On dynamic iterative algorithm and the loss of newsvendor problem

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Abstract

Based on the thinking and method of dynamic programming, this paper calculates the expected profit of every selling cycle of newsvendor by historical data and calculates the expected profit of the second selling cycle by that of the first selling cycle. We will get the optimal purchasing quantity as the expected profit begins to fall. Besides, this paper also discusses the stock loss and gets the optimal purchasing quantity by empirical examples.

Keywords: newsvendor problem, marginal income, dynamic iterative algorithm, optimal purchasing quantity

1 Introduction

Stocking is important for production and sales. It is divided into random stock model and definite stock model. Newsvendor problem is a typical random stock model. Traditional algorithms for the newsvendor model are common in textbooks and journals [1-5]. Many researchers have probed into this problem. Literature [6-10] discuss object function, strategic variation and model parameters respectively. Other researches [11-13] study the problem from time restriction and financial benefits. And more apply it to purchasing and inventory management of the supply chain [14-16] rather than focus on algorithms. This paper will adopt the principle and idea of dynamic programming, does some calculation and works out the optimal formula for newsvendor problem.

2 Traditional dynamic iterative algorithm

Traditional newsvendor problem model is shown in Literature [14]. The newsvendor gets the newspaper wholesale at 0.3 Yuan from the news agency and sells it at 0.45 Yuan. If the newspaper cannot be sold out, he will lose 0.3 Yuan. If out-of-stock, then the expected loss is 0.15 Yuan. Historical sales record is as Table 1, then how many pieces of newspapers should the newsvendor purchase to reduce the loss to the minimum?

TABLE 1 Historical sales record of the newspapers

Demand (pieces)	120	130	140	150	160
Demand probability P(D)	0.15	0.2	0.3	0.25	0.1

Solution: If calculated by traditional algorithm, we can get:

If purchasing 120 pieces, the average loss will be:

$$(120-120)\times 0\times 0.15+(130-120)\times 0.15\times 0.2+ (140-120)\times 0.15\times 0.3+(150-120)\times 0.15\times 0.25+(160-120)\times 0.15\times 0.1= 0+1.5\times 0.2+3\times 0.3+6\times 0.1$$

If purchasing 130 pieces, the average loss will be:

$$(130-120)\times 0.3\times 0.15+(130-130)\times 0\times 0.2+ (140-130)\times 0.15\times 0.3+(150-130)\times 0.15\times 0.25+(160-130)\times 0.15\times 0.1=3+0+1.5\times 0.3+3\times 0.25+4.5\times 0.1=2.1$$

We can get the average loss for other purchases, as is shown in Table 2.

TABLE 2 Loss for different numbers of newspapers' sales amount

D	120	130	140	150	160	**Average loss**
P(D)	0.15	0.2	0.3	0.25	0.1	
F / Q						
120	0	1.5	3	4.5	6	2.925
130	3	0	1.5	3	4.5	2.1
140	6	3	0	1.5	3	2.175
150	9	6	3	0	1.5	3.6
160	12	9	6	3	0	6.15

From Table 2, we can see that when the purchasing amount equals to the demand, the failure cost is 0. As a result, the loss in Table 2 can be divided into two types, one is the backlog of loss, on the left of 0; the other is out-of-stock loss, on the right of 0. The backlog of loss is real while out-of-stock loss talks about the probability. In the traditional algorithm, two losses weight the same and it is necessary to reduce their total amount to the

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minimum. The result is shown in Table 3, the same as in Table 2. We also get the curve, as in Figure 1.

TABLE 3 Contrast of two losses

D	120	130	140	150	160	backlog of loss	out-of-stock loss	Average loss
P(D)	0.15	0.2	0.3	0.25	0.1			
F \ Q								
120	0	1.5	3	4.5	6	0	2.925	2.925
130	3	0	1.5	3	4.5	0.45	1.65	2.1
140	6	3	0	1.5	3	1.5	0.675	2.175
150	9	6	3	0	1.5	3.45	0.15	3.6
160	12	9	6	3	0	6.15	0	6.15

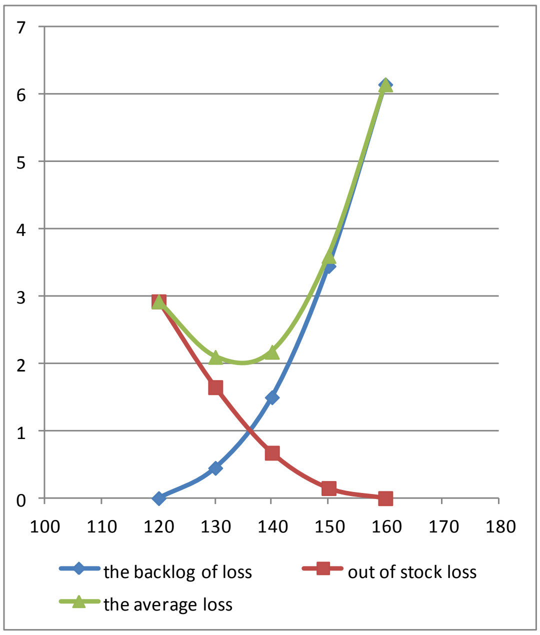

FIGURE 1 Failure cost for the newsvendor problem

If calculated by dynamic iterative algorithm, first we need to get the cumulative frequency for every sale amount, as in Table 3.

TABLE 4 Sales probability and cumulative frequency.

Demand (D)	120	130	140	150	160
Demand Probability P(D)	0.15	0.2	0.3	0.25	0.1
Cumulative frequency	1	0.85	0.65	0.35	0.1

Cumulative frequency means the frequency of the least amount of sales. For example, if the cumulative frequency for 120 pieces is 1, it means that 120 pieces of newspapers can be for 100% sure. In the same point, the cumulative frequency for 140 pieces is 0.65, indicating that 65% of 140 can be sold. It consists of 0.1 (cumulative frequency for 160 pieces), 0.25 (cumulative frequency for 150) and 0.3 (cumulative frequency for 140). And $1-0.65=0.35$, indicating that 0.35% of 140 pieces cannot be sold, either 120 can be sold with the possibility of 0.15 or 130 can be sold with the possibility of 0.2.

Then we are getting the expected profit.

The purchasing of 120 pieces consists of two parts: the profit and the loss. Then there is: $E[C(120)]=120\times0.15\times1=18$.

The purchasing of 130 pieces also consists of two parts: one is the expected profit for selling 120 pieces, the other is the expected profit higher than that of the 120. The above formula has got the profit for 120 pieces. As for the sales amount that is more than 120, for every step strength of 10, the probability of selling them is 85%. Then, the expected profit for purchasing 130 pieces is:

$$E[C(130)]=E[C(120)]+10\times0.15\times0.85-10\times0.3\times0.15=18+(1.275-0.45)=18+0.825=18.825.$$

In the same way, we can also get the expected profit for other purchasing amount.

For 140 pieces:

$$E[C(140)]=E[C(130)]+10\times0.15\times0.65-10\times0.3\times0.35=18.825+(0.975-1.05)=18.825-0.075=18.75$$

For 150 pieces:

$$E[C(150)]=E[C(140)]+10\times0.15\times0.35-10\times0.3\times0.65=18.75+(0.525-1.95)=18.75-1.425=17.325$$

For 160 pieces:

$$E[C(160)]=E[C(150)]+10\times0.15\times0.1-10\times0.3\times0.9=17.325+(0.15-2.7)=14.775$$

For the dynamic iterative algorithm, the optimal purchasing is also 130 pieces. In fact, we can stop by 140 pieces because increasing every 10 pieces, selling profit is smaller than the backlog of loss, which means the expected profit will fall. With the increase of the pieces, selling profit is smaller and smaller than the backlog of loss, meaning that the expected profit will also be small. That is the advantage of the dynamic iterative algorithm, in which the calculation stops once the sales profit is smaller than the backlog of loss. Compared to the traditional algorithm, the dynamic iterative algorithm reduces some steps with the same results and is able to deduce the optimal purchasing quantity.

TABLE 5 The process of iterate algorithm for the newsvendor problem

D	120	130	140	150	160	Selling profit	Increasing the unit backlog of loss	Expected profit
P(D)	0.15	0.2	0.3	0.25	0.1			
F \ Q								
120	18	0	0	0	0	18	0	18
130	18	1.275 – 0.45	0	0	0	19.275	0.45	18.825
140	18	1.275 – 0.45	0.975 – 1.05	0	0	19.8	1.05	18.75
150	18	1.275 – 0.45	0.975 – 1.05	0.525 – 1.95	0	19.275	1.95	17.325
160	18	1.275 – 0.45	0.975 – 1.05	0.525 – 1.95	0.15 – 2.7	17.475	2.7	14.775

The result is shown in Table 5 and Figure 2.

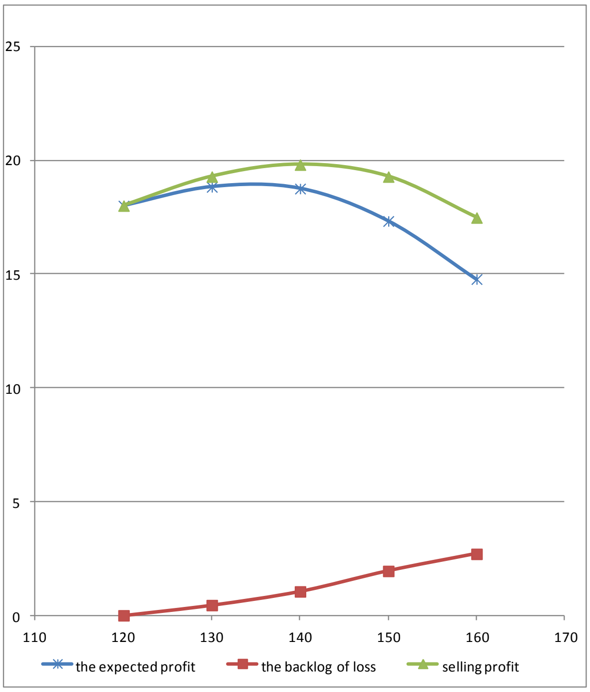

FIGURE 2 Iterative algorithm for the newsvendor problem

General deduction: Suppose the minimum sales amount is a, the maximum sales amount is b, $n \in [a,b]$ and the distributed probability $P(n)$ is known. Selling one piece of newspaper, the newspaper boy can get k Yuan, and loses h Yuan for every cumulative piece. d refers to the step length between No. x and No. $x + 1$ or No. $x – a$. It doesn't matter whether d is the average or not, or to say, it doesn't matter whether d is the well-distributed step length or not $\sum_{n=a}^{b} P(n) = 1$ the maximum expected profit is:

$$E(C(n)) = \begin{cases} n \times k, & n = a \\ E(C(n-1)) + dk \sum_{n=a}^{n} P(n) - dh \left(1 - \sum_{n=a}^{n} P(n) \right), & n > a \end{cases} . \quad (1)$$

Equation (1) is the traditional iterative algorithm for the maximum expected profit.

From the Equation (1), we can see $n \times k$ is above 0, which means the minimum sales profit. Generally speaking, $d \times k \times \sum_{n=a}^{n} P(n) - d \times h \left(1 - \sum_{n=a}^{n} P(n) \right)$ increases from the second and begins to fall later until it is below 0. Thus, the maximum expected profit is found.

When $d \times k \times \sum_{n=a}^{n} P(n) - d \times h \left(1 - \sum_{n=a}^{n} P(n) \right) < 0$, the expected profit will fall. So, when there is the first $d \times k \times \sum_{n=a}^{n} P(n) - d \times h \left(1 - \sum_{n=a}^{n} P(n) \right) < 0$, its previous value will be the maximum expected profit. That is to say, when $d \times k \times \sum_{n=a}^{n} P(n) - d \times h \left(1 - \sum_{n=a}^{n} P(n) \right) < 0$ is the last value above 0, it is the maximum profit.

The maximum profit n will get if we calculate:

$$d \times k \times \sum_{n=a}^{n} P(n) - d \times h \left(1 - \sum_{n=a}^{n} P(n) \right) < 0$$

and

$$d \times k \times \sum_{n=a}^{n} P(n) - d \times h \left(1 - \sum_{n=a}^{n} P(n) \right) > 0 .$$

When there is the last:

$$d \times k \times \sum_{n=a}^{n} P(n) - d \times h \left(1 - \sum_{n=a}^{n} P(n) \right) \geq 0 ,$$

the above formula will become:

$$k \times \sum_{n=a}^{n} P(n) - h \left(1 - \sum_{n=a}^{n} P(n) \right) \geq 0$$

or

$$k \times \sum_{n=a}^{n} P(n) \geq h \left(1 - \sum_{n=a}^{n} P(n) \right) \geq 0$$

as d is the step length and $d > 0$.

Obviously, $k, h \sum_{n=a}^{n} P(n), 1 - \sum_{n=a}^{n} P(n)$ are above 0, thus there is:

$$\frac{\sum_{n=a}^{n} P(n)}{1 - \sum_{n=a}^{n} P(n)} \geq \frac{h}{k} . \quad (2)$$

When there is:

$$d \times k \times \sum_{n=a}^{n+1} P(n+1) - d \times h \left(1 - \sum_{n=a}^{n+1} P(n+1) \right) \leq 0 ,$$

for the first time, the above formula will be:

as d is the step length and $d > 0$. Obviously, $k, h \sum_{n=a}^{n} P(n), 1 - \sum_{n=a}^{n} P(n)$ are above 0, thus there is:

$$\frac{\sum_{n=a}^{n+1} P(n)}{1-\sum_{n=a}^{n+1} P(n+1)} \le \frac{h}{k}. \quad (3)$$

From Equations (2) and (3), we can get the relationship between the optimal purchasing quantity n and $\frac{h}{k}$:

$$\frac{\sum_{n=a}^{n+1} P(n)}{1-\sum_{n=a}^{n+1} P(n+1)} \le \frac{h}{k} \le \frac{\sum_{n=a}^{n} P(n)}{1-\sum_{n=a}^{n} P(n)}. \quad (4)$$

From Equation (4), we can deduce the traditional optimal purchasing quantity for the newsvendor problem:

$$\sum_{n=a}^{n+1} P(n+1) \le \frac{k}{h+k} \le \sum_{n=a}^{n} P(n). \quad (5)$$

Substituting Equation (5) into the previous equation, we can get:

$$\sum_{n=a}^{n+1} P(n+1) \le \frac{0.15}{0.45} \le \sum_{n=a}^{n} P(n),$$

$$\sum_{n=a}^{n+1} P(n+1) \le 0.33 \le \sum_{n=a}^{n} P(n).$$

Then when the cumulative frequency is 0.33, the left side is the maximum profit, the right side is the second maximum profit with the corresponding frequency of 0.15 and 0.35 respectively. The cumulative frequency for the maximum profit is 0.35, indicating that 130 pieces is the optimal purchasing quantity.

3 Newsvendor problem with inventory loss

Many goods such as fresh agricultural products, food and perishable goods are experiencing losses either in the stock or in the process of sales. Thus, loss needs to be taken into consideration for the optimal purchasing quantity or the stock quantity. The formula is deduced as follows:

Suppose selling every piece of good can make k Yuan, and loses h Yuan if it is stocked, the cost is c Yuan, the loss rate of the stock or in the process of sales is d. the probability P of the everyday sales amount r is known.

Then what is the purchasing amount for the maximum expected profit or the minimum loss?

Suppose the everyday sales amount is r, the probability $P(r)$ is already known – $\sum_{r=0}^{\infty} P(r)=1$. The purchasing quantity is Q.

When the supply is bigger than the demand ($r \le Q$), there will be the loss for stocked newspapers and the expected loss is:

$$\sum_{r=0}^{Q} h(Q-r)P(r).$$

When the supply is smaller than the demand ($r > Q$), there will be the loss for out-of-stock newspapers and the expected loss is:

$$\sum_{r=Q+1}^{\infty} k(r-Q)P(r).$$

Thus, when the purchasing amount is Q, the expected loss is:

$$\sum_{r=0}^{Q} h(Q-r)P(r)$$

$$C(Q) = \sum_{r=0}^{Q} h(Q-r)P(r) + \sum_{r=Q+1}^{\infty} k(r-Q)P(r) + Q \times d \times c$$

We will then calculate the minimum Q for $C(Q)$. $C(Q)$ should meet the demand of the followings:

Condition (1): $C(Q) \le C(Q+1)$;

Condition (2): $C(Q) \le C(Q-1)$.

From condition (1), we can deduce:

TABLE 6 Demand and probability for a single season

Demand	100	200	300	400	500	600	700
Probability	0.17	0.2	0.25	0.12	0.1	0.08	0.08
Cumulative rate	1	0.83	0.63	0.38	0.26	0.16	0.08

$$Q \times d \times c + \sum_{r=0}^{Q} h(Q-r)P(r) + \sum_{r=Q+1}^{\infty} k(r-Q)P(r) \le$$

$$(Q+1) \times d \times c + \sum_{r=0}^{Q+1} h(Q+1-r)P(r) + \sum_{r=Q+2}^{\infty} k(r-Q-1)P(r) -$$

$$d \times c + h\left\{\sum_{r=0}^{Q} (Q-r)P(r) - \sum_{r=0}^{Q+1} k(Q-1-r)P(r)\right\} +$$

$$k\left\{\sum_{r=Q+1}^{\infty} (r-Q)P(r) - \sum_{r=Q+2}^{\infty} (r-Q-1)P(r)\right\} \le 0$$

and get:

$$-d \times c - h\sum_{r=0}^{Q} P(r) + k\sum_{r=Q+1}^{\infty} P(r) \le 0$$

there is:

$$-d\times c-h\sum_{r=0}^{Q}P(r)+k\sum_{r=Q+1}^{\infty}P(r)+k\sum_{r=0}^{Q}P(r)\le k\sum_{r=0}^{Q}P(r)-$$

$$d\times c+k\le (h+k)\sum_{r=0}^{Q}P(r).$$

We can deduce:

$$\sum_{n=0}^{Q}P(r)\ge \frac{k}{h+k}-\frac{d\times c}{h+k}. \quad (6)$$

From condition (2), we can also deduce:

$$\sum_{n=0}^{Q+1}P(r)\le \frac{k}{h+k}-\frac{d\times c}{h+k}. \quad (7)$$

Here we get the optimal purchasing quantity for the inventory loss d:

$$\sum_{n=0}^{Q+1}P(r)\le \frac{k}{h+k}-\frac{d\times c}{h+k}\le \sum_{r=0}^{Q}P(r). \quad (8)$$

From the above formula, the optimal purchasing quantity with inventory loss is larger than that without inventory loss. The increase amount depends on the sum of value and the loss rate and the ratio of the backlog of loss to out-of-stock loss. The bigger the ratio, the more the optimal purchasing amount will add. But at the same time, the expected profit will substantially decrease because the added purchasing amount is offset by the loss. Thus, it is important to reduce the loss rate.

4 Applications

The following example is derived from Reference [5]. Let's calculate it by dynamic iterative algorithm.

Part A sold in one store is 18 Yuan per each with the cost of 10 Yuan. The storage cost of every part for a single quarter is 4 Yuan. The probability distribution of demand for the first quarter (3 months) is shown in the following Table 6. We are calculating the optimal purchasing quantity (Stocked parts are not depreciated and can be sold at the original price).

$E(100)=100\times 8-100\times 2=600$,

$E(200)=(100)+100\times 8\times 0.83-100\times 0.17\times 4-$
$100\times 0.83\times 2=600+430=1030$,

$E(300)=(200)+100\times 8\times 0.63-100\times 0.37\times 4-$
$100\times 0.63\times 2=1030+230=1260$,

$E(400)=(300)+100\times 8\times 0.38-100\times 0.62\times 4-$
$100\times 0.38\times 2=1260-20=1240$.

At this time, there is no need to continue the calculation because the profit will fall. When the purchasing quantity is 300 pieces, the expected optimal profit is 1260 Yuan.

Substitute Equation (5) into the previous Equation, we can get:

$$\sum_{n=a}^{n+1}P(n+1)\le \frac{6}{6+4}\le \sum_{n=a}^{n}P(n).$$

Notice: k is the profit, h is the backlog of loss (here refers to as storage cost). The profit for every piece is $k=18-10-2=6$, and the storage cost h is 4 Yuan (it will be sold next month). Thus there is:

$$\sum_{n=a}^{n+1}P(n+1)\le 0.6\le \sum_{n=a}^{n}P(n).$$

The optimal purchasing quantity is 300 pieces. The result is equal to the result for using Equation (1).

If the rate of loss is 0.1, subsitute it into Equation (8):

$$\sum_{n=0}^{Q+1}P(r)\le \frac{6}{6+4}-\frac{0.1\times 10}{6+4}\le \sum_{r=0}^{Q}P(r),$$

$$\sum_{n=0}^{Q+1}P(r)\le 0.5\le \sum_{r=0}^{Q}P(r)$$

The optimal purchasing quantity is 400 pieces.

5 Conclusions

The dynamic iterative algorithm takes the advantage of the previous calculation and reduces some steps. Once the sales income of added sales is less than the backlog of loss, the calculation can be stopped and the optimal value is found. That is, when the marginal income in below 0, the corresponding purchasing quantity will be the optimal purchasing quantity. Here the marginal income refers to the total of sales income and the backlog of loss brought by added purchasing quantity. The step length d is a random number. It is well adaptive.

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