Calculation modelling of static constructions

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Abstract

The mathematic model of force loads calculations of static constructions in the framework of Theoretical Mechanics course is presented. The software to visualize and analyse force loads of static constructions is created.

Keywords: theoretical mechanics, informative systems of engineering analysis, visualization and analysis of static constructions, visualization of the force loading

1 Introduction

Computer-Aided Design (CAD) systems have spread widely in engineering to create solid models of real solid bodies as well as to simulate their motion and interaction. NX, Solid Edge, CATIA, SolidWorks, Inventor, Creo Elements/Pro, etc. [1, 2] are among the software products that are used for the purpose above. These CAD systems have numerous disadvantages preventing their usage for academic studies, for instance, within Theoretical Mechanics course. One of the drawbacks to utilize them for the course is CAD versatility, a wide range of interrelated tasks the software aims to support, which makes CAD to be a complex tool consuming considerable time and efforts to master. Another obstacle to use the software products above within academic curriculum is their focus on solid models meanwhile the types of the links the CADs consider differ from those accepted within conventional Theoretical Mechanics course.

CAD systems are generally licensed software products which makes them a black box for the end-user as there is no evidence of which mathematic principles they are based upon. A user can not estimate the validity of calculations without making some experiments. That is why some original customized software focused on academic course of Theoretical Mechanics might be a unique solution for the studies.

The mathematic model presented in the present paper is the foundation for this kind of software. This model is oriented at two dimensional system of active loads applied to a static construction, i.e. a body with some constraints. The aim of the research is to define indeterminate constraints as well as inertial forces. Subsequently dynamic model can be created on the basis of the model above.

2 General mathematic statement of the problem

The system of equations, equilibrium condition for a plane system of forces, can be described as follows:

$$\begin{cases} a_{11}\cdot x_1 + a_{12}\cdot x_2 + a_{13}\cdot x_3 = b_{11} \\ a_{21}\cdot x_1 + a_{22}\cdot x_2 + a_{23}\cdot x_3 = b_{21} \\ a_{31}\cdot x_1 + a_{32}\cdot x_2 + a_{33}\cdot x_3 = b_{31}, \end{cases}$$

where x_1, x_2, x_3 are indeterminate constraints and inertial forces; a_{11}, a_{12}, etc are coefficients depending on a type and direction of indeterminate constraints and inertial forces (Table 1); b_{11}, b_{21}, b_{31} are the values depending on applied active forces.

We shall understand that X-axis, an axis of abscissa, is always directed horizontally to the right and Y-axis, an axis of coordinates, is directed vertically up. We shall assign the axis *l* for each indeterminate constraint of a force which lays on this very axis; by convention we shall align the direction of the constraint and positive direction of the axis. Correspondingly we shall assign positive direction of the moment sought to act counterclockwise for each indeterminate constraint of a force couple.

b_{11}, b_{21}, b_{31} values can be determined as follows:

$$b_{11} = -\sum X_i\,;$$

$$b_{21} = -\sum Y_i\,;$$

$$b_{31} = -\sum M_{iO},$$

where X_i - the projection of *i*-th active force on X-axis; Y_i - the projection of *i*-th active force on Y-axis; M_{iO} - the moment of *i*-th active force with respect to a point *O*.

Inertial forces depend on the type of feasible motion of the construction [3, 4]. We shall carry out the calculations taking into the account that center-of-mass velocity, i.e. point *C*, is equal to zero and its acceleration is different to zero. Suppose under the rotation angular velocity is equal to zero but angular acceleration ε might occur.

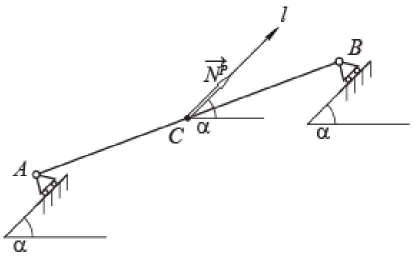

FIGURE 1 Translation motion

Resultant vector of forces $\overrightarrow{N^P}$ applied in the point of center of mass C acts upon the construction and it is of an indeterminate value. We shall determine the direction of l axis by virtue of rotation of the normal by the angle of -90º to the support surface of A support.

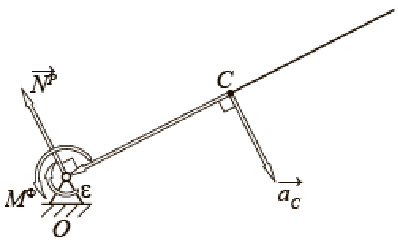

FIGURE 2 Rotation motion

The resultant moment of inertial forces M^{Φ} is an indeterminate value. The resultant vector of inertia forces can be determined as follows:

$$N^P = \frac{m}{I_O} \cdot CO \cdot M^{\Phi},$$

where m - mass of the body; I_O - moment of inertia with respect to the point O.

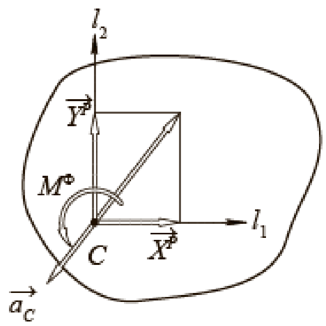

FIGURE 3 Plain motion of a free rigid body

Resultant moment of inertial forces and resultant vector of inertial forces projections onto x and y axes, i.e. X^P and Y^P values, are the indeterminate values here.

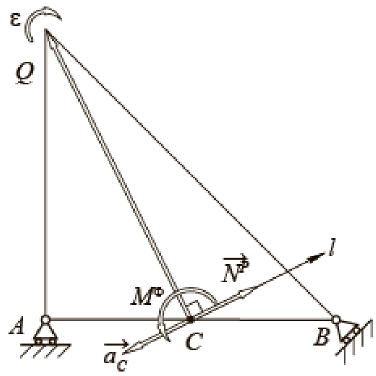

FIGURE 4 Plain motion of a free rigid body with respect of two points

The mass center has got the acceleration which aligns with the rotation acceleration. This very acceleration is perpendicular on $\overrightarrow{CQ}$ vector which joins the mass center with an instantaneous acceleration center and it is aligned with the direction of angular acceleration ε. The position of the instantaneous acceleration center can be determined as the intersection of perpendiculars on the support planes of A and B supports. The moment M^{Φ}, which is an indeterminate one, can be used by analogy with the rotation motion to find the value of N^P:

$$N^P = \frac{m}{I_C} \cdot CQ \cdot M^P,$$

where I_C is the moment of inertia with respect to the point C.

The direction of l axis which $\overrightarrow{N^P}$ force lies along can be determined by means of rotation of the $\overrightarrow{CQ}$ vector by the angle of -90º.

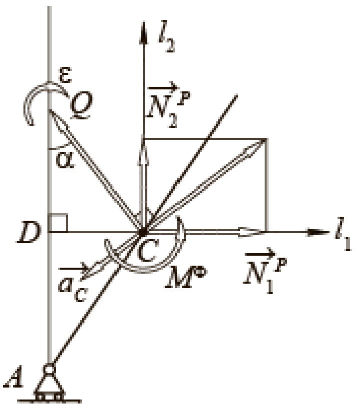

FIGURE 5 Plain motion of a rigid body with respect of one point

Moment M^{Φ} and force N_1^P are indeterminate ones. The axis l_1 of the latter can be determined by virtue of rotation of the normal by the angle of -90º to the support surface of A support. We shall obtain the value of force N_2^P with the help of an indeterminate moment M^{Φ} as below:

$$N_2^P = \frac{m}{I_C} \cdot CD \cdot M^{\Phi}.$$

The axis l_2 of N_2^P force is co-directional with the normal to the support surface of A support.

TABLE 1 Matrix A coefficients

N	Type of variables	Matrix A coefficients		
		a_{1i}	a_{2i}	a_{3i}
1	X, X^P	1	0	
2	Y, Y^P	0	1	$x \cdot a_{2i} - y \cdot a_{1i}$, where x and y are horizontal coordinates of the point of the force application with respect of the center where the moment is to be calculated.
3	N, N^P	cosα, where α is an angle between positive directions of the axes x and l.	sinα, where α is an angle between positive directions of the axes x and l.	
4	M^{Φ} at rotation motion of a construction	$\frac{m}{I_O} \cdot CO \cdot \cos\alpha$, where α is an angle between positive directions of the axes x and l.	$\frac{m}{I_O} \cdot CO \cdot \sin\alpha$, where α is an angle between positive directions of the axes x and l.	1
5	M^{Φ} at plain motion with respect of two points	$\frac{m}{I_C} \cdot CQ \cdot \cos\alpha$, where α is an angle between positive directions of the axes x and l.	$\frac{m}{I_C} \cdot CQ \cdot \sin\alpha$, where α is an angle between positive directions of the axes x and l.	1
6	M^{Φ} at plain motion with respect of one point	$\frac{m}{I_C} \cdot CD \cdot \cos\alpha$, where α is an angle between positive directions of the axes x and l.	$\frac{m}{I_C} \cdot CD \cdot \sin\alpha$, where α is an angle between positive directions of the axes x and l.	1

3 Conclusions

The mathematic model to determine force loads of static constructions simulating all types of their possible motions has been created on the basis of Theoretical mechanics laws.

This model can be implemented into an educational computer software product to visualize and analyse force loads of static constructions within the academic curriculum of Theoretical Mechanics course.

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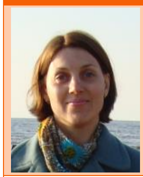

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