

# Bilinear model for ontology mapping

Jian-Zhang Wu<sup>1, 2\*</sup>, Yu Xiao<sup>3</sup>, Wei Gao<sup>4</sup>

<sup>1</sup>School of Computer Science & Engineer, Southeast University, Nanjing 210096, China

<sup>2</sup>Key Laboratory of Computer Network and Information Integration, Southeast University, Nanjing 210096, China

<sup>3</sup>School of Continuing Education, Southeast University, Nanjing 210096, China

<sup>4</sup>School of Information Science and Technology, Yunnan Normal University, Kunming, 650500, China

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## Abstract

As a model of concept representation, ontology has widely applied to various disciplines. Ontology mapping is used to create the link between different ontologies. In this paper, we present a new ontology mapping algorithm by virtue of bilinear model. The linear mapping pair is given by the iterative procedure. Two strategies are manifested to obtain the finally ontology mapping. The simulation experimental results show that the proposed new technologies have high accuracy and efficiency on ontology mapping in certain applications.

*Keywords:* ontology, ontology mapping, linear mapping, bilinear model, dimensionality reduction

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## 1 Introduction

As a knowledge representation and conceptual shared model, ontology has been applied in image retrieval, knowledge management and information retrieval search extension. Acting as an effective concept semantic model, ontology is also employed in disciplines beyond computer science, such as social science (for instance, see [1]), biology science [2] and geography science [3].

The ontology model is actually a graph  $G=(V,E)$ , each vertex  $v$  in an ontology graph  $G$  represents a concept and each edge  $e=v_i v_j$  on an ontology graph  $G$  represents a relationship between concepts  $v_i$  and  $v_j$ . The aim of ontology mapping is to bridge the link between two or more ontologies. Let  $G_1$  and  $G_2$  be two ontology graphs corresponding to ontology  $O_1$  and  $O_2$  respectively. For each  $v \in G_1$ , find a set  $S_v \subseteq V(G_2)$  where the concepts corresponding to vertices in  $S_v$  are semantically close to the concept corresponding to  $v$ . One method to get such mapping is, for each  $v \in G_1$ , to compute the similarity  $S(v, v_j)$  where  $v_j \in V(G_2)$  and to choose a parameter  $0 < M < 1$ . Then  $S_v$  is a collection such that the element in  $S_v$  satisfies  $S(v, v_j) \geq M$ . In this point of view, the essence of ontology mapping is to obtain a similarity function  $S$  and select a suitable parameter  $M$ . In our article, we focus on the technologies to yield an optimal similarity function  $S$  from dimensionality reduction standpoint. In fact, our approach for obtaining such similarity function is based on the linear mapping pair.

For ontology similarity measure, there are several effective learning tricks. Wang et al. [4] proposed to learn a score function which mapping each vertex to a real number, and the similarity between two vertices can

be measured according to the difference of real number they correspond to. Huang et al., [5] presented a fast ontology algorithm for calculating the ontology similarity in a short time. Gao and Liang [6] raised that the optimal ontology function can be determined by optimizing NDCG measure, and applied such idea in physics education. Gao and Gao [7] deduced the ontology function using the regression approach. Huang et al., [8] obtained ontology similarity function based on half transductive learning. Gao and Xu [9] explored the learning theory approach for ontology similarity computation using  $k$ -partite ranking method. Zhu and Gao [10] proposed a new criterion for ontology computation from AUC and multi-dividing standpoint. Gao et al., [11] presented a new ontology mapping algorithm using harmonic analysis and diffusion regularization on hypergraph. Very recently, Gao and Shi [12] proposed a new ontology similarity computation technology such that the new calculation model considers operational cost in the real implement.

In this paper, we determine the new ontology mapping algorithm based on dimensionality reduction idea and bilinear learning model. Using the optimization algorithm, we determine the linear mapping  $(L_1, L_2)$  to compute the similarity of vertices from two ontologies. The experiment is designed to show the efficiency of the algorithm.

## 2 Model and algorithm

For each vertex  $v$ , we use a vector to represent all its information. For two ontologies  $O_1$  and  $O_2$ , their structures can be determined by two ontology graphs

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\* Corresponding author e-mail: jzww@njnet.edu.cn

$G_1=(V_1,E_1)$  and  $G_2=(V_2,E_2)$  respectively. Suppose that  $V_1 \subset \mathbb{R}^{d_1}$  and  $V_2 \subset \mathbb{R}^{d_2}$ . That is to say, we use a vector with dimension  $d_1$  to represent the information of vertex in  $V_1$  and use a vector with dimension  $d_2$  to represent the information of vertex in  $V_2$ . For any  $v_i \in V_1$ , and  $v_j \in V_2$ ,  $S(v_i,v_j)=S_{ij}$  indicates the similarity between concepts corresponding to  $v_i$  and  $v_j$ . Our goal is to learn an optimal similarity function  $S$  based on the sample triple  $D=\{(v_i,v_j,S_{ij})\}$ , where  $v_i \in V_1$ ,  $v_j \in V_2$ . For such triple  $D$ , let  $V_1^D = \{v_i\} \subseteq V_1$ ,  $V_2^D = \{v_j\} \subseteq V_2$ ,  $n_1 = |V_1^D|$  and  $n_2 = |V_2^D|$ .

We are interested in searching a linear mapping pair  $(L_1,L_2)$  such that the corresponding images  $L_1^T v_i$  and  $L_2^T v_j$  are in the same  $d$ -dimensional latent space  $L$  with  $d \ll \min\{d_1,d_2\}$  and the degree of similarity between ontology vertices  $v_i \in V_1$  and  $v_j \in V_2$  can be reduced to  $L$ 's dot product:

$$D_{L_1,L_2}(v_i,v_j) = v_i^T L_1 L_2^T v_j.$$

By virtue of the trick used in [13] for kernel learning, we aim to maximize the following expected version:

$$E_{v_i,v_j} \{S(v_i,v_j) D_{L_1,L_2}(v_i,v_j)\} = E_{v_i} E_{v_j|v_i} \{S(v_i,v_j) v_i^T L_1 L_2^T v_j\}. \tag{1}$$

The Equation (1) could be estimated as follows:

$$\frac{1}{n_1 n_2} \sum_{v_i \in V_1^D} \sum_{v_j \in V_2^D} S_{ij} v_i^T L_1 L_2^T v_j.$$

Hence, the ontology mapping problem is boiled down to

$$\arg \max_{L_1,L_2} \frac{1}{n_1 n_2} \sum_{v_i \in V_1^D} \sum_{v_j \in V_2^D} S_{ij} v_i^T L_1 L_2^T v_j, \tag{2}$$

s.t.  $L_1 \in H_1$ ,  $L_2 \in H_2$ , where  $H_1$  and  $H_2$  are the hypothesis spaces for  $L_1$  and  $L_2$  respectively. Since the final computational model is linear in view of both ontology vertices  $v_i$  and  $v_j$ , learning model (2) is actually a bilinear model for calculating similarity in two spaces.

We apply  $l_1$  norm and  $l_2$  norm constraints on  $L_1$  and  $L_2$ . Let  $|\cdot|$  and  $\|\cdot\|$  be  $l_1$ -norm and  $l_2$ -norm respectively, and  $l_{v_i,x}$  and  $l_{v_j,y}$  be the  $x$ -th and  $y$ -th row of  $L_1$  and  $L_2$ . Specifically, we introduce two hypothesis spaces as:

$$H_1 = \{L_1 \mid |l_{v_i,x}| \leq \lambda_{v_i}, \|l_{v_i,x}\| \leq \theta_{v_i}, x = 1, \dots, d_1\},$$

$$H_2 = \{L_2 \mid |l_{v_j,y}| \leq \lambda_{v_j}, \|l_{v_j,y}\| \leq \theta_{v_j}, y = 1, \dots, d_2\},$$

where  $\{\lambda_{v_i}, \theta_{v_i}, \lambda_{v_j}, \theta_{v_j}\}$  are parameters selected by

experts. Here the constraints relying on  $l_1$ -norm will induce row-wise sparsity in  $L_1$  and  $L_2$ . Furthermore, the  $l_2$ -norm on rows with regularization can avoid degenerative solutions. By virtue of the definition of  $H_1$  and  $H_2$ , we infer the following program:

$$\arg \max_{L_1,L_2} \frac{1}{n_1 n_2} \sum_{v_i \in V_1^D} \sum_{v_j \in V_2^D} S_{ij} v_i^T L_1 L_2^T v_j, \tag{3}$$

s.t.:

$$|l_{v_i,x}| \leq \lambda_{v_i}, \|l_{v_i,x}\| \leq \theta_{v_i}, |l_{v_j,y}| \leq \lambda_{v_j}, \|l_{v_j,y}\| \leq \theta_{v_j}, \quad x \in \{1, \dots, d_1\}, \\ y \in \{1, \dots, d_2\}.$$

In practice reality, we solve the following variant version of Equation (3) for easier computation

$$\arg \min_{L_1,L_2} -\frac{1}{n_1 n_2} \sum_{v_i \in V_1^D} \sum_{v_j \in V_2^D} S_{ij} v_i^T L_1 L_2^T v_j + \tag{4}$$

$$\beta \sum_{x=1}^{d_1} |l_{v_i,x}| + \gamma \sum_{y=1}^{d_2} |l_{v_j,y}|,$$

$$\text{s.t. } \|l_{v_i,x}\| \leq \theta_{v_i}, \|l_{v_j,y}\| \leq \theta_{v_j}, \quad x \in \{1, \dots, d_1\}, \quad y \in \{1, \dots, d_2\},$$

where  $\beta > 0$  and  $\gamma > 0$  are the balance parameters to control the trade-off between objective term and penalty term. For given  $L_2$ , the objective mapping of Equation (4) can be re-represented as:

$$\sum_{x=1}^{d_1} \left\{ \left( -\frac{1}{n_1 n_2} \sum_{v_i \in V_1^D} \sum_{v_j \in V_2^D} v_i^x S_{ij} L_2^T v_j \right)^T l_{v_i,x} + \beta |l_{v_i,x}| \right\}.$$

By using  $\omega_x = [\omega_x^1, \omega_x^2, \dots, \omega_x^{d_1}]^T$  to represent the  $d$ -dimensional  $\frac{1}{n_1 n_2} \sum_{v_i \in V_1^D} \sum_{v_j \in V_2^D} v_i^x S_{ij} L_2^T v_j$ , we infer the optimal  $l_{v_i,x}$  as:

$$(l_{v_i,x}^k)^* = C_{v_i} \left( \max(|\omega_v^k| - \beta, 0) \text{sign}(\omega_v^k) \right), \quad k \in \{1, \dots, d\}, \tag{5}$$

where  $l_{v_i,x}^k$  is the  $k$ -th element of  $l_{v_i,x}$ :

$$\text{sign}(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$$

and  $C_{v_i}$  is a constant which makes  $\|l_{v_i,x}^*\| = \theta_{v_i}$  if there exist non-zero elements in  $l_{v_i,x}^*$ , and  $C_{v_i} = 0$  otherwise.

For given  $L_1$ , the objective mapping of Equation (4) can similarly re-written as:

$$\sum_{y=1}^{d_2} \left\{ \left[ -\frac{1}{n_1 n_2} \sum_{v_i \in V_1^D} \sum_{v_j \in V_2^D} v_j^y S_{ij} L_1^T v_i \right]^T l_{v_j, y} + \gamma |l_{v_j, y}| \right\}.$$

In terms of the same fashion, we use  $\eta_y = [\eta_y^1, \eta_y^2, \dots, \eta_y^d]^T$  to represent the  $d$ -dimensional

$\frac{1}{n_1 n_2} \sum_{v_i \in V_1^D} \sum_{v_j \in V_2^D} v_j^y S_{ij} L_1^T v_i$ , we yield the optimal  $l_{v_j, y}$  as:

$$(l_{v_j, y}^k)^* = C_{v_j} (\max(|\eta_{v_j}^k| - \gamma, 0) \text{sign}(\eta_{v_j}^k)), \quad k \in \{1, \dots, d\}, \quad (6)$$

where  $l_{v_j, y}^k$  is the  $k$ -th element of  $l_{v_j, y}$ , and  $C_{v_j}$  is a constant which makes  $\|l_{v_j, y}^*\| = \theta_{v_j}$  if there exist non-zero elements in  $l_{v_j, y}^*$ , and  $C_{v_j} = 0$  otherwise.

Let:

$$w_{v_i, x} = \frac{1}{n_1 n_2} \sum_{v_i \in V_1^D} \sum_{v_j \in V_2^D} v_i^x S_{ij} v_j$$

and

$$w_{v_j, y} = \frac{1}{n_1 n_2} \sum_{v_i \in V_1^D} \sum_{v_j \in V_2^D} v_j^y S_{ij} v_i,$$

which does not depend on the change of  $L_1$  and  $L_2$ , and can be pre-calculated. It is easy to verify that:

$$\frac{1}{n_1 n_2} \sum_{v_i \in V_1^D} \sum_{v_j \in V_2^D} v_i^x S_{ij} L_2^T v_j = L_2^T w_{v_i, x} =$$

and

$$\frac{1}{n_1 n_2} \sum_{v_i \in V_1^D} \sum_{v_j \in V_2^D} v_j^y S_{ij} L_1^T v_i = L_1^T w_{v_j, y}.$$

Let  $N_{v_i}$  be the average number of non-zeros in all  $v_i$  per dimension and  $N_{v_j}$  be the average number of non-zeros in all  $v_j$  per dimension,  $\tilde{n}_1$  be the average number of related  $v_i$  samples per  $v_j$  and  $\tilde{n}_2$  be the average number of related  $v_j$  samples per  $v_i$ ,  $c_1$  be the average number of non-zeros in each  $v_i$  sample and  $c_2$  be the average number of non-zeros in each  $v_j$  sample. Now, we present the following two algorithms:

**Algorithm 1.** Calculating  $w_{v_i, x}$  and  $w_{v_j, y}$

Input  $D = \{(v_i, v_j, S_{ij})\}$ , where  $1 \leq i \leq n_1$  and  $1 \leq j \leq n_2$ .

For  $x=1:d_1$ ,  $w_{v_i, x} \leftarrow 0$ ; For  $y=1:d_2$ ,  $w_{v_j, y} \leftarrow 0$ .

For  $x=1:d_1$ ,  $i=1:n_1, j=1:d_2$ ,  $w_{v_i, x} \leftarrow w_{v_i, x} + \frac{1}{n_1 n_2} v_i^x S_{ij} v_j$ .

For  $y=1:d_2$ ,  $i=1:n_1, j=1:d_2$ ,  $w_{v_j, y} \leftarrow w_{v_j, y} + \frac{1}{n_1 n_2} v_j^y S_{ij} v_i$ .

Output:  $\{w_{v_i, x}\}_{x=1}^{d_1}$  and  $\{w_{v_j, y}\}_{y=1}^{d_2}$ .

**Algorithm 2.** Calculating  $L_1^t$  and  $L_2^t$ .

Input  $\{w_{v_i, x}\}_{x=1}^{d_1}$ ,  $\{w_{v_j, y}\}_{y=1}^{d_2}$ ,  $d$ ,  $\beta$ ,  $\gamma$ ,  $\theta_1$ ,  $\theta_2$ , and set  $L_1^0$  and  $L_2^0$  randomly  $t \leftarrow 0$ .

While  $t \leq T$ . For  $x=1:d_1$ , compute  $\omega_x$  using  $(L_1^t)^T w_{v_i, x}$  and determine  $(l_{v_i, x})^*$  in terms of Equation (5), update  $L_1^{t+1}$ ;

For  $y=1:d_2$ , compute  $\eta_y$  using  $(L_2^t)^T w_{v_j, y}$  and determine  $(l_{v_j, y})^*$  in terms of Equation (6), update  $L_2^{t+1}$ ;  $t \leftarrow t+1$ .

Output  $L_1^t$  and  $L_2^t$ .

The complexities of Algorithm 1 and Algorithm 2 are  $O(d_1 N_{v_i} \tilde{n}_2 c_2 + d_2 N_{v_j} \tilde{n}_1 c_1)$  and  $O(d_1 W_1 d + d_2 W_2 d)$  respectively, where  $W_1$  is the number of non-zeros for each  $w_{v_i, x}$  on average and  $W_2$  is the number of non-zeros for each  $w_{v_j, y}$  on average.

After the similarity between vertices are determined by bilinear model, we select a strategy to derive finally ontology mapping. Following two strategies could be used for getting ontology mapping.

**Strategy 1.** For each  $v \in V(G_i)$ ,  $i=1,2$ . Let  $N \in \mathbb{N}$  be a parameter, and:

$$v_1 = \max_{v' \in V(G_i)} \{S(v, v')\},$$

$$v_2 = \max_{v' \in V(G_i), v' \neq v_1} \{S(v, v')\},$$

$$v_3 = \max_{v' \in V(G_i), v' \neq v_1, v' \neq v_2} \{S(v, v')\},$$

$$v_N = \max_{v' \in V(G_i), v' \neq v_1, v' \neq v_2, \dots, v' \neq v_{N-1}} \{S(v, v')\}.$$

Then, we deduce:

$$\text{map}(v) = \{v_1, v_2, \dots, v_N\}.$$

**Strategy 2.** For each  $v \in V(G_i)$ ,  $i=1,2$ . Let  $M \in \mathbb{R}^+$  be a parameter, and

$$\text{map}(v) = S\{(v, v') \geq M \mid v' \in V(G_i)\}.$$

### 3 Experiment

Experiment of relevance ontology mapping is designed below. In order to adjacent to the setting of ontology algorithm, we use a vector to express each vertex's information. Such vector contains the information of name, instance, attribute and structure of vertex. Here the instance of vertex refers to the set of its reachable vertex in the directed ontology graph.

We use physical education ontologies  $O_1$  and  $O_2$  (the structures of  $O_1$  and  $O_2$  are presented in Figures 1 and 2 respectively) for our experiment. The goal of this experiment is to determine ontology mapping between  $O_1$

and  $O_2$  via linear mapping  $L_1$  and  $L_2$  which is deduced by Algorithm 1 and Algorithm 2.  $P@N$  criterion (Precision Ratio, see Craswell and Hawking, [14]) is applied to measure the equality of the experiment. We first give the closest  $N$  concepts for each vertex on the ontology graph with the help of experts, and then we obtain the first  $N$  concepts for every vertex on ontology graph by the algorithm and compute the precision ratio. Also, ontology algorithms in [11, 5] and [6] are employed to “physical education” ontology, and we compare the precision ratio which we get from four methods. Several experiment results refer to Table 1.

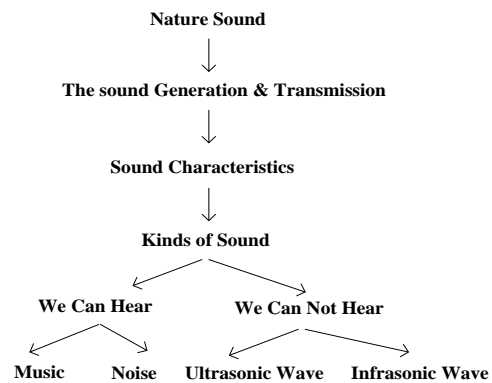


FIGURE 2 “Physical Education” Ontology  $O_2$

The experiment results in Table 1 reveal that the precision ratio in terms of our algorithm higher than the precision ratio determined by algorithms proposed in [11, 5] and [6] by taking  $N=1, 3$  or  $5$ . Specially, as  $N$  becomes large, such precision ratios in terms of our algorithm are increasing apparently. In this point of view, our algorithm is more efficient than algorithms raised in [11, 5] and [6] especially when  $N$  is sufficiently large.

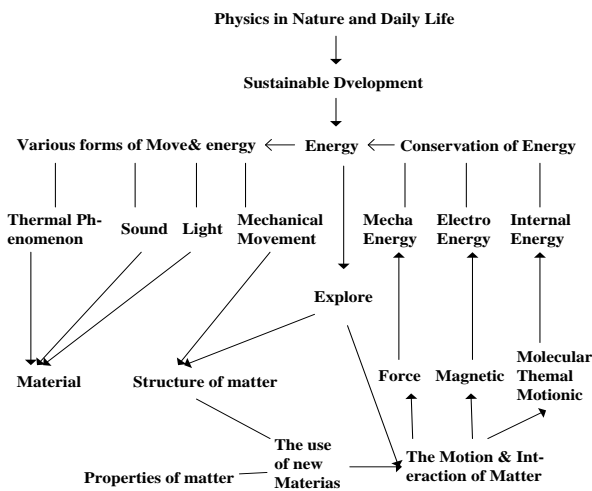


FIGURE 1 “Physical Education” Ontology  $O_1$

TABLE 1 The experiment data of ontology mapping

	$P@1$ average precision ratio	$P@3$ average precision ratio	$P@5$ average precision ratio
Algorithm presented in our paper	70.97%	79.37%	90.48%
Algorithm presented in [11]	67.74%	77.42%	89.68%
Algorithm presented in [5]	61.29%	73.12%	79.35%
Algorithm presented in [6]	69.13%	75.56%	84.52%

**4 Conclusions**

In this paper, we propose a new computation model for ontology mapping application. The model is bilinear and the algorithm is essentially a kind of dimensionality reduction algorithm which maps the high-dimensional ontology space into low-dimensional. At last, simulation data shows that our new algorithm has high efficiency in physics education ontologies. The algorithm achieved in our paper illustrates the promising application prospects for ontology mapping. The technologies raised in our paper contribute to the state of the art.

**References**

[1] Bouzeghoub A, Elbyed A 2006 Ontology mapping for web-based educational systems interoperability *Journal of Interoperability in Business Information Systems* 1(1) 73-84  
 [2] Hu B, Dasmahapatra S, Lewis P, Shadbolt N 2003 Ontology-based medical image annotation with description logics *Proceedings of*

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 [3] Fonseca F, Egenhofer E, Davis C, Camara G 2001 Semantic granularity in ontology-driven geographic information systems *AMAI Annals of Mathematics and Artificial Intelligence – Special Issue on Spatial and Temporal Granularity* 36 121-51

[4] Wang Y, Gao W, Zhang Y, Gao Y 2010 Ontology similarity computation use ranking learning Method *The 3<sup>rd</sup> International Conference on Computational Intelligence and Industrial Application* Wuhan China 20-2

[5] Huang X, Xu T, Gao W, Jia Z 2011 Ontology similarity measure and ontology mapping via fast ranking method *International Journal of Applied Physics and Mathematics* 1(1) 54-9

[6] Gao W, Liang L 2011 Ontology similarity measure by optimizing NDCG measure and application in physics education, *Future Communication, Computing, Control and Management* 142 415-21

[7] Gao Y, Gao W 2012 Ontology similarity measure and ontology mapping via learning optimization similarity function *International Journal of Machine Learning and Computing* 2(2) 107-12

[8] Huang X, Xu T, Gao W, Gong S 2011 Ontology similarity measure and ontology mapping using half transductive ranking *Proceedings of 4<sup>th</sup> IEEE international Conference on Computer Science and Information technology* Chengdu China 571-4

[9] Gao W, Xu T 2013 Stability analysis of learning algorithms for ontology similarity computation Abstract and Applied Analysis 2013 Article ID 174802




[10] Zhu L, Gao W Theoretical Analysis for New Multi-dividing Ontology Algorithm Based on AUC Criterion, Manuscript (in Chinese)

[11] Gao W, Gao Y, Liang L 2013 Diffusion and harmonic analysis on hypergraph and application in ontology similarity measure and ontology mapping *Journal of Chemical and Pharmaceutical Research* 5(9) 592-8

[12] Gao W, Shi L 2013 Ontology similarity measure algorithm with operational cost and application in biology science *An Indian Journal of BioTechnology (in Chinese)*

[13] Cristianini N, Shawe-taylor J, Elisseeff A, Kandola J 2011 On kernel-target alignment *Proceedings of the Neural Information Processing Systems NIPS'01* MIT Press 367-73

[14] Craswell N, Hawking D, Wilkinsin R, Wu M 2003 Overview of the TREC 2003 web track *Voorheer E (ed.) NIST Special Publication 500-255: The Twelfth Text REtrieval Conference (TREC 2003)* Gaithersburg MD 78-92

Authors	
	<p><b>Jian-Zhang Wu</b></p> <p><b>Current position, grades:</b> Associate Professor at the School of Computer Science &amp; Engineer (Southeast University).  <b>Scientific interests:</b> cloud computing, information retrieval, machine learning.</p>
	<p><b>Yu Xiao</b></p> <p><b>Current position, grades:</b> Lecturer at the School of Continuing Education (Southeast University).  <b>Scientific interests:</b> web technology, cloud computing, educational technology.</p>
	<p><b>Wei Gao</b></p> <p><b>Current position, grades:</b> Lecturer at School of School of Information Science and Technology (Yunnan Normal University).  <b>Scientific interests:</b> information retrieval, graph theory, machine learning.</p>