Lifetime forecasting for hemispherical resonator gyroscope with wavelet analysis-based GM(1,1)

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Abstract

Because of high cost and small batch of spacecraft like flywheel and gyroscope, how to estimate their reliability and lifetime becomes a tough task. A method to predict the lifetime of hemispherical resonator gyroscope (HRG) is put forward in this paper. This method utilizes grey correlation and mean absolute percent error (MAPE) to estimate the reliability of predictive data sequence. For reducing noise, Daubechies wavelet is used to decompose and reconstruct the test data in the paper as well. After pre-processing, predictive data sequences are gained by using GM(1,1) prediction model and then according to grey correlation and MAPE of each predictive data sequence, the threshold value meets conditions can be gained. Finally, the lifetime of HRG is predicted with using the threshold value. In this paper, the method is applied to the data of one type of HRG provided by a research institute in China and the result shows the gyroscope can normally run 4780 days at least, namely about 13.10 years.

Keywords: hemispherical resonator gyroscope (HRG), lifetime prediction; wavelet analysis, GM(1,1), MAPE, grey correlation

1 Introduction

With ever-developing space technologies of China, spacecraft need higher reliability, longer lifetime and higher efficiency now, say, satellites can work normally in orbit for 3 years or more, even 5-10 years. High cost and small number of flywheel and gyroscope, the indispensable parts in attitude control and attitude measuring unit of spacecraft makes it hard to estimate their lifetime which is a tough problem to solve. For gyroscope, it needs not only to detect the subtle angular displacement change and display rational response signal, but also to guarantee high reliability especially in unstaffed modern spacecraft, like satellites. A related survey [1] shows: 60% failure distribution of inertial system is from electronic circuit and 40% from inertial platform in which 60% is from gyroscope. Meanwhile, for domestic technological level, the failure rate of inertial platform and electronic circuit is half to half. Overall, researching reliability and lifetime prediction of gyroscope is important to inertial system. But worse still is that reports on the HRGs are not as many as other types of gyros, like dynamically tuned gyroscope (DTG) [2], MEMS gyro [3], fibre optic gyro [4]. For hemispherical resonator gyros, reference [5] analysed performances of HRGs with drift data and [6] researched the influences of temperature complement on navigation accuracy of hemispherical resonator gyros. In these studies, they didn't study the lifetime of the HRG, even no prediction methods mentioned in them. Therefore, we bring in grey system to analyse lifetime of the HRG.

The grey system theory is fairly appropriate for prediction. The accumulated generating operation [7] is the most important characteristic for the grey system theory and its purpose is to reduce the randomness of data. The main feature of grey theory is its capability of using as few as four data items to forecast the future data [7]. The grey prediction has been widely used in engineering sciences, social sciences [8, 9], power consumption [10, 11], as well as other fields [12, 13]. Moreover, various wavelets are also widely used to in signal processing and time series prediction, for example, reference [14] uses Haar filters to decompose time series data and predict, and reference [15] also adapts wavelet decomposition to time series prediction. Meanwhile, Daubechies wavelet is used to process biomedical signal in [16].

2 Lifetime prediction method with GM(1,1)

2.1 GREY PREDICTION MODEL

When modelling with grey system, pre-processing test data sequence is necessary and this process has two directions [12, 17]:

1) Provides intermediate information for modelling, i.e., finds out the law of raw data sequence.
2) Reduces the randomness of raw data sequence, that is, guarantees the correctness of the law.

Data producing mainly includes two operations: Accumulated Generating Operation (AGO) and Inverse Accumulated Generating Operation (IAGO).

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2.1.1 Accumulated generating operation

The AGO is a process that every data amasses its previous one and then gain a monotonically increasing data sequence.

Let $X^0 = [x^0(1), x^0(2), ..., x^0(n-1), x^0(n)]$ be the raw data sequence, where $x^0(i)$ is drift data at time i.

Do 1-AGO for X^0, then a new data sequence X^1 is generated, $X^1 = [x^1(1), x^1(2), \cdots, x^1(n-1), x^1(n)]$ and its derivation function is:

$$x^1(k) = \sum_{i=1}^{k} x^0(i), \quad k = 1, 2, ..., n . \tag{1}$$

If do m-AGO for X^0, its derivation function is

$$x^m(k) = \sum_{i=1}^{k} x^{m-1}(i), \quad k = 1, 2, ..., n . \tag{2}$$

Generally speaking, for nonnegative data sequence, more time it amasses, more remarkable the randomness reduces. Namely, when data amasses many times, the data sequence would become nonrandom. Also, 1-AGO for test data sequence is normally enough in grey model.

2.1.2 Inverse accumulated generating operation

IAGO is the inverse operation of AGO, that is, IAGO is a process that every collected data subtracts its previous one and then gain a new data sequence.

According to Equation (1), it's easy to regain X^0 from X^1 using 1-IAGO, and its process is

$$x^0(k) = x^1(k) - x^1(k-1), \quad 2 \le k \le n , \tag{3}$$

where $x^0(1) = x^1(1)$.
Accordingly, do m-IAGO:

$$x^{m-1}(k) = x^m(k) - x^m(k-1), \quad 2 \le k \le n , \tag{4}$$

where $x^{m-1}(1) = x^m(1)$.

2.2 GM(1,1) MODEL

1) Set sequence $X^0 = [x^0(1), x^0(2), ..., x^0(n-1), x^0(n)]$ denote the drift of the gyro, where $x^0(i)$ is the output at time i.
2) When X^0 is subjected to 1-AGO, then the following monotonically increasing sequence X^1 is obtained:

$$X^1 = [x^1(1), x^1(2), ..., x^1(n-1), x^1(n)] . \tag{5}$$

3) The whitening equation of X^1 is therefore, as follows:

$$\frac{dx'(k)}{dk} + ax'(k) = u , \tag{6}$$

in above, $\left[a \ u\right]^T$ is the parameters matrix that can be got as step 4) shows.
4) Parameters $\hat{a}$ can be obtained by using least square method:

$$\hat{a} = \begin{bmatrix} a \\ u \end{bmatrix} = (B^T B)^{-1} B^T y_N , \tag{7}$$

where

$$B = \begin{bmatrix} -\frac{1}{2}(x'(1) + x'(2)) & 1 \\ -\frac{1}{2}(x'(2) + x'(3)) & 1 \\ ... & 1 \\ -\frac{1}{2}(x'(n-1) + x'(n)) & 1 \end{bmatrix} \quad y_N = \begin{bmatrix} x^0(2) \\ x^0(3) \\ ... \\ x^0(n) \end{bmatrix} .$$

5) According to Equation (6), the solution of $\hat{x}^1(k)$ at time k is:

$$\hat{x}^1(k) = (x(1) - \frac{u}{a})e^{-ak} + \frac{u}{a} . \tag{1}$$

To obtain the predicted value of the primitive data, $\hat{X}^0$, the IAGO is used for $\hat{X}^1$, and then

$$\hat{X}^0 = \left[\hat{x}^0(1), \hat{x}(2), ..., \hat{x}^0(n)\right] . \tag{2}$$

where, $\hat{x}^0(k+1) = \hat{x}^1(k+1) - \hat{x}^1(k), \quad k = 1, 2, \cdots, n-1$, and $\hat{x}^0(1) = \hat{x}^1(1)$.

2.3 GREY CORRELATION

Grey correlation analysis can evaluate the compact degrees of two data sequences by using similarity degrees of their curves. The more similar the two curves they are, the larger correlation they have. And grey correlation is adopted to forecast short-term power in [18]. Grey correlation $\gamma(X_0, X_i)$ or γ_{0i} can be obtained as follows:
1) First of all, calculating range profile of each sequence:

$$X_i{}' = X_i / x_i(1) = (x_i{}'(1), x_i{}'(2), ..., x_i{}'(n)), \\ i = 0, 1, 2, ..., m. \tag{10}$$

2) Then calculating difference sequence:

$$\Delta_i(k) = | x_0{}'(k) - x_i{}'(k) |, \\ \Delta_i = (\Delta_i(1), \Delta_i{}'(2), ..., \Delta_i(n)), \quad i = 0, 1, 2, ..., m. \tag{11}$$

3) Next step: Calculating maximum difference and minimum difference:

$$M = \max_i \max_k \Delta_i(k), \quad m = \min_i \min_k \Delta_i(k) . \tag{12}$$

4) And calculating correlation coefficient $\gamma(x_0(k), x_i(k))$ or $\gamma_{0i}(k)$ at point k:

$$\gamma_{0i}(k) = \frac{m + \xi M}{\Delta_i(k) + \xi M}, \quad \xi \in (0,1), \quad k = 1,2,...,n, \quad i = 1,2,...,m. \tag{13}$$

5) At last, calculating grey correlation:

$$\gamma_{0i} = \frac{1}{n}\sum_{k=1}^{n}\gamma_{0i}(k) . \tag{14}$$

2.4 MEAN ABSOLUTE PERCENT ERROR AND PREDICTION ACCURACY

To guarantee the reliability of prediction data, mean absolute percent error (MAPE) is used to estimate the reliability and it can be gained:

$$MAPE = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{y_i - \hat{y}_i}{y_i}\right| \times 100\% . \tag{15}$$

where y is primitive data, $\hat{y}$ is predictive data, and N is length of $\hat{y}$.

Therefore, the prediction accuracy can be obtained:

$$P = 100\% - MAPE . \tag{16}$$

3 Data pre-processing with wavelet analysis

3.1 DATA

The test data of the HRG used in the paper is provided by China Electronics Technology Group Corporation 26th Research Institute. Besides, the duration of experiments is from June 26, 2009 to February 8, 2012, 956 days. And there is 1590 data points in total and all of them are positive with which GM(1,1) can be used to forecast.

3.2 DATA PREPROCESSING

To reduce randomness and noise of test data, Daubechies wavelet is adopted to pre-process test data. Because of low sampling frequency, test data is decomposed by 'db5' wavelet function in 3-scale, 6-scale, 9-scale respectively and then reconstructed low-frequency part respectively. The filtering results are shown in Figure 1.

Figure 1 shows Daubechies wavelet filters some high frequency away and expresses law of test data well. Result of 3-scale filtering is over fitting as well as that of 9-scale is under fitting, and 6-scale is properly fitting, therefore, the reconstruction data of 6-scale is adopted to predict lifetime of this HRG.

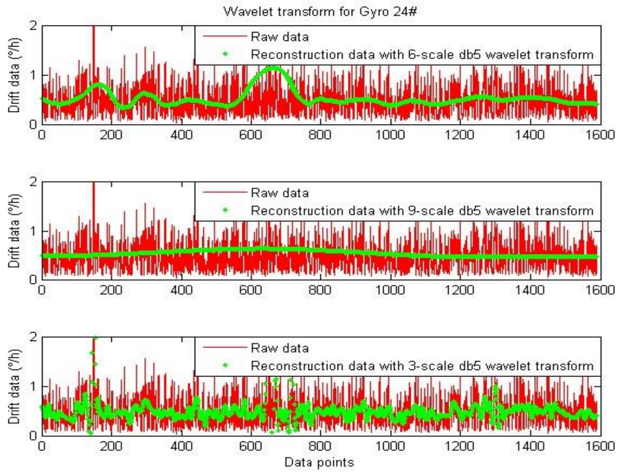

FIGURE 1 Filtering with Daubechies wavelet analysis

4 Predictive results and analysis

Predict 6 groups of data sequences using original data and pre-processed data with Daubechies wavelet respectively, and two residual sums of squares (RSS) are worked out. The results show that the residual sum of squares with Daubechies wavelet is 22.61 and MAPE is 11.3% as while as that of original data is 3421.33 and MAPE is 39.5% (Shown in Table 1).

TABLE 1 RSS and MAPE

	RSS	MAPE
Prediction with original data	3421.33	39.5%
Prediction with pre-processed data	22.61	11.3%

According to Table1, it's known that the prediction using Daubechies transform is much better than that without pre-processing, so in the paper, predictive data sequences got from pre-processed data are used in lifetime prediction of HRG, not using those of without pre-processing.

To predict lifetime of the gyro, grey correlation and mean absolute percent error are used to estimate whether the gyro is invalid or not. It is obvious the further prediction is away from samples, the lower the grey correlation is. Namely, grey correlation goes down while prediction data is generated. Therefore, when grey correlation suddenly raises, so the value could be the threshold which means the gyro is invalid after the point. Meanwhile, if the average accuracy at that value is not less than 60%, so the value is the threshold. Otherwise, let the former grey correlation compare with 60%, if it is also less than 60%, do this process until one grey correlation's average accuracy is equal to or more than 60%. Finally, the value meets the two conditions is the threshold.

4.1 GREY CORRELATIONS OF 24#HRG

6 groups of predictive data are obtained by using grey prediction model and each grey correlation is worked out as well (Shown in Table 2).

TABLE 2 Grey correlations of 6 groups of prediction data sequence

Prediction Data	1st time	2nd time	3rd time	4th time	5th time	6th time
Grey Correlation	0.814	0.766	0.741	0.723	0.749	0.768

4.2 AVERAGE ACCURACY

According to Equations (15) and (16), calculate MAPEs of 6 groups of predictive data sequences, and then work out average accuracy of every group, as Table 3 shows below.

TABLE 3 Average Accuracy of 6 groups of prediction data sequence

Prediction Data	1st time	2nd time	3rd time	4th time	5th time	6th time
Average Accuracy (%)	88.3	80.0	70.7	60.3	48.4	35.8

Moreover, grey correlations and average accuracies of 6 groups of prediction data sequence also shows in Figure 2.

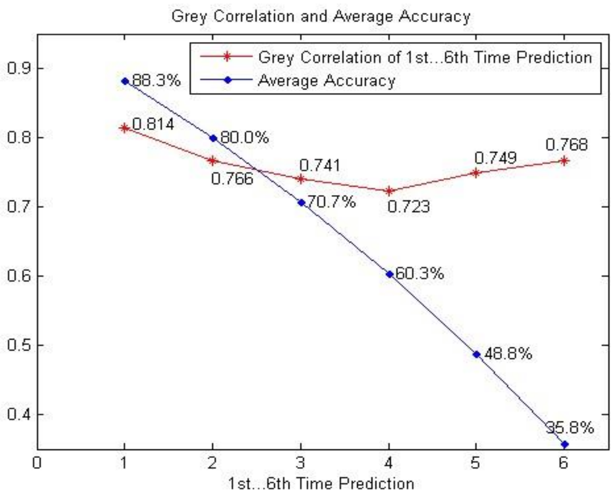

FIGURE 2 Grey Correlations and Average Accuracy of 6 Groups of Prediction Data Sequence

Figure 2 shows the average accuracy of 4th group is 60.3%. Both of 5th and 6th are less than 60%. Meanwhile, the grey correlation of 4th group predictive data sequence is under the threshold (0.723). Therefore, we predict that the gyro can normally work as 4 times long as it already ran for now. Namely, the gyro can at least run 3824 days (956·4=3824). And then add test period 956 days to predictive time, finally we calculate that the gyro can work at least 4780 days, that is, 13.10 years.

5 Conclusions

This paper uses Daubechies wavelet to decompose test data sequence and reconstruct it for reducing noise in it. And then GM(1,1) is applied to predict several sequences. Moreover, the predictive result of this method is much better than that of directly using original data. Meanwhile, grey correlation and MAPE are adopted to estimate prediction data and then find out the threshold value which helps determine the lifetime of the hemispherical resonator gyroscope. Applying the test data of one type of gyro experimented by one research institute to this prediction process, this type of gyro can normally work at least 4780 days, namely 13.10 years. According to the 10 global longest spacecraft: Voyager 2 (1977.8-), Voyager 1 (1977.9-), GOES 3 (1978.6-), ATS-3 (1967.11-2001), Mirasat F2 (1976.6-2008.10), Landsat 5 (1984.3-2012.12), TDRS-1 (1983.4-2009), GOES 7 (1987.2-2012.4), TDRS-3 (1988.9-), and GOES 2 (1977.6-2001), all of them can work more than 24 years, it means some gyroscopes, as the inertial unit in satellite can also run more than 24 years, therefore, in the paper, our predictive result is receivable and our method is reliable as well.

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