Volume integral equation-based electromagnetic inversion of 3D complex resistivity bodies

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Abstract

This study proposed the volume integral equation method to invert the complex resistivity parameters of 3D bodies in homogeneous half space. The partial derivative matrix of 3D bodies to complex resistivity was obtained with the use of electromagnetic field partial derivatives to the real resistivity combined with the complex resistivity spectrum partial derivatives of the Cole–Cole model. The classical damped least squares inversion method was applied. Results show that with at least four frequency data points, true complex resistivity parameters, as well as geometric parameters, can be inverted. The four parameters in the Cole–Cole model are interdependent. Chargeability and time constant are highly correlated, such that either one of these factors should be fixed prior to inversion. Furthermore, the proposed inversion algorithm we proposed has high efficiency because it only works in a split unit.

Keywords: volume integral equation, complex resistivity, 3D EM, damped least square method

1 Introduction

Spectral induced polarization (SIP), also known as complex resistivity (CR), was developed in the 1970s as a method of induced polarization. This approach uses the conventional resistivity method with a dipole-dipole device. In this method, the current passes through the power supply electrode to the underground in several frequencies, such that the difference in potential between the two electrodes can be observed. Apparent resistivity can then be calculated by using the traditional formula [1, 2] on the basis of a large number of measurement results for rock, ore, and outcrop samples. Pelton considered that the CR of rock and ore caused by IP effect rate varies with frequency (CR spectrum), which can be represented by the Cole–Cole model [3] as follows:

$$\rho(i\omega) = \rho_0 \left\{ 1 - m \left[1 - \frac{1}{1 + (i\omega\tau)^c} \right] \right\}, \tag{1}$$

where $\rho(i\omega)$ is the CR, ρ_0 is the direct resistivity, m is the chargeability, τ is the time constant, and c is the frequency dependence. These parameters are collectively known as the Cole–Cole model spectrum parameters.

In the SIP forward simulation, Hohmann calculated the 3D electromagnetic field of the CR body in homogeneous half-space by using an integral equation method [4]. Soininen simulated the CR spectrum of a rectangular solid with dipole and middle gradient arrangements in polarized and non-polarized earth [5, 6]. Weller studied the induced polarization response of a complex conductivity model by using the finite differential method [7].

As regards SIP inversion, Luo presented an analytical method in which the true spectrum is derived from the apparent spectrum [8] of the polarization body. Zhang proposed that the apparent spectrum can directly invert the true spectrum of the polarization body [9]. Wang introduced an algorithm through which the apparent spectrum directly inverts the true spectrum [10]. Liu reported that only through a plurality of apparent IP spectra measured in different positions can the true Cole–Cole parameters of a polarization body be determined. Therefore, inversion should also include a Cole–Cole parameter, true rock resistivity, geometry parameters, excitation parameters, measuring location, and other relevant parameters of the mathematical and physical field to achieve the real spectral parameter inversion [11, 12].

This paper proposes and implements a direct inversion of 3D CR parameter algorithm. First, the electromagnetic field of the 3D CR body is calculated. Then, the partial derivative matrix of the electromagnetic field to 3D CR body is deduced according to the partial derivative equation given by Eaton [13], combined with the partial derivative of CR spectrum on the parameters of the Cole-Cole model. Finally, the damped least square method is used to directly inverse each Cole-Cole model parameter.

2 Basic theory of forward integral equation

Forward calculation is the foundation of inverse calculation. Thus, we first introduce forward theory.

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According to the theory of integral equation, the tensor function of electromagnetic fields introduced by Green and the equations used by Maxwell are used to obtain an integral equation of a 3D CR body in a homogeneous earth [14].

$$\mathbf{F}(\mathbf{r}) = \mathbf{F}_1(\mathbf{r}) + \int_V \Delta\sigma \cdot \tilde{G}^F(\mathbf{r};\mathbf{r}') \cdot \mathbf{E}(\mathbf{r}')dv', \quad (2)$$

where $\mathbf{F}\ \mathbf{r}$ denotes total electromagnetic field at $\mathbf{r}$, $\mathbf{F}_1\ \mathbf{r}$ denotes primary electromagnetic field at $\mathbf{r}$, $\Delta\sigma = \sigma - \sigma_0$, σ denotes complex conductivity of 3D body, σ_0 denotes conductivity of homogeneous earth, $\tilde{G}^F(\mathbf{r};\mathbf{r}')$ denotes electromagnetic the tensor function, and $\mathbf{E(r')}$ denotes the electric field inside the 3D body. For convenience of numerical calculation, the 3D CR body is divided into N cubic cells. Assuming that resistivity is uniformly distributed in each cell, Equation (2) then becomes:

$$\mathbf{F}(\mathbf{r}) = \mathbf{F}_1(\mathbf{r}) + \sum_{n=1}^{N} \Delta\sigma_n \int_{V_n} \tilde{G}^F(\mathbf{r},\mathbf{r}') \cdot \mathbf{E}(\mathbf{r_n})dv'. \quad (3)$$

From the above Equation (3), we can obtain the electric field in each cell of 3D CR body, namely:

$$\mathbf{E}(\mathbf{r_m}) = \mathbf{E_1}(\mathbf{r_m}) + \sum_{n=1}^{N} \Delta\sigma_n \cdot \left(\int_{V_n} \tilde{G}^E\ \mathbf{r_m},\mathbf{r}'\ dv' \right) \cdot \mathbf{E}\ \mathbf{r_n}\ . \quad (4)$$

First, we obtain $\mathbf{E(r_m)}$ by solving Equation (4).

3 Theory of inversion

In inversion, we use the damped least squares method to fit the forward simulation data with the measured data. The CR parameters are then steadily changed. Finally, the best fitting of data is achieved. The inversion procedure can be easily expressed as follows: $\mathbf{f}_{si}$ represents the measured field, $\mathbf{F_i(r)}$ represents forward simulation data, and $\bar{X}$ represents CR parameters of each cell. The fitting degree of the field between forward simulation and that measured by the relative deviation of $\delta_i(\bar{X})$ is given by:

$$\delta_i(\bar{X}) = [\mathbf{f}_{si} - \mathbf{F}_i(\mathbf{r})] / \mathbf{f}_{si}. \quad (5)$$

Therefore, the inversion fitting error $\phi(\bar{X})$ is:

$$\phi(\bar{X}) = \sum_{i=1}^{k} \left[\delta_i\ \ \bar{X}\ \right]^2. \quad (6)$$

In Equation (6), the measured and simulation fields both have three components, and the equation *i*=1,2,…,*l* represents every observation location or frequency point.

The forward and deviation functions, as well as the fitting error, are nonlinear. Thus, approximate linearization processing should be applied to the deviation function to overcome the difficulties in solving nonlinear equations. Given a set of initial values $\bar{X}^0$ of model parameters, Taylor expansion at $\bar{X}^0$ is used, and each order over second-order partial derivatives is omitted, i.e.:

$$\delta_i(\bar{X}) \approx \delta_i(\bar{X}^0) + \sum_{k=1}^{n} \frac{\partial \delta_i(\bar{X}^0)}{\partial x_k} \Delta x_k, \quad (7)$$

where *k* is the *k*-th model parameter, and $\Delta x_k = x_k - x_k^0$ is the model modification. We then let $p_{ik} = \frac{\partial \delta_i(\bar{X}^0)}{\partial x_k}$. Substituting Equation (7) into Equation (6) yields

$$\phi(\bar{X}) = \sum_{i=1}^{l} [\delta_i(\bar{X}^0) + \sum_{k=1}^{n} p_{ik} \cdot \Delta x_k]^2, \quad (8)$$

$\phi(\bar{X})$ is expressed as a multiple function of the conductivity model variables $\Delta x_1, \Delta x_2, ..., \Delta x_n$, with the minimum condition given by:

$$\frac{\partial \phi(\bar{X})}{\Delta x_j} = 2\sum_{i=1}^{l} \left[\delta_i(\bar{X}^0) + \sum_{k=1}^{n} p_{ik} \cdot \Delta x_k \right] \cdot p_{ij} = 0. \quad (9)$$

Equation (9) is further derived as:

$$\sum_{i=1}^{l} \sum_{k=1}^{n} p_{ij} p_{ik} \Delta x_k = -\sum_{i=1}^{l} \delta_i(\bar{X}^0) \cdot p_{ij}. \quad (10)$$

Subsequently, we can derive the following equation with the use of $j = 1,2,...,n$:

$$(P^T P + \lambda) \cdot \Delta X = S, \quad (11)$$

where P is a Jacobian matrix with elements p_{ik}; P^T is the transpose matrix of *P*; λ is the damping factor, which is a positive constant; $\Delta X = (\Delta x_1, \Delta x_2, ..., \Delta x_n)^T$; and S is the right-side vector with elements given by $s_j = -\sum_{i=1}^{m} \delta_i(\bar{X}) \cdot p_{ij}$. We use Equation (11) to calculate the model modification value ΔX and then take $\bar{X} = \bar{X}^0 + \Delta X$ as a new parameter. The fitting error is then recalculated. Numerous iterations are required to achieve a fitting error that is less than the small positive number ε, with $\bar{X}$ as the most appropriate inversion result.

4 Partial derivative equation of CR parameters

$\bar{X}$ is set as an array of complex coal-coal model parameters, with element x_{kl} denoting the *l-th* $(l = 1,2,3,4)$ CR parameter of the *k-th* ($k = 1,2,...,N$) cell. $\Delta\bar{\sigma}$ is an array of complex conductivity differences, with element $\Delta\sigma_k$ denoting the differences between the complex conductivity of the *k-th* cell and the conductivity of the surrounding rock. $\bar{\rho}$ is an array of CR, and its element ρ_k denotes CR of the *k-th* cell.

According to the foregoing expression, the Jacobian matrix element is

$$p_{ikl} = \frac{\partial \delta_i(\bar{X}^0)}{\partial x_{kl}} . \quad (12)$$

Substituting (5) into (12) yields

$$p_{ikl} = -\frac{1}{f_{si}}\frac{\partial \mathbf{F}_i}{\partial x_{kl}} . \quad (13)$$

According to the derivative rule of compound functions,

$$\frac{\partial \mathbf{F_i}}{\partial x_{kl}} = \frac{\partial \mathbf{F}_i}{\partial \Delta\sigma_k}\cdot\frac{\partial \Delta\sigma_k}{\partial x_{kl}} = \frac{\partial \mathbf{F_i}}{\partial \Delta\sigma_k}\cdot\frac{\partial \Delta\sigma_k}{\partial \rho_k}\cdot\frac{\partial \rho_k}{\partial x_{kl}} \quad (14)$$

we simplify the above equation as

$$\frac{\partial \mathbf{F}_i}{\partial x_{kl}} = \frac{\partial \mathbf{F}_i}{\partial \Delta\sigma_k}\cdot\frac{-1}{(\rho_k)^2}\cdot\frac{\partial \rho_k}{\partial x_{kl}} = -(\Delta\sigma_k)^2\cdot\frac{\partial \mathbf{F_i}}{\partial \Delta\sigma_k}\cdot\frac{\partial \rho_k}{\partial x_{kl}}, \quad (15)$$

where $\frac{\partial \rho_k}{\partial x_{kl}}$ is the partial derivative of CR to the *l-th* CR parameter of the *k-th* cell, which can be calculated by using Equation (1) [15].

Equation (1) is decomposed into the real and imaginary parts:

$$\rho(i\omega) = \rho_0\cdot\left[1-m+\frac{mR}{R^2+I^2}-i\frac{mI}{R^2+I^2}\right], \quad (16)$$

where

$$R = 1+(\omega\tau)^c\cos\frac{\pi c}{2}, \quad (17)$$

$$I = (\omega\tau)^c\sin\frac{\pi c}{2}, \quad (18)$$

$$\frac{\partial \rho(i\omega)}{\partial m} = \rho_0\cdot\left[\frac{R}{R^2+I^2}-1-i\frac{I}{R^2+I^2}\right], \quad (19)$$

$$\frac{\partial \rho(i\omega)}{\partial c} = m\cdot\rho_0\cdot\left\{\begin{array}{l}\left[\frac{I^2-R^2}{(R^2+I^2)^2}\cdot\frac{\partial R}{\partial c}-\frac{2RI}{(R^2+I^2)^2}\cdot\frac{\partial I}{\partial c}\right]\\ +i\cdot\left[\frac{2RI}{(R^2+I^2)^2}\cdot\frac{\partial R}{\partial c}+\frac{I^2-R^2}{(R^2+I^2)^2}\cdot\frac{\partial I}{\partial c}\right]\end{array}\right\}, \quad (20)$$

$$\frac{\partial \rho(i\omega)}{\partial \tau} = m\cdot\rho_0\cdot\left\{\begin{array}{l}\left[\frac{I^2-R^2}{(R^2+I^2)^2}\cdot\frac{\partial R}{\partial \tau}-\frac{2RI}{(R^2+I^2)^2}\cdot\frac{\partial I}{\partial \tau}\right]\\ +i\cdot\left[\frac{2RI}{(R^2+I^2)^2}\cdot\frac{\partial R}{\partial \tau}+\frac{I^2-R^2}{(R^2+I^2)^2}\cdot\frac{\partial I}{\partial \tau}\right]\end{array}\right\}, \quad (21)$$

$$\frac{\partial \rho(i\omega)}{\partial \rho_0} = 1-m+\frac{mR}{R^2+I^2}-i\frac{mI}{R^2+I^2}, \quad (22)$$

$$\left.\begin{array}{l}\frac{\partial R}{\partial c} = (\omega\tau)^\tau\left[\ln(\omega\tau)\cdot\cos\frac{\pi c}{2}-\frac{\pi}{2}\sin\frac{\pi c}{2}\right]\\ \frac{\partial I}{\partial c} = (\omega\tau)^\tau\left[\ln(\omega\tau)\cdot\sin\frac{\pi c}{2}+\frac{\pi}{2}\cos\frac{\pi c}{2}\right]\\ \frac{\partial R}{\partial \tau} = \frac{c}{\tau}(\omega\tau)^\tau\cdot\cos\frac{\pi c}{2}\\ \frac{\partial I}{\partial \tau} = \frac{c}{\tau}(\omega\tau)^\tau\cdot\sin\frac{\pi c}{2}\end{array}\right\}. \quad (23)$$

We then solve the partial derivative $\frac{\partial \mathbf{F_i}}{\partial \Delta\sigma_k}$, which is the total field on earth relative to the conductivity difference of each cell of the underground abnormal body. Assuming that background resistivity is known, we take $\mathbf{F}_1(r)$ as a constant in Equation (3) relative to CR. That is:

$$\frac{\partial \mathbf{F_i}}{\partial \Delta\sigma_k} = \frac{1}{\partial \Delta\sigma_k}\left(\sum_{n=1}^{N}\Delta\sigma_n\int_{V_n}\tilde{G}^f(r,r')\cdot\mathbf{E}(r_n)\right). \quad (24)$$

We let $p^f{}_{in} = \int_{V_n}\tilde{G}^f(r,r')\cdot\mathbf{E}(r_n)$, where $\tilde{G}^f$ is the electromagnetic tensor function of the *n-th* cell of an abnormal body on the ground observation points. Through further derivation, we obtain:

$$\frac{\partial \mathbf{F_i}}{\partial \Delta\sigma_k} = p^f{}_{ik}+\left(\sum_{n=1}^{N}\Delta\sigma_n\frac{\partial p^f{}_{in}}{\partial \Delta\sigma_k}\right). \quad (25)$$

Two steps are required to solve Equation (25). First, the partial derivatives of the electric field of each cell relative to the conductivity difference are solved. Second, the partial derivatives of the total field relative to the conductivity difference are determined by superimposing the sum. $\mathbf{F_i}$ has three components, namely these are *x*, *y*, and *z*. Taking the *x*-component as an example, we derive:

$$\frac{\partial (p^f{}_{in})_x}{\partial \Delta\sigma_k} = \frac{\partial E_x(r_n)}{\partial \Delta\sigma_k}\Gamma^f_{xixn}+\frac{\partial E_y(r_n)}{\partial \Delta\sigma_k}\Gamma^f_{xiyn}+\frac{\partial E_z(r_n)}{\partial \Delta\sigma_k}\Gamma^f_{xizn}, \quad (26)$$

where $\frac{\partial E_x(r_n)}{\partial \Delta\sigma_k}, \frac{\partial E_y(r_n)}{\partial \Delta\sigma_k}, \frac{\partial E_z(r_n)}{\partial \Delta\sigma_k}$ are partial derivatives of the electric field of the *n-th* cell relative to the conductivity difference of the abnormal *k-th* cell. We also consider $\Gamma^f_{xixn} = \int_{V_n} \tilde{G}_{xix'}(r_i, r')dV'$, $\Gamma^f_{xiyn} = \int_{V_n} \tilde{G}_{xiy'}(r_i, r')dV'$, and $\Gamma^f_{xizn} = \int_{V_n} \tilde{G}_{xiz'}(r_i, r')dV'$.

We can obtain $\frac{\partial E_x(r_n)}{\partial \Delta\sigma_k}, \frac{\partial E_y(r_n)}{\partial \Delta\sigma_k}, \frac{\partial E_z(r_n)}{\partial \Delta\sigma_k}$ by using the same method employed to solve Equation (4).

After discretization, matrix equations can be expressed as:

$$\frac{\partial E(r_m)}{\partial \Delta\sigma_k} = \int_{V_k} \tilde{G}(r_m, r')\ E(r_k) + \sum_{n=1}^{N} \Delta\sigma_n \int_{V_n} \tilde{G}(r_m, r')dv' \cdot \frac{\partial E(r_n)}{\partial \Delta\sigma_k} . \quad (27)$$

Therefore, by solving n times n linear equations, we derive

$$\frac{\partial E_x(r_n)}{\partial \Delta\sigma_k}, \frac{\partial E_y(r_n)}{\partial \Delta\sigma_k}, \frac{\partial E_z(r_n)}{\partial \Delta\sigma_k}, \ (n = 1, 2..., N; k = 1, 2..., N).$$

Substituting these parameters into Equation (26) and then substituting Equation (26) into Equation (25), we can obtain the partial derivative matrix of the observation field on the earth relative to the conductivity difference of each cell of the abnormal body. $\frac{\partial \mathbf{F}_i}{\partial \Delta\sigma_k}$ is substituted by solving Equation (25) and the results were obtained by solving Equations (19)–(22) into Equation (15). Thus, we obtain the partial derivative matrix of total observation field on the earth to the CR parameters of Cole-Cole model.

5 Inversion examples of 3D CR body

As shown in Figure 1, a geoelectric model is designed for the inversion of CR parameters. We find 3D CR anomalies in homogeneous earth. These anomalies have the following parameters: centre coordinate of (0, 0, 200); size of 200×200×50*m*; excitation source coordinate of (800, 0, 0); frequencies of 0.1, 1, 10, and 100 Hz; electric dipole moment of 1 A·m; non-polarization of surrounding rock resistivity of 100 $\Omega\cdot m$; and CR parameters of $\rho_0 = 10\Omega\cdot m$, $m = 0.6, c = 0.1$, $\tau = 10s$. The abnormal body is divided into $4\times4\times1$ cells. Resistivity is assumed to be uniformly distributed in each cell and is equal to the value of the centre.

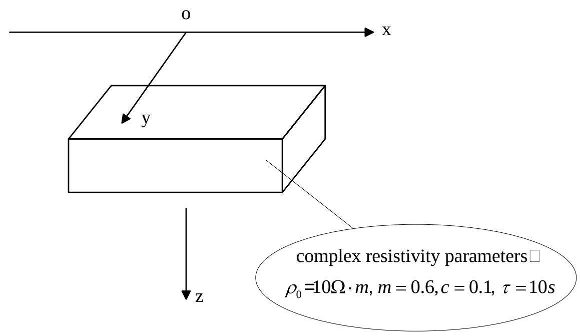
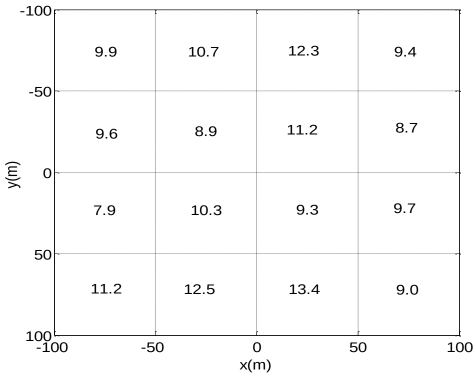


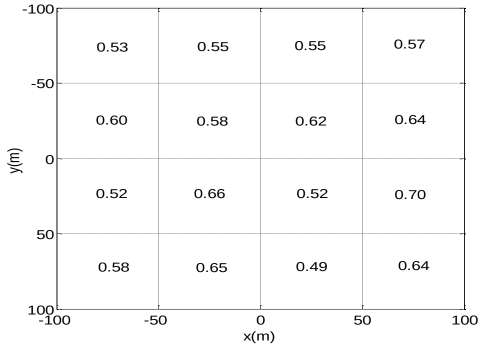
FIGURE 1 3D complex resistivity model in homogeneous earth

The four parameters of CR are inverted. The initial CR parameters of inversion are $\rho_0 = 30\,\Omega\cdot m$, $\tau = 100s$, $c =$ 0.25 and $m = 0.3$. The damping factor is 1, whereas its scaling factor is 5. This factor is calculated by using the inversion algorithm. Results after eight iterations are shown in Figure 2. The figures show that ρ_0 , c and m converge close to the true value. However, the results of time constant τ is poor and is far from the true value. Moreover, fitting error decreases as iterations increases. As shown in Figure 3, fitting error is 1.40×10^{-3} , which is significantly greater than the given threshold 1×10^{-7} . Thus, this inversion process has failed.

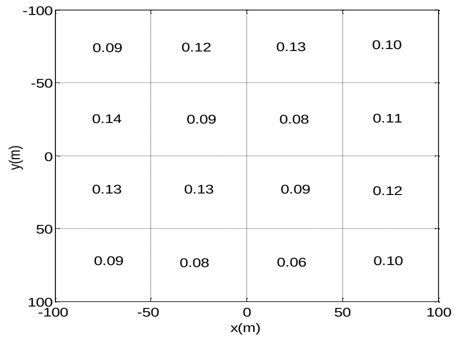
According to analysis, accurate results cannot be obtained if the four parameters of each cell are inverted simultaneously because of the very strong correlation between chargeability m and time constant τ [16]. m is one of the key parameters of CR inversion, whereas τ can be determined through physical measurements or estimated from prior work experience. Therefore, we fix the time constant τ of each cell and then insert the other three parameters. In the subsequent inversion, we fix τ as a true value 10, and the initial value of the other three parameters are the same as described above. Results after 16 iterations are shown in Figure 4. For this inversion, the fitting error is 1.40×10^{-3} (Figure 5), which is less than the given threshold 1×10^{-7} . Thus, the inversion process is successful.



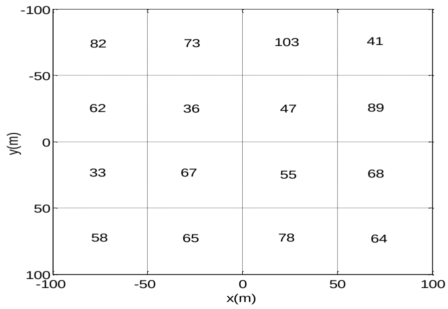
a) ρ_0



b) c



c) m



d) τ

FIGURE 2 Inversion results of complex resistivity 200 m underground

As shown in Figure 4, although the difference between the initial and true values is significant, the CR parameters converge to the true value after inversion. This result indicates that the algorithm is correct and effective.

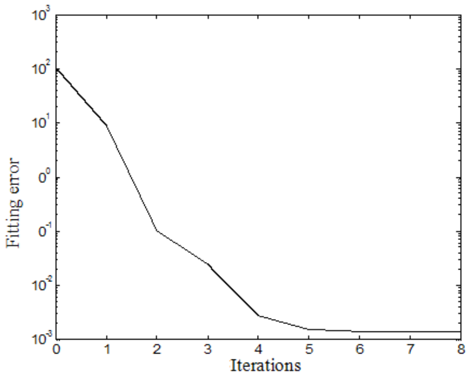
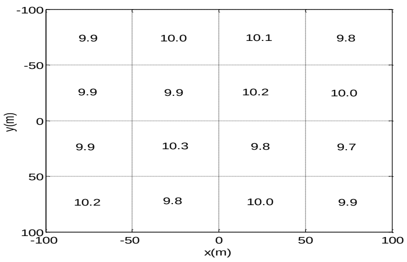
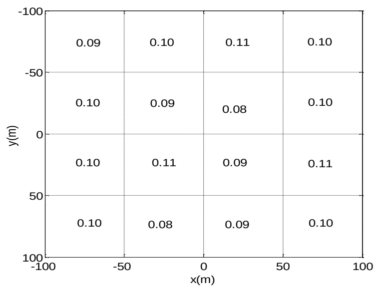


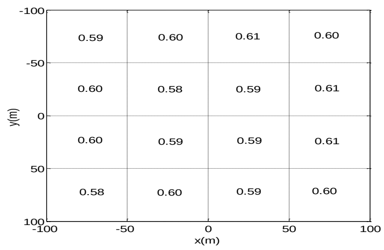
FIGURE 3 Relative error curve with the number of iterations



a) ρ_0



b) c



c) m

FIGURE 4 Inversion results of complex resistivity 200 m underground at a fixed time constant

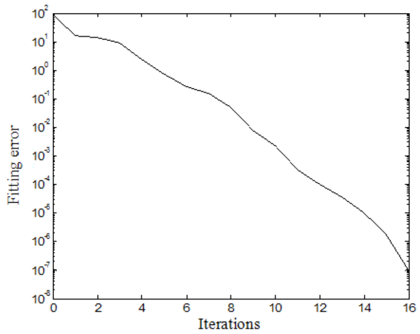


FIGURE 5 Relative error curve with the number of iterations at a fixed time constant

6 Conclusion

This study proposed and implemented a CR inversion algorithm. Through model simulation, we obtain the following conclusions:

1) The four parameters of the CR Cole–Cole model are not completely independent. in particular, a strong correlation exists between chargeability and time constant. Therefore, one of the two parameters should be fixed during the inversion process.

2) By using at least four frequencies of multiple-point observation data, the CR parameters of a 3D abnormal body can be determined.

3) The proposed in this paper has satisfactory convergence, and inversion results are unaffected by the initial value.

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