Frequency domain minimum error probability medical CT image blind equalization algorithm

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Abstract

A frequency domain minimum error probability medical CT image blind equalization algorithm was proposed. Blind image equalization is implemented by minimizing a cost function consisting of estimated image and blur. The steepest descent method was adopted to solve the proposed cost function. Computer simulation experiments show that the new algorithm reduces mean square error and improves restoration effect, peak signal to noise ratio and improving signal to noise ratio.

Keywords: Blind equalization algorithm, minimum error probability, medical CT image

1 Introduction

Transform domain or complex value transform is an important research spot of blind image restoration. Such methods obtain image smooth domain, border characteristics and other information mainly through the transform domain or orthogonal equivalent conversion to be conducive to signal analysis and be helpful to simplify the analysis. The common transform include Fourier transform, Z transform, wavelet transform, Curvelet and Contourlet sparse decomposition, etc. A frequency domain iterative blind restoration algorithm was proposed in [1]. The approach involves processing the projection data with Bellini's method for attenuation compensation followed by an iterative deconvolution technique which uses the frequency distance principle (FDP) to model the distance-dependent camera blur. Modelling of the camera blur with the FDP allows an efficient implementation using fast Fourier transform (FFT) methods. Reference [2] proposed to improve convergence toward global minima by single-site updating on the wavelet domain. For this purpose, a new restricted DWT space is introduced and a theoretically sound updating mechanism is constructed on this subspace.

An efficient algorithm is proposed for blind image restoration based on the discrete periodic Radon transform (DPRT) in [3]. The discrete periodic Radon transform is utilized to transform two-dimensional image into one dimensional signal sequence, then image restoration can reduce storage space and improve work efficiency. Partial differential equation is a method commonly used in image restoration. In [4], the partial differential equation method combined with a sampling wavelet transform to improve the ability of image edge preservation. [5] utilized a sparse representation transformation to realize image fusion and recovery. The implement of blind image restoration is equal to the solution and update of the sparse coefficient. Compare with the traditional wavelet transform, discrete wavelet transform and curvelet transform, the method has better feasibility [6] deals with nonconvex nonsmooth minimization methods for image restoration and reconstruction. The main goal of this method is to develop fast minimization algorithms to solve the nonconvex nonsmooth minimization problem. This method proposed the largest characteristic that completes image regularization in the frequency domain. Fast Fourier Transform, Fast Fourier Transform and FFT, improve the working efficiency, and improve the recovery effects.

In this paper, we utilized frequency domain transformation to convert two-dimensional medical CT image into one dimensional signal sequence, and found a corresponding complex sequence cost function. The cost function was solved by optimization strategy and the optimal estimation of complex sequence was obtained. We made use of the corresponding inverse transformation to complete image restoration. This paper is organized as follows. A description of the proposed algorithm is given in Section 2, together with implementation details. The experimental section, Section 3, empirically validates the proposed method, and Section 4 concludes this paper.

2 Principle of Frequency Domain Minimum Error Probability Medical CT Image Blind Equalization Algorithm

In order to describe the dimension reduction medical CT blind equalization algorithm in the frequency domain. The signal model is shown as

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$$G(k) = \mathrm{DFT}[g(n)] = \begin{cases} \sum_{n=0}^{N-1} g(n)\omega_N^{kn} & 0 \le k \le N-1, \\ 0, & otherwise \end{cases} \quad (1)$$

where, N is pixel point of image to be processed. According to the principle of the convolution of two signal sequences in time domain being equal to the product of their respective conversion in the frequency domain. Two sides of the image degradation equation was realized by FFT simultaneously. We obtain

$$G(k) = H(k)F(k) + N(k), \quad k = 0,1,\cdots,N-1, \quad (2)$$

where, $F(k)$ and $N(k)$ are the discrete Fourier transform of $f(n)$ and $n(n)$, respectively, $f(n)$ and $n(n)$ respectively represent real image sequence and noise sequence of the equivalent dimension reduction. $H(k)$ is the frequency response function of the point spread function (PSF) of the equivalent dimension reduction. In the other word, $H(k)$ is the discrete Fourier transform of $h(n)$.

Similarly, the eliminate form of dimension reduction point spread function can be represented as in the frequency domain

$$\tilde{F}(k) = G(\omega)W(\omega), \quad (3)$$

where, $W(\omega)$ denotes a frequency response function of blind equalizer, and it is the discrete Fourier transform of $w(n)$, namely $W(\omega) = \mathrm{DFT}[w(n)]$.

In order to establish a suitable cost function of blind equalization algorithm for medical CT image, here, medical CT images were simply pre – whitened and the transform of dimension reduction medical CT image is a smooth white sequence. Noise sample sequence is independent of the source signal and is a white noise sequence, thus we can get

$$\mathrm{E}[F(k)] = \mathrm{E}[N(k)] = 0, \quad (4)$$

$$\mathrm{E}[F_n(\omega)F_m(\omega)] = 0, n \ne m, \quad (5)$$

$$\mathrm{E}[F_n(\omega)N_n(\omega)] = 0, \quad (6)$$

$$\mathrm{E}[N_n(\omega)N_m(\omega)] = N\sigma_n^2\delta(n-m), \quad (7)$$

$$\mathrm{E}\left[F_n(\omega)F_m^*(\omega)\right] = 2A\delta(n-m). \quad (8)$$

According to image acquisition and restoration process, we know that

$$G(\omega) = H(\omega)F(\omega) + N(\omega). \quad (9)$$

By equation (3) and (9), we can derive the total characteristics $H_{\sum}(\omega)$ the equalization system

$$H_{\sum}(\omega) = \frac{\tilde{F}(\omega)}{F(\omega)} = \frac{1}{2A}W(\omega)\mathrm{E}\left[G(\omega)F^*(\omega)\right]. \quad (10)$$

In order to get the best estimate of the equalizer, namely $\hat{F}(\omega) = F(\omega)$, the frequency characteristics of the equalizer must achieve $W_{opt}(\omega)$, then

$$\tilde{F}(\omega) = G(\omega)W_{opt}(\omega) = F(\omega)H_{\sum}(\omega). \quad (11)$$

Blind equalization cost function is defined as

$$J(W^{(n)}(\omega), p^{(n)}) = \left|\frac{1}{2A}W^{(n)}(\omega)\mathrm{E}\left[G(\omega)\hat{F}^*(\omega)\right] - H_{\sum}(\omega)\right|^2 + \mathrm{E}\left[\left|G(\omega)W^{(n)}(\omega) - \hat{F}(\omega)H_{\sum}(\omega)\right|\right]. \quad (12)$$

Among them, (n) is the number of iterations; p is the error probability of the blind equalizer. In the whole process of iterative calculation, error probability changes slowly, namely $p^{(n)} \approx p^{(n-1)}$. In algorithm operation process, we must ensure that the decision device output $\hat{F}(\omega)$ and the source signal $F(\omega)$ propose the same statistical properties, and it can be expressed as

$$\mathrm{E}\left[|F(\omega)|^2\right] = \mathrm{E}\left[\left|\hat{F}(\omega)\right|^2\right], \quad (13)$$

$$\mathrm{E}\left[\hat{F}(\omega)G^*(\omega)\right] = \left(1-p^{(n)}\right)\mathrm{E}\left[F(\omega)G^*(\omega)\right]. \quad (14)$$

We set

$$\mathrm{R_a} = \mathrm{E}\left[|G(\omega)|^2\right], \quad (15)$$

$$R_b = \frac{1}{4A}\mathrm{E}\left[F(\omega)G^*(\omega)\right]\mathrm{E}\left[F^*(\omega)G(\omega)\right], \quad (16)$$

$$z = \left(1 + \frac{1}{2A}\right)H_{\sum}(\omega)\mathrm{E}\left[F(\omega)G^*(\omega)\right]. \quad (17)$$

By equation (15) and (16), $\mathrm{R_a}$, R_b are real number, and $\mathrm{R_a} > 0$, $R_b > 0$; z is a complex value number. In view of Equation (13) - (17), we have

$$\begin{aligned} J(W^{(n)}(\omega), p^{(n)}) &= \left|W^{(n)}(\omega)\right|^2\left[R_a + \left(1-p^{(n)}\right)^2 R_b\right] \\ &- W^{*(n)}(\omega)\left(1-p^{(n)}\right)z \\ &- W^{(n)}(\omega)\left(1-p^{(n)}\right)z^* + \left|H_{\sum}(\omega)\right|^2(1+2A) \end{aligned}. \quad (18)$$

Minimum frequency error probability of medical CT image blind equalization algorithm is that the equalization problem is transformed into solving the optimal solution of equation (18).

The derivative of the cost function equation (18) with respect to equalizer frequency characteristic $W(\omega)$ can be evaluated. Under the condition of minimum error probability, we can get the best value of equalizer

$$W_{opt}(\omega)=\frac{(1-p_{\min})z}{R_a+(1-p_{\min})^2R_b}. \tag{19}$$

We adjust the parameters of the equalizer by the steepest descent method, and the iterative update formula of frequency response function can be obtained as

$$\begin{aligned}&W^{(n+1)}(\omega)\\&=W^{(n)}(\omega)-\frac{1}{2}\mu\nabla J(W^{(n)}(\omega),p^{(n)})\\&=W^{(n)}(\omega)-\mu\left[R_a+(1-p_{\min})^2R_b\right]\\&\times\left[W^{(n)}(\omega)-W_{opt}(\omega)\right]\\&+\mu\left[(1-p_{\min})^2-(1-p^{(n)})^2\right]R_bW^{(n)}(\omega)\\&+\left[p_{\min}-p^{(n)}\right]z\end{aligned}, \tag{20}$$

where, μ is step-size.

Using equation (20), we can obtain the equalization effect of frequency adaptive blind equalization algorithm.

3 Simulation results

We illustrate the performance of the proposed method on handling noisy degraded medical CT images in the experiment. The sectional CT image of chronic inflammation of nasopharyngeal is utilized to illustrate the effectiveness of the proposed algorithm. The original images of size 256 × 256 shown in Figure 5(a) were degraded by a 25×25 Gaussian blur with a standard deviation of 0.002, followed by an additive white Gaussian noise with 0 mean and deviation of 0.05 to form the noisy blurred images shown in Figure 1(b). In the simulation, $\mu=5\times10^{-6}$, $p=0.05$. We compare the results of the proposed algorithm with iterative blind deconvolution algorithm, dispersion minimization algorithm, and reduction dimension constant module algorithm. Figure1(c) is the restoration effect of the maximum likelihood method. The number of iterations of IBD is 100 and the restored image is shown in Figure 1(d).The restored images of dispersion minimization algorithm [7] is depicted in Figure 1(e). Figure 1(f) shows the result of the proposed algorithm.

As shown in Figure 1, compare with divergence minimization blind restoration algorithm, maximum likelihood algorithm and IBD algorithm, the minimum frequency domain error probability medical CT image blind equalization algorithm obtain better recovery results.

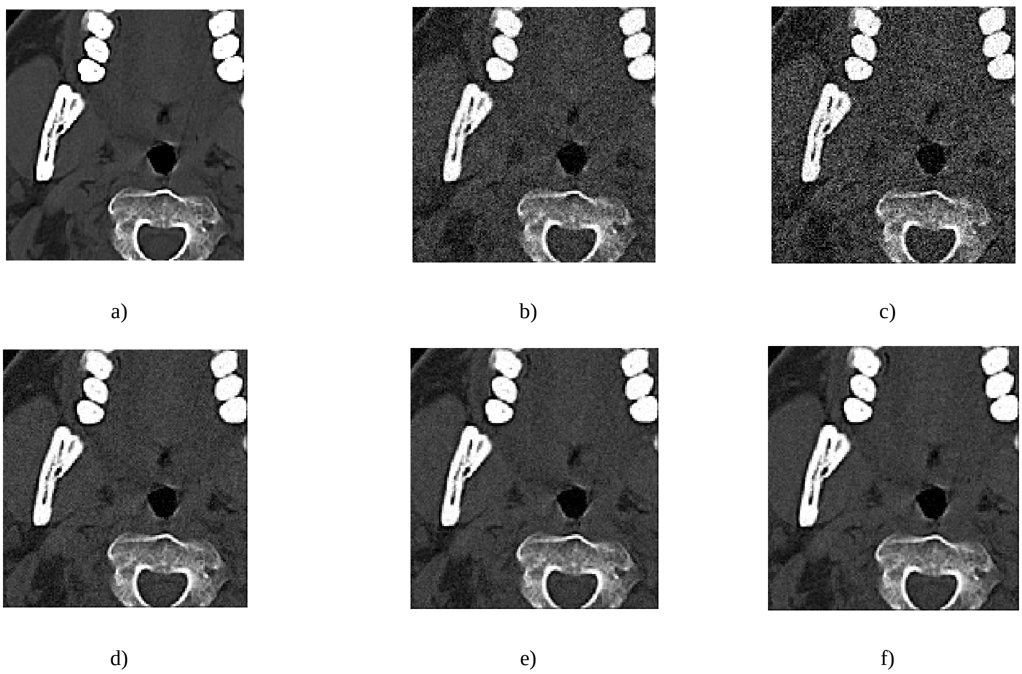

FIGURE 1 Experimental images and restoration results a) CT image (chronic recovery inflammation of nasopharyngeal), b) degraded images, c) maximum likelihood image d) IBD method image, e) divergence minimization image, f) the proposed method

Table 1 shows the PSNR [8], MSE [9], and ISNR [10] values achieved by the four algorithms mentioned above. The minimum frequency domain error probability image blind equalization algorithm improve the peak signal to noise ratio and the signal to noise ratio comparing with divergence minimize blind restoration algorithm. The new algorithm completes image blind equalization in the frequency domain. This method is essentially that a two-dimensional image signal is transform into a form of complex signal sequence. Image restoration is equivalent to minimizing a complex cost function. Compared with divergence minimization restoration algorithm, the new method is no longer competitive in the aspect of calculation performance, but it improves the signal to noise ratio PSNR improvement and reduce the minimum mean square error. IBD blind restoration algorithm and maximum likelihood methods are sensitive to noise. In particular, maximum likelihood algorithm can reduce calculate complex, but is all susceptible to noise. Computer simulation show that maximum likelihood method may yield a worse restored image than the degraded image, and even yields a negative ISNR value in low SNR.

TABLE 1 All kinds of algorithm performance

	The proposed method	**Dispersion minimization algorithm**	**IBD algorithm**	**Maximum likelihood**
PSNR	23.9758	23.7871	23.4624	18.7743
MSE	38.2247	40.6308	43.8088	69.9812
ISNR	1.1622	0.9736	0.6488	----

4 Conclusion

In this paper by frequency domain transformation, minimum medical CT images in frequency domain error probability of blind equalization algorithm, the algorithm of iterative formula is deduced, analysed the iteration step length selection principles and convergence performance, the computer simulation, the simulation shows that the new algorithm improves the peak signal-to-noise ratio and improve the signal-to-noise ratio, reducing the minimum mean square error. Line drawings should be drawn in black ink on drawing or tracing paper or should be glossy prints of the same, if they have not been prepared on your computer facility. All illustrations should be clearly displayed by leaving at least a single line of spacing above and below them.

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References

[1] Glick S J, Weishi Xia 1997 Iterative Restoration of SPECT Projection Images *IEEE Trans. Nuclear Science* **44**(2) 204-11

[2] Robini M C, Magnin I E 2003 Stochastic Nonlinear Image Restoration using the Wavelet Transform *IEEE Trans. Image Processing* **12**(8) 890-905

[3] Lun D P K, Chan T C L, Tai-Chiu Hsung, Feng D D, Yuk-Hee Chan 2004 Efficient Blind Image Restoration using Discrete Periodic Radon Transform *IEEE Trans. Image Processing* **13**(2) 88-200

[4] Junmei Zhong, Huifang Sun 2008 Wavelet-Based Multiscale Anisotropic Diffusion With Adaptive Statistical Analysis for Image Restoration *IEEE Transactions on Circuits and Systems-I* **55**(9) 2716-25

[5] Bin Yang, Shutao Li 2010 Multifocus Image Fusion and Restoration with Sparse Representation *IEEE Transactions on Instrumentation and Measurement* **59**(4) 884-92

[6] Nikolova M, Ng M K, Chi-Pan Tam 2010 Fast Nonconvex Nonsmooth Minimization Methods for Image Restoration and Reconstruction *IEEE Trans .Image Processing* **19**(12) 3073-88

[7] Vural C, Sethares W A 2006 Blind Image Deconvolution via Dispersion Minimization *Digital Signal Processing* **26** 137-48

[8] Dalong Li, Mersereau R M, Simske S 2010 Blind Image Deconvolution Through Support Vector Regression *IEEE Trans. Neural Networks* **18**(3) 931-5

[9] Wu Y D, Sun Y, Zhang H Y, Sun S X 2007 Variational PDE based image restoration using neural network *IET Image Process* **1**(1) 85–93

[10] Almeida M S C, Almeida L B 2010 Blind and Semi-Blind Deblurring of Natural Images *IEEE Trans .Image Processing* **19**(1) 36-52

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