

A novel method for identifying system modal parameters using stabilization diagram

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Abstract

Modal parameters of a structure are important for system identification. In order to identify modal parameters of a structure more accurately, this paper proposes a parameter identification method combined with stabilization diagram. Stochastic subspace identification (SSI) is a recently developed method for identifying a linear system. Combining SSI and the proposed method can easily confirm system order. However, the proposed method has difficulty in distinguishing spurious modals. Therefore, the proposed method must be revised to ensure that the spurious modal can be detected and the SSI can be used to improve identification accuracy. Finally, a simulation is conducted on a fracturing pump truck, when the damping ratio increases from 10% to 40%, those spurious modals disappear. The results indicate that this method performs precise identification.

Keywords: Parameter Identification, Modal Analysis, Stochastic Subspace Identification, Stabilization Diagram, Fracturing Pump Truck

1 Introduction

Determining modal parameters has become important for system identification in the past decades. Results of experimental modal analysis (EMA) are used in practice; an overview of the EMA method can be found in [1–3]. In some cases, performing the vibration test on large structures is difficult or impossible because some excitations, such as wind or traffic, cannot be measured. In addition, using artificial excitation, such as hammer or drop weight, is impractical or, in some cases, expensive.

Therefore, output-only stochastic system identification methods have been developed. In these methods, ambient forces cannot be ignored and should be regarded as stochastic quantities with some unknown parameters. Stochastic subspace identification (SSI) is one of the methods for identifying system parameters. SSI has two implementation procedures: covariance-drive (SSI-cov) and data-drive (SSI-data) implementation [4]. Given that these methods need only the outputs of the structure for measurement, artificial excitation is unnecessary.

Estimating the modal parameters of the structures according to the measured data involves three steps: data collection, system identification, and determination of modal parameters [5–7]. This paper focuses on data collection. Thus, system identification should be treated as an important problem and is defined as construction of the system model according to the measured data. The SSI method is used in the time domain because of its convenience [8]. The modal parameters can be

determined according to a free vibration analysis of the identified system model.

In this paper, the stabilization diagram can be used to determine the system order, which is an important step for system identification. Other studies [9–11] confirmed the stabilization diagram method based on singular value decomposition. However, the obtained results are insufficient because the stabilization diagram method is a comparatively new method of determining system order. The stabilization diagram method can be used to distinguish real modals and modals in cases with excess noise. The stabilization diagram can delete certain system poles that meet the condition, but cannot be treated as real poles because they may belong to noise modals and not to the system. Thus, these poles can be distinguished and deleted by using the stabilization diagram.

This paper is organized as follows: Section 2 shows how the vibration structure can be modeled according to stochastic state-space models and modal analysis. Section 3 discusses the subspace identification method used for system identification. The use of a stabilization diagram to determine the system order is discussed in Section 4. Section 5 shows the application of this method to a real structure.

2 Stochastic state-space model for vibrating structures

For a linear dynamical system model, the following system of ordinary differential equations can be obtained:

$$M \frac{d^2 \mathbf{u}(t)}{dt^2} + C_2 \frac{d\mathbf{u}(t)}{dt} + K\mathbf{u}(t) = \mathbf{B}_2 \mathbf{f}(t), \quad (1)$$

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where M represents the mass matrices, C_2 represents the stiffness matrices, K represents the damping matrices, $f(t)$ and $u(t)$ represent the nodal forces and nodal displacement, respectively, B_2 is the selection matrix, and t is the time. This equation can be converted into the state-space model as follows:

$$\frac{dx(t)}{dt} = A_c x(t) + B_c f(t), \tag{2}$$

where $x(t) = \begin{bmatrix} u(t) \\ \frac{du(t)}{dt} \end{bmatrix}$, $A_c = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C_2 \end{bmatrix}$,

and

$$B_c = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix} B_2, \tag{3}$$

where $x(t)$ is the state of the structure. The quantities of interest can be grouped in an output vector (t) as follows:

$$\begin{aligned} y(t) &= C_a \frac{du(t)}{dt} + C_v \frac{du(t)}{dt} + C_d u(t) \\ &= [C_d - C_a M^{-1} K C_v - C_a M^{-1} C_2] x(t) + C M^{-1} B_2 f(t), \tag{4} \\ &= Cx(t) + Df(t) \end{aligned}$$

when they are the linear combination of nodal displacements, velocities, or accelerations.

In these equations, C_a , C_v , and C_d are the selection matrices. The discrete-time state-space model can be obtained after discretization in time:

$$x_{k+1} = Ax_k + Bf_k, \tag{5}$$

$$y_k = Cx_k + Df_k. \tag{6}$$

From the relationship above, the system matrices on continuous-time and the discrete-time can be obtained as follows: $A = e^{A_c(\Delta t)}$ and

$$\begin{aligned} A &= e^{A_c(\Delta t)} B = \int_{k\Delta t}^{(k+1)\Delta t} e^{A_c((k+1)\Delta t - \tau)} d\tau B_c \\ &= (A - I)A_c^{-1} B, \tag{7} \end{aligned}$$

where Δt is the discrete-time step.

When system matrices A , B , C , and D are known, the outputs y_k are measured. However, the inputs cannot be known; thus, f_k remains unknown. In the state-space equation, the measurement noise on the measured outputs should not be neglected.

The state-space equation can be written as follows:

$$x_{k+1} = Ax_k + w_k, \tag{8}$$

$$y_k = Cx_k + v_k, \tag{9}$$

$$w_k = Bf_k \text{ and } v_k = Df_k + n_{y,k}, \tag{10}$$

where $N_{y,k}$ can be considered the measurement noise. The stochastic terms w_k and v_k are unknown in the above equation. However, these variables are assumed to have a white noise nature and an expected value of zero. The covariance matrices can then be defined as follows:

$$E \left(\begin{bmatrix} w_p \\ v_p \end{bmatrix} \begin{bmatrix} w_p^T & v_p^T \end{bmatrix} \right) = \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \cdot \delta(p-q). \tag{11}$$

The states and the output can be separated into a purely stochastic part as follows:

$$\begin{aligned} x_k &= x_k^d + x_k^s, \quad x_{k+1}^d = Ax_k^d + Bf_k, \quad x_{k+1}^s = Ax_k^s + w_k, \\ y_k &= y_k^d + y_k^s, \quad y_k^d = Cx_k^d + Df_k, \quad y_k^s = Cx_k^s + v_k. \end{aligned} \tag{12}$$

The state cannot be calculated exactly because of the stochastic terms. However, x_{k+1}^d can be estimated when the output vector y_k can be measured. The Kalman filter offers a method of determining the optimal linear estimate because of the unbiased and minimum variance of the estimator.

3 Reference-based deterministic-stochastic subspace identification

3.1 IDENTIFICATION OF SYSTEM MATRICES

In some vibration tests, the sensors are less adequate than the test spots in the structures. Hence, several steps may be needed to complete the tests. Several test spots are selected as reference spots to unify every test step. Sensors in the reference spots are stabilized and sustained. In the state-space equation, the system matrices A , B , C , D , Q , R and S are all unknown. The outputs can be grouped into the following block Hankel matrix:

$$Y_{0|2i-1} = \frac{1}{\sqrt{j}} \begin{bmatrix} y_0^{ref} & y_1^{ref} & y_2^{ref} & \dots & y_{j-1}^{ref} \\ y_1^{ref} & y_2^{ref} & y_3^{ref} & \dots & y_j^{ref} \\ \dots & \dots & \dots & \dots & \dots \\ y_{i-1}^{ref} & y_i^{ref} & y_{i+1}^{ref} & \dots & y_{i+j-2}^{ref} \\ y_i & y_{i+1} & y_{i+2} & \dots & y_{i+j-1} \\ y_{i+1} & y_{i+2} & y_{i+3} & \dots & y_{i+j} \\ \dots & \dots & \dots & \dots & \dots \\ y_{2i-1} & y_{2i} & y_{2i+1} & \dots & y_{2i+j-2} \end{bmatrix} = \begin{bmatrix} Y_P^{ref} \\ Y_f \end{bmatrix}. \tag{13}$$

The inputs can also be grouped into the following block Hankel matrix:

$$F_{0|2i-1} = \frac{1}{\sqrt{j}} \begin{bmatrix} f_0 & f_1 & f_2 & \dots & f_{j-1} \\ f_1 & f_2 & f_3 & \dots & f_j \\ \dots & \dots & \dots & \dots & \dots \\ f_{i-1} & f_i & f_{i+1} & \dots & f_{i+j-2} \\ f_i & f_{i+1} & f_{i+2} & \dots & f_{i+j-1} \\ f_{i+1} & f_{i+2} & f_{i+3} & \dots & f_{i+j} \\ \dots & \dots & \dots & \dots & \dots \\ f_{2i-1} & f_{2i} & f_{2i+1} & \dots & f_{2i+j-2} \end{bmatrix} = \begin{bmatrix} F_P \\ F_f \end{bmatrix}. \quad (14)$$

According to the subspace identification theorem, the following equation can be obtained:

$$a.s. \lim_{j \rightarrow \infty} \mathcal{G}_i = \lim_{j \rightarrow \infty} \Gamma_i \hat{X}_i, \quad (15)$$

where \mathcal{G}_i is the oblique projection of the row space of Y_f onto the joint row space of F_P and Y_P^{ref} in the direction of the row space of F_f ,

$$\mathcal{G}_i = Y_f / F_f \begin{bmatrix} F_P \\ Y_P^{ref} \end{bmatrix}, \quad (16)$$

where Γ_i is the extended observability matrix:

$$\Gamma_i = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{i-1} \end{bmatrix}, \quad (17)$$

where \hat{X}_i is the sequence of reference-based Kalman filter states: $\hat{X}_i = [\hat{x}_i \ \hat{x}_{i+1} \ \dots \ \hat{x}_{i+j-1}]$.

The theorem states that the rank of \mathcal{G}_i is equal to the system order n . The matrix Γ_i can be calculated according to the following singular value decomposition:

$$W_1 \mathcal{G}_i W_2 = [U_1 \ U_2] \begin{bmatrix} S_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} = U_1 S_1 V_1^T, \quad (18)$$

where $rank(P_i^{ref}) = n, U_1 \in R^{li \times n}, S_1 \in R^{n \times n}, V_1 \in R^{j \times n}$.

The state sequence of the Kalman filter can be obtained as follows:

$$\begin{aligned} O_i &= U_1 S_1^{1/2}, O_{i-1} = O_i(1:l(i-1),:) \\ \hat{X}_i &= O_i^+ P_i^{ref}, \hat{X}_{i+1} = O_{i-1}^+ P_{i-1}^{ref} \end{aligned} \quad (19)$$

The stochastic state-space model equations can be calculated as follows:

$$\begin{pmatrix} \hat{X}_{i+1} \\ Y_{ii} \end{pmatrix} = \begin{pmatrix} A \\ C \end{pmatrix} (\hat{X}_i) + \begin{pmatrix} w_i \\ v_i \end{pmatrix}. \quad (20)$$

The output sequence is represented as follows:

$$Y_{ii} = \begin{pmatrix} R_{21} & R_{22} & 0 \\ R_{31} & R_{32} & R_{33} \end{pmatrix}. \quad (21)$$

The system and output matrices of the structures have the least squares solution:

$$\begin{pmatrix} A \\ C \end{pmatrix} = \begin{pmatrix} \hat{X}_{i+1} \\ Y_{ii} \end{pmatrix} \hat{X}_i^+. \quad (22)$$

The noise sequence is given by

$$\begin{pmatrix} w_i \\ v_i \end{pmatrix} = \begin{pmatrix} \hat{X}_{i+1} \\ Y_{ii} \end{pmatrix} - \begin{pmatrix} A \\ C \end{pmatrix} (\hat{X}_i). \quad (23)$$

3.2 DETERMINATION OF MODAL PARAMETERS

The modal parameters of the system (eigenfrequencies, damping ratios, and mode shapes) can be obtained from the identified system description (A, C) . An eigenvalue decomposition of A obtains the diagonal matrix Λ of discrete-time system poles λ_i and corresponding right eigenvectors Ψ_i :

$$A = \Psi \Lambda \Psi^{-1}, A \Psi_i = \lambda_i \Psi_i. \quad (24)$$

The continuous-time system poles λ_{ci} can be calculated by the discrete-time system poles λ_i as follows:

$$\lambda_{ci} = \frac{\ln(\lambda_i)}{\Delta t}. \quad (25)$$

The undamped eigenequencies f_i and damping ratios ξ_i can be calculated from the continuous-time system poles λ_{ci} by

$$f_i = \frac{|\lambda_{ci}|}{2\pi} \text{ and } \xi_i = -100 \frac{\lambda_{ci}^R}{|\lambda_{ci}|}. \quad (26)$$

The experimental mode shape Φ_i can be calculated as follows:

$$\Phi = C \Psi, \Phi_i = C \Psi_i. \quad (27)$$

4 Stabilization diagram

As the true system order is often unknown, a practical method is to calculate the model orders n . The true

system poles can be detected by comparing the modal parameters for different model orders. Thus, weakly excited system poles can be detected. This procedure is called stabilization diagram and is one of the novel methods for distinguishing system modals. The basic concept of the stabilization diagram is shown in Figure 1.

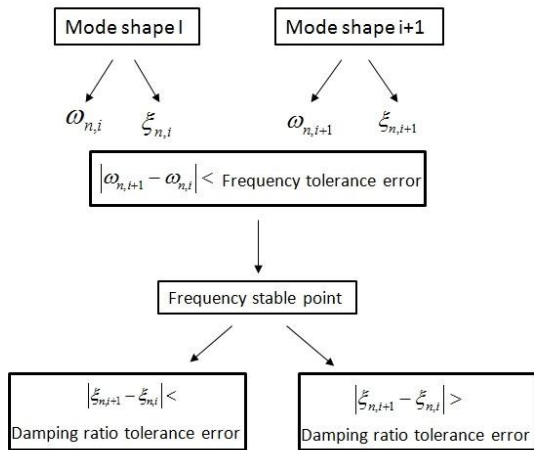


FIGURE 1 Theory of the stabilization diagram

Certain matrices, such as frequency, damping, and mode shape matrices, should be established to obtain accurate results. According to the frequency and damping matrices, every model order frequency and damping can be confirmed because they are both the average of each matrix.

Every point should then be judged according to whether they are stable or not based on the following:

- (1) The deviation between frequency and average frequency.
- (2) The deviation between damping and average damping.

In practice, when the assumed deviation of the damping ratio is under 10%, numerous mode shapes have similar frequencies. Thus, many false mode shapes are eliminated when the deviation of the damping ratio increases.

5 Sample analyses

One of the applications of the SSI modal analysis method is the fracturing pump truck, which has become increasingly important with the development of shale gas. Shale gas has a crucial role in fracturing work. Thus, learning the vibration characteristics of fracturing pump trucks and determining abnormal vibrations is important.

5.1 EXPERIMENT SETUP

An experiment on truck vibration characteristics is carried out to simulate fracturing pump truck vibration characteristics under the support boundary condition, as shown in Figure 2. This experiment consists of 14 reference channels, which can collect all acceleration data.

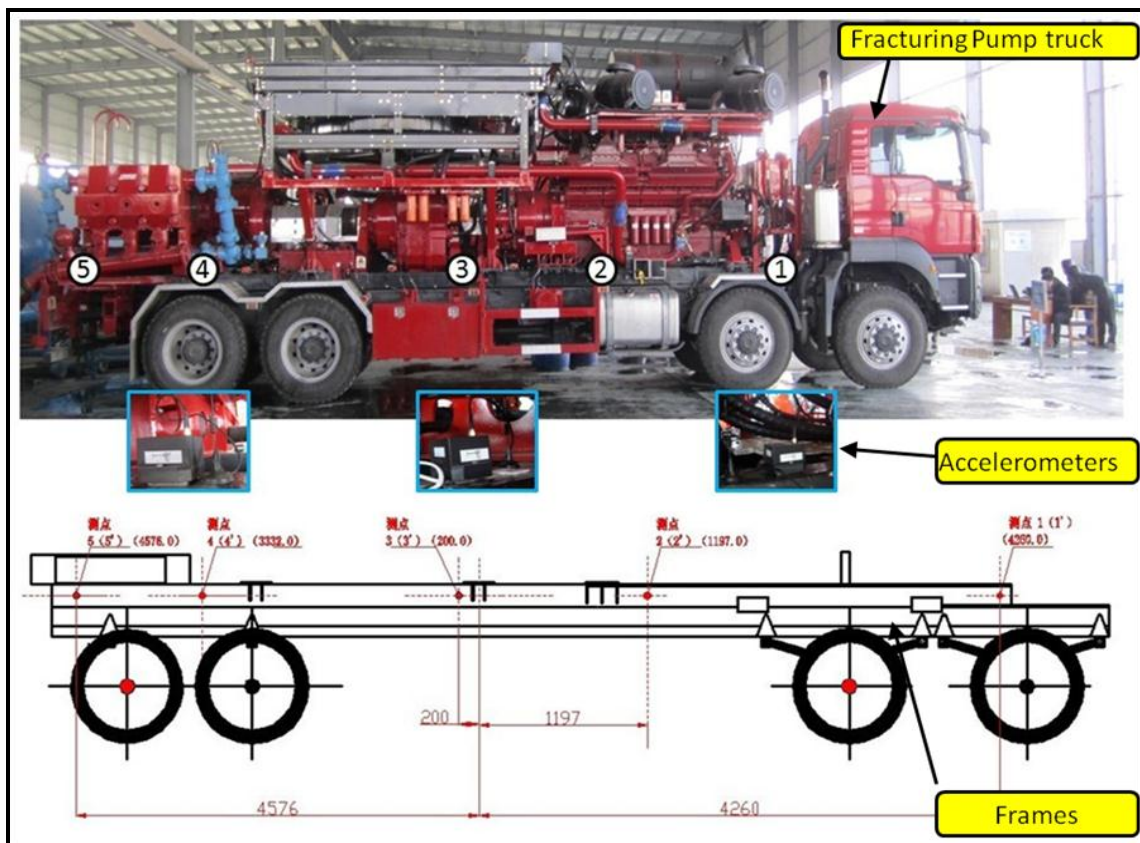


FIGURE 2 Experiment system: fracturing pump truck

5.2 SIMULATION OUTPUT

Simulations are performed to illustrate the function of the stabilization diagram. In these simulations, f_k is white noise, and v_k is a white noise vector. The only assumption of SSI is the infinite amount of measurement data. The stabilization diagram for this simulation is shown in Figure 3. Certain mathematical poles can be removed based on the following criteria: difference in

two consecutive eigenfrequencies $df_i < 1\%$; difference in two consecutive damping ratios $d\xi_i < 5\%$; and the highest modal transfer norms $N_n = 3$. The modal transfer norm n_i is the contribution of each mode to the total positive power spectral density. Given that the system and measurement noise terms are white noises, the contribution of the spurious modes is low enough that the modes are equal to an infinite number of samples.

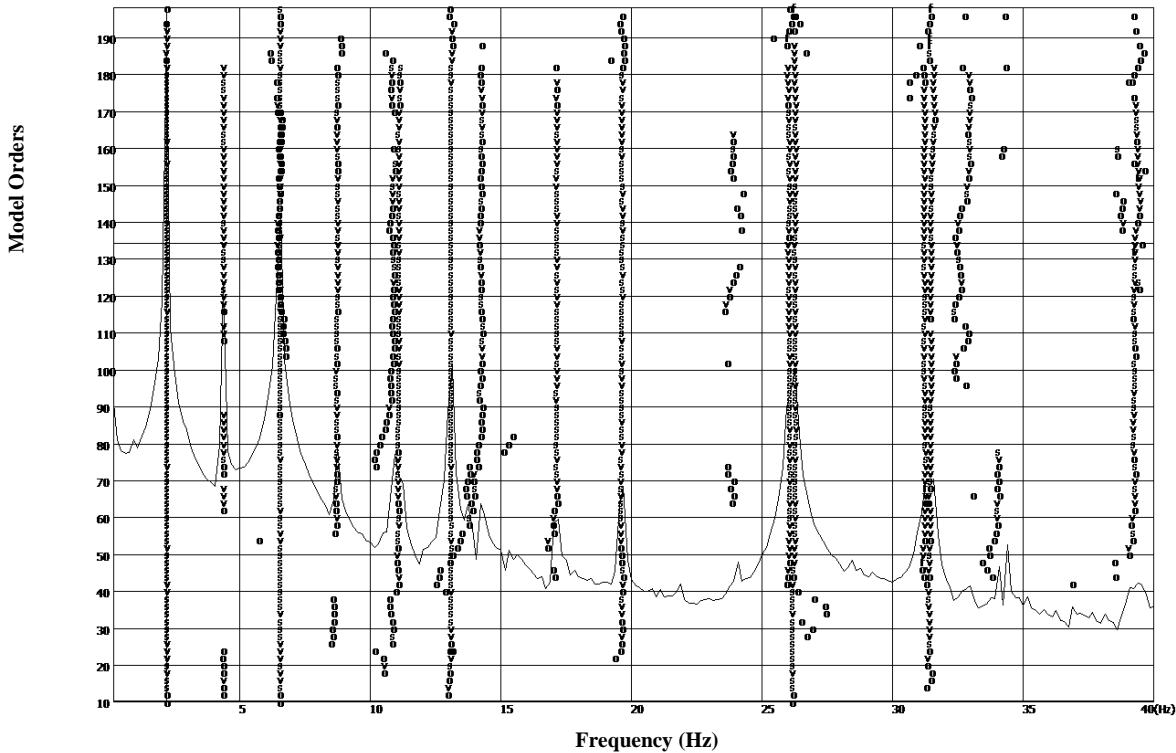


FIGURE 3 Stabilization diagram obtained by applying SSI. Stabilization criteria: 2% for frequencies, 10% for damping ratios, 2% for mode shape correlations, and $df_i < 1\%$, $d\xi_i < 5\%$, $N_n = 3$

Spurious modes are removed as shown in Figure 4. The simulation shows that the stabilization criteria are similar to those of the first simulation, except for the damping ratio deviation. In this simulation, the mode shape shows the operational deflection shapes. The spurious modes that pass the stabilization criteria can be easily detected based on the nature of their mode shapes. The SSI method can distinguish the system mode shape accurately, as shown in Table 1. When the damping ratio is 10%, spurious modals occur and the frequency of these modals and mode shapes are similar, except for the obvious difference in the damping ratio. When the damping ratio is 40%, these spurious modals can be eliminated.

TABLE 1 Dynamic parameters of the fracturing pump truck frame

Number	Frequency/Hz		Measurement
	Damping criteria:10%	Damping criteria:40%	Damping ratio/%
1	2.186	2.186	0.24
2	4.369		0.46
3	4.378	4.378	0.14
4	6.449		0.34
5	6.515		0.27
6	6.539	6.539	0.18
7	8.711	8.711	0.61
8	8.840		0.92
9	10.492		1.15
10	13.037	13.037	0.21
11	15.505		
12	15.535	15.535	0.73
13	15.552		

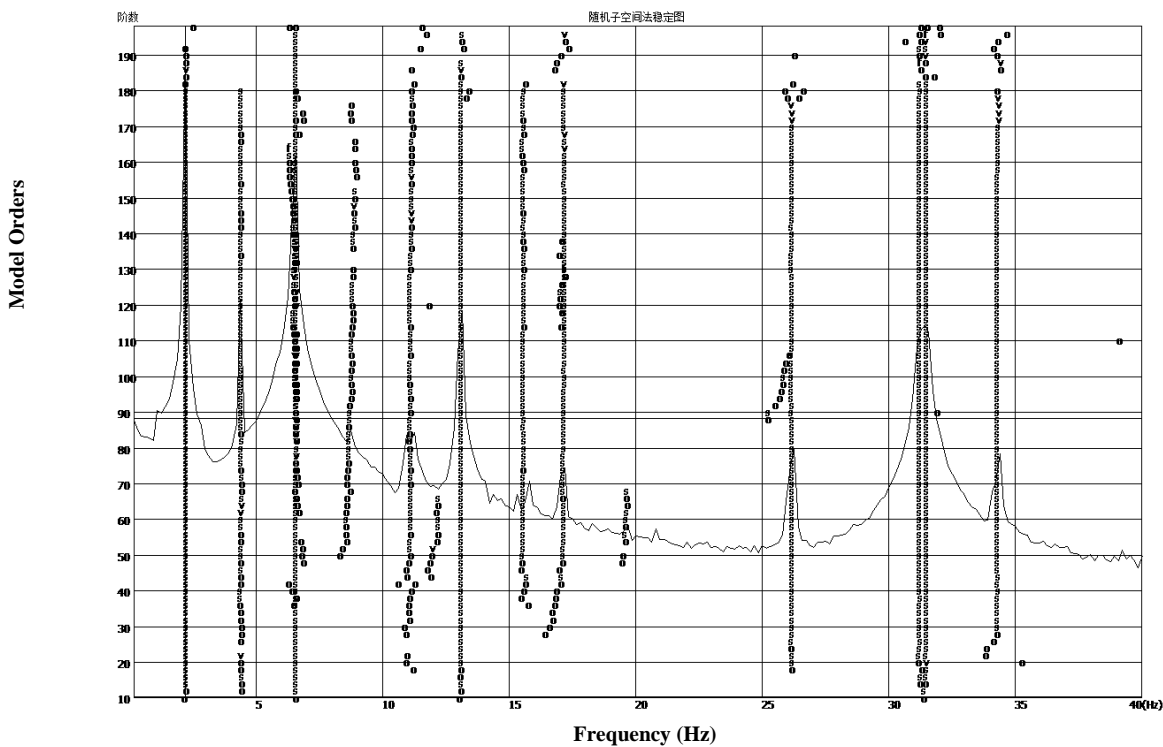


FIGURE 4 Stabilization diagram obtained by applying SSI. Stabilization criteria: 2% for frequencies, 40% for damping ratios, 2% for mode shape correlations, and $df_i < 1\%$, $d\xi_i < 5\%$, $N_n = 3$

5 Conclusions

This paper presents a modal parameter identification method that combines SSI and stabilization diagram. The proposed method is used to evaluate a fracturing pump truck system, and it obtains ideal results. A simulation of the fracturing pump truck shows that the damping ratio can affect the accuracy of the results. In the stabilization diagram, most points can meet the demand for frequency and stability of mode shapes. Therefore, this method can effectively identify system parameters.

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