

Operational risk quantification for loss frequency using fuzzy simulation

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Abstract

The estimation of the frequency parameter of operational risk quantification has received increased attention under the new Basel proposal. This paper proposes an advanced measurement approach using fuzzy point estimation. In this approach, prior membership function could be obtained through fuzzy maximum entropy rule. When operational risk loss data is given, posterior membership function can be easily calculated by using fuzzy point theorem. After posterior mean is exploited as fuzzy point estimate, loss frequency distribution is gotten. Finally, an empirical analysis on this model is conducted based historical data obtained from a Chinese commercial bank. The result shows that economical can reduce the complexity and communication cost.

Keywords: Operational Risk, Basel II Advanced Measurement Approach, Fuzzy Point Estimation, Loss Distributional Approach, Fuzzy Variable

1 Introduction

Operational Risk is an important quantitative topic in the banking world. Under the Basel II requirements [5,6], many banks intend to use the Advanced Measurement Approaches (AMA) for the quantification of operational risk. Through the Advanced Measurement Approach, the banks are permitted significant flexibility over the approaches that may be used in the development of operational risk models. There are various quantitative operational risk models including extreme value theory [2, 26], Bayesian inference [4, 17, 24, 29, 32, 33], dynamic Bayesian networks [30], maximum likelihood [12] and EM algorithms [3], VAR techniques [11, 13, 14], other approaches [1, 7]. Of the methods developed to model operational risk, the majority follow the Loss Distributional Approach (LDA) [25].

The idea of LDA is to fit frequency distributions over a predetermined time horizon, typically annual. The financial institutions use a wide variety of frequency and severity distributions for their operational risk data, including exponential, weibull, lognormal, generalized Pareto, and g-and-h distributions [8]. There are potentially many deferent alternatives [16, 15] for the choice of severity and frequency distributions. Several researchers [12, 8, 28, 31] have experimented with operational loss data by Basel II business line and event type over the past few years. Maryam Pirouz[34] discuss several statistical methods for modeling truncated data, and suggest the best one for modeling truncated loss data, the approach can be useful for increasing accuracy of estimating operational risk capital charge in E-banking. Fengge Yao[35] used Conditional value-at-risk (CVaR) models based on the peak value method of extreme value theory to measure operational risk. Younès, Moutassim[36] used separately a

lognormal distribution and a gamma distribution in the mixture models for the zeros losses. an operational risk assessment model of distribution network equipment based on rough set and D-S evidence theory was built[38]. Ahmed Barakat[39] investigates the direct and joint effects of bank governance, regulation, and supervision on the quality of risk reporting in the banking industry. Pjotr Dorogovsa[40] discussed new tendencies of management and control of operational risk in financial institutions.

Liu [21] proposed credibility measure and credibility theory, and introduced random fuzzy variable as a measurable function defined on a credibility space valued random variables. Chance measure was proposed by Li and Liu [22] to measure the chance of a random fuzzy event. The conditional chance measure was introduced by Li and Liu [23] to measure the chance of a random fuzzy event after it has been learned that some other event has occurred.

For the considered bank, the unknown parameters (for example the Poisson parameter or the Pareto tail index) of these distributions have to be quantized. Our approach to estimate the parameter of the loss distribution is based on fuzzy point inference. The idea is to use the banks collective losses and expert opinions to improve the estimates of the parameters of loss distributions. We demonstrate how the parameter uncertainty can be taken into account by bank internal data and expert opinions and study the impact on the capital charge. In any risk cell, we model the loss frequency and the loss severity by distributions where the lognormal and Pareto distributions are used for modelling severity distributions and Poisson distributions for frequency distributions, respectively. The model might be very useful at this stage when the data are very limited and it may also have educational impact. Financially, we analysis the results of an empirical study with external operational loss data of some Chinese commercial banks.

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2 Fuzzy variables

Definition1. [21] A Fuzzy variable is defined as a function from a credibility space $(\Theta, P(\Theta), Cr)$ to the set of real number.

Definition 2. [21] Let ξ be a fuzzy variable with membership function μ . Then for any set B of real numbers,

$$Cr\{\xi \in \beta\} = \frac{1}{2} (1 + \sup_{x \in \beta} \mu(x) - \sup_{x \in \beta^c} \mu(x)). \tag{1}$$

Definition 3. [21] Let ξ be a fuzzy variable on the credibility space $(\Theta, P(\Theta), Cr)$. The expected value $E[\xi]$ is defined as

$$E[\xi] = \int_0^{+\infty} Cr\{\xi \geq r\} dr - \int_{-\infty}^0 Cr\{\xi \leq r\} dr \tag{2}$$

Definition 4. [21] Suppose that ξ is the continuous fuzzy variable, then its entropy is defined by

$$H[\xi] = \int_{-\infty}^{+\infty} S(Cr\{\xi = x\}) dx \tag{3}$$

where $S(t) = -t \ln t - (1-t) \ln(1-t)$

Example 1. [21] Let ξ be a trapezoidal fuzzy variable (a, b, c,d), then the expected value of ξ is

$$E[\xi] = \frac{a + b + c + d}{4}.$$

Its entropy is defined by

$$H[\xi] = (\ln 2 - 0.5) - (c - d) + \frac{d - a}{2}$$

$$Ch\{\tilde{a} = a | X(\tilde{a}) = x\} = \begin{cases} 0, & \text{if } \min_{1 \leq i \leq m} Cr\{a_i = a\} = 0 \\ \frac{\prod_{i=1}^n f(x_i, a)}{\sup_{a \in \mathcal{R}^m} \prod_{i=1}^n f(x_i, a)}, & \text{if } \frac{\prod_{i=1}^n f(x_i, a)}{\sup_{a \in \mathcal{R}^m} \prod_{i=1}^n f(x_i, a)} < 0.5 \text{ and } \min_{1 \leq i \leq m} Cr\{\tilde{a}_i = a_i\} \neq 0 \\ y(\text{where } \geq 0.5), & \text{otherwise} \end{cases} \tag{5}$$

3 Fuzzy point estimation for loss frequency

Suppose that the frequencies of operational risk losses is modeled by Poisson distribution $P(\lambda)$ with a density

$$f(N | \lambda) = \frac{\lambda^N}{N!} e^{-\lambda}, N = 0, 1, \dots \tag{6}$$

In this section, λ is viewed as a non-negative fuzzy variable on the credibility space $(\Theta, P(\Theta), Cr)$ with membership function $\mu_{\tilde{\lambda}}$, which is called prior membership functions. The parameters of prior membership functions are called hyper-parameters (parameters for parameters). In a more general framework the parameters of the prior membership function $\gamma_1, \gamma_2 \dots \gamma_k$ are estimated by maxi-

Definition 5. [34] Let $X_1(\tilde{a}), X_1(\tilde{a}), \dots, X_1(\tilde{a})$ random fuzzy variables where $\tilde{a} = (\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_m)$ is fuzzy vector such that for each $\tilde{a} = (\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_m) \in \tilde{a}_i(\Theta^+)$, $X_1(\tilde{a}), X_1(\tilde{a}), \dots, X_1(\tilde{a})$ are iid random variables with function (pdf) or probability mass function (pmf) $f(x, \tilde{a})$. Then, given the sample, the way to get the posterior membership function of prior membership function is called fuzzy point estimation.

Theorem 1. [34] Let $X_1(\tilde{a}), X_1(\tilde{a}), \dots, X_1(\tilde{a})$ be iid continuous random variables, where $\tilde{a} = (\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_m)$ is fuzzy vector with prior membership functions $\mu_{\tilde{a}_i}(a), i = 1, 2, \dots, m$, such that for each $\tilde{a} = (\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_m) \in \tilde{a}_i(\Theta^+)$, $X_1(\tilde{a}), X_1(\tilde{a}), \dots, X_1(\tilde{a})$ are iid random variables with pdf $f(x, \tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_m)$, let x be a sample. If $f(x, \tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_m)$ is continuous with respect to (x, a) then the posterior membership function of \tilde{a} can be deduced by

$$\mu_{\tilde{a}}(a | X(\tilde{a}) = x) = \left(2 \sup_{a_{-i} \in \mathcal{R}^{m-1}} Ch\{\tilde{a} = (a_{-i}, a) | X(\tilde{a}) = x\} \right) \wedge 1, \tag{4}$$

where

num entropy rule and expert opinions. Expert opinions modify this characteristic according to the actual experience. Then $P(\lambda)$ can be considered as random fuzzy variable on the space $(\Theta, P(\Theta), Cr) \times (\Omega, A, Pr)$. Let \tilde{x} denote the observations sample $x_1, x_2 \dots x_k$, the sample can be observed and take crisp values. According to Equation (4), the posterior membership function can be deduced as

$$\mu_{\tilde{\lambda}}(\lambda | X(\tilde{\lambda}) = x) = (2Ch\{\tilde{\lambda} = \lambda\}) \wedge 1. \tag{7}$$

where

$$Ch\{\tilde{\lambda} = \lambda | x\} = \begin{cases} \frac{Ch\{\{\tilde{\lambda} = \lambda\} \cap \{\eta = x\}\}}{Ch\{\eta = x\}}, & \text{if } \frac{Ch\{\{\tilde{\lambda} = \lambda\} \cap \{\eta = x\}\}}{Ch\{\eta = x\}} < 0.5 \\ y(\geq 0.5), & \text{otherwise} \end{cases}$$

Then $Pr\{\eta = x | \tilde{\lambda} = \lambda\} < 0.5$,

$$\frac{Cr\{\{\tilde{\lambda} = \lambda\} \cap \{\eta = x\}\}}{Ch\{\eta = x\}} < 0.5,$$

$$\sup_{\lambda} (Cr\{\tilde{\lambda} = \lambda\} \wedge Pr\{\eta = x | \tilde{\lambda} = \lambda\}) < 0.5,$$

We can get

$$Ch\{\{\tilde{\lambda} = \lambda\} \cap \{\eta = x\}\} = Cr\{\tilde{\lambda} = \lambda\} \wedge \{\eta = x | \tilde{\lambda} = \lambda\} \tag{8}$$

$$Ch\{\eta = x\} = \sup_{\lambda} (Cr\{\xi_{\lambda} = \lambda\} \wedge Pr\{\eta = x | \xi_{\lambda} = \lambda\}) \tag{9}$$

Then posterior membership function is formulated as

$$\mu_{\tilde{\lambda}}(\lambda | x) = \left(\frac{2Cr\{\tilde{\lambda} = \lambda\} \wedge \prod_{i=1}^n e^{-\lambda} \frac{\lambda^{x_i}}{x_i!}}{\sup_{\lambda} (Cr\{\tilde{\lambda} = \lambda\} \wedge \prod_{i=1}^n e^{-\lambda} \frac{\lambda^{x_i}}{x_i!})} \right) \wedge 1, \lambda = 0, 1, \dots \tag{10}$$

4 Fuzzy point estimation of loss frequency parameter

The sample of loss frequency in corporate finance is $x = (2480, 964)$, the experts give the range of loss frequency of corporate finance is 500 ~ 3500. The loss frequency will be decreased by 100 times in order to prevent from the probability of poison distribute in positively infinite, then will be 5 ~ 35, $x = (25, 10)$.

Let $\tilde{\lambda}$ be trapezoidal fuzzy variable $(5, a, b, 35)$, then the prior membership of $\tilde{\lambda}$ is.

$$\mu_{\tilde{\lambda}}(\lambda) = \begin{cases} \frac{\lambda - 5}{a - 5}, & \text{if } 5 \leq \lambda \leq a \\ 1, & \text{if } a \leq \lambda \leq b \\ \frac{\lambda - 35}{b - 35}, & \text{if } b \leq \lambda \leq 35 \\ 0, & \text{otherwise} \end{cases} \tag{11}$$

According by equation (2) and (3), the expectation of $\tilde{\lambda}$ is

$$E[\tilde{\lambda}] = \frac{a + b + 40}{4}, \tag{12}$$

the entropy of $\tilde{\lambda}$ is $H[\tilde{\lambda}] = (\ln 2 - 0.5)(b - a) + 15$. (13)

It follows from fuzzy maximum entropy rule equation

$\max_{\gamma_1 \dots \gamma_k} H[\tilde{\lambda}, \gamma_1 \dots \gamma_k] - M | E[\tilde{\lambda}] - \bar{\mu} |$, where $\bar{\mu}$ is the mean of those experts estimate. We can get

$$\max_{a,b} (\ln 2 - 0.5)(b - a) + 15 - M \left| \frac{a + b + 40}{4} - 13 \right|, \tag{14}$$

s.t. $5 \leq a \leq b \leq 35$,

where M is a sufficiently larger number.

By applying the graphic method, then the value of a and b can be obtained: $a = 7, b = 17$ is a trapezoidal fuzzy variable $(5, 7, 17, 35)$ then the prior membership is

$$\mu_{\tilde{\lambda}}(\lambda) = \begin{cases} \frac{\lambda - 5}{2}, & \text{if } 5 \leq \lambda \leq 7 \\ 1, & \text{if } 7 \leq \lambda \leq 17 \\ \frac{35 - \lambda}{18}, & \text{if } 17 \leq \lambda \leq 35 \\ 0, & \text{otherwise} \end{cases} \tag{15}$$

According to equation (1), the posterior membership of $\tilde{\lambda}$ is

$$\mu_{\tilde{\lambda}}(\lambda | x) = \left(\frac{2Cr\{\tilde{\lambda} = \lambda\} \wedge \prod_{i=1}^2 e^{-\lambda} \frac{\lambda^{x_i}}{x_i!}}{\sup_{\lambda} (Cr\{\tilde{\lambda} = \lambda\} \wedge \prod_{i=1}^2 e^{-\lambda} \frac{\lambda^{x_i}}{x_i!})} \right) \wedge 1 \tag{16}$$

$$= \left(9.885e^{-2\lambda} \lambda^{35} \times 10^{-29} \right) \wedge 1, 5 \leq \lambda \leq 35$$

$$= 0, \text{ otherwise}$$

The figure for the posterior membership of $\tilde{\lambda}$ can be depicted as FIGURE 1.

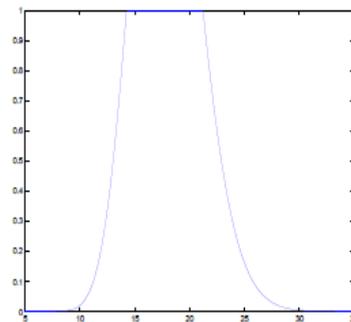


FIGURE 1 The posterior membership function of $\tilde{\lambda}$

5 Fuzzy simulation for posterior mean $E[\tilde{\lambda}]$ of the $\tilde{\lambda}$

By the fuzzy simulations technique we can calculate $E[\tilde{\lambda}]$, $E[\tilde{\lambda}]$, then posterior mean $E[\tilde{\lambda}]$ of the $\tilde{\lambda}$, is exploited as the fuzzy point estimation.

For simplicity, A fuzzy simulation will be designed to estimate $E[\tilde{\lambda}]$ by the following procedure.

1) Set $e = 0$.

2) Randomly generate $\tilde{\lambda}(\theta_k)$ from the ε -level set of $\tilde{\lambda}$ and write $v_k = \mu_{\tilde{\lambda}}(\lambda|x)$ for $k=1,2,\dots,N$, where μ the membership of function of $\tilde{\lambda}$.

3) Set $a = \tilde{\lambda}(\theta_1) \wedge \dots \wedge \tilde{\lambda}(\theta_N)$, $b = \tilde{\lambda}(\theta_1) \vee \dots \vee \tilde{\lambda}(\theta_N)$

4) Uniformly generate r from $[a,b]$.

Set $e = e + Cr\{\tilde{\lambda} \geq r\}$, where

$$\text{Cr}\{\tilde{\lambda} \in \beta\} = \frac{1}{2} (1 + \sup_{x \in \beta} \mu_{\tilde{\lambda}}(\lambda|x) - \sup_{x \in \beta^c} \mu_{\tilde{\lambda}}(\lambda|x)).$$

5) Repeat the fourth step for N times.

6) Compute $E[\tilde{\lambda}] = a \vee 0 + b \wedge 0 + e \cdot (b - a) / N$, then output $E[\tilde{\lambda}]$.

By fuzzy simulation technique, we can take the posterior mean $E[\tilde{\lambda}]$ of $\tilde{\lambda}$, as fuzzy point estimation of $\tilde{\lambda}$ is 18.0635, to amplitude the result by 100 times fuzzy point estimation of is 1806. Then density function of frequencies in operational risk losses is

$$\pi(\eta = m) = \frac{1806^m}{m!} e^{-1806}, m = 0, 1, \dots \quad (17)$$

6 Conclusions

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