Finite time control for probe descent near small bodies

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Received 1 July 2014, www.cmnt.lv

Abstract

For the purpose of probe soft landing on small bodies safely, this paper focuses on the orbital dynamics and describes a new control algorithm for the probe descent near a rotating small body. The general formulation of the probe equations of motion in the Body-Fixed Coordinate system is obtained through Newton's second law firstly. Then the nominal polynomial trajectory of vertical direction is planned in the condition of fuel consumption suboptimum. Considering uncertainties and perturbations, the control laws based on Adaptive Terminal Sliding Mode with compensation term are developed to track the desired trajectory finally. Suppose the initial conditions presented in this paper, position and velocity errors tend to zero in the finite time in the phase of sliding mode motion so it will make the task succeed during descent phase near small bodies. Finally, the effectiveness of the guidance and control algorithm is verified by MATLAB simulations. The proposed algorithm can fast and accurately track the planned trajectory in the finite time and is robust to parameter uncertainty, feedback state error and external disturbances. The validity is conformed by computer simulation.

Keywords: adaptive terminal sliding mode control, soft landing, descent near small body, guidance and control

1 Introduction

In recent years our solar system is small bodies (e.g., mainly asteroids and comets) have received increased attention because of the insight they can give about the history of the solar system. In this respect, several studies have been conducted and some missions have been flown and planned [1]. The missions flown include: On 12 February 2001, Near Earth Asteroid Rendezvous (NEAR) Shoemaker became the first spacecraft landing on a small body 433 Eros after a busy 4-year cruise [2-4]. ISAS had planed to launch a sample and return spacecraft MUSES-C in 2002 and the spacecraft arrived at near earth asteroid 1989ML in 2003 [5, 6]. And Rosetta - ESA Comet Mission, will fly by asteroids Steins and Lutetia in 2004.

The small size, irregular shape, mass distribution as well as the state of rotation of an asteroid, have significant effects on the evolution of the orbit of the spacecraft. The high precise guidance, navigation, and control (GNC) system is needed for safe and precise asteroid landing [7]. Several papers have addressed the problems of orbital dynamics, autonomous guidance and control for landing on small body about asteroids. PD and Sliding Mode Controller are designed to tracking the fuel consumption suboptimal nominal trajectory [8]. The authors provide scope for precision landing of payloads on minor solar system bodies using a probe as a reference to generate a line of sight to the landing location [9, 10]. In these methods the probe commanded acceleration is collinear with the vehicle velocity, and the spacecraft trajectories are restricted to a plane. With gravitational effects taken into account, a new solution to the fuel optimal vertical landing on an asteroid was obtained. The open-loop guidance based on a pseudo way-point generation algorithm is provided that uses a discrete linear time-varying model of the dynamics that incorporates required thruster silent times. Then it is transferred to second-order cone programming problem. Feedback control is implemented to track the pseudo way-point trajectory [11]. Since the gravitation on an asteroid is low, a wheeled vehicle would likely bounce back from hitting the surface, and be difficult to control. A first order model of the dynamics of hopping robots is developed to estimate the total time and distance covered from an initial bounce to a stop due to friction and restitution coefficients. Then sliding-mode control is applied to discrete formation control [12].

The rapidity and robustness of tracking the desired trajectory are not ideal in condition of uncertainties and perturbations for normal Sliding Mode Control. The Terminal Sliding Mode Control with compensation term is proposed to deal with MIMO system [13, 14]. The nonlinear term is introduced into the design of the sliding mode surface such that the state variable of the sliding mode tends to zero in the finite time in the phase of sliding mode motion.

The body of this paper consists of four main sections followed by a conclusion. Section 2 describes the probe descent orbital dynamic model through Newton's second law referring to lunar soft landing as well as the irregular gravitation potential model of small body through harmonic series expansion method. The nominal trajectories considering fuel suboptimal are designed in

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section 3. In section 4, an Adaptive Terminal Sliding Mode Controller with compensation term is designed to track the desired trajectory so the probe can land on the surface of small body quickly and safely while the boundary of uncertainties and perturbations are unknown. Computer simulations are carried out to illustrate the effectiveness and robustness of the control laws.

2 Small body and probe models

2.1 PROBE ORBITAL DYNAMICS MODEL

The probe orbital dynamic model is built in the three dimensional coordinates referring to model of soft landing on lunar [15], as shown in Figure 1. The small body has the instantaneous rotation angular velocity vector of ω. Affected by the irregular and weak gravitational potential, the perturbation force accelerations such as solar radiation pressure and solar gravitational, the probe cannot always retain the same plane during landing.

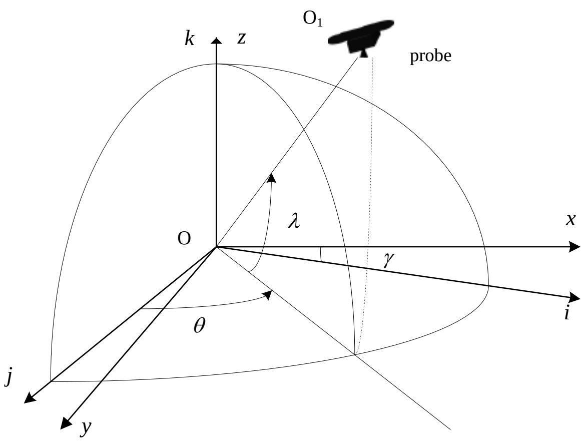

FIGURE 1 Viewing geometry of coordinate systems

For further understanding of equations of motion, it is convenient to define the following coordinate frames. The small body Central Inertial Coordinate system *oijk* with the body equator as the reference plane, coordinate system origin *o* is the centre of mass of the small body. The asteroid Body-Fixed Coordinate system *oxyz* is defined. Let the coordinate system origin *o* is fixed with the asteroid mass centre, and the *z*-axis coinciding with the asteroid's maximum moment of inertia, the x-axis and the y-axis coinciding with the minimum and intermediate moments of inertia, respectively. For simplicity, we assume that the spin axis of the target asteroid is coincided with the asteroid's maximum moment of inertia. All the coordinates are satisfied with right handed systems. Without lose of generality, we assume that *oijk* and *oxyz* coincide with each other at the beginning of descending and soft landing. And o_1 is the centre of mass of the probe. Supposed the small body is a unit mass and the fuel consumption is omitted, system orbital dynamic equations in the Body Central Inertial Coordinate can be derived based on Newton's second law. The detail process is presented as below:

$$\left[\frac{d\vec{V}}{dt}\right]_{ijk} = P_1\begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix}_{ijk} + \vec{G}_{ijk} + \vec{\Delta}_{ijk}, \tag{1}$$

$$P_1 = \begin{bmatrix} \cos\lambda\cos\theta & \sin\theta & -\sin\lambda\cos\theta \\ -\cos\lambda\cos\theta & \cos\theta & \sin\lambda\sin\theta \\ \sin\lambda & 0 & \cos\lambda \end{bmatrix},$$

where $\vec{V}$ and U are velocity vector and gravity function of probe in *oijk* coordinate, and Δ is perturbation force acceleration which includes solar radiation pressure perturbation and solar gravitational perturbation. u_x, u_y, u_z are components of control acceleration, and u_z coinciding with the vector from the small body mass centre to the probe, u_x perpendicular with u_z and pointing to the motion of probe, u_x, u_y and u_z complied with the right handed system. We define position and velocity vector of probe $\vec{R}$ and $\vec{V}_L$ in the Body-Fixed Coordinate, and $\vec{\omega}$ is the Body-Fixed Coordinate system *oxyz* rotating velocity relative to Central Inertial Coordinate *oijk*.

The following equation is derived based on Koch law:

$$\vec{V} = \vec{V}_L + \vec{\omega}\times\vec{R}. \tag{2}$$

Taking the derivative of the Equations (2), the acceleration of probe in the Body Central Inertial Coordinate is deduced as Equation (3):

$$\left[\frac{d\vec{V}}{dt}\right]_{ijk} = \left[\frac{d\vec{V}_L}{dt}\right]_{ijk} + \vec{\omega}\times\left[\frac{d\vec{R}}{dt}\right]_{ijk} + \left[\frac{d\vec{\omega}}{dt}\right]_{ijk}\times\vec{R}. \tag{3}$$

Similarly, Equation (4) is derived based on Koch law

$$\begin{aligned} \left[\frac{d\vec{V}_L}{dt}\right]_{ijk} &= \left[\frac{d\vec{V}_L}{dt}\right]_{xyz} + \vec{\omega}\times\vec{V}_L \\ \left[\frac{d\vec{R}}{dt}\right]_{ijk} &= \vec{V}_L + \vec{\omega}\times R \end{aligned}. \tag{4}$$

Suppose the small body rotation speed is uniform with the constant rate of revolution ω:

$$\left[\frac{d\vec{\omega}}{dt}\right]_{xyz} = 0. \tag{5}$$

Substituting Equation (4) and Equation (5) into Equation (3):

$$\left[\frac{d\vec{V}_L}{dt}\right]_{xyz} = \left[\frac{d\vec{V}}{dt}\right]_{ijk} - 2\vec{\omega}\times\vec{V}_L - \vec{\omega}\times\left(\vec{\omega}\times\vec{R}\right). \tag{6}$$

Therefore, the acceleration of probe is in the Body-Fixed Coordinate:

$$\left[\frac{d\vec{V}}{dt}\right]_{xyz} = P_1 P_2 \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} + \vec{G}_L - 2\vec{\omega}\times\vec{V}_L - \vec{\omega}\times\left(\vec{\omega}\times\vec{R}\right), \quad (7)$$

$$P_2 = \begin{bmatrix} \cos\gamma & 0 & -\sin\gamma \\ 0 & 1 & 0 \\ \sin\gamma & 0 & \cos\gamma \end{bmatrix}.$$

Small body gravitation acceleration projection formation in the Body-Fixed Coordinate is indicated as:

$$\vec{U}_L = \vec{G}_L - \vec{\omega}\times(\vec{\omega}\times\vec{R}).$$

Substituting above equation into Equation (3),

$$\begin{bmatrix} \dot{V}_{xL} \\ \dot{V}_{yL} \\ \dot{V}_{zL} \end{bmatrix} = P_1 P_2 \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} - \begin{bmatrix} U_{xL} \\ U_{yL} \\ U_{zL} \end{bmatrix} - \begin{bmatrix} 2\omega_L V_{zL} \\ 0 \\ -2\omega_L V_{xL} \end{bmatrix}.$$

V_{xL}, V_{yL} and V_{zL} are probe velocity projection in the Body-Fixed Coordinate, ω_L is the rotation rate of small body, U_{xL}, U_{yL}, U_{zL} are gravity acceleration projection in the Body-Fixed Coordinate. Quadratic component is omit due to ω_L is very small, then the orbital dynamics of the probe in the Body-Fixed Coordinate is:

$$\begin{aligned} &\dot{x}_L = V_{xL}, \dot{y}_L = V_{yL}, \dot{z}_L = V_{zL}, \\ &\dot{V}_{xL} = u_{xL} - U_{xL} + 2\omega V_{yL} + \Delta_{xL}, \\ &\dot{V}_{yL} = u_{yL} - U_{yL} - 2\omega V_{xL} + \Delta_{yL}, \\ &\dot{V}_{zL} = u_{zL} - U_{zL} + \Delta_{zL} \end{aligned}, \quad (8)$$

where x_L, y_L, z_L are probe velocity projections in the Body-Fixed Coordinate.

$$U_{xL} = \frac{\partial V(R)}{\partial x}, \; U_{yL} = \frac{\partial V(R)}{\partial y}, \; U_{zL} = \frac{\partial V(R)}{\partial z}.$$

$V(R)$ indicates gravitational potential function, which is expressed using spherical harmonics. And:

$$\left[u_{xL} \quad u_{yL} \quad u_{zL}\right]^T = P_1 P_2 \left[u_x \quad u_y \quad u_z\right]^T,$$

is the component of acceleration in the Body-Fixed Coordinate. Δ_{xL}, Δ_{yL} and Δ_{zL} are perturbation force accelerations such as solar radiation pressure and solar gravitational.

2.2 GRAVITY FIELD MODEL OF SMALL BODY

The two main methods calculating the gravitational potential include harmonic series expansion method and polyhedral method. To simplify the calculation, the second method which has the analytical form is adopted in this paper. Considering the special performance during probe descent and landing, we assume the small body is a homogeneous tri-axial ellipsoid with axes a,b,c. The gravitational potential can be expanded into a series of spherical harmonics [16], that is:

$$V(R) = \frac{GM}{R}\left\{ \begin{aligned} &1 + \sum_{n=1}^{\infty}\sum_{m=0}^{n}\left(\frac{R_0}{R}\right)^m P_{nm}(\sin\theta) \\ &\times[C_{nm}\cos m\lambda + S_{nm}\sin m\lambda] \end{aligned} \right\}, \quad (9)$$

where GM equals to the product of the gravitational constant and the mass of small body, n is the degree and m is the order. R_0 is the reference radius(generally the largest equatorial radius), R, θ and λ are the distance from the mass centre of small body to the spacecraft, the latitude and longitude respectively in a Body-Fixed Coordinate system whose origin is at the centre of mass of the body. P_{nm} is the associated Legendre polynomials, C_{nm} and S_{nm} are coefficients of the potential determined by the mass distribution with in the small body. We can calculate the coefficients taking into account the above suppose, for such a symmetric body. $S_{nm} = 0$ for all n or m, $C_{nm} = 0$ for n or m odd and while other conditions:

$$C_{nm} = \frac{3}{R_0^{\,n}}\frac{(n/2)!(n-m)!}{2^m(n+3)(n+1)!}(2-\delta_{0m})\times$$

$$\sum_{i=0}^{\mathrm{int}(\frac{n-m}{4})}\frac{(a^2-b^2)^{\frac{m+4i}{2}}\left[c^2-\frac{1}{2}(a^2+b^2)\right]^{\frac{n-m-4i}{2}}}{16^i(\frac{n-m-4i}{2})!(\frac{m+2i}{2})!i!},$$

where δ_{0m} is Kronecker symbol, and the value of δ_{0m} is

$$\delta_{0m} = \begin{cases} 0, m = 0 \\ 1, m = 1 \end{cases}.$$

For our purposes we have stopped the expansion of Equation (9) to the second order, so we get the following coefficient:

$$C_{20} = \frac{2c^2 - (a^2+b^2)}{10R_0^{\,2}}, C_{22} = \frac{a^2-b^2}{20R_0^{\,2}}.$$

So, components of small body gravity are obtained as follow:

$$U_{x_L} = \frac{\partial V(R)}{\partial x_L} =$$

$$-\frac{GMx_L}{R^3}\begin{bmatrix} 1 + \frac{3}{2}C_{20}\left(\frac{R_0}{R}\right)^2\left(5\frac{z_L^{\,2}}{R^2} - 1\right) \\ +3C_{22}\left(\frac{R_0}{R}\right)^2\left(5\frac{x_L^{\,2} - y_L^{\,2}}{R^2} - 2\right) \end{bmatrix}, \quad (10)$$

$$U_{y_L} = \frac{\partial V(R)}{\partial y_L} = -\frac{GMy_L}{R^3}\begin{bmatrix} 1+\frac{3}{2}C_{20}\left(\frac{R_0}{R}\right)^2\left(5\frac{{z_L}^2}{R^2}-1\right) \\ +3C_{22}\left(\frac{R_0}{R}\right)^2\left(5\frac{{x_L}^2-{y_L}^2}{R^2}+2\right)\end{bmatrix}, \quad (11)$$

$$U_{z_L} = \frac{\partial V(R)}{\partial z_L} = -\frac{GMz_L}{R^3}\begin{bmatrix} 1+\frac{3}{2}C_{20}\left(\frac{R_0}{R}\right)^2\left(5\frac{{z_L}^2}{R^2}-3\right) \\ +15C_{22}\left(\frac{R_0}{R}\right)^2\left(\frac{{x_L}^2-{y_L}^2}{R^2}\right)\end{bmatrix}, \quad (12)$$

$$R = \sqrt{{x_L}^2+{y_L}^2+{z_L}^2}$$

3 Guidance and control

3.1 NOMINAL TRAJECTORY GUIDANCE

The desired descent altitude and velocity is planned in order to satisfy the requirements of soft landing on the surface of small bodies. We suppose the nominal trajectory is fuel suboptimal polynomial trajectory employed by Apollo landing lunar. The acceleration of one axis is planned as $a = c_0 + c_1 t$, and then desired descent altitude and velocity are shown after the integral.

$$v = c_0\left(t-t_0\right)+\frac{1}{2}c_1\left(t-t_0\right)^2+v_0, \quad (13)$$

$$r = \frac{1}{2}c_0\left(t-t_0\right)^2+\frac{1}{6}c_1\left(t-t_0\right)^3+v_0(t-t_0)+r_0. \quad (14)$$

The undetermined parameters c_0 and c_1 are obtained if the boundary condition is given by on the condition of the limited descend time.

$$c_0 = -\frac{2v_t+4v_0}{t_{go}}+6\frac{r_t-r_0}{{t_{go}}^2}, \quad (15)$$

$$c_1 = \frac{6v_t+6v_0}{{t_{go}}^2}-12\frac{r_t-r_0}{{t_{go}}^3}, \quad (16)$$

The parameters with subscript t stand for final states of the probe, and the parameters with subscript 0 stand for initial states of the probe, t_{go} is the descent time.

3.2 TERMINAL SLIDING MODE CONTROL WITH COMPENSATION TERM

Substituting $x_{11} = x_L$, $x_{12} = V_{xL}$, $x_{21} = y_L$, $x_{22} = V_{yL}$, $x_{31} = z_L$ and $x_{32} = V_{zL}$ into Equation (8):

$$\begin{cases} \dot{x}_{11} = x_{12} \\ \dot{x}_{12} = u_{xL} - U_{xL} + 2\omega_L x_{22} + \Delta_{xL} \\ \dot{x}_{21} = x_{22} \\ \dot{x}_{22} = u_{yL} - U_{yL} - 2\omega_L x_{12} + \Delta_{yL} \\ \dot{x}_{31} = x_{32} \\ \dot{x}_{32} = u_{zL} - U_{zL} + \Delta_{zL} \end{cases}, \quad (17)$$

$$f(X,t) = \begin{bmatrix} -U_{xL}+2\omega_L x_{22} \\ -U_{yL}-2\omega_L x_{12} \\ -U_{zL} \end{bmatrix}, \; b(X,t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Modelling uncertainties related to irregular gravitational potential and perturbations as below:

$$\Delta f(x) = \begin{bmatrix} 0.2\sin(\omega_L t)\cdot U_{xL} \\ 0.2\sin(\omega_L t)\cdot U_{yL} \\ 0.2\sin(\omega_L t)\cdot U_{zL} \end{bmatrix}, \; d(t) = \begin{bmatrix} 0.02\sin(2\pi t) \\ 0.02\sin(2\pi t) \\ 0.02\sin(2\pi t) \end{bmatrix}.$$

So the model is similar with the following nonlinear system:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(X,t)+\Delta f(X,t)+b(X,t)u+d(t) \end{cases}. \quad (18)$$

$$X = [{x_1}^T {x_2}^T]^T = [{x_1}^T {\dot{x}_1}^T]^T$$

Suppose: Uncertainty $\Delta f(X,t)$ and external disturbance $d(t)$ are bounded and satisfy $\left|\Delta f(X,t)\right| \le F(X,t)$, $\left|d(t)\right| \le D(t)$, $F(X,t)$ and $D(t)$ are nonnegative functions. Definition error vector:

$$E = X - X_d = [e^T \quad \dot{e}^T]^T, \; e = x_i - x_{id} = [e_1 \quad e_2 \quad e_3]^T.$$

Sliding surface is given as:

$$\sigma(X,t) = CE - W(t), \quad (19)$$

$C = [C_1 \quad C_2]$ is normal constant matrix.

$$W(t) = CP(t), \quad (20)$$

$$P(t) = [p(t)^T \quad \dot{p}(t)^T]^T, \; p(t) = [p_1(t) \quad p_2(t) \quad p_3(t)]^T.$$

and satisfy the following conditions: for a constant $T > 0$, $p_i(t)$ is bounded in $[0,T]$, and

$$p_i(0) = e_i(0), \dot{p}_i(0) = \dot{e}_i(0), p_i^{(2)}(0) = e_i^{(2)}(0), i = 1,2,3,$$

$$p_i(t)=\begin{cases}\sum_{k=0}^{n}\frac{1}{k!}e_i(0)^{(k)}t^k+\sum_{j=0}^{n}\left(\sum_{l=0}^{n}\frac{a_{jl}}{T^{j-l+n+1}}e_i(0)^{(l)}\right)\cdot t^{j+n+1}, 0\le t\le T\\ 0, t>T\end{cases}, \quad (21)$$

the order of this system $n=2$.

The following coefficients are obtained by solving the equation:

$$\begin{cases}a_{00}=-10\\ a_{10}=15\\ a_{20}=-6\end{cases}, \begin{cases}a_{01}=-6\\ a_{11}=8\\ a_{21}=-3\end{cases}, \begin{cases}a_{02}=-1.5\\ a_{12}=1.5\\ a_{22}=-0.5\end{cases}.$$

Sliding surface is finally gained.

The terminal sliding mode controller is deduced by Lyapunov function as follows:

$$\begin{aligned}&\dot{\sigma}(X,t)=CE-C\dot{P}(t)=\\ &C\left[\dot{e}^T\ddot{e}^T\right]^T-C\left[\dot{p}^T(t)\ddot{p}^T(t)\right]^T=\\ &C_2\left[\ddot{e}-\ddot{p}(t)\right]+\sum_{k=1}^{n-1}C_k\left[e^{(k)}-p^{(k)}(t)\right]=\\ &C_2\left[\ddot{x}_1-\ddot{x}_{1d}-\ddot{p}(t)\right]+C_1\left[\dot{e}-\dot{p}(t)\right]=\\ &C_2\left[\dot{x}_2-\ddot{x}_{1d}-\ddot{p}(t)\right]+C_1\left[\dot{e}-\dot{p}(t)\right],\end{aligned}$$

$$\begin{aligned}&\dot{\sigma}(X,t)=C_2[f(X,t)+\Delta f(X,t)+b(X,t)u+\\ &d(t)-\ddot{x}_{1d}-\ddot{p}(t)]+C_1\left[\dot{e}-\dot{p}(t)\right]\end{aligned}.$$

Lyapunov function is:

$$\begin{aligned}&\dot{V}=\sigma^T\dot{\sigma}=\\ &\begin{bmatrix}\sigma^TC_2\{f(X,t)-\ddot{x}_{1d}-\ddot{p}(t)+C_2^{-1}C_1\left[\dot{e}-\dot{p}(t)\right]\}\\ +\sigma^TC_2b(X,t)u+\sigma^TC_2\left[\Delta f(X,t)+d(t)\right]\end{bmatrix}\le\\ &\begin{bmatrix}\sigma^TC_2\left\{f(X,t)-\ddot{x}_{1d}-\ddot{p}(t)+C_2^{-1}C_1\left[\dot{e}-\dot{p}(t)\right]\right\}\\ +\sigma^TC_2b(X,t)u+\|\sigma^TC_2\|\cdot\|\Delta f(X,t)+d(t)\|\end{bmatrix}.\end{aligned}$$

The final terminal controller is:

$$u(t)=\begin{Bmatrix}-b(X,t)^{-1}\{f(X,t)-\ddot{x}_{1d}-\ddot{p}(t)\\ +C_2^{-1}C_1(\dot{e}-\dot{p}(t))\}\\ -b(X,t)^{-1}\frac{C_2^T\sigma}{\|C_2^T\sigma\|}\{F(X,t)+D(t)+K\}\end{Bmatrix}. \quad (22)$$

K is positive constant.

$$\begin{aligned}&\dot{V}\le\begin{Bmatrix}\|C_2^T\sigma\|\cdot\{\|\Delta f(X,t)+d(t)\|-\left[F(X,t)+D(t)\right]\}\\ -K\|C_2^T\sigma\|\end{Bmatrix}=\\ &\|C_2^T\sigma\|\left\{\left[\|\Delta f(X,t)\|-F(X,t)\right]+\left[\|d(t)\|-D(t)\right]\right\}\\ &-K\|C_2^T\sigma\|\le-K\|C_2^T\sigma\|<0\end{aligned}.$$

Based on the suppose and sliding surface Equation (19),

$$\begin{aligned}&\sigma(X,0)=CE(0)-W(0)=CE(0)-P(0)\\ &=C\left\{\left[e(0)^T\dot{e}(0)^T\right]^T-\left[p(0)^T\dot{p}(0)^T\right]^T\right\}=0\end{aligned}.$$

The initial state of the system reach sliding surface, and global stability is guaranteed since the reach phase is removed. Therefore, it makes the systems stability and strong global robustness.

3.3 ADAPTIVE TERMINAL SLIDING MODE CONTROL WITH COMPENSATION TERM

The uncertainties such as small body rotating, solar radiation pressure and solar gravitational are more complex, so it is hard to obtain the boundary. The efficiency is not well if parameters in the control law are too large, on the contrary, the sliding mode is not conformed to exit if they are too small. Controller with the simple parameter adaptive law is effective for estimating the unknown parameters. This section presents a novel adaptive Terminal Sliding Mode Control law combined with variable structure and adaptive control for probe descent near the small body when the boundary of the uncertainties and external disturbances are unknown.

Suppose: The boundary of uncertainty $\Delta f(X,t)$ and external disturbance $d(t)$ are unknown, and satisfy the following inequality:

$$\|\Delta f(X,t)+d(t)\|\le\delta_0+\delta_1\|X\|, \quad (23)$$

δ_0 and δ_1 are unknown nonnegative constants.

The adaptive laws are proposed as follows to estimate the uncertainty and disturbance:

$$\begin{aligned}&\dot{\hat{\delta}}_0(t,X)=\rho_0^{-1}\left\|C_n^T\sigma\right\|\\ &\dot{\hat{\delta}}_1(t,X)=\rho_1^{-1}\left\|C_n^T\sigma\right\|\cdot\|X\|\end{aligned}, \quad (24)$$

$\hat{\delta}_0(t,X)=\bar{\delta}_0(t,X)-\delta_0, \hat{\delta}_1(t,X)=\bar{\delta}_1(t,X)-\delta_1$, $\bar{\delta}_0(t,X)$ and $\bar{\delta}_1(t,X)$ are the adaptive estimation of unknown parameters δ_0 and δ_1, ρ_0 and ρ_1 are positive adaptive gain. δ_0 and δ_1 are constants, so the Equation (24) can be rewritten as follow:

$$\begin{aligned}&\dot{\bar{\delta}}_0(t,X)=\rho_0^{-1}\left\|C_n^T\sigma\right\|\\ &\dot{\bar{\delta}}_1(t,X)=\rho_1^{-1}\left\|C_n^T\sigma\right\|\cdot\|X\|\end{aligned}. \quad (25)$$

Considering Lyapunov function:

$$2V(\sigma,\hat{\delta}_0,\hat{\delta}_1)=\sigma^T\sigma+\rho_0\hat{\delta}_0^{\,2}+\rho_1\hat{\delta}_1^{\,2},$$

$$\dot{V}(\sigma,\hat{\delta}_0,\hat{\delta}_1)=\sigma^T\dot{\sigma}+\rho_0\hat{\delta}_0\dot{\hat{\delta}}_0+\rho_1\hat{\delta}_1\dot{\hat{\delta}}_1= \left\{\begin{matrix}\sigma^T C_2\begin{bmatrix} f(X,t)+\Delta f(X,t)+b(X,t)u+d(t)\\ -\ddot{x}_{1d}-\ddot{p}(t)\end{bmatrix}\\ +C_1\left[\dot{e}-\dot{p}(t)\right]+\rho_0\hat{\delta}_0\dot{\hat{\delta}}_0+\rho_1\hat{\delta}_1\dot{\hat{\delta}}_1\end{matrix}\right\}\le \left\{\begin{matrix}\sigma^T C_2[f(X,t)+b(X,t)u-\ddot{x}_{1d}-\ddot{p}(t)]\\ +C_1\left[\dot{e}-\dot{p}(t)\right]+\left\|C_n^{\,T}\sigma\right\|\cdot(\delta_0+\delta_1\|X\|)\\ +\rho_0\hat{\delta}_0\dot{\hat{\delta}}_0+\rho_1\hat{\delta}_1\dot{\hat{\delta}}_1\end{matrix}\right\}. \quad (26)$$

The final adaptive terminal controller is:

$$u(t)=-b(X,t)^{-1}\left\{\begin{matrix} f(X,t)-\ddot{x}_{1d}-\ddot{p}(t)\\ +C_2^{-1}C_1(\dot{e}-\dot{p}(t))\end{matrix}\right\}- b(X,t)^{-1}\frac{C_2^{\,T}\sigma}{\left\|C_2^{\,T}\sigma\right\|}\left\{(\bar{\delta}_0+\bar{\delta}_1\|X\|)+K\right\}, \quad (27)$$

K is positive constant.

$$\dot{V}(\sigma,\hat{\delta}_0,\hat{\delta}_1)\le\left\{\begin{matrix}\|C_2^{\,T}\sigma\|\cdot\left\{(\bar{\delta}_0+\bar{\delta}_1\|X\|)+K\right\}\\ -\|C_2^{\,T}\sigma\|(\delta_0+\delta_1\|X\|)\\ +\rho_0\hat{\delta}_0\dot{\hat{\delta}}_0+\rho_1\hat{\delta}_1\dot{\hat{\delta}}_1\end{matrix}\right\}= \left\{\begin{matrix}-K\|C_2^{\,T}\sigma\|+\hat{\delta}_0(\rho_0\dot{\hat{\delta}}_0-\|C_2^{\,T}\sigma\|)\\ +\hat{\delta}_1(\rho_1\dot{\hat{\delta}}_0-\|C_2^{\,T}\sigma\|\cdot\|X\|)\end{matrix}\right\}\le -K\|C_2^{\,T}\sigma\|\le 0, \quad (28)$$

Based on Equations (26) and (28):

$$\dot{V}(\sigma,\hat{\delta}_0,\hat{\delta}_1)=\sigma^T\dot{\sigma}+\rho_0\hat{\delta}_0\dot{\hat{\delta}}_0+\rho_1\hat{\delta}_1\dot{\hat{\delta}}\le -K\|C_2^{\,T}\sigma\|$$

That is:

$$\sigma^T\dot{\sigma}+\rho_0\hat{\delta}_0\rho_0^{-1}\|C_2^{\,T}\sigma\|+\rho_1\hat{\delta}_1\rho_1^{-1}\|C_2^{\,T}\sigma\|\|X\|\le -K\|C_2^{\,T}\sigma\|,$$

$$-\sigma^T\dot{\sigma}\ge\hat{\delta}_0\|C_2^{\,T}\sigma\|+\hat{\delta}_1\|C_2^{\,T}\sigma\|\cdot\|X\|+K\|C_2^{\,T}\sigma\|\ge 0$$

$$\|\sigma\|\cdot\|\dot{\sigma}\|\ge\hat{\delta}_0\|C_2^{\,T}\sigma\|+\hat{\delta}_1\|C_2^{\,T}\sigma\|\cdot\|X\|+K\|C_2^{\,T}\sigma\|$$

$$\|\dot{\sigma}\|\ge\frac{\hat{\delta}_0\|C_2^{\,T}\sigma\|+\hat{\delta}_1\|C_2^{\,T}\sigma\|\cdot\|X\|+K\|C_2^{\,T}\sigma\|}{\|\sigma\|}(\|\sigma\|\ne 0)$$

without loss of generality, let $C_2=\begin{bmatrix}1&0\\0&1\end{bmatrix}$, and:

$$\|\dot{\sigma}\|=\frac{\hat{\delta}_0\|\sigma\|+\hat{\delta}_1\|\sigma\|\cdot\|X\|+K\|\sigma\|}{\|\sigma\|} = \hat{\delta}_0+\hat{\delta}_1\cdot\|X\|+K>K(\|\sigma\|\ne 0) \quad (29)$$

So the sliding mode σ convergences to zero in the finite time.

Meanwhile in order to reducing buffeting, $C_2^{\,T}\sigma/\left\|C_2^{\,T}\sigma\right\|$ is replaced by continuous function vector S_ε.

$S_\delta=\dfrac{C_2^{\,T}\sigma}{\left\|C_2^{\,T}\sigma\right\|+\varepsilon}$, $\varepsilon=\varepsilon_0+\varepsilon_1\|e\|$, ε_0 and ε_1 are two positive constant.

4 Simulation and analysis

The initial conditions and other simulation parameters of the target small body are listed in Table 1 referring to references [17, 18].

The trajectory of z axis is planning as Polynomial method mentioned in section 3.1 in order to landing on the surface of small body safely. The position curve $z_c(t)$ and velocity curve $\dot{z}_c(t)$ of z axis are as follows if the above initial condition and descent time $t_{go}=4000s$ are given. Based on the Equations (13) - (16), z-axis is planed Nominal Polynomial trajectory.

$$z_c(t)=\left\{\begin{matrix}z_0+\dot{z}_0t+(3z_n-3z_0-2\dot{z}_0t_{go})(t/t_{go})^2\\ +(2z_0+\dot{z}_0t_{go}-2z_n)(t/t_{go})^3\end{matrix}\right\}, \quad (30)$$

$$\dot{z}_c(t)=\left\{\begin{matrix}\dot{z}_0+(6z_n-6z_0-4\dot{z}_0t_{go})t/t_{go}^{\,2}\\ +(6z_0+3\dot{z}_0t_{go}-6z_n)t^2/t_{go}^{\,3}\end{matrix}\right\}. \quad (31)$$

z_0 and $\dot{z}_0$ are the initial position and velocity of z axis, z_n and $\dot{z}_n$ is landing position and velocity of z-axis. The landing position and velocity of x-axis, y-axis and z-axis is desired as shown in table 1 in the finite time, and x-, y-axes are expected to zero but not the nominal polynomial trajectory.

TABLE 1 Parameters and Simulation value

Parameters	Simulation value
GM(m^3/s^2)	4.842×10^5
Spin period(h)	10.54
Reference radius R0(m)	1138.5
C20	-0.003
C22	0.0396
Initial position	[350,300,9000]
Landing site	[0,0,7000]
Initial velocity	[-1.2,0.2,-1.0]
Landing velocity	[0,0,0]

Equation (18) and ATMSC with compensation term Equation (27) described above are incorporated into the simulation software that is used to generate time history of the probes dynamical state during the probe descent phase near rotating small body. The simulation results are shown in Figures 2-13 and all data are described in the Body-Fixed Coordinate *oxyz*.

The controller parameters are $K = 10$, $T = 10$, $\varepsilon_0 = 0.03$, $\varepsilon_1 = 1,\ \rho_0 = 1,\ \rho_1 = 1$:

$$C = [C_1 \quad C_2] = \begin{bmatrix} 1 & 0 & 0 & 4 & 0 & 0 \\ 0 & 1 & 0 & 0 & 4 & 0 \\ 0 & 0 & 1 & 0 & 0 & 4 \end{bmatrix}.$$

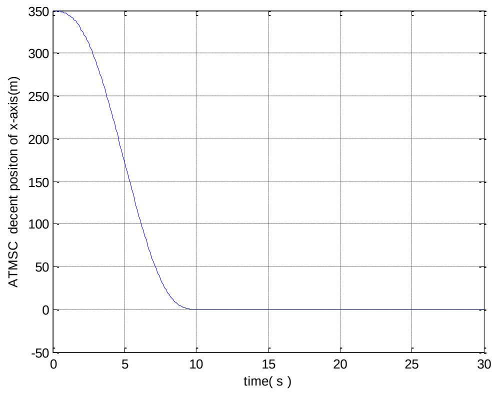

FIGURE 2 ATSMC descent altitude time history of *x*-axis

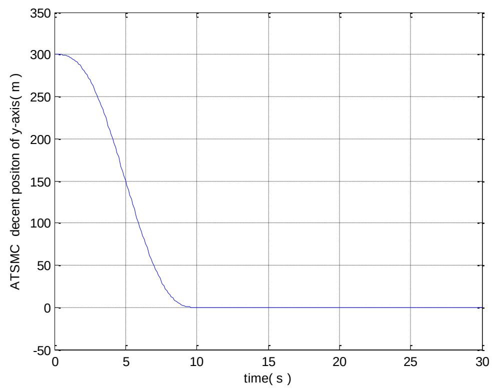

FIGURE 3 ATSMC descent altitude time history of *y*-axis

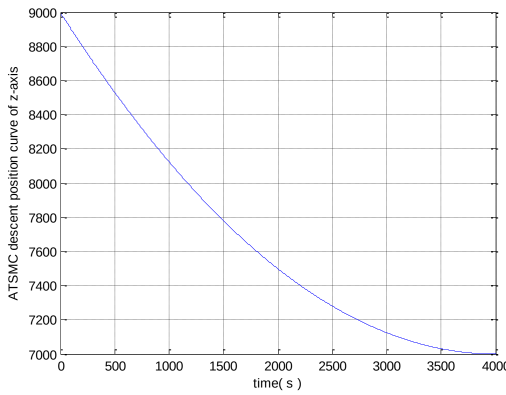

FIGURE 4 ATSMC descent altitude time history of *z*-axis

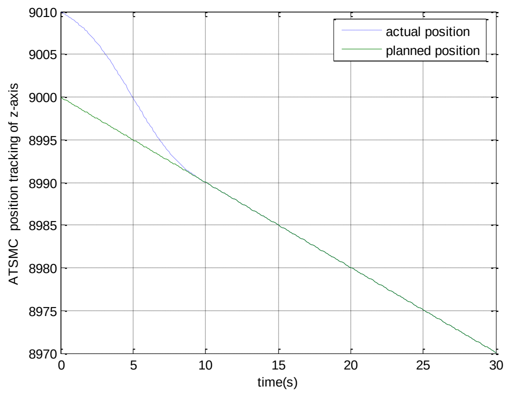

FIGURE 5 ATSMC descent position tracking of *z*-axis

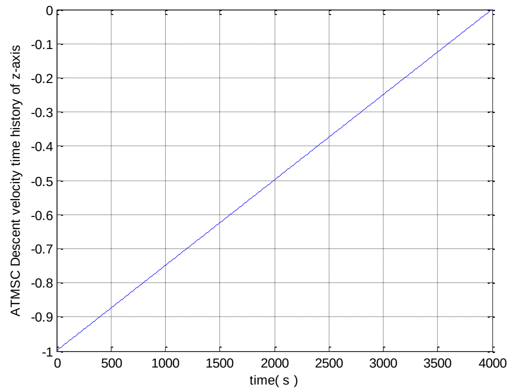

FIGURE 6 ATSMC descent velocity time history of *z*-axis

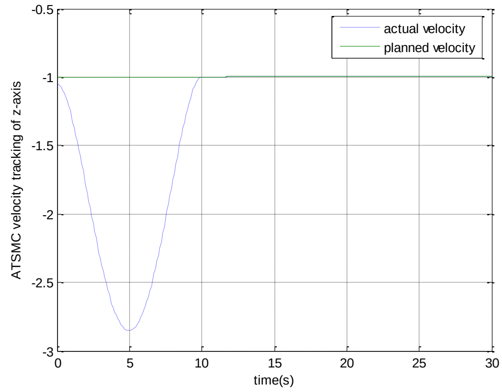

FIGURE 7 ATSMC descent velocity tracking of *z*-axis

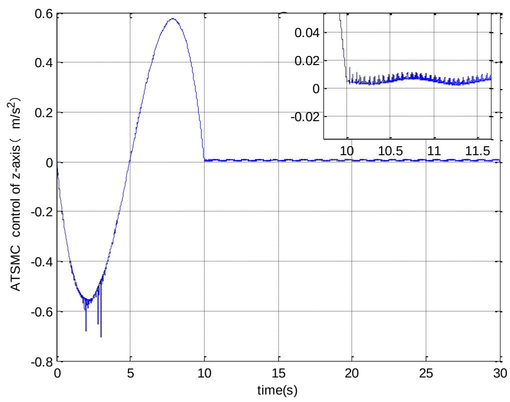

FIGURE 8 Control acceleration of *z*-axis

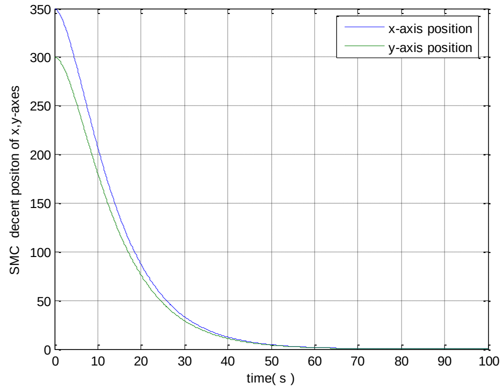

FIGURE 9 SMC descent altitude time history of *x*, *y*-axes

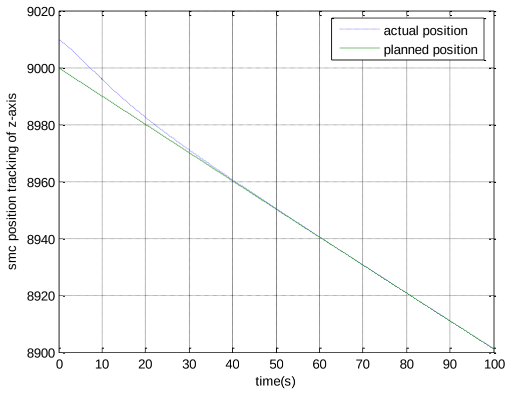

FIGURE 10 SMC descent position tracking of z-axis

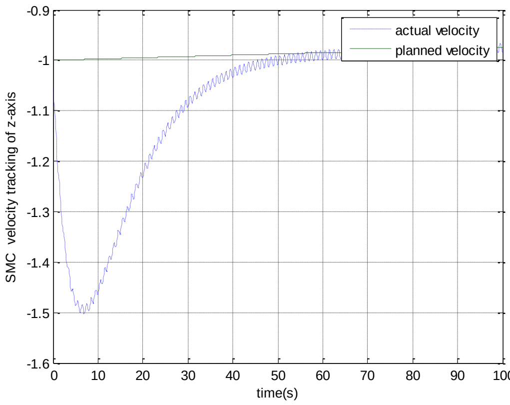

FIGURE 11 SMC descent velocity tracking of *z*-axis

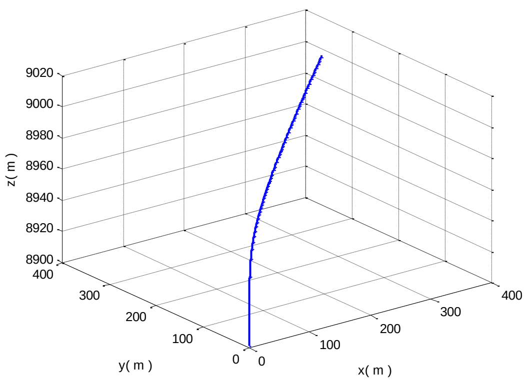

FIGURE 12 SMC three-dimensional trace

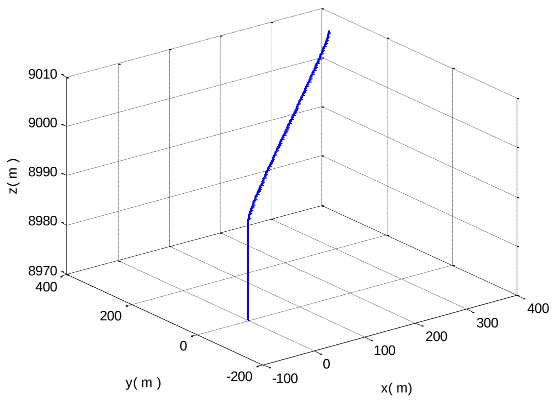

FIGURE 13 ATSMC three-dimensional trace

Figures 2-4 give the descent position simulation time history of *x*-, *y*-, and *z*-axes respectively under the controller Equation (27) and here all data are described in the Body-Fixed Coordinate system. Figure 6 gives the velocity simulation time history of *z*-axis under controller Equation (27). The descent positions of *x*- and *y*- axes approach zeros quickly, meanwhile descent position and descent velocity of z-axis conform to the planed Nominal Polynomial trajectory Equations (23) and (24). Figures 5 and 7 give the descent position and velocity tracking curves of *z*-axis. These results show that positions and velocity of *x*-, *y*- and *z*-axes can reach desired site in the finite time about 10s. Control force in Figure 8 can satisfied engineering requirements. Figures 9, 10 and 11 give the descent position and velocity time simulation curve of *x*-, *y*-, and *z*-axes based on normal Sliding Mode Control. The tracking time is about 50s and the curves are affected by perturbation obviously. We can see the guidance and control algorithm based on Adaptive Terminal Sliding Mode Control can fast and accurately track the planned trajectory in the finite time and is more robust to parameter uncertainty, feedback state error and external disturbances than the normal Sliding Mode Control. The performance of tracking the desired trajectory fast is also verified from three-dimensional curves in Figure 12 and 13.

5 Conclusions

The present paper examines the three dimensional orbital dynamics and control aspects of the probe descent near rotating small bodies. The relative kinetics equations for the system are derived using Newton's second law in the Body-Fixed Coordinate system with the assumption that the small body is a homogeneous tri-axial ellipsoid. The nominal polynomial trajectory is employed for *z*-axis taking account of the fuel suboptimal. Terminal Sliding Mode Control law with compensation term is derived and the efficacy of the controller performance is tested via numerical simulation. The Adaptive Terminal Sliding Mode Control law with compensation term is then derived in the case the boundary of uncertainties and external disturbances are unknown. Results of the numerical simulation match with closed-form solutions and demonstrate the proposed algorithm can ensure fast and accurately respond in conditions of parameter uncertainty, feedback state error and external disturbances compared with SMC, and the system can ensure the descent accuracy requirement and a safe landing velocity.

6 Further Work

The gravitational potential function adopted in the present paper is the second order harmonic series expansion method which may generate inaccuracy when the probe closing to the small body. Segmentation method is intended to adopted for calculating the gravitational potential function when the probe landing on the surface of small body. Harmonic series expansion method is adopted when the probe is far from the small body and polyhedral approach is adopted when the distance is close.

Nominal polynomial trajectory in the present paper has limitations only considering fuel suboptimal. The future work is focused on descent trajectory planning taking fully account into the fuel and irregular gravitation etc.

Acknowledgments

This work is supported by National Basic Research Program of China (973 Program) under Grant 2012CB720000.

References

[1] Kowal C T 1996 Asteroids Their nature and utilization *Wiley*, Chichester UK

[2] Veverka J, Farquhar B, Robinson M, Thomas P, Murchie S, Harch A, Antreasian PG, Chesley SR, Miller JK, Owen WM Jr, Williams BG, Yeomans D, Dunham D, Heyler G, Holdridge M, Nelson RL, Whittenburg KE, Ray JC, Carcich B, Cheng A, Chapman C, Bell JF 3rd, Bell M, Bussey B, Clark B, Domingue D, Gaffey MJ, Hawkins E, Izenberg N, Joseph J, Kirk R, Lucey P, Malin M, McFadden L, Merline WJ, Peterson C, Prockter L, Warren J, Wellnitz D 2001 *Nature* **413**(6854) 390-3

[3] Holdridge M E 2002 *Johns Hopkins APL technical digest* **23**(1) 58-70

[4] David W D, Robert W F, Farquhar R W, McAdams J V 2002 *Icarus* **159**(2) 433-8

[5] Kubota T, Hashimoto T, Sawai S, Kawaguchi J I, Ninomiya K, Uo M, Baba K 2003 *Acta Astronautica* **52**(2) 125-131

[6] Kubota T, Otsuki M, Hashimoto T, Bando N, Yano H, Uo M, Kawaguchi J I 2006 *Proceedings of AIAA/AAS Astrodynamics Specialist Conference and Exhibit 2006* AIAA-2006-6539

[7] Varatharajoo R, Kahle R 2005 *Aircraft Engineering and Aerospace Technology* **77**(2) 131-41

[8] Li S, Cui H T, Cui P Y 2005 *Aircraft Engineering and Aerospace Technology* **77**(4) 317-23

[9] Cui H T, Shi X Y, Cui P Y 2002 *Chinese Journal of Space Science* **22**(3) 256-60

[10] Colin R M, Gianmarco R 1996 *Journal of Guidance, Control and Dynamics* **19**(3) 740-2

[11] Carson J M, Açıkmese B 2006 A model predictive control technique with guaranteed resolvability and required thruster silent times for small-body proximity operations *Proceedings of the AIAA Guidance, Navigation, and Control Conference and Exhibit* Keystone CO AIAA-2006-6780

[12] Bellerose J, Girard A, Scheeres D J 2009 Optimization and Cooperative Control Strategies, *Springer Berlin Heidelberg* 135-150

[13] Liu J K 2005 *MATLAB Simulation for sliding mode control* Tsinghua Press: Beijing *(in Chinese)*

[14] Hu J B, Shi M H 2005 *Control Theory &Applications* **22**(3) 540-8 *(in Chinese)*

[15] Zhou J Y, Zhou D 2007 *Journal of Astronautics* **28**(6) 1462-6 *(in Chinese)*

[16] Werner R A, Scheeres D J 1996 *Celestial Mechanics and Dynamical Astronomy*, **65**(3) 313-44

[17] Zhang Z X, Wu W D, Li L T, Huang X Y, Cui H T, Li S, Cui P Y 2012 *Journal of the Franklin Institute* **349**(2) 493-509

[18] Huang X Y, Cui H T 2004 *Acta Astronautica* **54**(10) 763-71

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