Fractal model of electric conductivity of aging Al alloys

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Abstract

Theoretical study of anisotropy changes of crystal structure in Al alloys as a result of senescence is performed. The appropriate model is developed that allowed to explain experimental results concerning the influence of tempering and cold rolling on the anisotropy of electrical conductivity of Al alloy D16.

Keywords: crystal anisotropy, plastic deformation, diffusion, dislocations, electrical conductivity list

1 Introduction

Aluminium (Al) alloys are the most important construction materials in the modern industry. Further development of nuclear energetics, all transport modes, space vehicles demand an application of these materials.
The typical alloying elements in these materials are copper, magnesium, manganese, silicon and zinc. There are two principal classifications of Al alloys, namely casting alloys and wrought alloys, both of which are further subdivided into the categories heat-treatable and non-heat-treatable. About 85% of aluminium is used for wrought products, for example rolled plate, foils and extrusions. Cast aluminium alloys yield cost-effective products due to the low melting point, although they generally have lower tensile strengths than wrought alloys. The most important cast Al alloy system is Al–Si, where the high levels of silicon (4.0–13%) contribute to give good casting characteristics. Al alloys are widely used in engineering structures and components where light weight or corrosion resistance is required [1, 2].

Different factors are responsible for the anisotropy of physical properties of Al alloys. Texture, or crystallographic anisotropy, is probably the most important one, but also the grain shape, precipitates and dislocation structures may influence the properties. While a single crystal is highly anisotropic, an aggregate of completely differently oriented grains might be also isotropic. This relates to the variation in strength of different orientations, which for a tensile test is given by the Schmidt factor for each grain. In textured polycrystals a large number of the grains have approximately the same orientation, causing anisotropy.

Physical characteristics of Al alloys (electric and heat conductivity, diffusion, thermal expansion) are described by second-order tensors and very structurally - sensitive [3, 4]. Studies of the anisotropy properties of Al alloys are important to understand their defect structure and their influence on service characteristics of these materials.

2 Description of the model and obtained results

As a result of plastic deformation the metals with homogeneously distributed lattice defects form a defect fractal structure [5]. It has been found that after considerable deformation the metals with cubic
lattice exhibits anisotropy of physical properties that can be described by a second-order tensor.

Rectangular samples with sizes 10x100 mm were investigated. Samples were cut at various angles relative to the rolling direction (see Figure 1).

Figure 1 shows changes of electric resistivity of freshly deformed D16 alloy (as a result of quenching and rolling). One can see anisotropy of resistivity. The analogous results were observed previously in sheets of copper alloys Cu. [6].

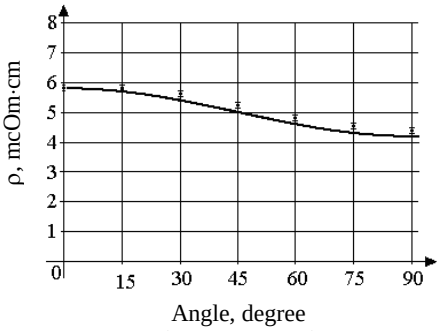

FIGURE 1 Changes of the resistivity of D16 alloys a result of quenching and rolling; the points correspond to experimental results; the solid line is obtained theoretically

In the process of significant plastic deformation (more than 50 % in thickness) as a rule, cellular dislocation structure is formed in metals and alloys. In metals with pure texture deformation cellular dislocation structure is

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different for different textural components. The walls of cells with large concentration of dislocations have a fractal nature with unequal fractal dimension [7 - 9].

Present study of anisotropic electric conductivity for alloy D16 is based on the accounting a fractal structure that can be depicted as in Figure 2. To compose such fractal model the details of the dislocation structure must be accounted.

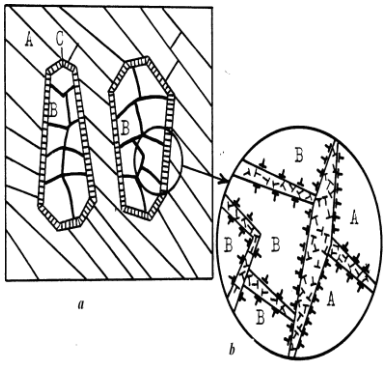

FIGURE 2
a) Fractal model of the structure of deformed sample: A, B corresponds to a highly disoriented meso- region with small intrinsic misorientation of dislocation cells, C is a boundary between strongly disoriented meso - regions (texture components) consisting of dislocations;
b) Fragment shows the accommodation between cells of meso – areas A–B (large dislocation density) and between cells A–A and B–B (low dislocation density).

It is known that aluminium has high stacking fault energy (~ 0.2 J/m^2). Therefore, a cellular dislocation structure at earlier stages of deformation is formed unlike the situation in copper, which has a lower stacking fault energy (0.04-0.05 J/m^2).

As mentioned, alloy D16 after rolling by 50% in thickness has a texture similar to the texture of the copper after rolling by 95 % in thickness. The structure of Al alloys after deformations is characterized by rectangular grains. Within the relative strain ε <1 the grain boundaries consist of a plexus of high density of dislocations. So it can be assumed that in D16 alloy the formed cellular dislocation structure similar to the structure of the deformed copper. Therefore, the model calculations of anisotropic electric conductivity for deformed alloy D16 can be based on the data of the structure of the deformed copper.

Developing the model we selected a representative volume of heterogeneous material. The inclusion matrix is a parallelepiped with edges of length l_i. The problem reduces to the calculation of the effective electric conductivity in micro-volume of homogeneous material. The details of the model are illustrated in Figure 3.

Since the orientation of sub-boundaries in general case does not coincide with the axes of the laboratory coordinate system, we introduce the crystallographic coordinate system axis Ox_1, Ox_2 and Ox_3, which are oriented along the principal axes of the electric conductivity tensor σ_{ii} (i = 1, 2, 3). Then, in accordance with the appropriate types of the dislocation structure, the axes of the electric conductivity tensor (the component {110} <112>) and the switched (component {112} <111>) are oriented as shown in Figure 3.

Taking into account the anisotropy of the properties, the first iteration step can be written as a function of the angle φ in the plane xOy of the laboratory coordinate system:

$$\sigma(\phi)=\left[\frac{1-m}{\sigma_{22}^{M}+\left(\sigma_{11}^{M`}-\sigma_{22}^{M}\right)\cos^2\phi}+\frac{m}{\sigma_{22}^{B`}+\left(\sigma_{11}^{B`}-\sigma_{22}^{B`}\right)\sin^2\phi}\right]^{-1}, \quad (1)$$

where m is a volume content of inclusions (meso - region texture components {112} <111>).

$$\sigma_{11}^{M`}=\sigma_{33}^{M}+\left(\sigma_{11}^{M}-\sigma_{33}^{M}\right)\cos^2\psi_1, \quad (2)$$

σ_{22}^{M} is a part of the conductivity tensor matrix along Ox_2, which coincides with O_y, that is $\sigma_{22}^{M`}=\sigma_{22}^{M}$,

$$\sigma_{11}^{в`}=\sigma_{22}^{в}\sin^2\varphi_0+\left(\sigma_{11}^{в}\cos^2\psi_2+\sigma_{33}^{в}\sin^2\psi_2\right)\cos^2\varphi_0, \quad (3)$$

$$\sigma_{22}^{в`}=\sigma_{22}^{в}\cos^2\varphi_0+\left(\sigma_{11}^{в}\cos^2\psi_2+\sigma_{33}^{в}\sin^2\psi_2\right)\sin^2\varphi_0 \quad . \quad (4)$$

Then we transform the experimental values of electric conductivity in relative units as the ratio $\sigma(\varphi)=\rho_0/\rho(\varphi)$, where $\rho_0$is resistivity of the aged alloy D16 equal to 5.2 10^{-4} ohms-cm.

In the expression (1) $\sigma_{11}^{M`}$, $\sigma_{22}^{M}$, $\sigma_{22}^{в`}$ and $\sigma_{22}^{в}$ are the electric conductivity values of the matrix and inclusion, reduced to x and y axes of the laboratory system.

The calculated contribution of dislocations, in the laboratory frame, to electric conductivities relatively texture components are presented in Table 1. As we see, the values of the electric conductivity tensor expressed in the laboratory coordinate system are markedly different for the main texture components for D16 alloy. Expressions (3) - (4) show these differences.

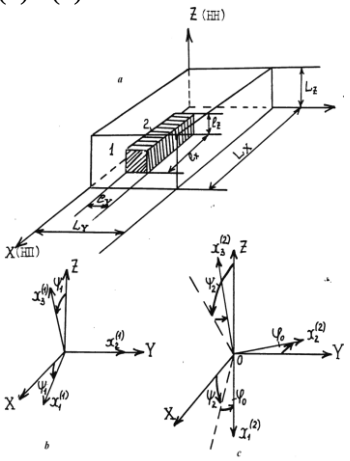

FIGURE 3 Model structure: 1 is an external matrix; 2 is an inclusion, b, c are the principal axes of the conductivity tensor matrix (texture component {110} <112>) and inclusion (texture component {112} <111>),$\psi_1=30^0$, $\psi_2=35^0$, $\varphi_0=39^0$.

TABLE 1 Dislocation contribution to electric conductance of texture components of alloy D16

Texture components	Components of the electric conductivity tensor
{110}<112>	$\sigma_{11}^{M'} = 0{,}769$ $\sigma_{22}^{M'} = 1{,}755$
{112}<111>	$\sigma_{11}^{B'} = 1{,}755$ $\sigma_{22}^{B'} = 0{,}672$

We calculate the anisotropic resistivity of D16 alloy using the data in Table 1 and experimental values of electric conductivity.

For computer simulation of the fractal model of electric conductivity of deformed alloy, we use a method of renormalization – group transformations [10]. On the first iteration step in formula (3 and 4) were substituted for the original values of the parameters of matrix and inclusion with a given volume concentration m_1= 0, 4 and calculate σ_{11}^{ef} and σ_{22}^{ef} conductivity. At the second step iteration in the appropriate formula (3), (4) instead of σ_{11}^{δ} and σ_{22}^{δ} tripped conductivity coefficients σ_{11}^{ef} and σ_{22}^{δ}, calculated in the first step, and the new concentration m2, the ratio found in the interval [0, 1]:

$$m_k = m_{k-1}^2\left(4 + 8m_{k-1} - 14m_{k-1}^2 - 40m_{k-1}^3 + 16m_{k-1}^4 + 288m_{k-1}^5 - 655m_{k-1}^6 + 672m_{k-1}^7 - 376m_{k-1}^8 + 112m_{k-1}^9 - 14m_{k-1}^{10}\right), \quad (5)$$

The function (5) is shown in Figure 4.

As a result equation (1) gives a curve of the ellipse form relatively the principal axes. Thus, we obtained that the electric conductivity of deformed Al alloy D16 with a cubic lattices described by a second-order tensor.

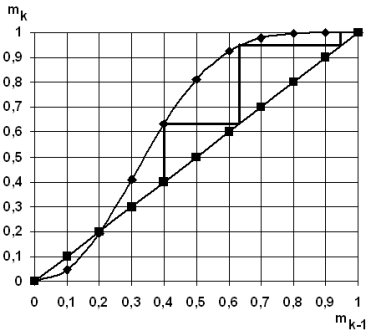

FIGURE 4 The function $m_k = f\left(m_{k-1}\right)$ determined at the interval [0, 1]. Solid line reflects the change of dislocation concentration and their inclusion in the calculation σ_{11}^{ef} and σ_{22}^{ef}.

3 Conclusions

1) The anisotropy of electric conductivity was measured for D16 Al alloy with cube lattice after tempering and cold rolling.
2) It was established that the electric conductivity of studied alloy in the direction of rolling is 30 % less than in transversal direction of sheet. The anisotropy of electric conductivity can be described by the second-order tensor, which is consistent with the fractal model and experimental results.
3) Using a developed fractal model of the structure of deformed metallic materials it was determined the dislocation role in formation of the main components of texture that explain the nature of the general anisotropy of electric conductivity of alloy D 16.

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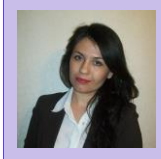

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