Detecting trap region with assortativity measurement in temporal networks

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Abstract

Based on the threshold model of Watts, the effect on cascade dynamics induced by temporal shuffling according to assortative structure was investigated in this paper. Two assortative rewiring schemes were introduced and explored, by considering the topological parameter of nodal degree and the average degree of nodal neighbors. Temporal behaviors are generated by edge breaking and rewiring, according to the assortativity coefficient of links. Analysis shows that the trap region on cascade dynamics identified by edge assortativity based on degree of the neighbors is better than the nodal degree-based one. The correctness of the analysis is validated with simulations on scale-free module networks.

Keywords: cascade threshold model, trap region, module network, assortative links

1 Introduction

With the increasing demand of technical application and social interaction, technology and social networks have been becoming more and more complex and interdependent. Individuals in such networks prefer to give consideration to every aspect of the information collected from different ways, for their own decisions. On one hand, it is still a comprehensive and complex task in theoretical research to capture the human behavior in the cascade modeling due to the different evaluation criteria. On the other hand, it is still a tough job to find a model that can express the topology of social networks reasonably and precisely. Therefore, it is meaningful to find a generalized way to capture the priority of human behavior in the adoption cascading process on the social network [1]. The discipline mentioned above has not been widely researched until the emergence of the well-known cascade model introduced by Watts [2]. In threshold model of watts, some assumptions referred to are as follows: an innocent node became an informed and spreading one when the proportion of its adopted friends exceeded the threshold; each node contacted with all of its neighbors in one time step, and to every neighbor, the probability of contact is equal. Under these assumptions, the bigger the value of degree is, the more neighbors in adoption state are needed for the state change from innocent to adopted state on that specific node. Therefore, the probability that information cascading between nodes depends on two constrained relationships: nodes with high degree are easy to contact information, but hard to change their state [3-5]. These two simple elements give rise to a rich dynamics behavior that also inspires wide interest in network community.

In basic Watts' threshold model, it assumes that people contact and perceive with all their neighbors in a consequent way. However, due to various social or technical reasons, an individual always treats information from different sources with different priorities, and almost in a discrete way. The discrete way means a node will not contact with all of its neighbors in each time step, even the node is free. The event-driven contact is a common example, and widely exists in human interaction, wireless sensor networks and technical networks. The sparseness in contacting behavior are studied as *burstiness* in temporal network in related research community [6-9].

To capture the temporal property mentioned above, temporal networks are proposed and widely researched in cascade modeling. By considering the time-dependent interactions within the networks of real-world, the author in reference [6] have encoded two real temporal data， City ware project and Enron email community, into graphs. They analyzed individuals' link numbers, appearing times and active time sequences under three different temporal measurement, and obtained some surprising and interesting results, for example, high-degree individuals have a lower ability of finding the route of information propagation, which causes that they are relatively slower when propagating information through the network. However, these two communication data exhibit a striking difference in terms of some temporal measurements, and specific analysis of concrete networks is needed to find their temporal characteristics hidden in the data. A kind of simple and generalized model is still absent. In most of previous research, temporal networks

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were built based on real event data. However, in the complex networks with very lager scale, it's hard to record and establish every user's contact event sequence in a long time duration. For example, when rumor spreads on twitter, it's hard to record the time stamp of every online communicate behavior for all user. And the structure of many temporal networks are not correspond perfectly with the artificial network models, such as small-world graph, scale-free networks, thus increasing the difficulty to examine the dynamic variance to determine the factors that may have caused it. To solve the problem, some models are proposed recently.

In reference [8], four null models have been generated for exploring the temporal correlations. According to the structure of static networks and the characteristics of human behavior, shuffling of event sequence between edges to destroy the correlation of temporal event and weight-topology of the collected temporal data, which including the call, SMS, Email and conference networks. In reference [9], random time shuffle model and random offset model were analyzed, in the former model, event time stamp are random shuffled on every node, and the latter shifted the initial event sequence in each node with a various time delay. Both of the models mentioned above have provided a framework to explore the dynamic effect induced by some focused temporal and topological correlations, with destroying other correlations between time sequences and between links at the same time.

The cluster made by nodes with high-degree and the sparse links between clusters are the trap regions for cascade dynamics, which have been discovered in previous research papers. However, the researches specialized in this theme on temporal networks with excluding other elements is not common. In this paper, we resolved this problem into simple elements as follows: first, we built a module network follows scale-free degree distribution as the static network platform, by using the method proposed by Yan [10], in which the numbers of intra- and inter-cluster links are adjustable. Second, a random event sequence is generated and assigned to links, with number of events evenly distributed in a set of number [1, 4], and the memory window model is used to highlight the effect of assortative links on cascade dynamics. Third, a new breaking and rewiring scheme is proposed based on the assortativity of the degree of neighbors of the node, which means a node with high average degree of its neighbors has high probability to break the link with its low-degree neighbor and rewire with a high-degree node, and vice versa. The effect on dynamics induced by this assortativity are compared with the traditional degree-based assortative mating, with various memory window length and threshold value. The parameters of static networks, such as global population, the clustering coefficient and initial seed size, which play an important role in cascades, are fixed. The discussion of the effect of these elements are not given here due to the main purpose of our work is focused on the temporal network.

2 Models

In our work, binary decisions framework was used to explore the effect of our proposed assortivity scheme on module network. The *binary decisions with externalities* is a simple but widely accepted model in research fields of cultural fashions, collective synchronization, the diffusion of new invention and product, and rumors [1]. In binary state dynamics, there are only two opponent choices for individuals, and the way to make a decision depends on the contest of different choices made by their neighbors. The model corresponds to the scene where individuals do not have sufficient information for decision, then they have to form the reference by the inclination of their neighbors.

$$\sigma(\sigma_{\Gamma(i)},k)=\begin{cases}1, & \sum_{j\in\Gamma(i)}\sigma_j>\gamma k\\ 0, & \sum_{j\in\Gamma(i)}\sigma_j\le\gamma k\end{cases}, \quad (1)$$

where σ denotes nodal state, state 1 represents the state of being adopted or infected, and 0 represents the state of being innocent or susceptive; γ denotes threshold of adoption, $\Gamma(i)$ denotes neighbors' set of node *i*.

For the temporal network, the bursty nature of human behavior needs to be considered, and randomized event sequences are assigned to links. The original Watts' model only considers the effect on dynamics induced by topological property of the network, and such graphs without time-event behaviour are refered to as static networks [2]. To capture the common property of human behavior, there many models proposed to explore the correlation between topology and event sequence. Four null models are proposed by authors of refenrence [8], in which some correlations are retained and others are destroyed by shuffling the event sequence among links with different correlation criteria, for example, the same degree, the same event number, etc. The null models give us a good platform for further research. Karimi' model [7] considered the accumulative effect of nodal contacts within a memory window, that is to say, contacts from a singal node enables a innocent node to become a adopted one, if the window is long enough and nodal degree is very low. When these conditions are reached, memory window model actually discards the effect of topology [9].

$$\sigma(\sigma_{\Gamma(i)},k,t\mid\tau)=\begin{cases}1, \sum_{j\in\Gamma(i)}\sigma_j(t')>\gamma k_i,\ t'\in[t-\tau,t]\\ 0, \sum_{j\in\Gamma(i)}\sigma_j(t')\le\gamma k_i,\ t'\in[t-\tau,t]\end{cases}. \quad (2)$$

In our work, two measures are adopted to ensure the temporal property: first, random event sequences are generated and assigned to links. The max event number evenly distributed on [1,4], for example, (0 0 1 1 0 0 0 1 0 1) is an event sequence with 4 event number in 10 time steps; second, memory window model are used to amplify the effect of our proposed rewiring and shuffling scheme.

We designed two rewiring strategies: the neighbor-based assortativity scheme and degree-based assortativity scheme. The first scheme compares the ratio of degree of the two end nodes to the sum of the degree of their neighbors, while the second one uses the degree of nodes to judge the edge assortativity. The details are as follows:

$$F(ji)=\begin{cases}1, & \left|\dfrac{k_i k_j}{\sum\limits_{m\in\Gamma_i, m\neq j} k_m}-1\right|>\varepsilon \\ 0, & \left|\dfrac{k_i k_j}{\sum\limits_{m\in\Gamma_i, m\neq j} k_m}-1\right|\leq\varepsilon\end{cases}, \quad (3)$$

where ε is a nonnegative constant number, acts as an adjustment parameter, Γ_i denotes the neighbors' set of node *i*, and $j\in\Gamma_i$, *i* and *j* are end nodes of the link. F(*ji*) denotes the level of absolute deviation of the degree of node j compared with other neighbors.

$$Q(i,j)=F(i,j)\cap F(j,i)\ , \quad (4)$$

where *Q* is the judgment measurement for neighbor assortativity, by using AND operation. When *Q* is true, the edge (*i*,*j*) is defined as assortative link.

In degree-based model, the assortativity denoted by nodal degree is widely researched in cascade models. The aasortativity is measured as the correlation between two nodes [11]. The measurement of assortativity of modal degree is as follows:

$$\left\langle k_{NN}\right\rangle=\sum_{k'}k'P(k'/\mathrm{k})\ , \quad (5)$$

where *N* is the population of the graph, *P*(*k'*/*k*) denots the probability that a node with degree k has a neighbor with degree *k*', k_{NN} defines the correlation between a node with *k* degree with the average degree of its neighbors.If k_{NN} is an ascending curve with *k* increases, then the graph is assortative.

In context of trap region detecting, we made a simple method for judging edge assortativity for nodal degree:

$$Q(i,j)=\begin{cases}1, & \left|k_j-k_i\right|\leq\max(k_i,k_j)\delta \\ 0, & \left|k_j-k_i\right|>\max(k_i,k_j)\delta\end{cases}, \quad (6)$$

where δ is a nonnegative constant number, acts as an adjustment parameter, k_i and k_j are the degree of the endnodes.

An example of detecting of trap regions in cascade model is shown in Figure 1. In Figure 1a, edge (*i* ,*j*) is an obstacle for propagation to pass through, even nodes in one side are all occupied, the cascade still have hard path to reach other side because of the high degree of the node *i* and *j*. This trap can be detected by both schemes: first, node *i* and *j* both have high degree. The edge linked them is found as an assortative edge under $\delta=0.3$. Second, both node *i* and *j* are friends with high degree to each other, with other friends having lower degree, edge (*i*,*j*) is an assortative link under the neighbor assortativity criterion with $\varepsilon=0.3$. While all other edges are not assortative links under these two value in both scheme; In Figure 1b, node *i* and *j* are friends with low degree to each other, with other friends having higher degree, edge (*i*,*j*) acts as a bridge linked two cluster regions. The cascade is still hard to be disseminated from one side to other side. The trap in here can be detected by neighbor assortativity criterion but degree assortativity one, because the latter cannot tell the difference of Figures 1b and 1c .And in Figure 1c, there is no trap for cascade in graph.

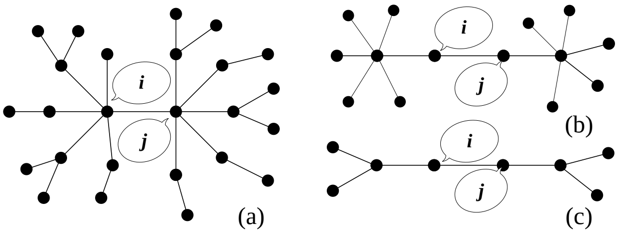

FIGURE 1 An example of trap region (a) the trap caused by edge (*i*,*j*), which can be detected by degree assortativity and neighbor assortativity. (b), (c) a trap found in (b), while not in (c), which can be detected by neighbor assortativity scheme only

To validate the reasonable and effective of our proposed method for trap detecting, we generated a temporal network with breaking and rewiring behavior. The breaking and rewiring are integrated into temporal behaviors with steps as shown in Figure 2: The random event sequence on edge (A, B) is generated with time window length $\tau=12$ and event number equals 3. Then at current simulation time step *t*, if edge (A, B) is detected as an assortative link, we break the edge and rewire node A with node C and node B with D, while node C and D are end nodes of an random chosen edge. After that, the event sequence shuffling is performed according to the average degree of the node pair. If node A and C have higher mean degree, then event sequence with more event is given to the edge linked them. The event sequences are right shifted with Δt steps before inheriting, while $\Delta t=\mathrm{mod}(t,\tau)$, and $\Delta t=2$ in Figure 2. The shifting in here is working for destroying the event-event correlation, inspired by the method in reference [9].

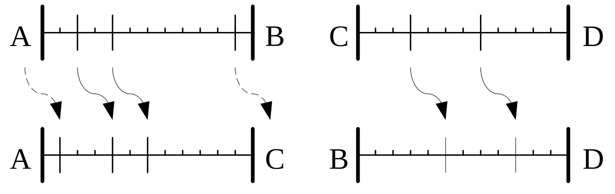

FIGURE 2 An example of breaking the assortative edge and rewiring with a random chosen edge

Our cascade model with shuffling is as follows: First, a scale-free module network is constructed with randomized event sequence assigned to every link at begin. All initial states are assigned to nodes in whole graph with a fraction of nodes chosen to be seeds. Second, in each time step, function *Q* of all edges is computed. If *Q* of a random chosen edge are true, and the random number generated here less than shuffling threshold λ ($\lambda \in [0,1]$), then shuffling is performed by breaking and rewiring as method mentioned above, with event sequence transferred from old edges to new links . Third, updating the state of all nodes according to the threshold rule of watts, combined with the active state of each link. Fourth, repeated step second and step third until the cascade process ended.

3 Results

In our cascade model, the degree distribution and connectedness are maintained and only assortativity correlation added, which made the simulation results reasonable for explore the effect of assortative links on cascades [8,12]. The reason for using scale-free module network as our original graph is the heterogeneous nature of degree and the adjustable community strength in its structure. Scale-free property is defined as the degree distribution in network follows a power law. For the continually increasing new nodes in the network are prone to connect with the node that has larger connectivity. The module networks with adjustable community strength are generated by model proposed by Yan as follows [10]: first, build c isolated cores， with a number of nodes are placed and linked with each other in each core, thus totally g complete graph are generated, g is the number of communities in whole network; Second, add a new node to each core with m edges linking with existing nodes. And n edges are linked with the nodes in same community, $l=m-n$ edges are inter-community. By this way, isolated communities generated in first step are connected; the adding and growing process is performed until the whole population gets N. In the growth process, new node always links existing nodes following the preferential attachment rule, thus the power law property of the degree distribution is guaranteed. By changing the ratio of intra- and inter-community edges, the community strength Q is controlled. In our work, a simple parameter c is used for capture the level of dense links intra-community and sparse links inter-community:

$$Q = \sum_{1}^{c}\left[\frac{l_s}{L} - \left(\frac{d_s}{2L}\right)^2\right] = \frac{n}{m} - \frac{1}{g}, \tag{7}$$

where c is the number of the modules, L is the overall total number of edges, l_s is the total number of edges in the module, d_s is cumulative sum for the node degree within the module.

$$c = \frac{n}{m}. \tag{8}$$

In this section, we investigate our proposed shuffling strategies in a scale-free module network with population $N=10^4$, fraction of initial seeds in whole population is $\rho=5\times10^{-3}$. The uniform cascade threshold is $R=0.16$ and length of memory window is $\tau=8$. The results are averaged over 10 realizations with a randomized event sequence assigned to every link. In our work, a high level of initial seeds size is used for highlighting the effect on cascade size induced by our shuffling scheme t. The population of graph, degree distribution, cluster structure and initial seeds sizes all play important roles on final cascade sizes. This discipline is verified by previous researches and our simulation results. The effect on cascade dynamics induced by ingredients mentioned above are not discussed here. The main purpose of this paper is to find the variations in cascade dynamics induced by different shuffling scheme, and the cooperation effect induced by degree and shuffling is discussed.

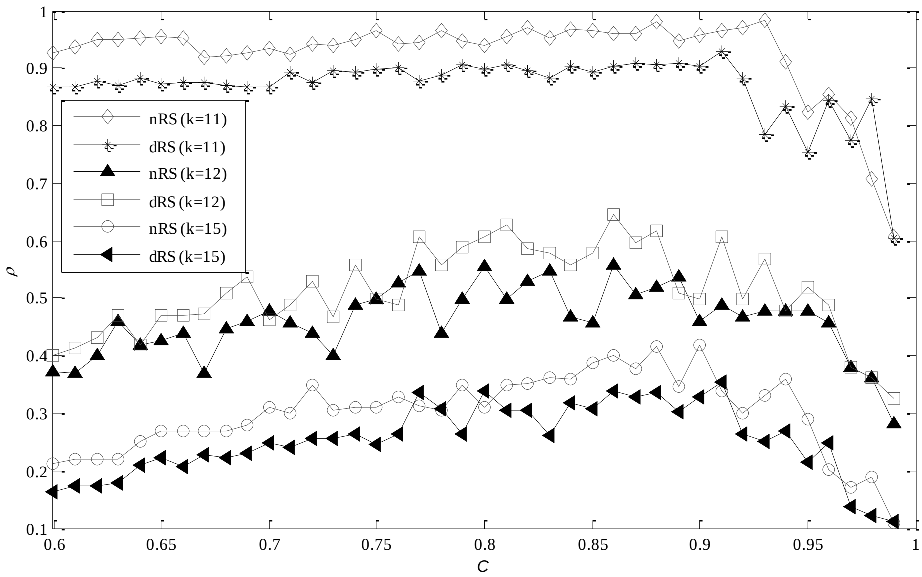

FIGURE 3 Average fraction of adopted nodes ρ versus community strength c under three different mean degree of the network, and with two schemes: neighbor assortivity rewiring and shuffling (nRS) and degree assortivity rewiring and shuffling (dRS)

Figure 3 displays the result caused by different assortativity scheme combined with various community strength on cascade size ρ. It is clear that average fraction of adopted nodes increased with the *c* in the interval [0.6, 0.9], which is a counterintuitive phenomenon. That means, with the decreasing of inter-community edges, and the increasing of intra-community edges, there are more dense links in community and sparse links between communities. Results in Figure 3 show that there existed a critical point in interval $c \in [0.9, 0.93]$, below which the increasing of *c* promoted the level of cascade, and above which the increasing of *c* stunted the cascade. It is reasonable to conclude that the sparse links between communities are not the more the better, there exists a minimize number for these links to satisfy the network connectivity and maximizing the cascading level. The rewiring and shuffling scheme according to the assortativity of degree of neighbors has the stronger promotion effect on cascade than the nodal degree assortativity one, under all conditions.

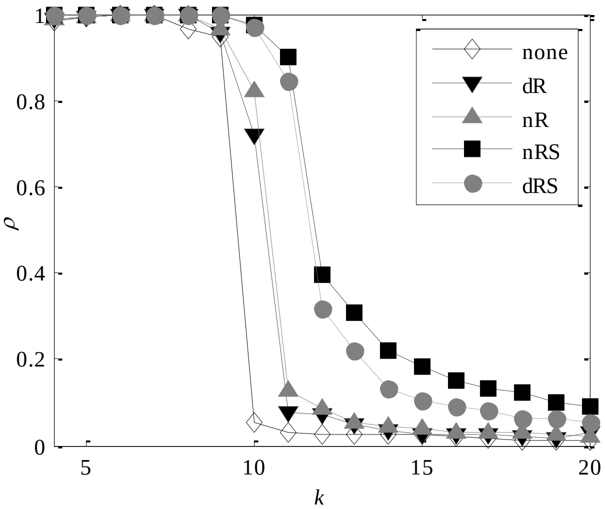

FIGURE 4 Average fraction of adopted nodes *ρ* versus mean degree *k*, with three breaking and rewiring scheme: none shuffling (diamond), degree-based rewiring only (▼), neighbors-based rewiring only (▲), degree-based rewiring and shuffling (circle), and neighbor-based rewiring and shuffling (square).

The change law of prevalence level of cascade under the effect of mean degree combined with shuffling is explored in Figure 4. The first regular pattern is that the shuffling action made the curve fell in a series of cascades down that is deferent with the vertical drop in curve of none shuffling one. The critical point of degree has right shifted due to the effect of rewiring and shuffling, degree 9 for none shuffling process, degree 10 for rewiring only process and degree 11 for rewiring and shuffling process, which shown the promotion effect on the cascading level. Under the same conditions about cumulative effect induced by memory window and shuffling effect, the power to find and destroy trap links of method based on neighbor assortativity is still stronger than the degree one.

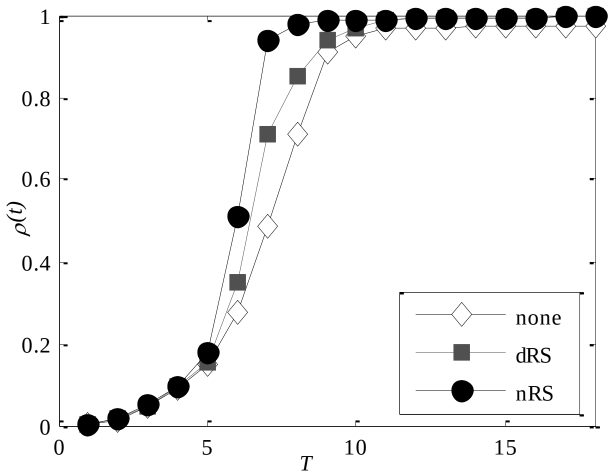

FIGURE 5 Average fraction of adopted nodes ρ(t) acts as the function of time step T, with three breaking and rewiring scheme: none shuffling (diamond), degree-based rewiring and shuffling (square) and neighbor-based rewiring and shuffling (circle).

In Figure 5, the change rule curves of average fraction of adopted nodes which change along with variation of time, under the effect of three scheme, are investigated. With the rewiring action, the trap region have been destroyed which speeded the spreading up on early stage. At the late stage before hitting fully cascading, the acceleration is maintained in nRS, while slowing down in dRS.

The results shown in simulation completely agree with the analysis of our model. The cascade level is very sensitive to the mean degree on the scale-free network. With destroying the trap region by rewiring and shuffling, the sudden drop happened on critical point is changed to a sloped changing, which show the reduced sensitivity to high degree.

4 Conclusion

We investigated the influence of the effect by edge rewiring and event shuffling on global cascade accordng to the assortativity. Our proposed method can be argued to find the trp link better than the degree one, due to that the design of neighbor assortativity is applicable to search more wide area. In particular, the reinforcing effect on links inter-cluster in a module structure network induced by our method is recovered as special cases.

In this paper we define the assortativity of an edge, which are the relative similar degree of node pairs in tradition degree assortativity scheme and the same role compared with other neighbors in neighbor assortativity scheme. Trap links are founded if their end nodes plays same role compared with neighbors of each other in the latter model, and the definition of their assortative level suggests the method of shuffling in temporal networks will be a reasonable way to promote the cascade. In this framework, we adopt three measures to ensure the breaking and rewiring action are utilized properly. We generate a temporal network with random event sequence assigned to each link, with different event number in sequences. The breaking and rewiring action happened on an assortative edge with the event sequence shifted and exchanged between edges at that time stamp. The memory window model is utilized to highlight the effect of the shuffling. At the microscopic level, we illustrate with Figure 1 how the proposed method effectively finds the trip link, while the tradition degree assortativity method fails. In simulations, the cascade level is measured as the average fraction of adopted nodes, thus allows for a comparison of the two different model on propagation coverage rate and spreading speed. The simple and effective feature of our model indicates the potential for further applications in related research fields.

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