Research on the laser transmission simulation based on random phase screen in atmospheric turbulent channel

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Abstract

On the basis of collimated Gaussian beams, the paper focused on the modelling and simulating of the transmission of laser beams using two-dimension random phase screens in the atmospheric turbulence channel. Firstly, with the analysis of the transmission model of Gaussian beams through the phase screens, the simulation theory of random phase screens and the depth range model of the phase screens were proposed. Then, In accordance with Kolmogorov atmospheric turbulence theories, a two-dimension random phase screen was built using Fourier transform. Numerical simulation experiments were conducted with low frequency compensation to simulate the propagation of Gaussian collimated beam in Kolmogorov turbulence. Finally, the two-dimension random phase screen was testified by the phase structure function. The results showed that the approach of simulating the random phase screen using Fourier transform was appropriate after compensating the low frequency.

Keywords: random phase screen, atmospheric turbulence, Gaussian beam, Fourier transform, Kolmogorov

1 Introduction

Atmospheric turbulence is one of the important factors affecting the beam propagation. Numerical simulation method for beam propagation is an effective way to study the atmospheric turbulence besides experimental and theoretical research. Several numerical simulation methods have been proposed to generate the random phase screen for numerically simulating the atmospheric turbulence [1, 2]. Numerical simulating methods can be basically divided into two categories. The first one was proposed by Mc Glamery, which was indirect simulation of the frequency field using Fourier transform. The other was direct simulation of the spatial domain, which can represent the phase front using an orthogonal complete set of Zernike polynomial [3]. Moreover, Yan put forward a random numerical simulation method of the atmospheric turbulence based on fractals, Wang et al, proposed a simulating model on laser transmission in the atmosphere through any thick random phase screen, and Andrews et al, studied the statistical characteristics of the transmission in thin random phase screen [4].

In order to study effect of atmospheric turbulence on the propagation properties of the laser beam, in the paper, the random phase screen established by the Fourier transform is simulated in compliance with Kolmogorov atmospheric turbulence theories. A new method for laser beam propagation research in atmospheric turbulence is put forward to overcome the limitations of experimental and theoretical approaches.

2 Simulating theories on random phase screen

As to the collimated Gaussian beams propagation through the atmospheric turbulence, let ω_0 be the beam-waist radius at the input end. And denote bean-waist radius as ω after the beams transmit a distance of Lkm. In this process, if the changes caused by the fluctuation of atmospheric refractivity is sufficiently small, the continuous atmospheric turbulence can be divided into a series of phase screens (sampling grid) with Δ_z per thickness. The collimated Gaussian beams located in the front surface of Z_i screen will be transmitted to the back surface of the screen through the atmospheric turbulence with Δz thickness. Then the phase modulation caused by the phase screen in the atmospheric turbulence forms the ultimate optical field distribution E_i. After Field E_i passes through the same atmospheric turbulence and is modulated, it arrives at the back surface of Z_{i+1}. There are three steps to generate the two-dimension phase screens using Fourier transform. First, a matrix with random numerals obeying the Gaussian distribution is generated. Second, the air power spectral function adhering to the Kolmogorov turbulence distribution filters the matrix generated in the first step. Finally, the new filtered

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complex Gaussian matrix is computed with the inverse Fourier transform to obtain the random phase. In this process, the phase distortion occurring on each phase screen is accumulated on Gaussian optical field E, and the optical fielding after passing through i phase screens is expressed as follows [5, 6]:

$$E_{i+1} = F^{-1}\{F[E_i \times \exp(i\phi)] \times \exp(-is_i)\}. \quad (1)$$

In order to ensure that the phase changes caused by each phase screen is sufficiently small and meanwhile the propagation distance L in the atmospheric turbulence can be substituted by calculus of Δz, Thus, the thickness of the two-dimension random phase screen should be infinitely thin so that the generated optical waves will only affect the phase of the Gaussian optical waves while with no obvious influence on the amplitude. Therefore, the following condition must be satisfied.

$$\Delta z << \lambda / \sigma_n, \quad (2)$$

where, λ is the wavelength of Gauss beam and $\sigma^2{}_n$ is the average variance of the refraction rates fluctuation [5].

The adjacent phase screens should be mutually independent and meanwhile spatially connected. And the front and back phase screen should have some extent of correlation. Therefore, the depth of the phase screen Δz should exceed the outer scale of the turbulence. That is:

$$\Delta z > L_0, \quad (3)$$

where L_0 is the outer scale of the turbulence [5].

In Fourier transform and inverse Fourier transform, the thickness Δz of a random phase screen replaced the thickness calculus of the whole screen. However, the prerequisite to do so is that the refraction in the phase screen is evenly distributed and the transmission of optical lights follows the principles of geometric optics. Hence, the scale of Fresnel should be smaller than that inside the turbulence. That is:

$$\Delta z < l_0^2 / \lambda, \quad (4)$$

where l_0 is the inner scale of the turbulence.

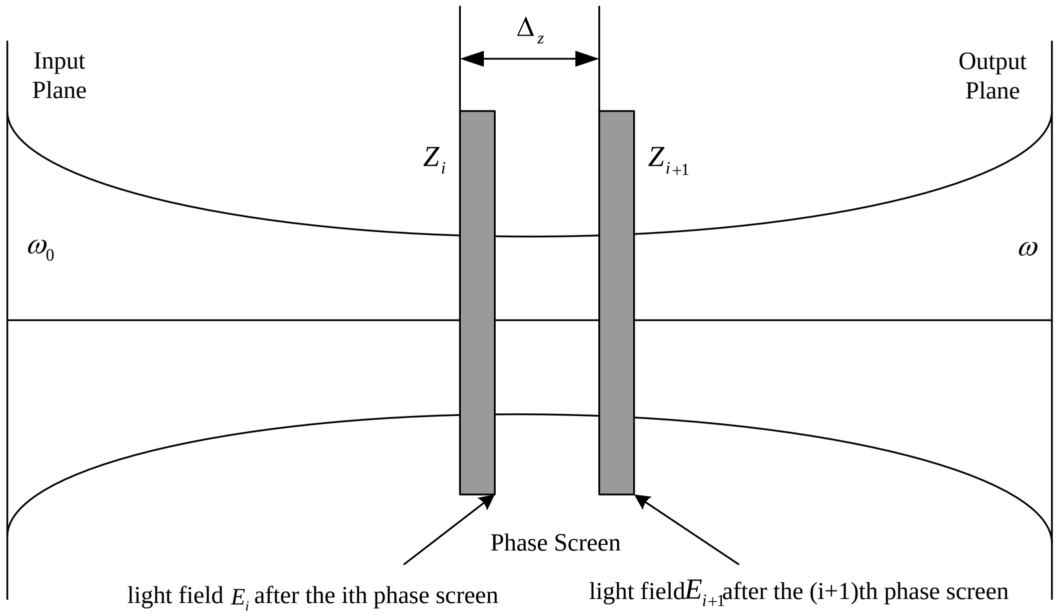

FIGURE 1 Model of the transmission of Gaussian beams through the phase screen

3 The construction of the random phase screens

Along the propagation direction of the Gaussian beams, the atmospheric turbulence $(0, L)$ is uniformly divided into N phase screens. Thus, the thickness of each phase screen is the same as $\Delta z = L/N$. First, using the Fourier transform and the Kolmogorov turbulence distribution theories, each phase screen grid with no spatial correlation is calculated to obtain the complex Gaussian random matrix. Then the modified Von Karman model is used for filtering. Moreover, the random phase is obtained based on the spatially correlated phase screens after inverse Fourier transformation. Finally, the same method is used to generate the next new phase screen with the same spatial distribution. The newly generated random numbers of the screens are not completely new and thus the new screens have a spatial correlation with both the front and the back screens [7]. The above process can be defined as [8, 9]:

$$\mathrm{X}(m\Delta_x, n\Delta_y) = \sum_{m'=-(Nx)/2}^{(Nx/2)-1} \sum_{n'=-(Ny)/2}^{(Ny/2)-1} [a(m,n) + jb(m,n)] \times \exp[2\pi j(\frac{m'm}{N_x} + \frac{n'n}{N_y})], \quad (5)$$

where N_x and N_y represent the dimensions in the direction of x, y in the matrix, Δ_{xk} and Δ_y are the intervals of the sampling grid in the direction of x, y. $a(m,n)$ and $b(m,n)$ are mutually independent Hermitian

Gaussian random numbers with the mean zero. The variance is [10]:

$$< a^2(m,n) >=< b^2(m,n) >= \frac{\sqrt{0.00058 r_0^{-5/6}}}{\sqrt{G_x G_y}} \times \Delta k_x \Delta k_y F(m\Delta k_x, n\Delta k_y, z)\Phi_n(\Delta k_x, \Delta k_y), \quad (6)$$

where $F(m\Delta k_x, n\Delta k_y, z)$ is the filtering function and can be rewritten as the following equation:

$$F(m\Delta k_x, n\Delta k_y, z) = [(2\pi)^3 / \lambda^2 \times \Delta z \times 0.33 C_n^{\ 2}(k^2 + k_0^{\ 2})^{-11/6} \exp(-k^2 / k_m^{\ 2})]^{1/2}, \quad (7)$$

where $k = 2\pi[1/(m\Delta k_x)^2 + 1/(m\Delta k_y)^2]^{-1/2}$, and $<>$ refers to the overall average. $F(\cdot)$ is the spatial filtering function of the phase screen, and it is also the function of the propagation distance z. Δk_x and Δk_y are the grid intervals on the phase screens. G_x and G_y represent the size of the phase screen, r_0 is the atmospheric coherence length and Δz is the depth of the turbulence layer. $\Phi_n(\cdot)$ is the function calculating the refraction power density. Here the power spectral density function derived from the Kolmogorov model is adopted.

If the modified Von Karman power spectral density is substituted for power spectral density function $\Phi_n(\cdot)$. And the plane wave can be given by the following Equation [11]:

$$\Phi_n(\cdot) = 0.49 r_0^{-5/3} \frac{\exp(-k^2 / k_m^{\ 2})}{(k2 + k_0^{\ 2})^2}, \quad (8)$$

where $k = 2\pi f$, $k_0 = 2\pi f_0$ and $k_m = 2\pi f_m$.

As seen from above equations, it is easy to use inverse Kolmogorov to construct the two-dimension random phase screens, but the lack of samples on the spatially low frequency part leads to the absence of power spectrum at the low frequency components in this phase screen, thus resulting in a relatively low accuracy of the generated phase screen. Hence low frequency compensation is necessary for improving the accuracy. With the help of Lane's ideas, interpolation merge is conducted based on the re-sampling of the Fourier low frequency subharmonics so as to make low frequency compensation to the subharmonics in this screen. The equation can be rewritten as follows:

$$\Phi_{SH}(m\Delta_x, n\Delta_y) = \sum_{m'=-1}^{1} \sum_{n'=-1}^{1} \sum_{p=1}^{N_p} [a(m,n) + jb(m,n)] \times \exp[2\pi j \times 3^{-p}(\frac{m'm}{N_x} + \frac{n'n}{N_y})], \quad (9)$$

where p refers to the subharmonic series.

The inverse transform method is adopted to simulate the Kolmogorov spectrum phase screens under the conditions that the wave length is 1.06×10^{-5}m, the size of the phase screen is 4.8m×4.8m, $W_0 = 0.8\times10^{-6} m$, the propagation distance is $L_0 = 10km$, the sampling points is $N_x = N_y = 1024$, and the interval between each phase screen is $\Delta z = 500m$.

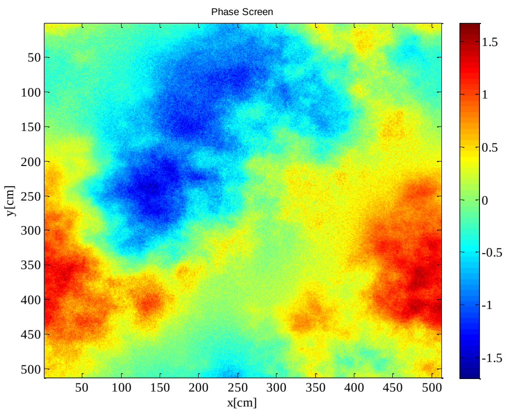

FIGURE 2a Two-dimension figure of the random phase screen after the harmonics are added

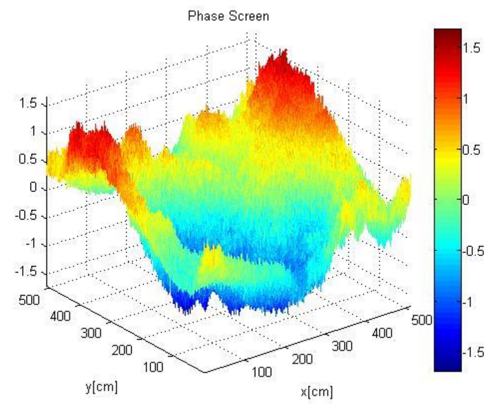

FIGURE 2b Three-dimension figure of the random phase screen after the harmonics are added

From Figure 2a and 2b the low frequency part of phase screen is more apparent after the overlay of sub-harmonics, which shows the overlay of harmonics can effectively compensate the lack of low frequency caused by the Fourier transform.

4 Propagation step by step

As illustrated in Figure 1, in the beam propagation process, the collimated Gaussian beams are quite similar to the Gaussian beams at the transmitting terminal (the input plane) with the same beam-waist radius ω_0. After the beams transmit for a distance of L in the atmospheric turbulence, they remain similar to Gaussian beams at the receiving terminal with the beam-waist radius ω.

According to the Gauss equation, the optical field distribution in the input plane is [12]:

$$U_G(x,y,z) = \frac{A}{q(z)} \exp ik\left(\frac{x^2 + y^2}{2q(z)}\right). \quad (10)$$

We can obtain the following equation from Collins integral equation:

$$\frac{1}{q(z)}=\frac{1}{R(z)}-\frac{i\lambda M^2}{\pi W^2(z)}, \quad (11)$$

were:

$$W^2(z)=W_0^2(z)\left[1+\left(\frac{\lambda z}{\pi W_0^2}\right)^2\right], R(z)=z\left[1+\left(\frac{\lambda z}{\pi W_0^2}\right)^2\right].$$

are the isophase surface curvature radius and the beam radius of the Gaussian beam respectively. M^2 is defined as the beam quality factor and for the fundamental-mode Gaussian beam, its value is 1. W_0 is the beam waist radius, λ is the wavelength of the laser being transmitted and z is the position where the laser is located on its transmitting route.

From the equation of $R(z)$, we can find $R(z)\to\infty$. Substituting $R(z)\to\infty$ into Equation (11), the result is:

$$q(z)=\frac{i\pi W_0^2}{\lambda M^2}. \quad (12)$$

If q(z) is substituted into Equation (10), the optical field distribution is as follows:

$$U_G(x,y,z)=-\frac{iA\lambda M^2}{\pi W_0^2}\exp\left(M^2\frac{x^2+y^2}{W_0^2}\right). \quad (13)$$

Diffraction occurs after the beams are collimated, and the diffracted beam continue to transmit for a distance of Δz, in the end we can represent the optical field distribution as follows based on Fresnel diffraction integral:

$$U_G(x_2,y_2,z_2)=-\frac{\exp(ikz_2)}{i\lambda\Delta z}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}U_G(x_1,y_1,z_1)\times \exp\left[\frac{i\pi M^2}{\lambda\Delta z}\frac{(x_2-x_1)^2+(y_2-y_1)^2}{W_0^2}\right]dx_1dx_2 . \quad (14)$$

5 Simulating

When the quasi-Gaussian beam propagates some distance, beam expander will occur. For expanded beam, the emission aperture and receiving aperture will change, its new diffraction diameter and receiving aperture is defined as [13]:

$$D_1'=D_1+c\frac{\lambda\Delta z}{r_{0,rev}}, \quad (15)$$

$$D_2'=D_2+c\frac{\lambda\Delta z}{r_{0,rev}}, \quad (16)$$

where $r_{0,rev}$ is atmospheric coherence diameter for back-propagation and c is an adjustment factor of turbulence sensitive.

In order to simulate the beam propagation process with more accuracy, sampling points of the transmit and receive aperture plane aperture plane, sampling interval, maximum allowable interval between planes and minimum number of transmission steps should be selected [14]. The phase difference between two adjacent points on the phase screen should be smaller than π according to the Nyquist law. In other words, the grid sampling intervals Δk_x and Δk_y on the phase screen satisfy the conditions [7]:

$$\left|\Psi(k_x+\Delta k_x,k_y,z)-\Psi(k_x,k_y,z)\right|<\pi, \quad (17)$$

$$\left|\Psi(k_x,k_y+\Delta k_y,z)-\Psi(k_x,k_y,z)\right|<\pi. \quad (18)$$

The parameters of the system are selected as follows for the purpose of simulation:

TABLE 1 The values of the simulation parameters and their physical significance

Parameter	Value	Physical significance
W_0	0.05m	Beam waist
λ	1.06μm	The wavelength of the reflected laser
Δz	500m	Interval between the phase screens
N_x, N_y	1024	Sampling points
L_0	20m	Outer turbulence scale
l_0	5m	Inner turbulence scale
C_n^2	$10^{-17}m^{-2/3}$	Atmosphere structure parameters
M^2	1	Gaussian beam quality factors
G_x, G_y	4.8m	Size of the phase screen
r_0	0.810mm	Atmospheric coherence length
D_1	0.1m	Transmitter aperture diameter
D_2	0.2m	Receiver calibre diameter

After the Kolmogorov atmospheric turbulence is introduced into the optical propagation route, the light intensity and phase distribution in receiving aperture are shown in Figures 3a and 3b.

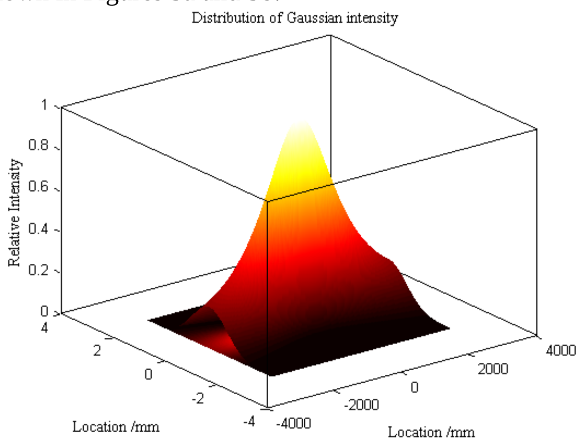

FIGURE 3a Three-dimension figure of Optical field distribution of collimated Gaussian beams in the receiving plane

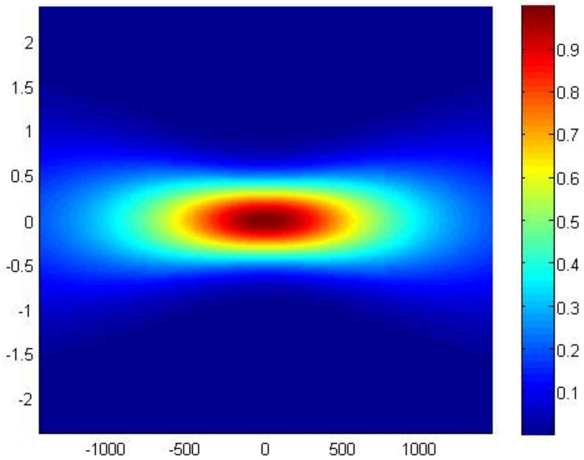

FIGURE 3b Two-dimension figure of Optical field distribution of collimated Gaussian beams in the receiving plane

6 Testifying the results after simulating the turbulent atmosphere

The statistical characteristics of the atmospheric turbulence phase can be depicted by the phase structure function, thus the structure function can be used as a benchmark to testify if the simulated phase screen is correct. Thus, Fried offered the definition equation of the structure function corresponding to Kolmogorov spectrum [15]:

$$D(r) = 6.88(r / r_0)^{5/3} \tag{19}$$

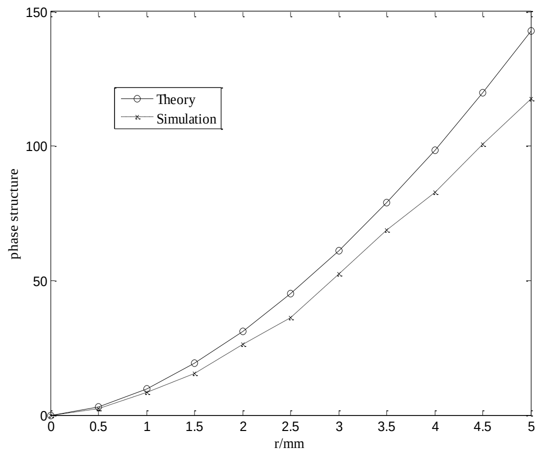

FIGURE4 Comparison between values of the phase structure function

In the experiment, r_0=0.810mm and Y-axis are the values of the phase structure function. It can be found from Figure 4 that due to the sampling frequency of the Fourier transform, part of the low frequency is lost in the phase screen generated. Thus the structure function of the phase screen obviously lacks low frequency part compared to the theoretical situation, while the performance is the same in the high frequency parts.

7 Conclusions

Based on Kolmogorov turbulence theory and power spectral inversion method, the model and simulation method on laser beams propagation through two-dimension random phase screens in the atmospheric turbulence channel were proposed and testified by phase structure function. In the method, the random phase screen was established by the Fourier transform, and step-by-step transmitting approach was used to simulate the propagation of collimated Gaussian beams in Kolmogorov turbulence. According to the divergence between phase structure function and theoretical results, the accuracy of simulated phase screen was analysed. Simulation results showed the proposed method in the paper was appropriate after compensating the low frequency and can be used to calculate the light propagation, which will be more practical meaningful to evaluate and test the phase screen.

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