

# Risk-aversion revenue sharing contract in financial industry chain

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## Abstract

Revenue sharing contract is one of the effective ways in the coordination of entity industrial chain. However, the specificity of financial industry chain leads to more risks and more complicated coordination. Therefore, this paper constructs a risk-aversion revenue sharing contract to analyze the coordination method of financial industry chain, and also makes empirical analysis on it. The result shows that the risk-aversion revenue sharing contract can play a coordinating role in the financial industrial chain, and the more strengths of peculiarity on risk aversion, the larger benefit share of the enterprise should be.

*Keywords:* Financial Industrial Chain, Revenue Sharing Contract, Risk Aversion

## 1 Introduction

The financial industry chain is a business chain, which means the financial industries of banking, securities and trust afford a financial products and services to the entity enterprises. This business chain works the financial input and output as a link, works the added value as an orientation, and works the financial needs of entity enterprise as a goal. Moreover, the specificity of financial industry chain is not only shown that financial products are currency which is a special commodity, not the generally material goods, but also shown that its motion characteristics are different from those of entity industry chain, where there is a quick innovation on financial product and a various types on financial risk. Thus, it is difficult to coordinate the financial industry chain rather than the entity industry chain. Therefore, it is of a certain significance to study how to coordinate the financial industry chain.

There are rich research literatures on industry chain coordination at home and abroad. Many of them are mainly focus on the entity industrial chain coordination. Chauhan & Proth studied the application of risk-neutral revenue sharing contract on the industry chain relationship management, and found that the method can not only match the coordination between industry chain cooperation, but also reach a better performance than wholesale price contract[1,2,3]. Van-der-Veen & Venugopal also confirmed the above conclusion by studying DVD rental industry chain[4]. Chao et al. used super modular game model to study the coordination of entity industry chain, and found that there are many risks in the industry chain, such as bad quality, and information asymmetry, so "Design by Contract" can be used to avoid this risk and maximize the industrial chain benefit [5,6,7]. However,

the researches on the coordination of financial industry chain are relatively small. Xu & Birge constructed a single period newsvendor model to analyze the interaction between the company operation and financial decisions, the result shows that the entity corporate should coordinate company operation and financial decisions because the entity enterprise exist the tax cost and the risk of bankruptcy [8,9]. Chen & Wan used wholesale contract and loan contract to study the coordination of financial industry chain, and found that the regardless of the enterprise financial risk and financial market risk will lead to the mismatch of financial services and the operation decision of entity enterprise [10].

To sum up, recent researches on the coordination of financial industry chain are relatively less, and mainly study the risks in the coordination of financial industry chain, not focusing on the coordination method of financial industry chain. Meanwhile, most of recent researches on the entity industry chain use risk-neutral revenue sharing contract as main research method, which neglect the risk. Therefore, this paper constructs a risk-aversion revenue sharing contract which takes the risk factors of real economy into account in order to analyze the coordination method of financial industry chain.

## 2 Model construction

### 2.1 MODEL HYPOTHESIS

It is assumed that the financial industry chain composed of a risk neutral financial provider  $M$  and a risk aversion firm  $N$ . Both of them are independent to each other and the financial provider of financial industry chain can meet the demand of the market. Meanwhile, the dominant financial

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provider is the market leader and has the supreme position in the market place. Therefore, as to participation enterprises, their leader position in the enterprise is according to the contract parameters to make the optimal decision. It is suppose that the market demand is  $Q \geq 0$ . The distribution function and probability density function are  $F(x)$  and  $f(x)$ , where  $F(x)$  is differentiable and monotone increasing, plus  $F(0) = 0, \bar{F}(x) = 1 - F(x)$ . The parameter settings are as follows: The risk neutral financial provider is  $M$ . The risk aversion firm is  $N$ . The market demand is  $Q$ . The unit costs of production is  $a$ . The wholesale price from financial providers is  $t$ . The retail price of unit product is  $q$ . The order of entities is  $L$ . The sales revenue is entities are  $y$ . The share of financial provider is  $1-y$ .

It is going to take Loss-averse model to analyze, and the formula goes to:

The utility of business entity function is

$$T_N = \begin{cases} E_N, E_N \geq 0 \\ hE_N, E_N \leq 0 \end{cases}, \tag{1}$$

where  $E_N$  is the entity enterprise profit,  $h$  is the risk aversion factor, so the centralized control of the supply chain profit is:

$$E = q\{L[1 - F(L)] + \int_0^L xf(x)dx\} - aL. \tag{2}$$

The quantity of supply chain optimal order is:

$$L^* = F^{-1}\left(\frac{q-a}{q}\right). \tag{3}$$

The expected utility of enterprise is:

$$T_N(L, t, y) = N_N(L, t, y) + (h-1)S_N(L, t, y). \tag{4}$$

The expected profit of enterprise is:

$$N_N(L, t, y) = q\{L[1 - F(L)] + \int_0^L xf(x)dx\} - tL. \tag{5}$$

The expected profit loss of enterprise is:

$$S_N(L, t, y) = \int_0^{\frac{tL}{qy}} [yqx - tL]f(x)dx. \tag{6}$$

The expected profit of financial provider is:

$$T_M = (1-y)q\{L[1 - F(L)] + \int_0^L xf(x)dx\} + (t-a)L. \tag{7}$$

According to formula 9, the existence and uniqueness of  $t^*/y^*$  makes the supply chain coordination, and:

- 1)  $t^* < y^*$
- 2)  $d(t^*/y^*)dh < 0$

## 2.2 MODEL HYPOTHESIS

Based on formula 4,  $T_N$  is a concave function of  $L$ , so there is only one  $L^*$ . And it convenes the first-order conditions:

$$yq - t - yqF(L^{**}) - yqF(L^{**}) - (h-1)tF\left(\frac{tL^{**}}{yq}\right) = 0. \tag{8}$$

It is going to set  $L^{**} = L^*$ . And set it into the above formula.

$$\frac{t}{y} + \frac{t}{y}(h-1)F\left[\frac{tL^*}{yq}\right] = a. \tag{9}$$

It is obvious that on the left side of formula 9, and it is an increasing function of  $t/y$ ; when  $t/y \rightarrow 0$ , the left side became 0. However, when  $t/y \rightarrow \infty$ , the left side tends to infinity. Therefore, if there is only  $t^*/y^* > 0$ , and it matches formula 9. And the result is  $t^*/y^* < a$ , which means  $t^* < ay^*$ .

It is set that:

$$I\left[\frac{t^*}{y^*}, h\right] = \frac{t^*}{y^*} + \frac{t^*}{y^*}(h-1)F\left[\frac{t^*L^*}{y^*q}\right] - a = 0. \tag{10}$$

And use the implicit function get the result:

$$d(t^*/y^*)/dh = -I'_h / I'_{t^*/y^*} < 0$$

The optimal expected utility of enterprise is:

$$T_N(L, t, y) = N_N(L^*, t^*, y^*) + (h-1)S_R(L^*, t^*, y^*). \tag{11}$$

The optimal expected profit of financial provider is:

$$E_M = (1-y)q\{[L^*[1 - F(L^*)] + \int_0^{L^*} xf(x)dx]\} + (t^* - a)L^*. \tag{12}$$

Based on the above analysis, through the proper design of contract parameter, the revenue sharing contract can play a coordinating role in the financial industry chain. And the financial service providers can make small profits on providing the physical firm with financial products, which comes from the profit creating by the physical firms. Therefore, it shows that with the strengths of the effect on risk aversion by the physical firms, the financial service providers must take further steps to attract more physical firms to buy their financial products via lowering the price of the financial products or lowering their profit margin. It will also urge the financial service providers to improve their competitiveness to offer the financial products to physical firms well, while physical firms improve their competitiveness by using those financial products. It is crucial for whole financial industrial chain.

## 3 Further analysis

Under the constraint of rational thinking, among the financial industrial chain, all parties should take its benefits into consideration at the very beginning, because only on the premise of satisfying its own interests, should all parties

take a look at profit maximization in the whole financial industrial chain. Thus, in order to guarantee that both parties can enjoy the revenue sharing contract, the precondition is that the gains from both parties must be greater than the money earned respectively. So it should match the following inequality.

$$\begin{aligned}
 &1) T_N(L, t, y) \geq T_N^Q(L, t^{**}), \\
 &2) E_M \geq E_M^Q, \tag{13}
 \end{aligned}$$

where  $T_N^Q(L, t^{**})$  and  $E_M^Q$  were dispersed under the condition of the optimal expected utility function of enterprise and the optimal expected profit of financial provider.

When  $t$  is fixed down, the optimal expected utility function of enterprise is:

$$T_N^Q(L, t) = N_N(L, t) + (h-1)S_N^Q(L, t). \tag{14}$$

After analyzing the above formula, the result is:

$$\frac{\partial T_N(L, t)}{\partial L} = q - t - qF(L) - (h-1)tF\left(\frac{tL}{q}\right), \tag{15}$$

If set formula 15 is 0, the result is:

$$q - tqF(L) - (h-1) + F\left(\frac{tL}{q}\right) = 0. \tag{16}$$

The optimal expected profit of financial provider is:

$$E_M^Q = (t-a)L. \tag{17}$$

After analyzing the above formula, setting it as 0, and get the formula of optimal pricing of financial provider The optimal expected utility function of enterprise is:

$$T_N^Q(L, t^{**}) = N_N(L, t^{**}) + (h-1)S_N^Q(L, t^{**}). \tag{18}$$

The optimal expected profit of financial provider is:

$$E_M^Q(L, t^{**}) = (t^{**} - a)L. \tag{19}$$

#### 4 Empirical analysis

In order to illustrate the effectiveness of financial industrial chain profit contract mentioned in this article, the concrete analysis is as follows:

From formula 2, it is found that under the centralized control, the profit of financial industry chain is,

$$\begin{aligned}
 E &= q\{L[\bar{F}(L)] + \int_0^L xf(x)dx\} - aL \\
 &= L(q-a)\bar{F}(L) + \int_0^L xf(x)dx
 \end{aligned}$$

Therefore, the expected utility of enterprise is

$$T_N = N_N + (h-1)S_N.$$

And the result shows that, from formula 4, the concave function of  $L$  is  $T_N$ , and  $L$  in formula 8 can match

$$yq - t - yqF(L^{**}) - (h-1)tF\left(\frac{tL^{**}}{yq}\right) = 0$$

$$q[1 - F(L^*)] - \frac{t}{y} - \frac{t}{y}(h-1)F\left(\frac{tL^*}{yq}\right) = 0$$

Duo to  $F(0) = 0, \bar{F}(x) = 1 - F(x)$ , and it is get that:

$$\frac{t}{y} + \frac{t}{y}(h-1)F\left(\frac{tL^*}{yq}\right) = a.$$

From the above formula, the left side of it is the increasing function of  $t/y$ , when  $t/y \rightarrow 0$ , the left side became zero. On the other hand, when  $t/y \rightarrow \infty$ , the left side tends to infinity. Therefore, when  $t^*/y^* > 0$ , it can match formula 9, and the result is  $t^*/y^* < a$ , which means  $t^* < ay^*$  in formula 9.

Moreover, according to formula 10, it is going to set

$$\frac{t^*}{y^*} = y, h = x, A = \frac{L^*}{q}, u = yA. \text{ And formula 10 will be:}$$

$$I(y, x) = y + y(x-1)F(u) - a = 0.$$

Setting a derivation of implicit function  $y + y(x-1)F(u) - a = 0$ , and

$$\frac{dy}{dx} + \frac{dy}{dx}(x-1)F(u) + yF(u) + y(x-1)f(u)A \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx}[1 + (x-1)F(u) + y(x-1)f(u)A] = -yF(u)$$

$$\Rightarrow \frac{dy}{dx} = -\frac{yF(u)}{1 + (x-1)F(u) + y(x-1)f(u)A}$$

Nevertheless, when  $I(y, x) = y + y(x-1)F(u) - a$ , the  $x$  derivative will be:

$$\frac{d[I(y, x)]}{dx} = \frac{d[y + y(x-1)F(u)]}{dx},$$

$$\frac{d[I(y, x)]}{dx} = yF(u) \frac{dy}{dx}.$$

And the  $y$  derivative will be:

$$\frac{d[I(y, x)]}{dy} = \frac{d[y + y(x-1)F(u)]}{dy}$$

$$\Rightarrow \frac{d[I(y, x)]}{dy} = [1 + xF(u) + yxf(u)A - F(u) - yf(u)A] \frac{dy}{dx}$$

$$\Rightarrow \frac{d[I(y, x)]}{dy} = [1 + (x-1)F(u) + y(x-1)f(u)A] \frac{dy}{dx}.$$

And the result is:

$$\frac{dy}{dx} = -\frac{d[I(y, x)]}{dx} \bigg/ \frac{d[I(y, x)]}{dy},$$

where  $d(t^*/y^*)/dh = -I'_h / I'_{t^*/y^*} < 0$ .

Based on the above analysis, the financial industrial chain matches

$$1: t^* < ay^*$$

$$2: \frac{dy}{dx} = -\frac{d[I(y, x)]}{dx} \bigg/ \frac{d[I(y, x)]}{dy} = -I'_h / I'_{t^*/y^*} < 0$$

In order to meet the above two type financial industry chain, it can construct financial industry chain coordination. For further analysis, when  $t$  is given. Formula 14 can be identified as

$$T_N^Q(L, t) = N_N(L, t) + (h-1)S_N^Q(L, t).$$

Analyzing the partial derivative on the above formula, and

$$\frac{\partial [T_N^Q(L, t)]}{\partial L} = \frac{\partial [N_N(L, t)]}{\partial L} + (h-1) \frac{\partial [S_N^Q(L, t)]}{\partial L},$$

$$\frac{\partial [N_N(L, t)]}{\partial L} = q - t - qF(L),$$

$$\frac{\partial [S_N^Q(L, t)]}{\partial L} = tF\left(\frac{tL}{q}\right).$$

Therefore, the outcome is:

$$\frac{\partial T_N^Q(L, t)}{\partial L} = q - t - qF(L) - (h-1)tF\left(\frac{tL}{q}\right),$$

which matches formula 15. If setting formula 15 is 0, and formula 16 can be obtained as

$$q - t - qF(L) - (h-1)tF\left(\frac{tL}{q}\right) = 0.$$

If acquire the representation of  $F(x)$ , it is easy to get the representation of  $L'$ . As shown above,  $L'$  is the order quantity of financial commodities. Meanwhile, according to the above analysis, the profits of financial provider is the wholesale price of financial providers minus the unit cost of financial product, and then times the order quantities. After that, formula 17 can be acquired. Under the same process, the optimal expected profit of financial provider in formula 19 is  $E_M^Q(L, t^{**}) = (t^{**} - a)L'$ .

And the optimal expected utility function of enterprise of formula 18 goes to:

TABLE 1 Data parameters

$h$	$t^{**}$	$L^{**}$	$T_N^Q(L, t^{**})$	$E_M^Q$	$y$ (Value Range)	$y$ (Assumed Value)	$t^*$	$T_N(L, t, y)^*$	$E_M^*$
2	7.75	341	1489.1	2012.7	(0.31, 0.52)	0.43	1.3	2150	2500
3	8.16	294	1876.2	1872.4	(0.32, 0.57)	0.49	1.5	2450	2200
4	8.4	287	1632.1	1892.2	(0.38, 0.58)	0.52	1.7	2600	1950
5	8.5	245	1998.2	1789.4	(0.39, 0.59)	0.59	1.9	2680	1860

### 5 Conclusions

Taking the risk factors of real economy into account, this paper has constructed a risk-aversion revenue sharing contract which to discuss the coordination method of financial industry chain, and the conclusion has been the following: firstly, the risk-aversion revenue sharing contract can play a coordinating role in the financial industrial chain. Concretely, risk-aversion revenue sharing contract will not only develop the core competitiveness of the financial ser-

$$T_N^Q(L, t^{**}) = N_N(L, t^{**}) + (h-1)S_N^Q(L, t^{**}).$$

In order to construct the financial industry chain to congregate two inequalities for Pareto improvement, so

1)  $T_N(L, t, y) \geq T_N^Q(L, t^{**})$

2)  $E_M \geq E_M^Q$ .

Putting the retailer's optimal expected utility and the supplier optimal expected profit into the above two inequality, where can obtain the value range of parameter  $y$ . On the other hand, in order to ensure the benefits and allow the business entity able to accept the contract. Financial providers in the design of revenue sharing contract should make the range of  $y$  in the above two formulas, which is the only way to make the financial industry chain coordination. The value depends on the parameters of  $y$  in the financial providers and enterprise in the supply chain as well as the status of negotiations between each other.

In order to further illustrate an effective coordination of the financial industry chain, it is going to design a specific example. According to an observation of a bank and an enterprise, there is a random variables distribution  $x \sim B(i, j)$ ,  $i=190, j=900, q=11, c=2$ . Please see Table 1.

Table 1 shows that comparing with the disperse state, the financial service providers, the member firms and the whole supply chain make a fortune under the condition of the revenue sharing contract. With the enlargement of the factors in risk aversion, the value range of the contract parameter will be larger, which means that with the strength of the effect on risk aversion by the physical firm, the financial service providers only can attract the customers by lowering its price. Therefore, in order to make huge profits, the financial service providers only can improve its sales by attracting more customers to buy. This is conformed to the rules under the market economic system, which the financial service providers must improve the profit margin by competition. The specific parameter values of the contract depend on the positions and the negotiating abilities from both parties.

vice providers in the financial industrial chain, but also improve the capacities of their existences. Meanwhile, it can also offer the physical firms with high-class financial services. Therefore, it can advance the community superiority and the driving forces among each member in the whole financial industrial chain. Secondly, with the strength of risk aversion of the member firms, the value range of the contract parameter will be larger, their profit margin must increase as well. The empirical analysis has also confirmed the conclusion.

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