, and is not the C_2 type curve. The reference [5] gives the method of using the Clothoids curve to get path that the curvature is continuous, the curvature linearly change along the curve, but the path length is not easy to generate closed solution. Jolly [6] applies the Bezier curve to the path planning of mobile robot; the reference [7] uses the seven order Bezier curve to plan the UAV's flight path, considering constraints of system performance such as the curvature. At present, the Pythagorean Hodograph (PH) curve is widely used in aircraft path planning online [8-10], the planning path is C_2 type path that has continuous curvature, and has a lot of advantages compared to other curve. The overall curvature is small. The length, curvature and bending energy of the curve can be calculated in closed form. At the same time, the length and curvature of PH trajectory is easy to coordinate.

The references [8-10] give the method of generating PH curve path, and consider the kinematics constraint and safety constraint. When the path doesn't meet the requirements of curvature constraints, increase the tangent vectors' length of the start and the end control points of the PH curve by iteration, but this article doesn't study the optimization parameter' selection for the coordination of the curvature and the path length, and the iteration is time consuming.

In this paper, the effect of the key parameters on the trajectory generation are analysed, and the appropriate value range that satisfy the constrains are given. To overcome the blindness iteration, the method of improved estimation of distribution algorithm (EDA) is used to trajectory optimization. The concept of Estimation of Distribution Algorithm (EDA) is first proposed in 1996 and develops rapidly in about 2000 [11], which has become the cutting-edge research in the field of evolutionary computation. EDA presents a new evolutionary pattern. In traditional genetic algorithm,

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with the population representing a group of candidate solutions for optimization problems, each individual of the population has its own adaptive value, and then do the genetic manipulation of simulated natural evolution, such as choose, crossover and mutation, repeating the manipulation until the problem is solved. However, in EDA, there is no traditional genetic operation such as crossover and mutation, but the learning and sampling of the probabilistic model. EDA describe the distribution of the candidate solutions in space through a probability model, uses statistical learning tools from the perspective of the macro group to establish a probabilistic model to describe the distribution of solutions, and then take a random sampling of the probabilistic model to generate new populations, repeat it and achieve the optimization of the population. Traditional evolutionary algorithm achieves the population evolution based on the genetic manipulation for the every individual in the population (crossover and mutation, etc.) and establishes the mathematical model from the "micro" level. But EDAs directly describes the evolution trend of the population through the establishment of a mathematical model based on the whole population from the "macro" level in biological evolution. To satisfy the value range of the PH curve and select the optimum parameters, the probability selection mechanism of global elite individual based on interval selection is raised, which improve the speed and precision of the path generation.

2 PH Trajectory Optimization Generation

2.1 PROBLEM PRESENTATION

Let the starting pose of the i^{th} UAV is $pose_{si}(x_{si}, y_{si}, \theta_{si})$ and the final pose of the same UAV is $pose_{fi}(x_{fi}, y_{fi}, \theta_{fi})$, then the path of the UAV is defined as the parametric curve $r_i(t) = [x(t), y(t)]$ with curvature κ_i connecting the initial and final pose: $Pose_{si}(x_{si}, y_{si}, \theta_{si}) \xrightarrow{r_i(t)} Pose_{fi}(x_{fi}, y_{fi}, \theta_{fi})$.

In this case Pythagorean Hodograph curve in parametric form subjected to the following constraints:

1) $|\kappa_i(t)| \le \kappa_{max} 10$, where κ_{max} is the maximum curvature attainable by the UAVs.

2) The vehicles must keep safe distance to avoid inter collision while tracing their corresponding trajectories. This can be written in mathematical form as follow: $R_{si} \cap R_{sj} = \varphi$.

That is, the intersection of the safety circles of radius R_{si}, R_{sj} corresponding to *i*-th and *j*-th UAVs must be empty.

3) The vehicles must keep safe distance to avoid collisions with known obstacles while tracing their corresponding trajectories. This can be written in mathematical form as follow: $d(obstacle, UAV) \ge R_{obstacle} + R_s$, where $d(obstacle, UAV)$ is the distance between the centre of the circle enclosing the obstacle and the centre of the safety circle of the UAV. $R_{obstacle}$ is the radius of the circle enclosing the square obstacle.

2.2 USING PH TRAJECTORY GENERATION AND ITS ANALYSIS

Reference [10] show that the lowest order of the PH path that has a point of inflection is the fifth, called the quantic PH. The presence of an inflection point allows the path to have more flexibility so that the path can easily be manipulated. Hence, the quantic PH curve is used for path planning, in which six control points are needed.

Supposing the path is $s(t) = \int_{t_1}^{t_2} |r'(t)| dt = \int_{t_1}^{t_2} \sqrt{x'(t)^2 + y'(t)^2}\, dt$.

Selecting appropriate $\sigma(t)$ to satisfy $\sigma^2(t) = x'^2(t) + y'^2(t)$.

Then the length of the path is

$$s(t) = \int_{t_0}^{t_1} |\sigma(t)| dt \text{ .} \tag{1}$$

We can produce PH curve by selecting appropriate $u(t), v(t), w(t)$ to construct $\dot{x}(t), \dot{y}(t)$ and satisfy Equation (1). Therefore

$$x'(t) = w(t)[u^2(t) - v^2(t)],\ y'(t) = 2w(t)u(t)v(t),$$

$$\sqrt{x'^2(t) + y'^2(t)} = \sqrt{\left[w(t)\left\{u^2(t) + v^2(t)\right\}\right]^2}$$
$$\sigma(t) = w(t)\left[u^2(t) + v^2(t)\right] ,$$

where $w(t) = 1$. The Bernstein form of $u(t), v(t)$ is

$$u(t) = \sum_{k=0}^{2} u_k \binom{2}{k} t^k (1-t)^{2-k}, t \in [0\ 1] ,$$
$$u(t) = u_0(1-t)^2 + 2u_1(1-t)t + u_2 t^2 10.$$

$$v(t) = \sum_{k=0}^{2} v_k \binom{2}{k} t^k (1-t)^{2-k}, t \in [0\ 1],$$
$$v(t) = v_0(1-t)^2 + 2v_1(1-t)t + v_2 t^2.$$

Considering the numeric stability, PH curve can be give by Bézier curve. The curve is

$$r(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \sum_{k=0}^{5} P_k \binom{5}{k} (1-t)^{5-k} t^k \text{ ,} \tag{2}$$

where $P_k = (x_k, y_k), k$=0,1,2,3,4,5 are control points, which have the following relation:

$$P_1 = P_0 + \frac{1}{5}(u_0^2 - v_0^2, 2u_0v_0) . \quad (3)$$

$$P_2 = P_1 + \frac{1}{5}(u_0u_1 - v_0v_1, u_0v_1 + u_1v_0) . \quad (4)$$

$$P_3 = P_2 + \frac{1}{5}(u_1^2 - v_1^2, 2u_1v_1) + \frac{1}{15}(u_0u_2 - v_0v_2, u_0v_2 + u_2v_0) . \quad (5)$$

$$P_4 = P_3 + \frac{1}{5}(u_1u_2 - v_1v_2, u_1v_2 + u_2v_1) . \quad (6)$$

$$P_5 = P_4 + \frac{1}{5}(u_2^2 - v_2^2, 2u_2v_2) . \quad (7)$$

Giving the initial and final tangent vector length $(\Delta x_s, \Delta y_s)$, $(\Delta x_f, \Delta y_f)$, P_1, P_4 can be determinate.

$$\boldsymbol{r}(0) = \boldsymbol{P}_0 = (\boldsymbol{x}_s, \boldsymbol{y}_s) , \quad (8)$$

$$\boldsymbol{r}'(0) = 5(\boldsymbol{P}_1 - \boldsymbol{P}_0) = [\Delta \boldsymbol{x}_s, \Delta \boldsymbol{y}_s] , \quad (9)$$

$$\boldsymbol{r}(1) = \boldsymbol{P}_5 = (\boldsymbol{x}_f, \boldsymbol{y}_f) , \quad (10)$$

$$\boldsymbol{r}'(1) = 5(\boldsymbol{P}_5 - \boldsymbol{P}_4) = [\Delta \boldsymbol{x}_f, \Delta \boldsymbol{y}_f] , \quad (11)$$

$$\boldsymbol{P}_1 = \boldsymbol{P}_0 + \frac{1}{5}[\Delta \boldsymbol{x}_s, \Delta \boldsymbol{y}_s] , \quad (12)$$

$$\boldsymbol{P}_4 = \boldsymbol{P}_5 + \frac{1}{5}[\Delta \boldsymbol{x}_f, \Delta \boldsymbol{y}_f] . \quad (13)$$

According to reference [10]:

$$\begin{bmatrix} \boldsymbol{u}_0 \\ \boldsymbol{v}_0 \end{bmatrix} = \pm \begin{bmatrix} \sqrt{\dfrac{\varepsilon_s + \Delta \boldsymbol{x}_s}{2}} \\ \boldsymbol{sign}(5\Delta \boldsymbol{y}_s)\sqrt{\dfrac{(\varepsilon_s - \Delta \boldsymbol{x}_s)}{2}} \end{bmatrix} , \quad (14)$$

$$\begin{bmatrix} \boldsymbol{u}_2 \\ \boldsymbol{v}_2 \end{bmatrix} = \pm \sqrt{\frac{5}{2}} \begin{bmatrix} \sqrt{\varepsilon_f + \Delta \boldsymbol{x}_f} \\ \boldsymbol{sign}(\Delta \boldsymbol{y}_f)\sqrt{\varepsilon_f - \Delta \boldsymbol{x}_f} \end{bmatrix} , \quad (15)$$

$$\begin{bmatrix} u_1 \\ v_1 \end{bmatrix} = -\frac{3}{4}\begin{bmatrix} u_0 + u_2 \\ v_0 + v_2 \end{bmatrix} \pm \sqrt{\frac{1}{2}} \begin{bmatrix} c + a \\ sign\ (b)\sqrt{c - a} \end{bmatrix} , \quad (16)$$

where $\varepsilon_s = \sqrt{\Delta \boldsymbol{x}_s^2 + \Delta \boldsymbol{y}_s^2}$, $\varepsilon_f = \sqrt{\Delta \boldsymbol{x}_f^2 + \Delta \boldsymbol{y}_f^2}$, ,

$$a = \frac{9}{16}(u_0^2 - v_0^2 + u_2^2 - v_2^2) + \frac{5}{8}(u_0u_2 - v_0v_2) + \frac{15}{2}(x_4 - x_1),$$

$$b = \frac{9}{8}(u_0v_0 - u_2v_2) + \frac{5}{8}(u_0v_2 - u_2v_0) + \frac{15}{2}(y_4 - y_1),$$

$$c = \sqrt{a^2 + b^2}$$

Therefore P_2, P_3 can be determinate.

From the generation of the PH trajectory, it can be seen that if the start and the final pose are known, the tangent length ε_s, ε_f of the start and the end points can determine the UAV trajectory. The trajectory has continuous curvature. From equation (3) to (16) it can be seen that after the pose of the initial control point $pose_s(x_s, y_s, \theta_s)$ and the final control point $pose_f(x_f, y_f, \theta_f)$ and the tangent length ε_s, ε_f are given, four paths will be generated. Select the path with minimum energy, and justify whether the path is satisfy the curvature constrain. If not, justify the tangent vector length ε_s, ε_f . The bending energy is calculated by the following expression:

$$E = \int \kappa^2(t)\sigma(t)dt10 \quad (17)$$

The curvature of the UAV path is:

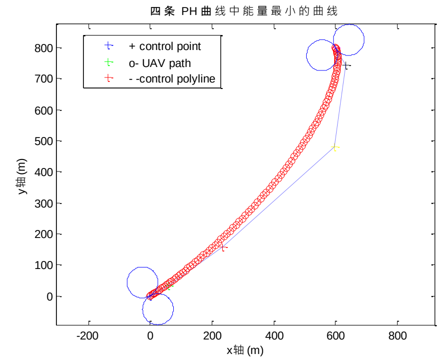
$$\kappa(t) = \frac{|\dot{r}(t) \times \ddot{r}(t)|}{|\dot{r}(t)|^3} = \frac{2[u(t)\dot{v}(t) - v(t)\dot{u}(t)]}{w(t)[u(t)^2 + v(t)^2]^2} . \quad (18)$$

In references [8-10], the parameter ε_s, ε_f are selected by iteration, which is time consuming and the optimize parameters cannot be get. On the following, through analysing the relationship between tangent vector length ε_s,ε_f and the path performance under the different initial and the final position and the tangent line direction, the parameters value range are given.

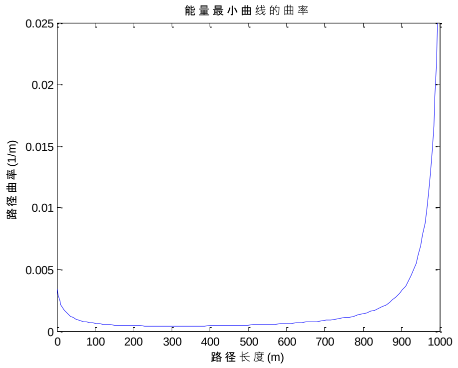
Suppose the initial and the final control points are: $(P_s, \theta_s) = (0, 0)$, $(P_f, \theta_s) = (600 , 800)$. The distance between the two points is: $|P_0P_5| = 1000$. Suppose the maximum curvature of the planning trajectory is 0.02.

TABLE 1 The effect of different tangent vector length and initial/final angle to the curvature

tangent vector length / initial/final angle (°)	$\frac{1}{15}d$	$\frac{1}{10}d$	$\frac{1}{5}d$	$\frac{1}{3}d$	$\frac{1}{2}d$	$\frac{2}{3}d$
0/120	0.0235	0.0118	0.0034	0.0038	0.0076	0.0157
30/120	0.0249	0.0128	0.0042	0.0056	0.0129	0.0273
60/120	0.0270	0.0143	0.0055	0.0074	0.0144	0.0215
90/120	0.0297	0.0161	0.0067	0.0079	0.0118	0.0141
120/120	0.0324	0.0180	0.0076	0.0075	0.0092	0.0099
150/120	0.0408	0.0221	0.0111	0.0113	0.0138	0.0154

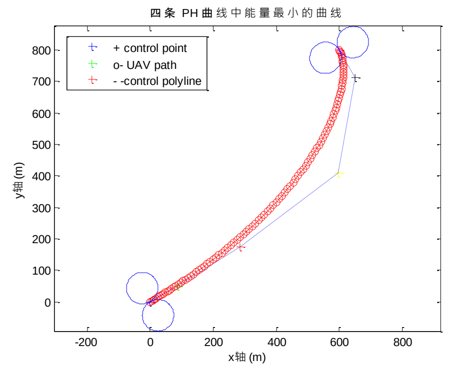


(a) The generated PH path

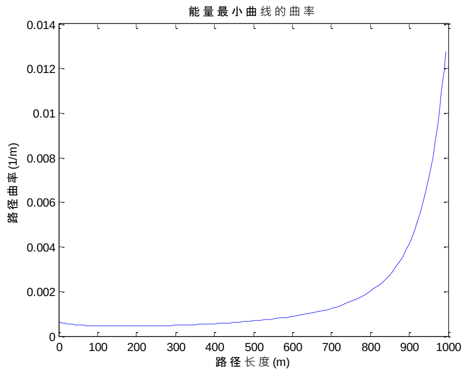


(b) The curvature of the PH path

FIGURE 1 PH path and its curvature when $\theta_{si} = \pi/6$, $\theta_{fi} = 2\pi/3$, $\varepsilon_0 = \varepsilon_1 = \frac{1}{15}d$

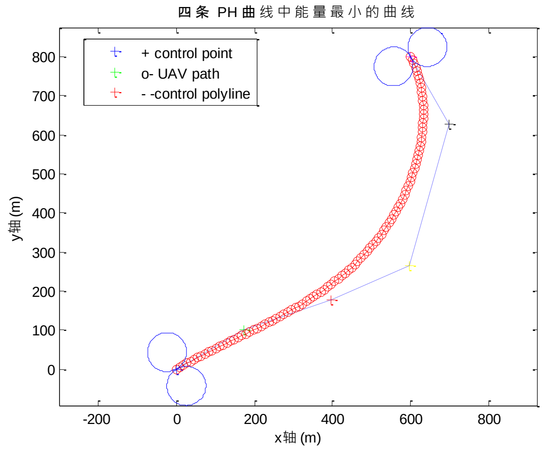


(a) The generated PH path

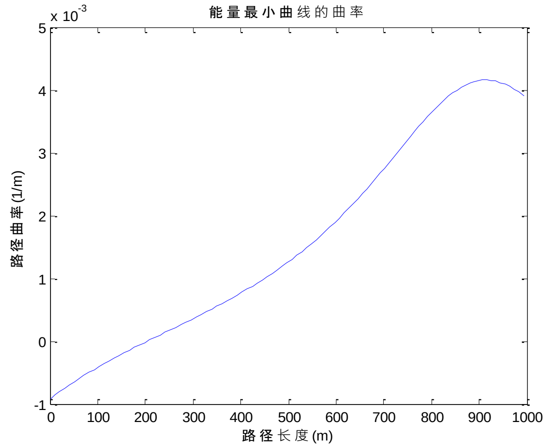


(b) The curvature of the PH path

FIGURE 2 PH path and its curvature when $\theta_{si} = \pi/6$, $\theta_{fi} = 2\pi/3$, $\varepsilon_0 = \varepsilon_1 = \frac{1}{10}d$

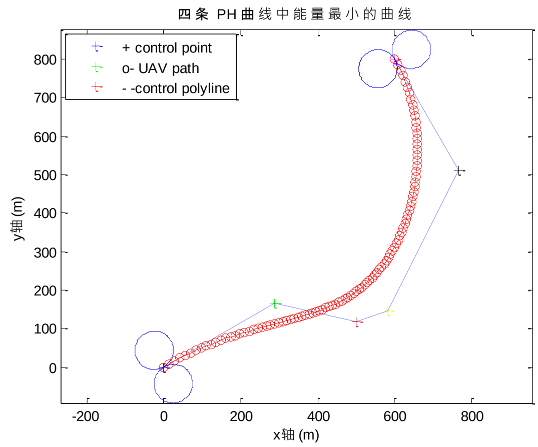


(a) The generated PH path



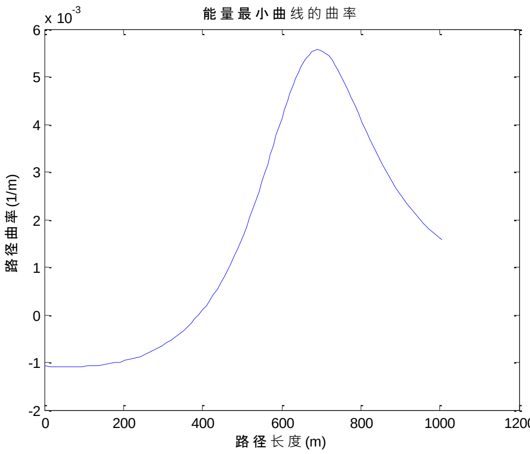
(b) The curvature of the PH path

FIGURE 3 PH path and its curvature when $\theta_{si} = \pi/6$, $\theta_{fi} = 2\pi/3$, $\varepsilon_0 = \varepsilon_1 = \frac{1}{5}d$



(a) The generated PH path

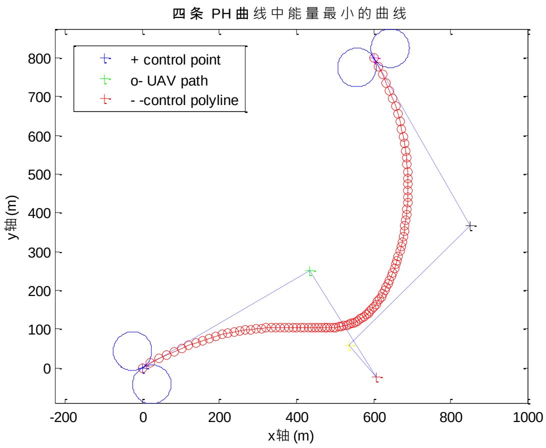
FIGURE 4 PH path and its curvature when $\theta_{si} = \pi/6$, $\theta_{fi} = 2\pi/3$, $\varepsilon_0 = \varepsilon_1 = \frac{1}{3}d$



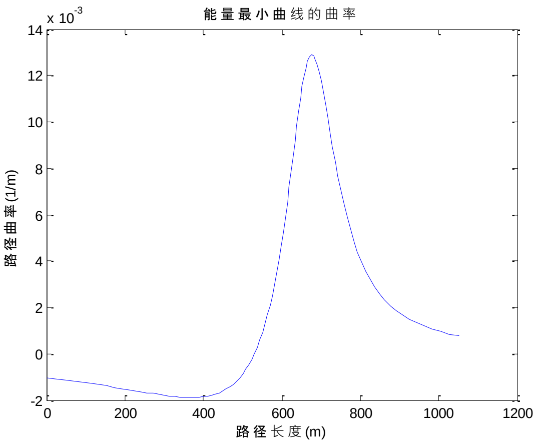
(b) The curvature of the PH path

FIGURE 4 PH path and its curvature when $\theta_{si} = \pi/6$, $\theta_{fi} = 2\pi/3$,

$$\varepsilon_0 = \varepsilon_1 = \frac{1}{3}d$$



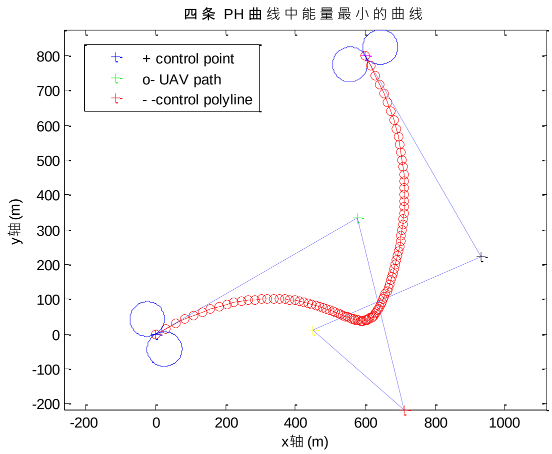
(a) The generated PH path



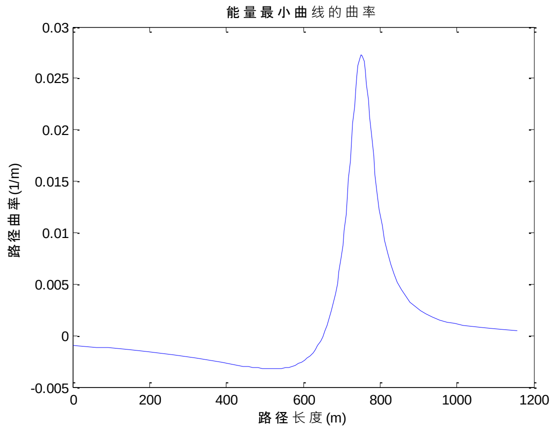
(b) The curvature of the PH path

FIGURE 5 PH path and its curvature when $\theta_{si} = \pi/6$, $\theta_{fi} = 2\pi/3$,

$$\varepsilon_0 = \varepsilon_1 = \frac{1}{2}d$$



(a) The generated PH path



(b) The curvature of the PH path

FIGURE 6 PH path and its curvature when $\theta_{si} = \pi/6$, $\theta_{fi} = 2\pi/3$,

$$\varepsilon_0 = \varepsilon_1 = \frac{2}{3}d$$

For convenience, set the tangent vector length $\varepsilon_0, \varepsilon_1$ is proportional to the distance d of the initial and the final points where $d = |P_0 P_5|$. Similarly, to get the affection of the initial and the final heading angle to the PH path curvature, set the heading angle at some step, and the simulation results are shown in table 1.

Suppose the initial/final angle is 30/120°, from Figures 1-5, the minimum bending energy PH curve with different tangent vector length and the corresponding curvature are shown.

It can be seen that when the tangent vector length is short, the curvature of the PH curve is very big and not easy to satisfy the constrains, which is shown in Figure 1 ($\varepsilon_0 = \varepsilon_1 = \frac{1}{15}d$). With the increase of the tangent vector length, such as $\varepsilon_0 = \varepsilon_1 = \frac{1}{5}d$ (in Figure 3), the curvature is small, which can satisfy the demand of the performance. When increase the tangent vector length continuously, such as $\varepsilon_0 = \varepsilon_1 = \frac{1}{3}d$ (in Figure 4), the peak appear in the curvature curve, accordingly, the bending energy and the length increase. From the above analysis, it shows that the tangent vector length $\varepsilon_s, \varepsilon_f$ should be selected appropriately in some interval. At the same time, with the different initial and final angle $\varepsilon_s, \varepsilon_f$ may be selected differently. From above it show that $\varepsilon_s, \varepsilon_f$ can be selected in the interval $\varepsilon \in \left[\frac{1}{15}d, \frac{2}{3}d\right]$.

Under the different combination of the initial and the final tangent vector length, the PH path maximum curvature change with the initial heading angle increase at the step $\pi/6$ is shown in Figure 7, where the final angle is $2\pi/3$. The initialization condition of the best PH curve can be found from Figure 7. At the same time, the heading angle that satisfy the curvature constrain can also be found from the different initial and the final tangent vector length.

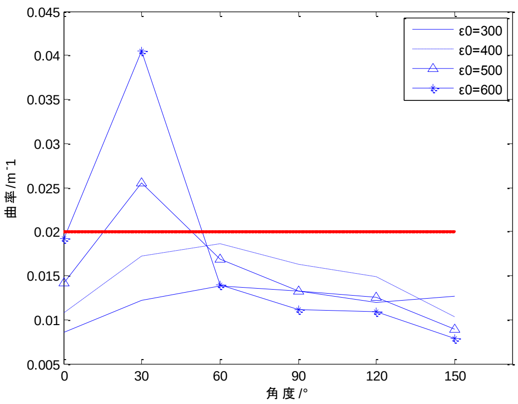


FIGURE 7 PH path maximum curvature change with the initial heading angle increase

2.3 PATH OPTIMIZATION GENERATION BASED ON ESTIMATION OF DISTRIBUTION ALGORITHM

In order to make full use of the performance of PH curve, and overcome the defects of blind iteration in the present literature, the method for generating and optimizing flight path based on distribution estimation algorithm is put forward.

EDAs use probabilistic modelling of promising solutions to estimate a distribution over the search space, which is then used to produce the next generation by sampling the search space according to the estimated distribution. An algorithmic framework of most EDAs can be described as follows:

Framework of EDA
Pop InitializePopulation( )
while Stopping criteria are not satisfied **do**
Popsel = Select(Pop)/*Selection*/
Prob =Estimate(Popsel) /*Estimation*/
Pop = Sample(Prob)/*Sampling*/
endwhile

An EDA starts with a solution population Pop and a solution distribution model Prob. The main loop consists of three principal stages. The first stage is to select the best individuals (according to some fitness criteria) from the population. These individuals are used in the second stage in which the solution distribution model Prob is updated or recreated. The third stage consists of sampling the updated solution distribution model to generate new solutions offspring.

Estimation of distribution algorithm is suitable especially for the situation of this article, which needs to find the feasible range of $\varepsilon_s, \varepsilon_f$ first, which meets the dynamic performance constraints and then optimize the parameters. On the following, the probability selection mechanism of global elite individual based on interval selection is introduced to the EDA to adapt the interval optimization of the PH trajectory planning. The improved EDA is applied to optimize parameters of the flight path, in order to complete the manoeuvre in high dynamic environment with the threats.

1) Performance Index

In order to get the shorter path and has strong manoeuvrability, considering both the length s and curvature κ, and the curvature of path reflects by bending energy E, so the performance index is defined as:

$$J = \min(\lambda_1 \times s + \lambda_2 \times E), \tag{19}$$

where the bending energy E is calculated by the equation (17), the length of path is calculated by the Equation (1).

2) Gene coding

The real values ε_s and ε_f are applied to coding the individual gene in estimation of distribution algorithm. Suppose the distance between initial point to final point is $|p_s p_f|$, the range of tangent vector length of the starting and ending points are set to $\varepsilon_s \in \left[\frac{1}{15}|p_s p_f|, \frac{2}{3}|p_s p_f|\right]$, $\varepsilon_f \in \left[\frac{1}{15}|p_s p_f|, \frac{2}{3}|p_s p_f|\right]$, which can satisfy the constraints of system dynamic performance, and doesn't make the length of path and bending energy too large.

Using this heuristic information, the number of iterations can be reduced when generate and optimize the trajectory.

3) Individuals Chaotic Initialization

In order to improve the population diversity, initialize the individuals using chaotic variables.

The Logistic mapping chaotic model is induced:

$$\boldsymbol{r}_{d+1} = \mu \boldsymbol{r}_d (1 - \boldsymbol{r}_d), \tag{20}$$

where $r_d \in (0,1)$, μ is the control parameter. When $\mu \in (3.56, 4.0)$, equation (20) is in a chaotic state. Take $\mu = 3.6$, the coverage of chaotic sequence can meet the requirements of the population diversity.

Mapping the chaotic variables to the range of the decision variables ($x_{j\_\min}, x_{j\_\max}$), the *j*-th component of the *i*-th individual X_{ij} can be get:

$$X_{ij} = x_{j\_\min} + (x_{j\_\max} - x_{j\_\min}) r_{d+1}, \tag{21}$$

where $j = 1, 2, \ldots, M$, M is the number of the variables, $i = 1, 2, \ldots, N$, N is the number of the individuals.

4) Establishment of the Probability Estimation Model and Generation of the New Population

Divide the range $[a_j, b_j]$ of all variables x_j into equal span $[a_{jk}, b_{jk}]$, where k is the kth search interval, $k = \{1, 2, \ldots, n\}$, n is the interval number of division. Select the elite individuals at the probability p_e . The next generation individuals' genes are generated according to the probability of the elite individual in each interval $[a_{jk}, b_{jk}]$. That is, suppose the probability of the elite individual in the k^th^ search interval of the j^th^ dimension search variable is p_{jk} (shown in Figure 8)

$$p_{jk} = \frac{\text{The number of elite individual contained in Kth intervalof locus } j}{N}. \tag{22}$$

When new individuals are generated, create random number p^j in (0, 1) for each gene. If $\sum_{k=1}^{s} p_{jk} < p^j \le \sum_{k=1}^{s+1} p_{jk}$ ($s = \{1, 2, \ldots, n-1\}$), select the values within the (s+1)th interval $[a_{js}, b_{js}]$ to generate the new genes. If $p^j \le p_{j1}$, select the values within interval $[a_{j1}, b_{j1}]$ to generate the new genes. Thus, the next generation population will be generated at a rather large probability in the interval with more excellent individual.

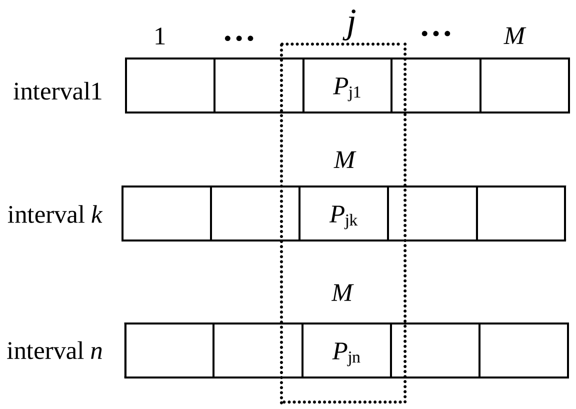


FIGURE 8 Probability value of the elite individual locus in every search interval

5) Individual Diversity Adjustment Based on Interval Information Entropy

a) Diversity calculation of each locus

The interval information entropy theory is used to define the population diversity. The *j*-th locus information entropy $H_j(N)$ is defined as

$$H_j(N) = -\sum_{k=1}^{n} p_{jk} \log p_{jk} \tag{23}$$

$H_j(N)$ is the overall information measure of the j^{th} locus. From the definition of information entropy, the greater of $H_j(N)$ the better of the locus diversity. Population information is abundant in the early evolutionary, which may achieve the maximum entropy $\log|n|$. With the increase of the evolution generation *t* and the improvement of the fitness, the population diversity will reduce gradually. When the individuals in the population achieve consistent, the diversity is zero.

b) Population diversity calculation and adjustment.

The threshold for diversity of each generation is defined as:

$$\varphi(t) = A \cdot (\log|n|) \cdot e^{-\frac{Bt}{G}}, \tag{24}$$

where $A \in (0,1)$ is regulating coefficient, $B > 0$ is accelerating factor, *t* is the current evolution generation, *G* is the total evolution generation.

Justify the population diversity in the process of evolution. If the diversity is below to the threshold given by equation (23), the chaos substitution needs to be adopted. The detail operations are:

i) *M* individuals are selected randomly from the population with probability P_α , where $P_\alpha \in (0,1)$.

ii) The chaos substitution ware adopted. Low-diversity gene loci j of X_i^t was chosen, a new gene was generated according to the Equation (20) after normalization, then chaos substitution was carried out with the probability p_c.

6) Vaccination Based on the System Information

In order to make full use of the system information, the immune evolutionary mechanism is added. Extract the vaccine at first, and according to the system information and the heuristic information, such as if initial boundary curvature does not meet the requirement of the curvature's constraints, the value of ε_S should be increased, and inoculate it with the probability P_u . Reserve the best individual of each generation.

7) Gauss Perturbation Optimization

In order to find the optimal individual, Gauss perturbation is adopted for the neighbourhood optimization of the best individual. The variable perturbation is carried out as follows:

$$x'_{b,j} = x_{b,j} + N(0, \sigma), \tag{25}$$

where $x_{b,j}$ is the j^{th} variable of the optimal individual, $x'_{b,j}$ is the individual values of variables after perturbation, $N(0,\sigma)$ is the amount of Gaussian distribution, σ is the variance, $N(0,\sigma)$ may be replaced by other random distribution. If the fitness of the individual $x'_{b,j}$ exceeds $x_{b,j}$, the perturbation is carried out. Retain the original optimal individual.

8) Performance analysis of the algorithm

a) The improved algorithm is convergent by probability 1, and the convergence speed is accelerated.

For the optimal reservation mechanism in the algorithm, although the other operators are added, the convergence of the algorithm is not affected. Based on the certification of literature [12], the convergence probability of this algorithm is 1. The global elite individual probability selection based on EDA can effectively improve the convergence speed. After the vaccination operation, the system information can be fully used to speed up the evolution towards the optimal solution.

b) The global optimal solution is appropriate to get.

The loci interval information entropy is used to judge the population diversity and it can more accurately reflect the information of the individual, and it can obtain the individual that satisfies the evolution rule by the adjustment of the threshold in the process of evolution, which is more meet the requirements of the evolution than using the fixed density individual choice. Chaos initialization and Gauss perturbation optimization make the search space increase and the probability of local optimum decrease.

9) Step of the Algorithm

a) Determine the range of the parameters and decoding them with real value. Divide the interval, using the chaotic initialization and perform individual vaccination with probability p_u.

b) Calculate the interval probability estimation model of elite individuals, and generate the new population. Calculate locus information entropy of each individual and adjust the low diversity genes using chaotic replace operation.

c) Using immune mechanics, perform individual vaccination with probability p_i.

d) For the individuals satisfying the dynamic constrains, calculate the fitness J and preserve the best individual X_b. Optimize the best individual by Gauss perturbation for the neighbourhood. If the best fitness keeps constant in g generations or arrives the max generation, then output the best individual and stop, else return to step b).

2.4 COLLISION AVOIDANCE PH TRAJECTORIES GENERATION

When UAV detects the unknown moving threat or obstacle and it need to be avoided, the interrupt point of the trajectory should be given. The UAV will replan the trajectory according to the velocity and the position of the obstacles. After the UAV velocity turn angle and the interrupt point coordinate are determined, the new trajectory can be generated between the obstacle detected point and the interrupt point. Then take the interrupt point as the initial point and the object or the next interrupt point as the final point, the next trajectory can also be generated. To keep the continuous of the trajectory, on the detect point and interrupt point, the direction of the tangent line keep unchanged. Through adjusting ε_s, ε_f of each trajectory, the flyable trajectory that satisfy the performance constrains can be given.

3 Simulation Results

1) The optimization PH trajectory generation based on EDA

Supposing UAV sets up from the initial point P_s $(0,0)$ to the object point P_f $(200,500)$. The initial angle θ_s is $\frac{\pi}{6}$ and the final angle θ_f is $\frac{5}{4}\pi$. The maximum curvature constrain is $k_{\max} = \pm 0.02$.

Estimation of distribution algorithm is used to generate the trajectory. After normalizing the curvature and the trajectory length, the weight coefficient is set to $\lambda_1 = 0.008, \lambda_2 = 40$. Determining the parameter range of every gene, the elite individuals selection probability $p_e = \frac{1}{3}$, the diversity select threshold coefficient are set to A=0.8, B=0.8. The low diversity individual probability $\boldsymbol{P}_\alpha$ =0.6. The chaotic replacement probability $p_c = 0.5$. The inoculation probability P_i =0.5. The initialization individuals gene inoculation probability p_u =0.6. The Gauss perturbation variance $\sigma = 0.1$.

Set the population individuals number N=10, the evolution generation G=10. The tangent vector length is selected as $\varepsilon_s \in \left[\frac{1}{15}|p_s p_f|, \frac{2}{3}|p_s p_f|\right]$, $\varepsilon_f \in \left[\frac{1}{15}|p_s p_f|, \frac{2}{3}|p_s p_f|\right]$. The optimization trajectory is generated and the corresponding optimized parameters $\varepsilon_s = 97.3$, $\varepsilon_f = 98.2$, the path length $\boldsymbol{s} = 548.63$, the maximum curvature $\bar{k}_{\max} = 0.0111$.

Set the evolution generation G=5, the simulation results are shown from Figure 9 to 11. The optimized parameter $\varepsilon_s = 88.3$, $\varepsilon_f = 99.8$, $\boldsymbol{s} = 546.6$, $\bar{k}_{\max} = 0.013$. Compared with the simulation results of G=10, the near optimal trajectory is already found. For the online planning, the optimal trajectory is not needed, while the iteration times is reduced largely, which show that the flyable trajectory can be generated in a short time.

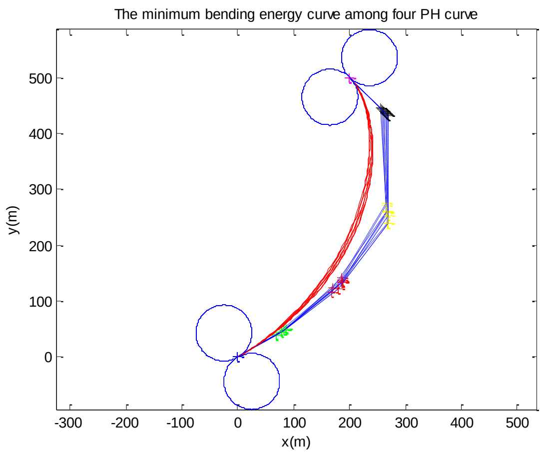


FIGURE 9 N Trajectories of the Last Generation Individuals

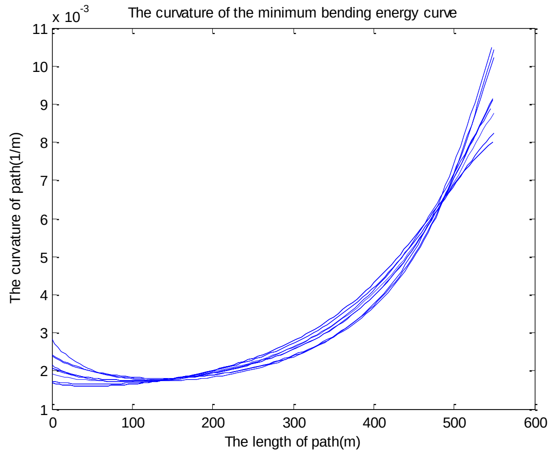


FIGURE 10 Curvature Change with the Length of the Trajectories in Figure 9

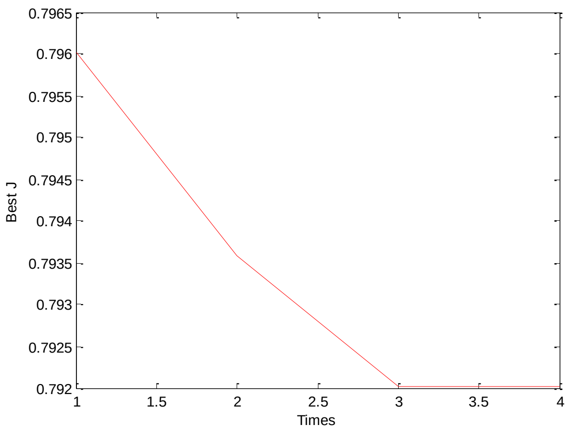


FIGURE 11 The Optimum Individual Fitness Evolution Curve

2) Collision Avoidance Trajectories Generation Online

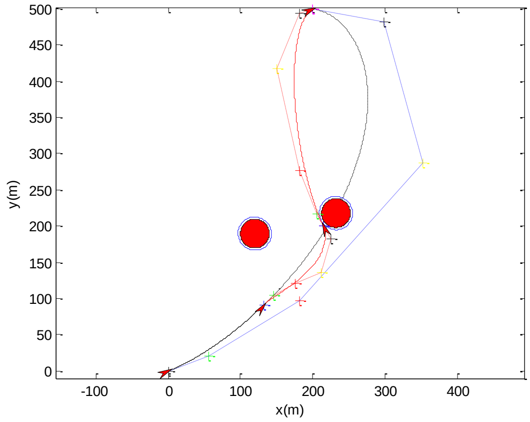


FIGURE 12 UAV planning Trajectory Based on PH Curve

It is assumed that the starting point P_s and the final point P_f are $(0,0)$ and $(200,500)$. The heading angle θ_s, θ_f are $\frac{\pi}{9}$ and $\frac{17}{18}\pi$. Suppose the detect distance S_t is 100. The average UAV speed V_u is 50. On the location $(132,90)$ of the PH path, the obstacle is detected. The radius of the barrier's circle is 20, the movement velocity is $V_o = 40$, the line-of-sight angle between UAV and obstacle is $\theta_o = 6.6^{\circ}$.

It can be judged whether UAV need to avoid obstacle according to the obstacle avoidance principle of velocity. The coordinates of the insertion point are $(215, 200)$ and the angle that UAV needs to counter clockwise turn is $\alpha = 76.4^{\circ}$ by calculation at the insertion point.

Using PH curve, UAV collision avoidance trajectory is generated, which is shown in Figure 12. The UAV successfully realize the obstacle avoidance online.

4 Conclusion

Pythagorean Hodograph (PH) curve is used to plan the UAV trajectory, which is flyable and satisfy the kinematics and dynamics constrains. The effect of the key parameters on the trajectory generation are analysed, and the appropriate value range that satisfy the constrains are given. EDA is used to optimize the trajectory overcome the blindness iteration. The proposed probability selection mechanism of global elite individual based on interval selection is appropriate to the PH trajectory interval optimization, which improve the speed and precision of the path generation. The method of PH optimization generation can be generalized to three-dimension trajectory generation.

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References

[1] Mangal Kothari, Ian Postlethwaite 2013 A Probabilistically Robust PathPlanning Algorithm for UAVs Using Rapidly-Exploring Random Trees *Journal of Intelligent and Robotic Systems* **71**(2) 231

[2] Xuan Zou, Bin Ge, Peng Sun 2012 Improved Genetic Algorithm for Dynamic Path Planning *International Journal of Information and Computer Science* **5** 28-31

[3] Li Xia, Wei Ruixuan, Wang Zhike 2011 Three-dimension path planning for UAV using improved A* algorithm in complicated threat environment *High Technology Letters* **1** 13-8

[4] Madhavan Shanmugave, Antonios Tsourdos, Brian White, Rafa Zbikowski 2009 Co-operative Path Planning of Multiple UAVs Using Dubins Paths With Clothoid Arcs *Control Engineering Practice* doi:10.1016.

[5] Dai R, John E Cochran Jr, 2009 Path Planning for Multiple Unmanned Aerial Vehicles by Parameterized Corno-Spiral *American Control Conference Hyatt Regency Riverfront St. Louis MO USA* June

[6] K G Jolly, R Sreerama Kumar, R Vijayakumar 2009 A Bezier Curve Based Path Planning in a Multi-Agent Robot Soccer System Without Violating the Acceleration Limits *Robotics and Autonomous Systems* **57** 23-33

[7] Armando Alves Neto, Douglas G Macharet, Mario F M Campos 2013 Feasible path planning for fixed-wing UAVs using seventh order Bézier curves *Journal of Braz Comput Soc* **19** 193-203

[8] M Shanmugavel, A Tsourdos, R Zbikowski, B A White, C A Rabath, N Lechevin 2006 A solution to simultaneous arrival of multiple UAVs using Pythagorean Hodograph curves *Proceedings of the 2006 American Control Conference Minneapolis Minnesota USA* June 2006 2813-8

[9] M Shanmugavel, A Tsourdos, R Zbikowski, B A White 2007 3D path planning for multiple UAVs using Pythagorean Hodograph curves *AIAA Guidance, Navigation and Control Conference and Exhibit 20 - 23 August 2007 Hilton Head South Carolina*: AIAA 2007-6455

[10] M A Shah 2011 Cooperative path planning and cooperative perception for UAVs swarm *Cranfield University PhD Thesis*

[11] Larrinaga P, Lozano J A 2002 Estimation of Distribution Algorithms *A New Tool for Evolutionary Computation. Boston: Kluwer Academic Publishers*

[12] Yi Hong, Qingsheng Ren, Jin Zeng, Yuchou Chang 2005 Convergence of estimation of distribution algorithms in optimization of additively noisy fitness functions *17th IEEE International Conference on Tools with Artificial Intelligence* 223-7

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