Error evaluation for IDW mineral reserve estimates

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Abstract

Inverse distance weighting (IDW) is a well-known method for computer aided mineral reserve estimation. In practice, however, how to evaluate errors of the estimates has always been a problem for this method. This paper proposes an alternative solution to this issue. Firstly, based on a series of discussion and analysis, implementation procedure of IDW mineral reserves estimation method in computer software was designed and explained in detail. Secondly, two methods, comparing with measured values in a developed or developing mine and with different IDW interpolation parameters in the framework of cross validation, were proposed to evaluate error of mineral reserves estimates. At last, the procedures of this method in detail has clearly been illustrated through a typical practical application, from which the practicability of the solution based on cross validation is clearly shown. Thus, it can be concluded that the proposed method is valid and practicable both in theory and practice.

Keywords: reserve estimation; IDW; error evaluation; spatial interpolation`

1 Introduction

Nowadays, mineral resources is crucial for improvement and development of human sociality and performance of mineral resource estimation methods selected is important for this matter [1,2]. However, determining a mineral resource estimation method is usually a difficult choice for researchers and practitioners [3].

On one hand, conventional mineral reserve estimation methods based on geometry usually need a mass of human-computer interactions and thus inconvenient to be implemented in computer. Additionally, geo-statistical reserve estimation methods which is of high efficient and precision [1, 4] have been well developed and convenient to make full use of the computer software technique. However, a proper spatial variation which is pre-requisite for geo-statistical technologies is hard to obtain for many geo-statistical practitioners [4-6].

On the other hand, as a typical spatial interpolation technology, inverse distance weighting (IDW) method [4, 7-9] of which the basic character is between geo-statistical and conventional methods is easy to be understood and convenient to be implemented in computer software. Compared with geo-statistical methods, a severe deficiency in this method is that error of IDW estimates is hard to be measured and evaluated [10-14].

In this context, this paper discusses and proposes an alternative method to evaluate the error of IDW mineral reserve estimates and the corresponding way to implement it in computer software.

To start with, this document will firstly introduce basic principle and method of common reserve estimation methods, among of which IDW method will be emphatically analysed. Additionally, based on the main principle of IDW method, the paper will discuss on the ways to implement the IDW mineral reserve estimation process in computer software. Subsequently, the way to evaluate the error of IDW mineral reserve estimates will be discussed in detail. Several different kind of solutions will be proposed and analysed in this section. What's more, a typical application based on a real mineral dataset will be carried out and illustrated, from which it can be concluded that the proposed method for implement IDW mineral reserve estimates and error evaluation is valid and practicable.

2 Theoretical Background

2.1 MINERAL RESERVES EVALUATION

Basic idea of mineral reserve estimation method is to divide the whole irregular mine body into a series of relative regular mineral blocks [1] (for instance, cuboid, prism, cone and so on). In a given industrial parameter, main process of mineral reserve estimation is to respectively calculate the reserve value for every mineral block and then summarize them.

For a certain mineral block, the reserve information will be control by three key factors as follows: (1) average mineral grade; (2) average weight or density; (3) total volume.

For a mineral block with a regular geometric sharp, the volume can usually be expressed by a simple mathematical formula. Additionally, ore weight can be regarded as a constant value in general for a certain mineral deposit. On the contrary, the third parameter, ore grade in an ore body or deposit will usually vary from one point to another.

Therefore, with regard to the three parameters illustrated above, the last two of them are relative simply to obtain a credible value in mineral reserve estimation process. Conversely, the first argument, ore grade, is most difficult to be well evaluated by a simple way. Geo-statistical interpolation and IDW method are just solutions to this issue.

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2.2 IDW MINERAL RESERVES ESTIMATION METHOD

Essentially, inverse distance weighting (IDW) is a weighted average spatial interpolation method. The basic idea of this method is that the relationship is strong between samples within a short distance. On the contrast, the relationship is weak between points within a long distance.

Therefore, IDW method supposes that participant samples around the point to be estimated will be assigned a larger weight value if they are far away from the estimated point, or vice versa[7]. Obviously, the parameter of distances between samples and estimated point is meaningful and vital to IDW interpolation process. Thus it is necessary to employ another parameter to reduce the impact of distance for IDW estimates and the power of IDW is the very one.

For a certain study area, suppose that $z(x)$ represents attribute value of the i-th estimated point (x); $z(x_i)$ represents attribute value of the i-th sample point (x_i); d_i represents the distance between points (x) and (x_i); λ_i denotes the weight value assigned to the i-th estimated point (x_i); n represents the number of the whole samples in estimation, the the key interpolation process of IDW can be represented by the following expression:

$$z^*(x) = \sum_{i=1}^{n} \left[\lambda_i \cdot z(x_i) \right], \qquad (1)$$

where the weight value λ_i for i-th participant sample can be obtain by:

$$\lambda_i = \frac{\frac{1}{d_i^{\mathrm{p}}}}{\sum_{i=1}^{n}\left(\frac{1}{d_i^{\mathrm{p}}}\right)}, \qquad (2)$$

where the power value p of distance parameter d_i can be a natural number or a real number.

In most cases, value of the parameter p in expression (2) is determined by experience or a constant value for every study area, for instance, 2.0.

It is worth noting that interpolation process of IDW mineral reserve estimation is of high speed in general. However, in the situations that the sample dataset is large the estimation speed will become very low. In this case, a searching neighbourhood which usually is constructed as an ellipsoid or cuboid is necessary to restrict the count of participant samples to be estimated to attend the interpolation calculation [15].

3 Principle and Mouthed

3.1 ERROR EVALUATION

The accuracy and credibility of mineral reserve estimation result has been always paid more and more attention [16]. Characters of mineral reserve estimation technology and the corresponding error estimation method adopted will directly infect the practicability of mineral reserve estimates.

Generally, one of the most common solution to this issue is to compare with other estimates resulted from other methods. And this is applicable for IDW mineral reserve estimation method. If estimates are similar with other results calculated from other methods, this implies that the current interpolation values are of high precision or believable. Besides this method, this section will discuss this issue in detail and propose several other kinds of feasible ways to assess the quality of IDW estimates.

2.2.1Compare with developed mineral reserve

This way is the most believable method to assess the real error of the estimated mineral reserve information. For a developing or developed mine, collecting the data in the developed part of the mine to compare with the relative IDW estimated results can produce reliable errors of full mineral reserve information including average grade, metal tonnage and ore tonnage.

2.2.2 Cross validation

This method mainly by estimating the sampled locations, can produce many kinds of useful parameters to examine the estimated error at every estimated point. In this procedure, all of the sampled locations are estimated and thus it can provide both a true value and an estimated value for every sampled point and based on these values. Then the interpolation errors can easily be calculated for evaluating the associated interpolation methods.

2.2.3 Compare with other estimates resulted from other IDW parameters

Based on the principle of IDW mineral alternate estimation process, estimates are relevant with different parameters, such as the size of mineral block, count of participant samples, searching radius and so on.

Generally, different interpolation coefficient will result in different estimates of which the difference varies from every interpolation point. Some estimated point may have similar interpolation values with different interpolation parameters while others may be in contrast. It is reasonable that the former kind of estimates which have lesser variation with different IDW parameters are more reliable than the latter ones which have more variation.

Specifically, if calculation results are similar in total under the conditions of same original datum and different calculation parameters, it means that the estimated error is small, or vice versa.

3.2 ALGORITHM IMPLEMENTATION

Based on the analysis of the basic principle of IDW mineral reserve estimation method, as the illustration in Figure 1, the implementation this method of estimating mineral reserve by IDW in computer software can be generalized as the following steps (Figure 1):

(1) Datum processing. Extract drill data from geological database and translate them into the needed format that

can be read easily by computer program while checking errors and correcting them and then visualize them in 3D environment for ease of statistics analysis on data;

(2) Statistics analysis. Calculate the summary statistics and cumulative distribution of the sample length and the associated assay value of ore grade;

(3) Equalized the length of samples. Observe and analyze the distribution principle of original samples in order to confirm the space variability of grade distributing of the mineral stones and transform samples with different lengths so that the length for every sample is equivalent;

(4) IDW grade interpolation. *Firstly*, assign basic information of mineral block units (length, width, height); *Secondly*, construct a block model according to the spatial stretch form of the mine body; *Thirdly*, calculate the grade value of every mineral unit in the block model by IDW and an grade model of the mine body is created;

(5) Block reserve evaluation. On the basis of grade model of the mine body under the condition of a certain mineral industrial index, calculate mineral reserve for every block unit according the assigned ore weight and the volume of every block unit.

(6) Totalize block reserve. Calculate the mineral reserve for the whole mine body by summarizing the mineral reserve estimates for all of the blocks in the block model.

(7) Output result and analysis. Output the IDW mineral reserve estimates by two dimensional maps and three dimensional models and so some analysis base on them, such as error evaluation, economic assessment and so on, if needed.

It is notable that if the used mineral industrial index has changed, extract the block model under the new industrial conditions and the new mineral reserve estimates for all block units can be regained by simply repeat the step (5)-(7).

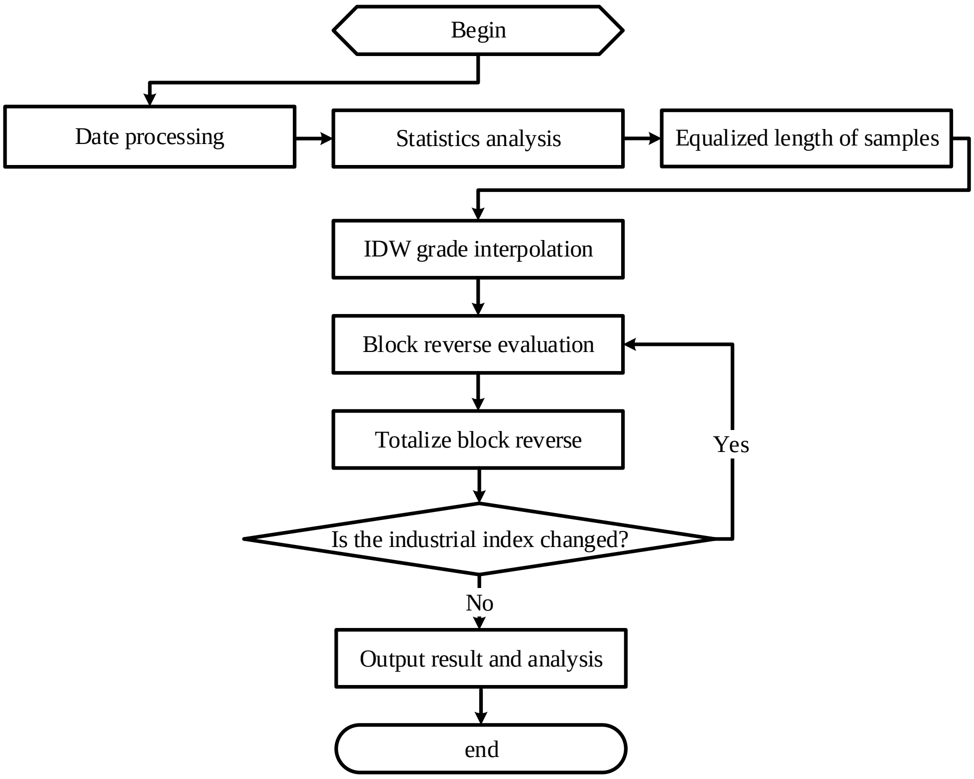

FIGURE 1 A flow chart to implement the mineral evaluation process based on IDW..

4 Case Study

This section will demonstrate a practical application procedure with a real developing mine in southeast China by the proposed method. It will be analysed and illustrated in detail in the way of a typical case study, from which the validity of calculation result from the proposed method can be obviously proven.

4.1 EXPERIMENT DATA

On the basis of discussion on principle and implementation flow of IDW mineral reserve estimation, this method is fully implemented in a digital mine system and succeed in this case study (Figure 2), which has shown a remarkable improvement on evaluation efficiency and quality compared with conventional mineral reserve estimation methods.

In this study, a typical mine body and the associated exploitation engineering consisted of 1840 drill samples are selected for the test. Distributions of attributes and spatial positions of these data are respectively described in Figure 2.

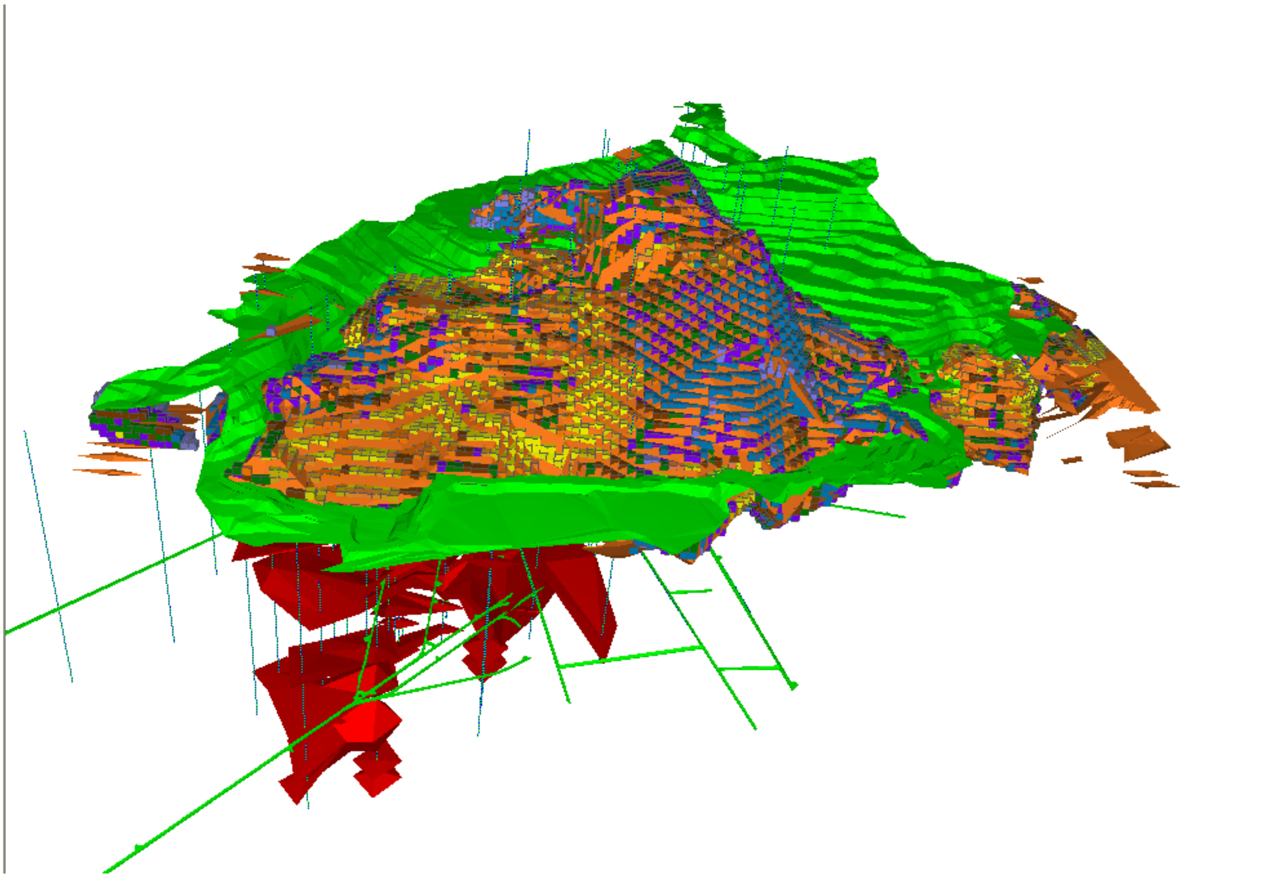

FIGURE 2 Spatial stretch status of the mine body, exploration engineering and the associated IDW mineral reserves estimates.

4.2 EXPERIMENT METHOD AND PROCEDURE

As stated above, the experiment procedure is carried out in the framework of cross validation. In this procedure, every original sample in the test dataset will be orderly selected as an unknown point to be estimated its nearby samples. In order to evaluate performance of the interpolation result corresponding to some certain power value, a measurement named Mean Relative Error (MRE) resulting from cross validation is employed, which is calculated by:

$$\mathrm{MRE} = \frac{1}{\mathrm{N}}\sum_{i=1}^{\mathrm{N}}\left|\frac{z(x_i)-z^*(x_i)}{z(x_i)}\right|, \quad (3)$$

where N represents the number of the points to be estimated, $z^*(x_i)$ and $z(x_i)$ represents the estimate and true attribute value of the interpolation point.

Additionally, a set of IDW estimation parameters consists of different searching parameters and sizes of block unit are designed in table 1 to examine the uncertainty of IDW mineral reserve estimates. In this table, the first group of parameters (in bold) is the determined parameters and others are prepared for evaluating error associated with those parameters.

TABLE 1 Test parameters set of IDW mineral reserve estimation designed to evaluate error of estimates.

NO.	Searching radius (m)			block unit size (m)		
	Radius 1	*Radius 2*	*Radius 3*	*Length*	*Width*	*Height*
1	**120**	**100**	**80**	**20**	**20**	**20**
2	80	120	80	30	30	30
3	100	120	80	40	40	40
4	80	100	120	50	50	50
5	100	150	80	60	60	60
6	120	1	80	20	30	40
7	100	120	80	40	50	60

4.3 EXPERIMENT RESULTS AND ANALYSIS

As stated above, cross validation is executed to examine the accuracy of the designed model corresponding to the first group parameters in TABLE 1. As a result of this procedure, the difference between the actual and estimated values can be used to evaluate the error of IDW mineral reserve estimates.

Relative error is an important index resulted from this procedure to measure estimated error and thus utilized in this text. Average ore grade estimates are compared with the associated average value of measured samples and the differences measured by MRS in expression (3) are clearly illustrated by the following scatter plot (Figure 3). In this figure, it is obvious that the range of the estimated error of average ore grade corresponding to the determined IDW parameter in table 1 is between 0.25% and 6.58%, which is enough small for geological estimation. Therefore, the estimates expressed by table 1 can not only prove that the designed parameters are validity for the current interpolation problem, but also the estimated results is of high reliability in this study.

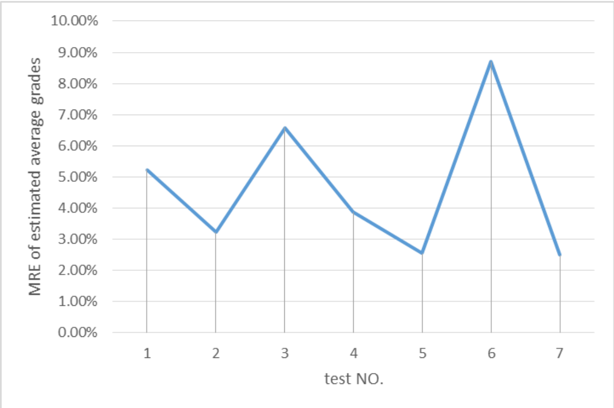

FIGURE 3 Relative mean error of estimated average grades by IDW from different parameters in TABLE 1

In order to further verify the validity of the current IDW mineral reserve estimates parameterized in table 1, the quantity of metal tonnage and ore tonnage resulted from the prepared seven pair of parameters are compared with each other and recorded in figure 4 and 5. In this two figures, the differences between these IDW mineral reserve estimates are not obvious and it is easy to infer that the associated average grade estimates are close too. As discussed above, similar estimates with different interpolation parameters can indicate that the uncertainty of estimates is small. In other words, the current mineral reserve estimates calculated in the first group of table 1 is acceptable in total.

Obviously, this application has taken satisfied effect and proves the proposed method is feasible and valid completely in practice.

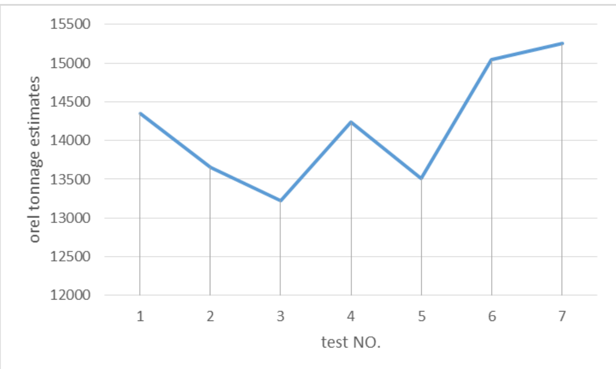

FIGURE 4 Estimated ore tonnage resulted from IDW with different parameters in TABLE 1

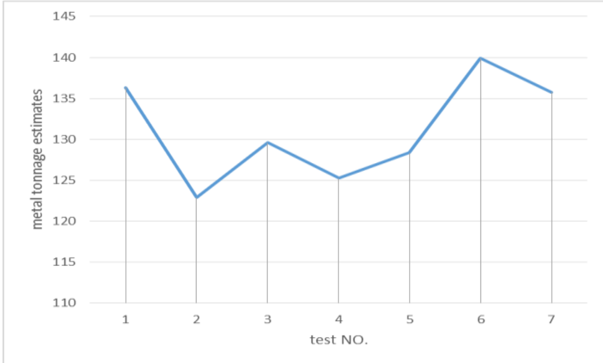

FIGURE 5 Estimated metal tonnage resulted from IDW with different parameters in TABLE 1

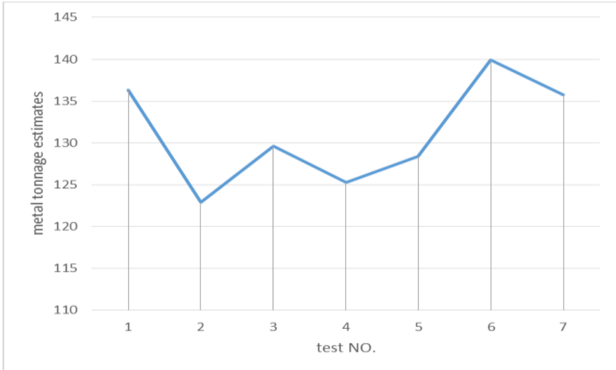

FIGURE 5 Estimated metal tonnage resulted from IDW with different parameters in TABLE1

5 Conclusions

Inverse distance weighting (IDW) is a significant and well-known mineral reserve estimation method. This paper explores the implementation procedure of this method in computer software in detail and proposes an alternative method to evaluate the estimated error of IDW mineral reserve estimates. In the proposed method, errors of IDW estimated mineral reserves can be evaluated by comparing with actual values in a developed mines, estimates resulted from other methods or other parameters in the framework of cross validation.

Based on the analysis of the basic principle of IDW mineral reserve estimation method, the complete process in the implementation of IDW mineral reserve estimation and error estimation has be discussed and implemented in computer software. Additionally, a typical case study which is carried out based on a real developing mine is illustrated in detail in this text. According to compare the IDW mineral reserve estimates with the actual developed situations, ore tonnage and metal tonnage estimates have shown clearly clear stability and robustness, which can strongly prove the validity and practicability of the proposed method.

Thus, both theoretical analysis and practical application can show the validity and practicability of the proposed method.

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